第７回J-PARCハドロンサロン
KEK東海、２０１３年３月１日

中性子過剰核 $^{6}_{\Lambda}$H, $^{5}_{\Lambda\Lambda}$H における YN,YNN,YY,YYN 有効相互作用

赤石　義紀

理研、日大理工

Hyper-heavy hydrogen ${}^{6}_{\Lambda}\mathrm{H}$

in collaboration with

Theingi & Khin Swe Myint

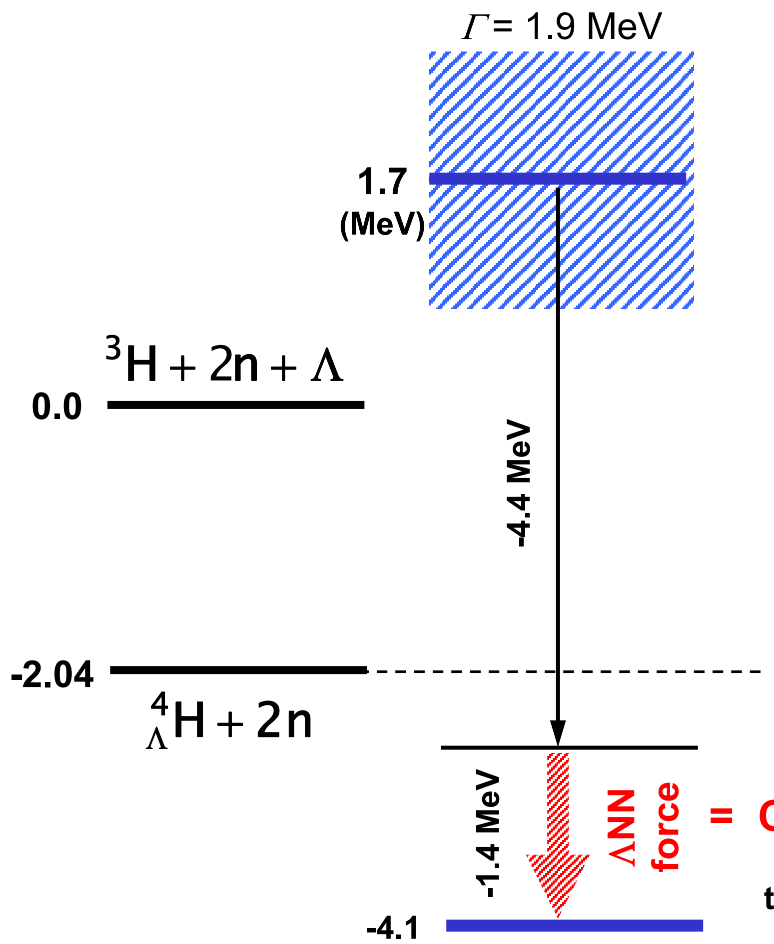
arXiv:1211.5719 [nucl-th]

最近話題になっている${}^{6}_{\Lambda}\mathrm{H}$の束縛のメカニズムを追究することによって、中性子過剰核特有の3体力効果を明らかにする。この効果は、ΛとΣが相互に転移する平均場を用いて記述できる。この新しい型の平均場は、中性子過剰のハイパー核に通常のハイパー核にはない際立った特質をもたらす。${}^{6}_{\Lambda}\mathrm{H}$で当面する課題について議論する。

$^{1}H\,(^{6}He,\,^{2}He)\,\boxed{^{5}H}$

Superheavy hydrogen

A.A. Korsheninnikov et al,
Phys. Rev. Lett. 87 (2001) 092501

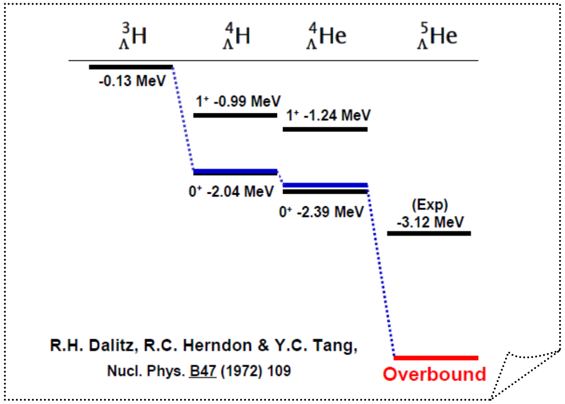


= Coherent Λ-Σ coupling
which solves
the $^{5}_{\Lambda}He$ overbinding problem.

$^{6}Li\,(\pi^{-},\,K^{+})\;\boxed{^{6}_{\Lambda}H}$

"Hyperheavy hydrogen"

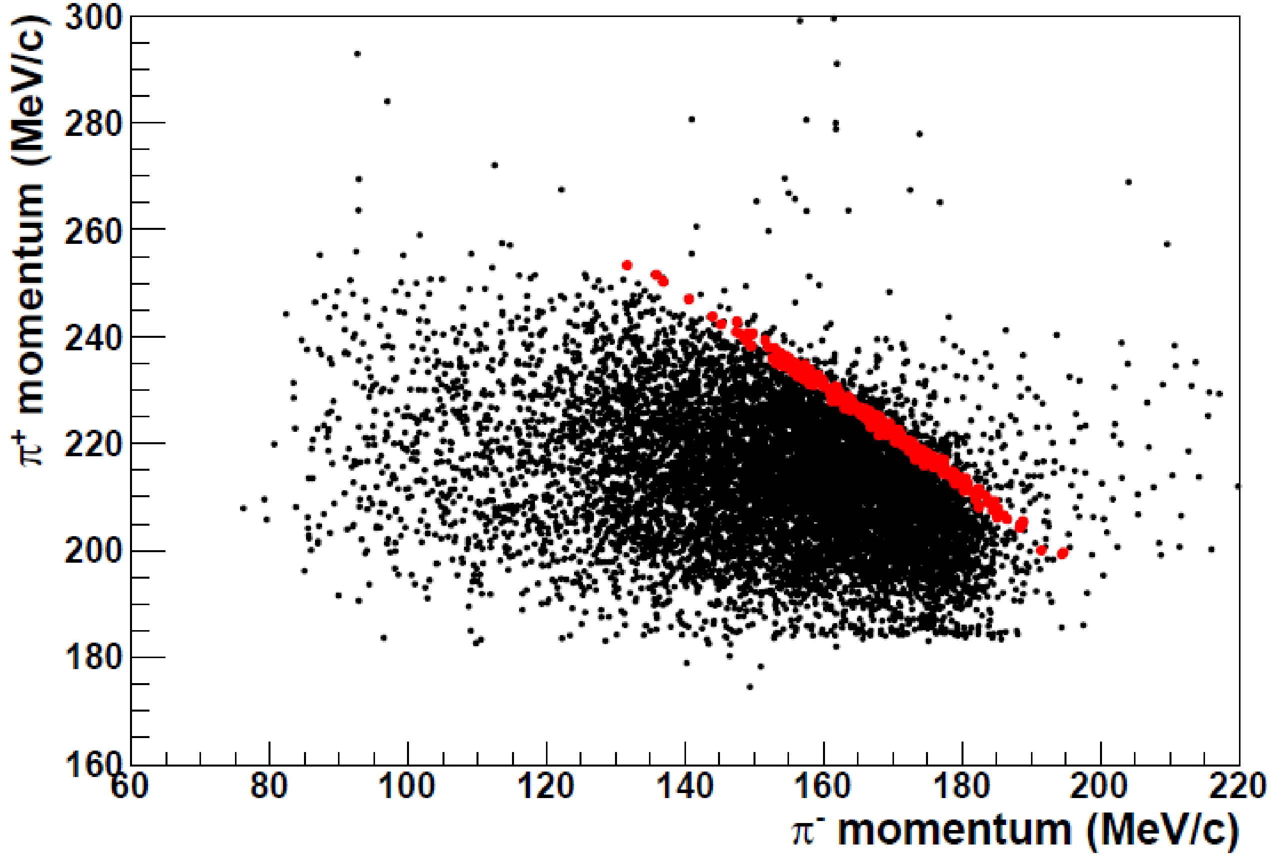
Khin Swe Myint & Y. Akaishi,
Prog. Theor. Phys. Suppl. 146 (2002) 599



R.H. Dalitz, R.C. Herndon & Y.C. Tang,
Nucl. Phys. B47 (1972) 109

Pions from stopped K^- on ^{6}Li

M. Agnello et al., Nucl. Phys. A 881 (2012) 269



$$M(K^-) + 3M(p) + 3M(n) - B(^6\mathrm{Li}) = M(^6_\Lambda\mathrm{H}) + T(^6_\Lambda\mathrm{H}) + M(\pi^+) + T(\pi^+),$$

$$M(^6_\Lambda\mathrm{H}) = 2M(p) + 4M(n) - B(^6\mathrm{He}) + T(^6\mathrm{He}) + M(\pi^-) + T(\pi^-),$$

$$\Downarrow$$

$$T(\pi^+) + T(\pi^-) = M(K^-) + M(p) - M(n) - 2M(\pi)$$
$$-B(^6\mathrm{Li}) + B(^6\mathrm{He}) - T(^6\mathrm{He}) - T(^6_\Lambda\mathrm{H}) = 203 \pm 1.3\ \mathrm{MeV}$$

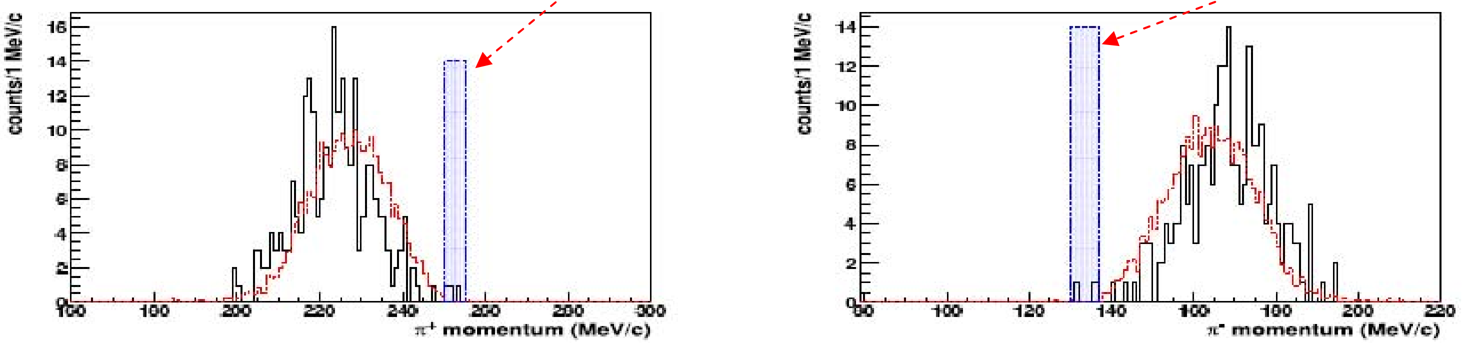
Evidence for heavy hyperhydrogen $^{6}_{\Lambda}$H

M. Agnello et al., Phys. Rev. Lett. 108 (2012) 042501

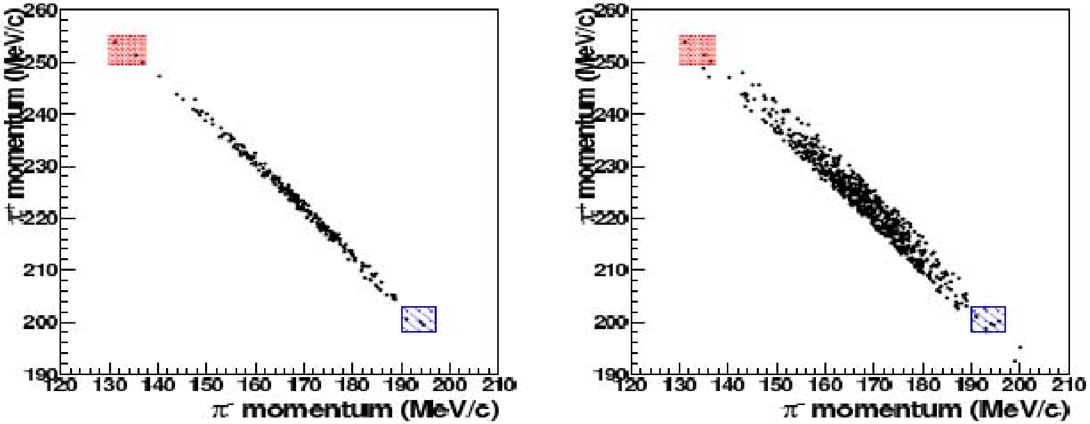
$\Lambda \rightarrow p + \pi^-$ Weak decay ; 2.6E-10 s

$K^-_{stop} + {}^6\mathrm{Li} \rightarrow {}^{6}_{\Lambda}\mathrm{H} + \pi^+$

${}^{6}_{\Lambda}\mathrm{H} \rightarrow {}^6\mathrm{He} + \pi^-$

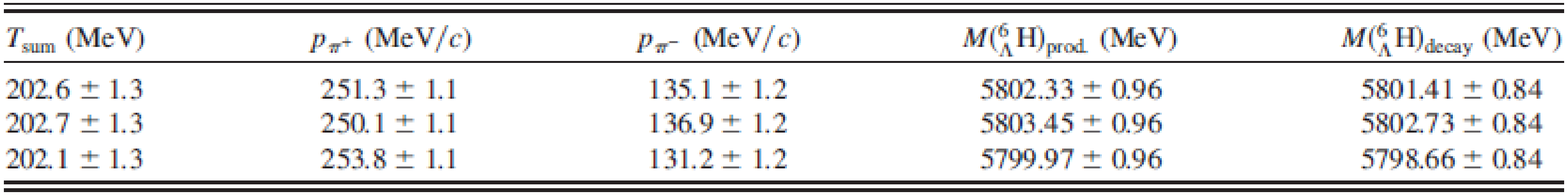


$\pi^{\pm}$ in coincidence (p_{π^+} left, p_{π^-} right), $T(\pi^+) + T(\pi^-) = 202 - 204$ MeV



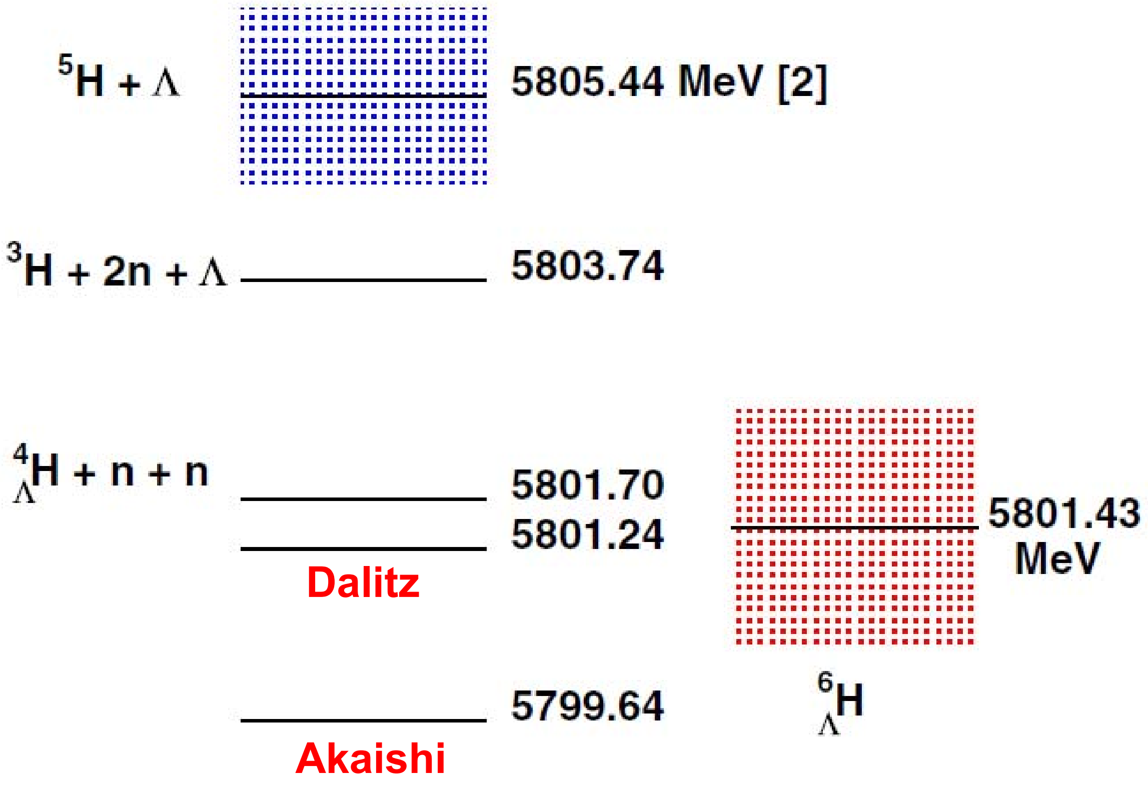
p_{π^+} vs. p_{π^-} for $T(\pi^+) + T(\pi^-) = 202 - 204$, $200 - 206$ MeV (left, right)

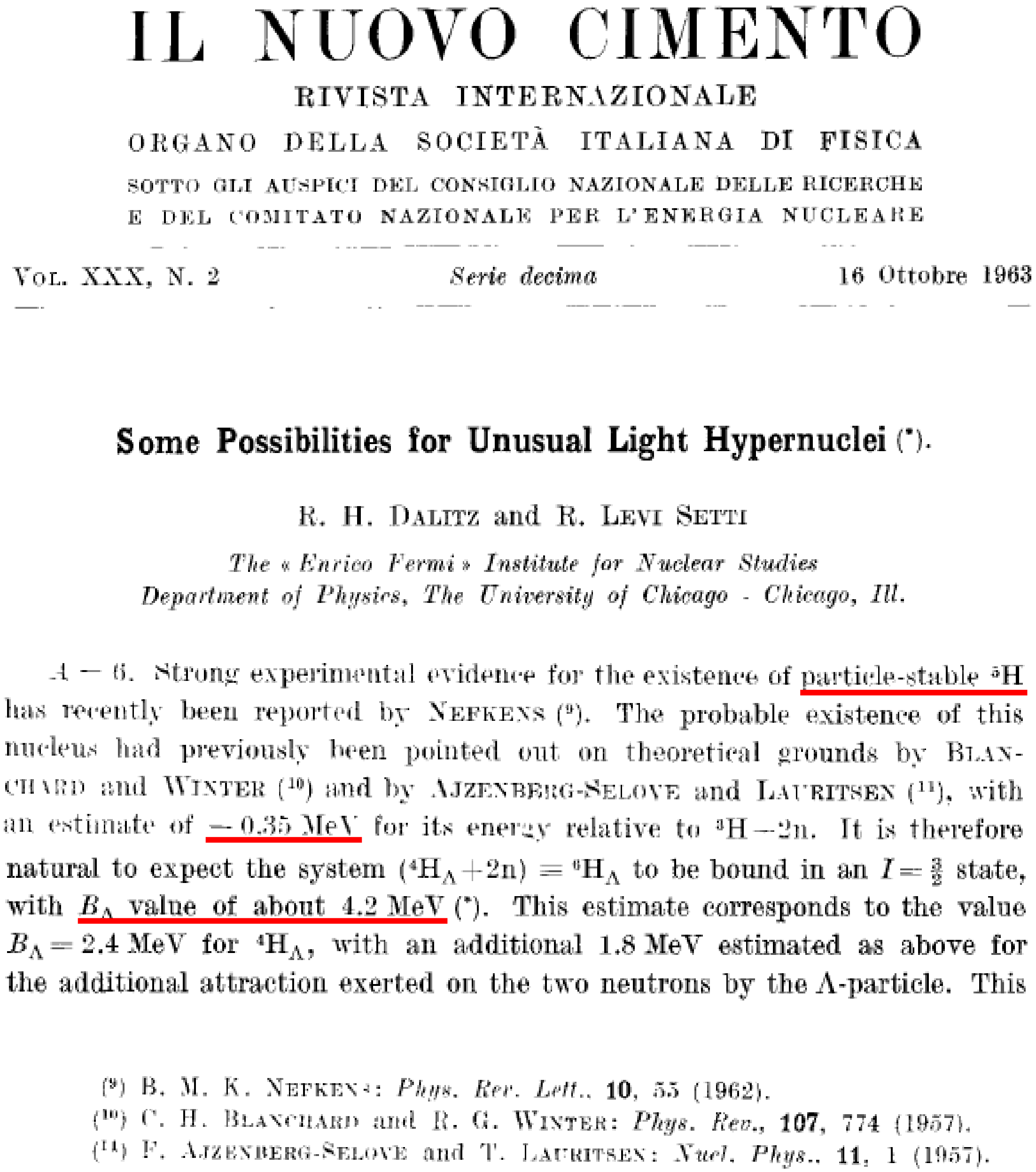
Candidates of $^{6}_{\Lambda}\mathrm{H}$



T_{sum} (MeV)	p_{π^+} (MeV/c)	p_{π^-} (MeV/c)	$M(^{6}_{\Lambda}\mathrm{H})_{\mathrm{prod.}}$ (MeV)	$M(^{6}_{\Lambda}\mathrm{H})_{\mathrm{decay}}$ (MeV)
202.6 ± 1.3	251.3 ± 1.1	135.1 ± 1.2	5802.33 ± 0.96	5801.41 ± 0.84
202.7 ± 1.3	250.1 ± 1.1	136.9 ± 1.2	5803.45 ± 0.96	5802.73 ± 0.84
202.1 ± 1.3	253.8 ± 1.1	131.2 ± 1.2	5799.97 ± 0.96	5798.66 ± 0.84

$^{4}_{\Lambda}\mathrm{H}$+2n threshold: 5801.71 MeV





IL NUOVO CIMENTO
RIVISTA INTERNAZIONALE
ORGANO DELLA SOCIETÀ ITALIANA DI FISICA
SOTTO GLI AUSPICI DEL CONSIGLIO NAZIONALE DELLE RICERCHE
E DEL COMITATO NAZIONALE PER L'ENERGIA NUCLEARE

VOL. XXX, N. 2 *Serie decima* 16 Ottobre 1963

Some Possibilities for Unusual Light Hypernuclei (*).

R. H. DALITZ and R. LEVI SETTI

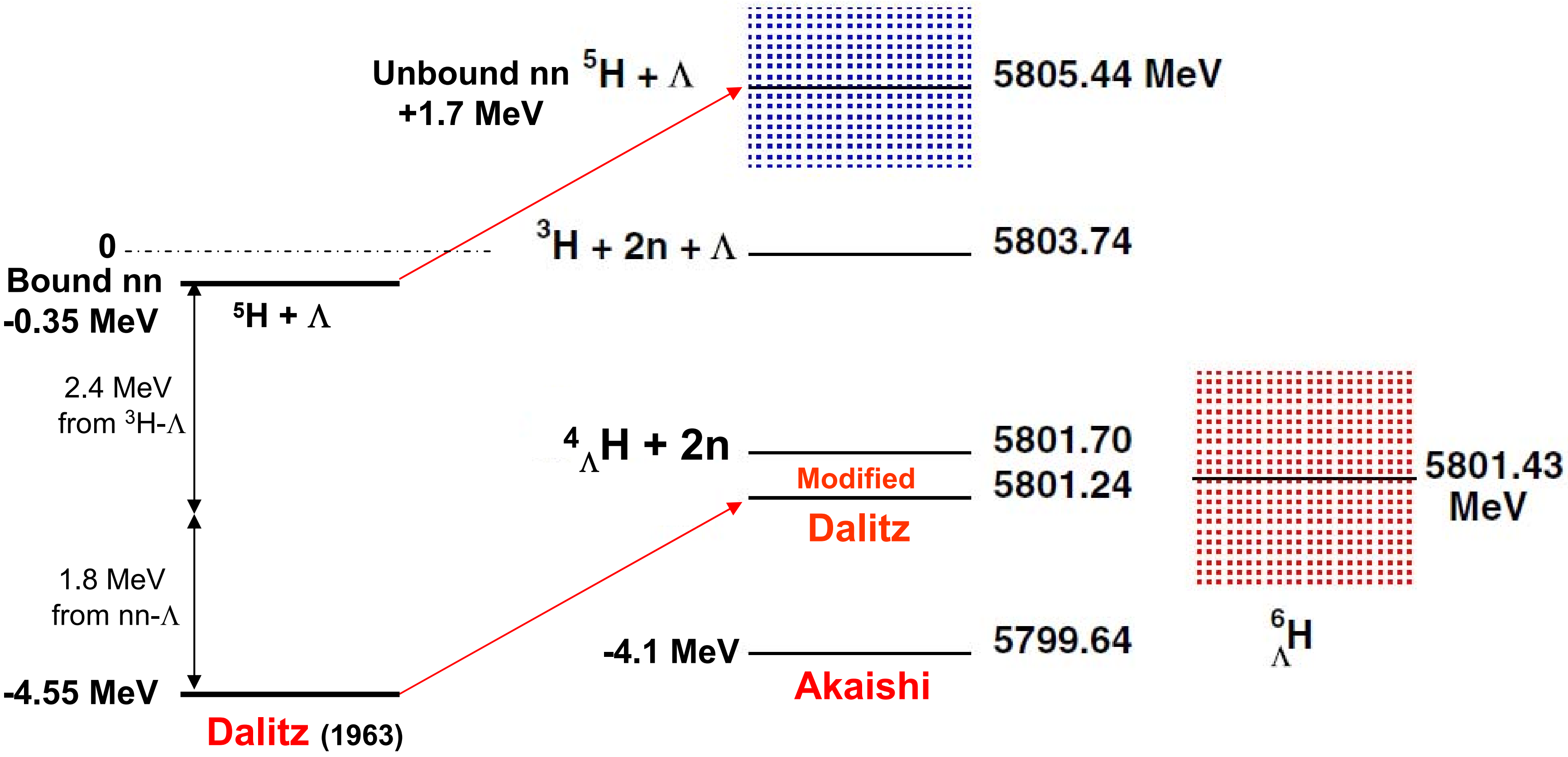
The « Enrico Fermi » Institute for Nuclear Studies
Department of Physics, The University of Chicago - Chicago, Ill.

$A = 6$. Strong experimental evidence for the existence of particle-stable ^{5}H has recently been reported by NEFKENS (9). The probable existence of this nucleus had previously been pointed out on theoretical grounds by BLANCHARD and WINTER (10) and by AJZENBERG-SELOVE and LAURITSEN (11), with an estimate of -0.35 MeV for its energy relative to $^3\mathrm{H}+2\mathrm{n}$. It is therefore natural to expect the system $(^4\mathrm{H}_\Lambda+2\mathrm{n}) \equiv {}^6\mathrm{H}_\Lambda$ to be bound in an $I=\frac{3}{2}$ state, with B_Λ value of about 4.2 MeV (*). This estimate corresponds to the value $B_\Lambda = 2.4$ MeV for $^4\mathrm{H}_\Lambda$, with an additional 1.8 MeV estimated as above for the additional attraction exerted on the two neutrons by the Λ-particle. This

(9) B. M. K. NEFKENS: *Phys. Rev. Lett.*, **10**, 55 (1962).
(10) C. H. BLANCHARD and R. G. WINTER: *Phys. Rev.*, **107**, 774 (1957).
(11) F. AJZENBERG-SELOVE and T. LAURITSEN: *Nucl. Phys.*, **11**, 1 (1957).

Evidence for $^{6}_{\Lambda}$H

Bressani et al.



Evidence for $^6_\Lambda$H

Bressani et al.

$$v_{t(nn)} = -13.3_{\rm MeV} \exp\left\{-\left(\frac{r}{2.2_{\rm fm}}\right)^2\right\}$$

$\boxed{E\text{=-0.35 MeV}}$

$$v_{t\Lambda} = -45.4_{\rm MeV} \exp\left\{-\left(\frac{r}{1.53_{\rm fm}}\right)^2\right\}$$

E=-2.4 MeV

$$v_{(nn)\Lambda} = -11.5_{\rm MeV} \exp\left\{-\left(\frac{r}{1.8_{\rm fm}}\right)^2\right\}$$

$E_{t(nn)\Lambda}$= -4.54 MeV

$$v_{t(nn)} = -10.5_{\rm MeV} \exp\left\{-\left(\frac{r}{2.2_{\rm fm}}\right)^2\right\}$$

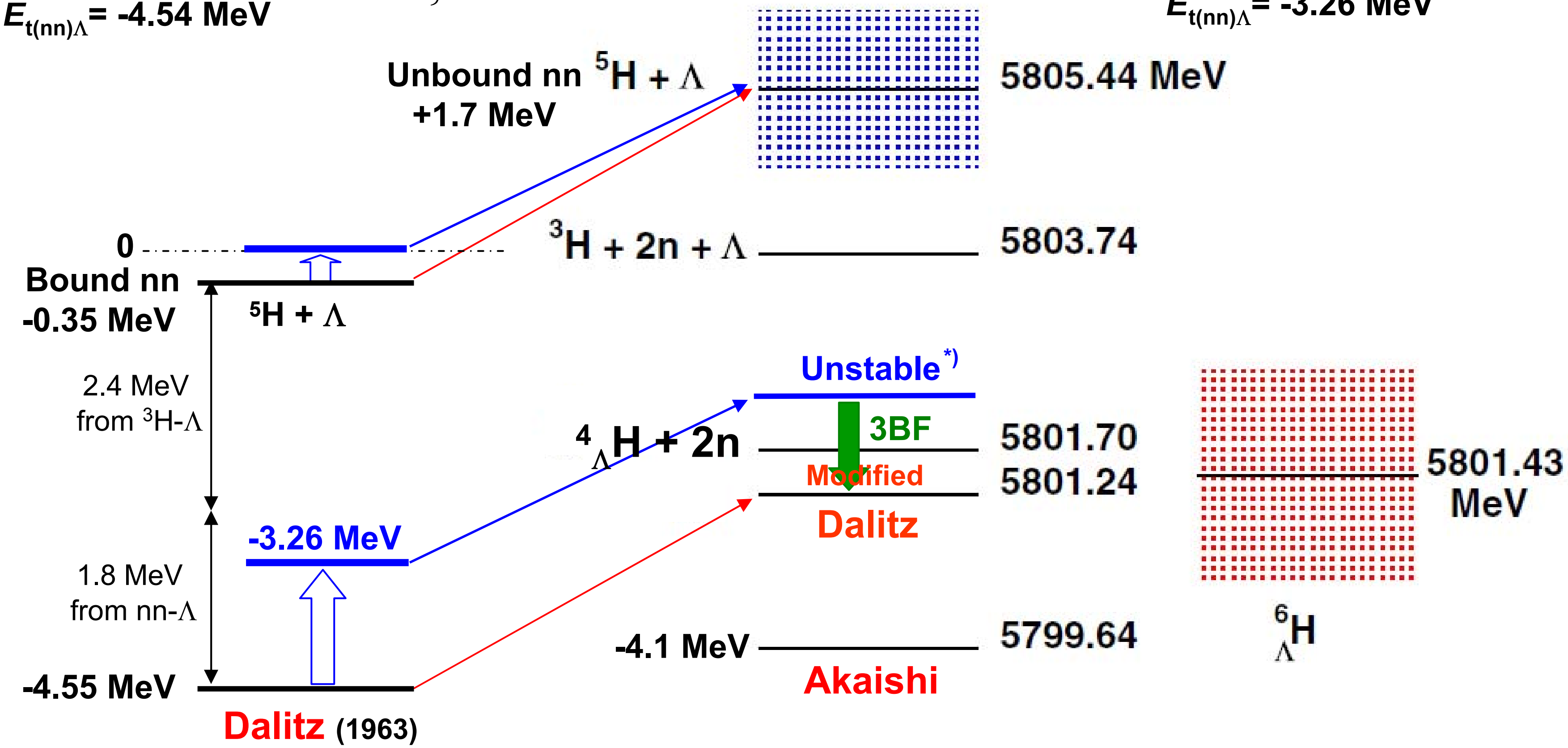
$\boxed{E\text{=0}}$

$$v_{t\Lambda} = -43.8_{\rm MeV} \exp\left\{-\left(\frac{r}{1.53_{\rm fm}}\right)^2\right\}$$

E=-2.04 MeV

$$v_{(nn)\Lambda} = -11.5_{\rm MeV} \exp\left\{-\left(\frac{r}{1.8_{\rm fm}}\right)^2\right\}$$

$E_{t(nn)\Lambda}$= -3.26 MeV



Unbound nn ^{5}H + Λ
+1.7 MeV

5805.44 MeV

0

Bound nn
-0.35 MeV

^{5}H + Λ

^{3}H + 2n + Λ

5803.74

2.4 MeV
from ^{3}H-Λ

Unstable$^{*)}$

3BF

$^4_\Lambda$H + 2n

5801.70

Modified

5801.24

Dalitz

5801.43 MeV

-3.26 MeV

1.8 MeV
from nn-Λ

-4.1 MeV

5799.64

Akaishi

$^6_\Lambda$H

-4.55 MeV

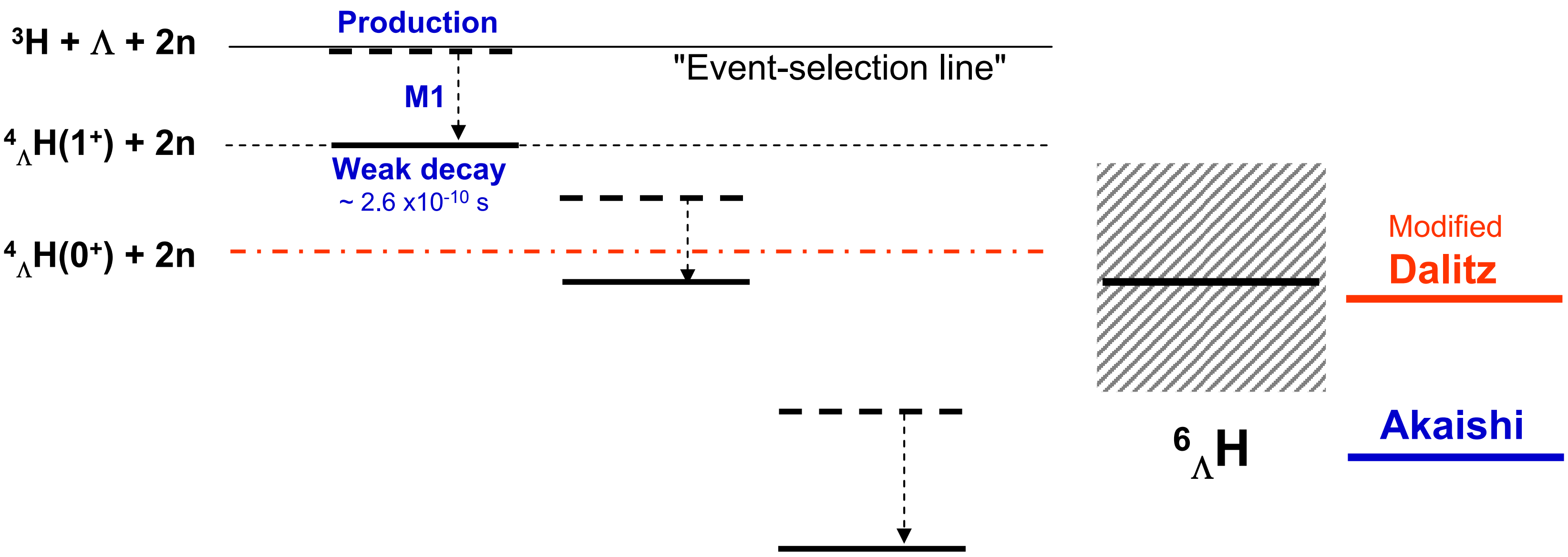
Dalitz (1963)

$^{*)}$This state comes above the threshold and cannot survive till weak decay.

Thus, the coherent Λ-Σ coupling is necessitated.

at best only
a rough estimate

3 candidate events of $^{6}_{\Lambda}$H



^{3}H + Λ + 2n

Production

"Event-selection line"

M1

$^{4}_{\Lambda}$H(1^{+}) + 2n

Weak decay

~ 2.6 x10^{-10} s

$^{4}_{\Lambda}$H(0^{+}) + 2n

Modified

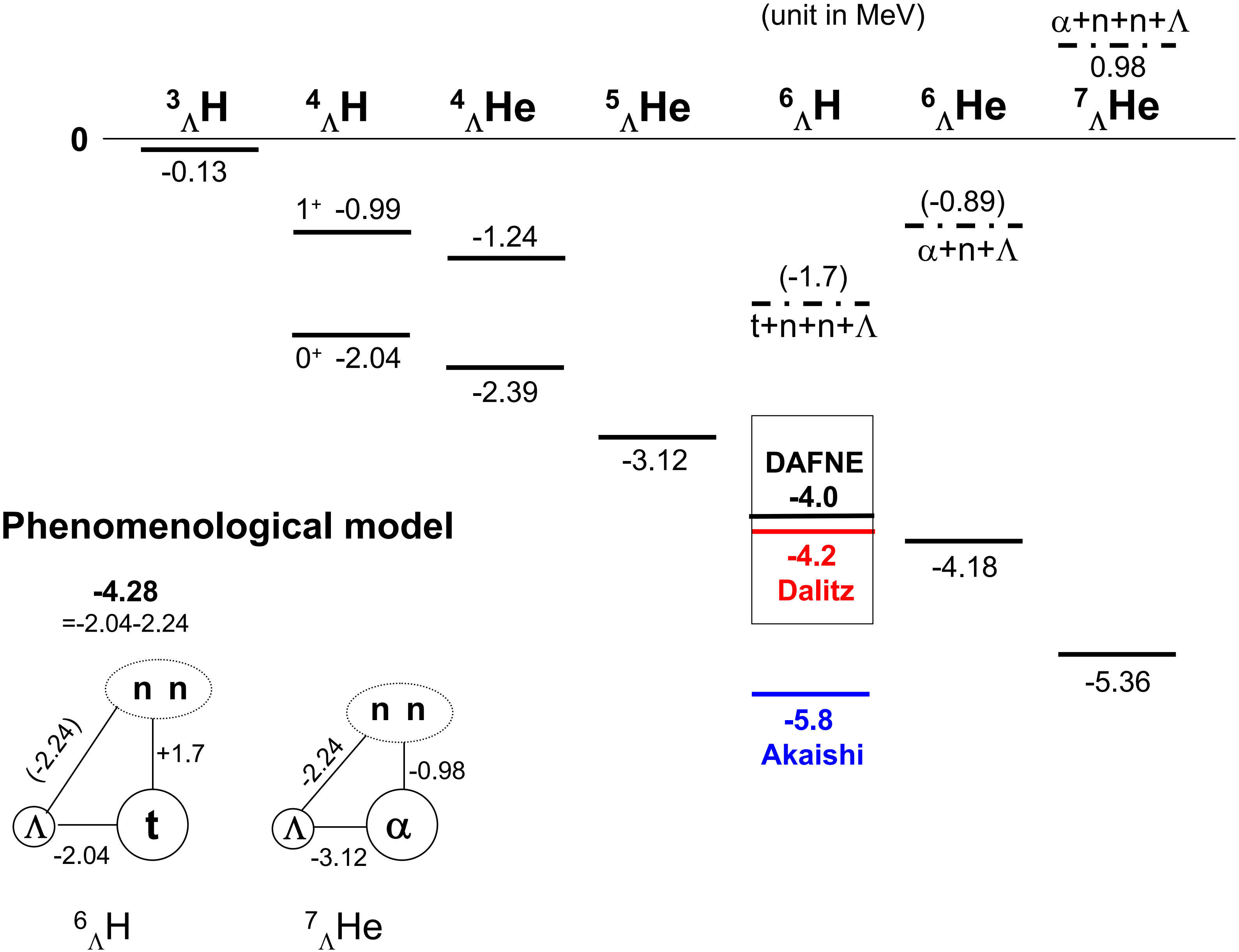
Dalitz

Akaishi

$^{6}_{\Lambda}$H

Λ separation energy

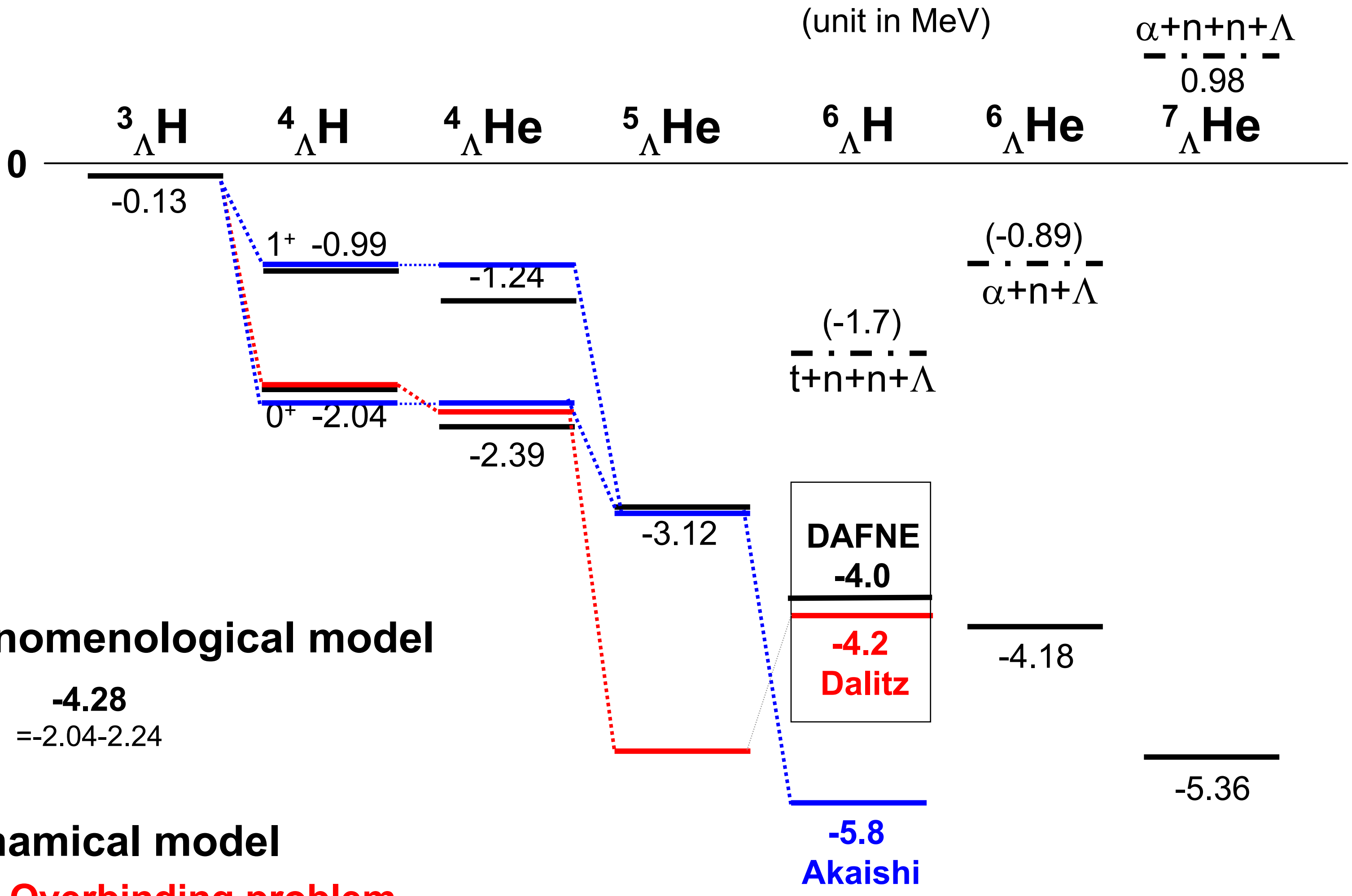
(unit in MeV)



☆ **Phenomenological model**

-4.28
=-2.04-2.24

Λ separation energy



☆ **Phenomenological model**

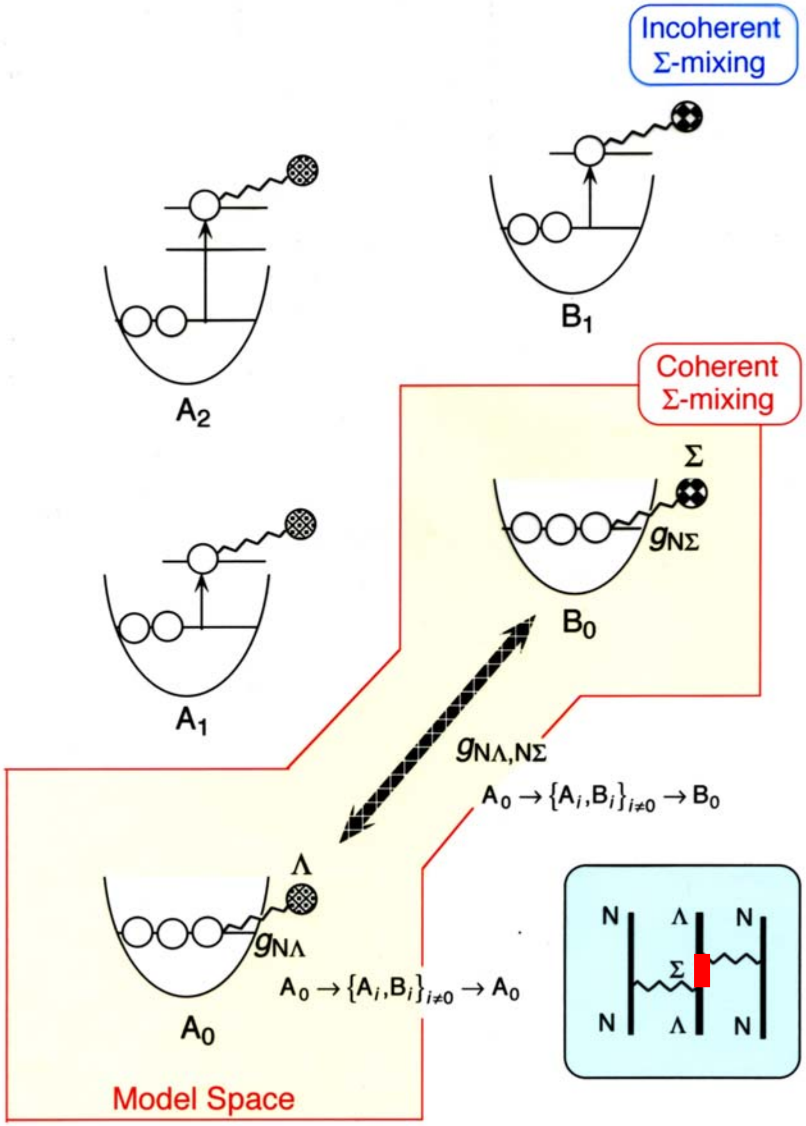
-4.28
=-2.04-2.24

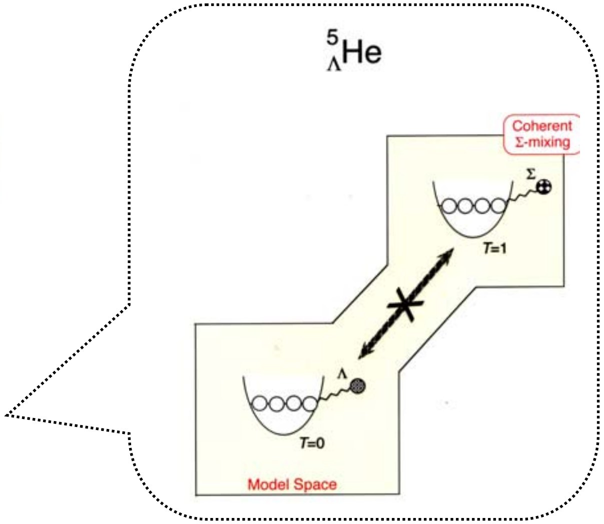
☆ **Dynamical model**

Overbinding problem

☆ **Dynamical model with coherent Λ-Σ coupling**

Coherent Λ-Σ coupling

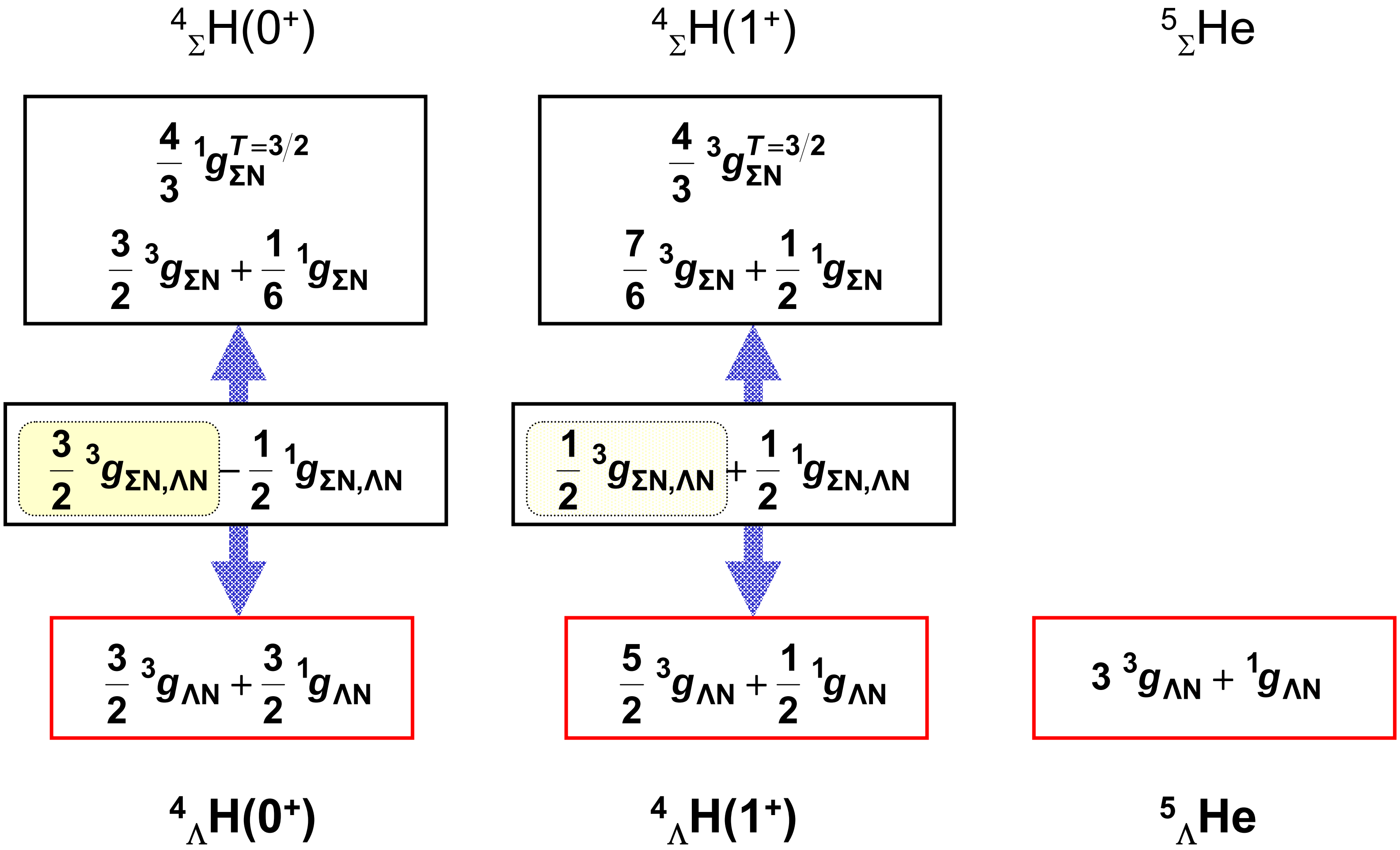




Λ T=0 1116 MeV/c²

Σ T=1 1193 MeV/c²

YN interaction weights from *s*-shell nucleons



${}^{4}_{\Sigma}\mathrm{H}(0^{+})$

$$\frac{4}{3}\,{}^{1}g^{T=3/2}_{\Sigma N}$$

$$\frac{3}{2}\,{}^{3}g_{\Sigma N}+\frac{1}{6}\,{}^{1}g_{\Sigma N}$$

$$\frac{3}{2}\,{}^{3}g_{\Sigma N,\Lambda N}-\frac{1}{2}\,{}^{1}g_{\Sigma N,\Lambda N}$$

$$\frac{3}{2}\,{}^{3}g_{\Lambda N}+\frac{3}{2}\,{}^{1}g_{\Lambda N}$$

${}^{4}_{\Lambda}\mathrm{H}(0^{+})$

${}^{4}_{\Sigma}\mathrm{H}(1^{+})$

$$\frac{4}{3}\,{}^{3}g^{T=3/2}_{\Sigma N}$$

$$\frac{7}{6}\,{}^{3}g_{\Sigma N}+\frac{1}{2}\,{}^{1}g_{\Sigma N}$$

$$\frac{1}{2}\,{}^{3}g_{\Sigma N,\Lambda N}+\frac{1}{2}\,{}^{1}g_{\Sigma N,\Lambda N}$$

$$\frac{5}{2}\,{}^{3}g_{\Lambda N}+\frac{1}{2}\,{}^{1}g_{\Lambda N}$$

${}^{4}_{\Lambda}\mathrm{H}(1^{+})$

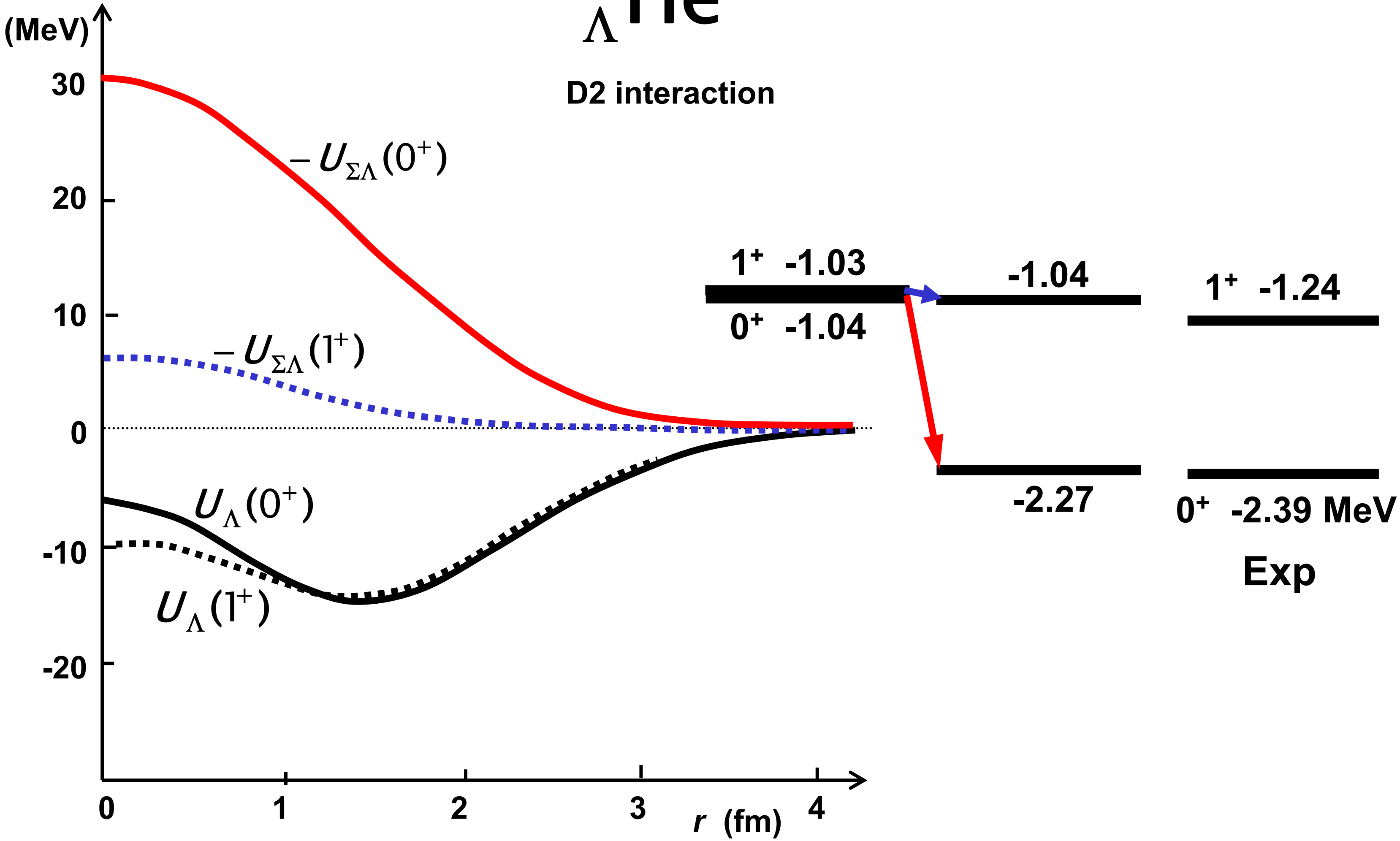
${}^{5}_{\Sigma}\mathrm{He}$

$$3\,{}^{3}g_{\Lambda N}+{}^{1}g_{\Lambda N}$$

${}^{5}_{\Lambda}\mathrm{He}$

$^{4}_{\Lambda}\mathrm{He}$

D2 interaction



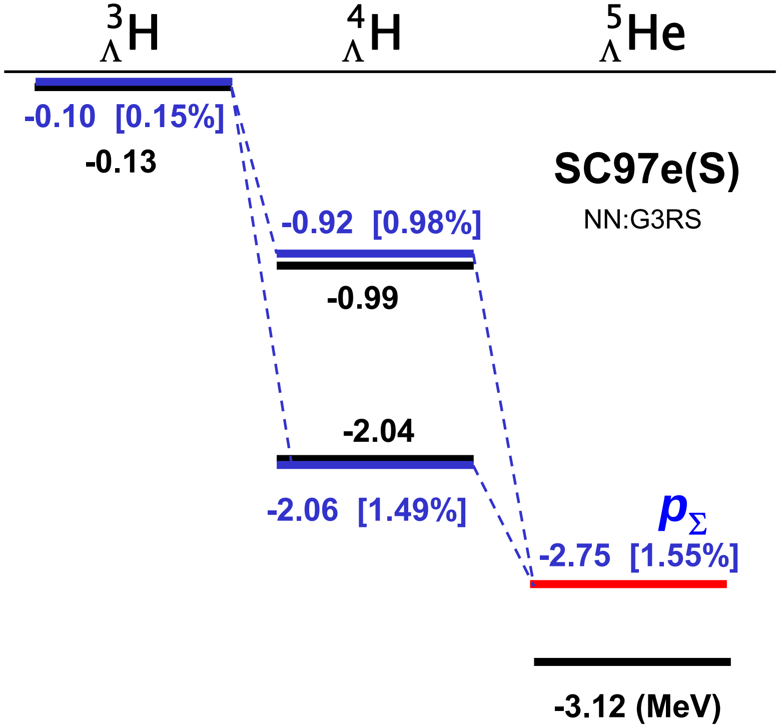
Y. Akaishi, T. Harada, S. Shinmura and Khin Swe Myint, Phys. Rev. Lett. 84 (2000) 3539

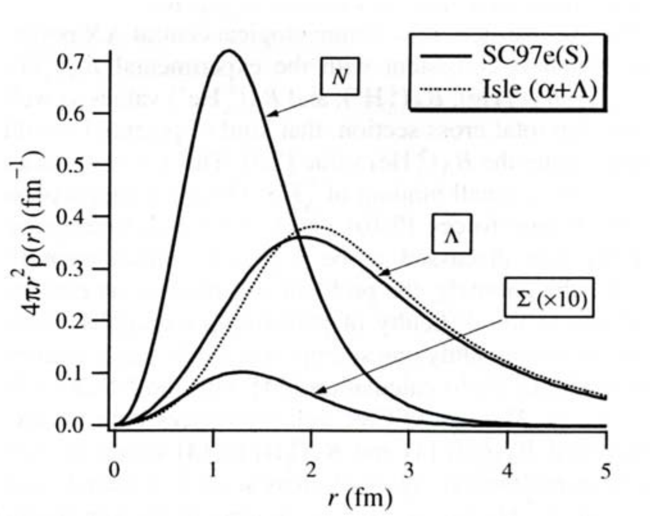
Stochastic variational calculation of ${}^{5}_{\Lambda}$He

H. Nemura, Y. Akaishi & Y. Suzuki,
Phys. Rev. Lett. 89 (2002) 142504

The first successful *ab initio* 5-body calculation including Σ degrees of freedom

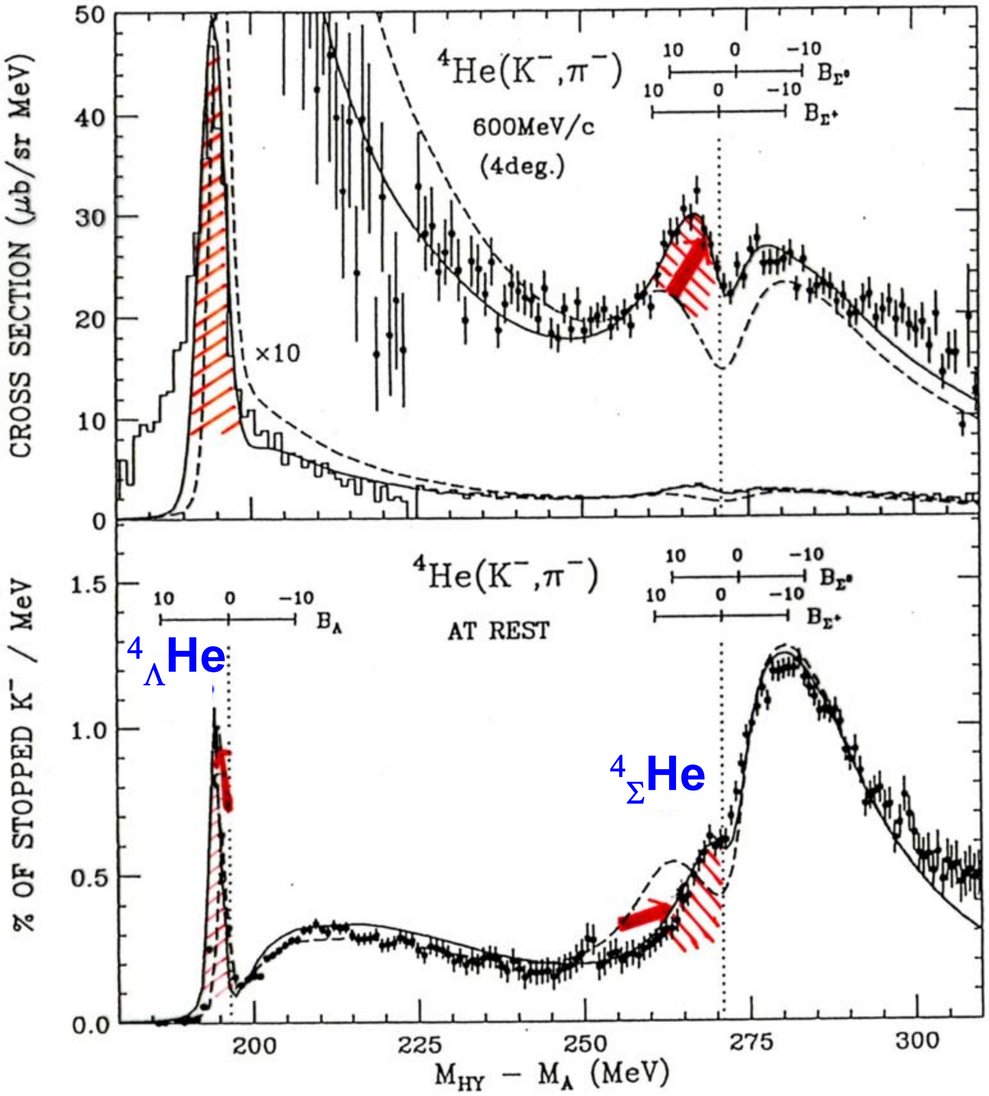
J.A. Carlson,
AIP Conf. Proc. 224 (1991) 198
SC89: unbound

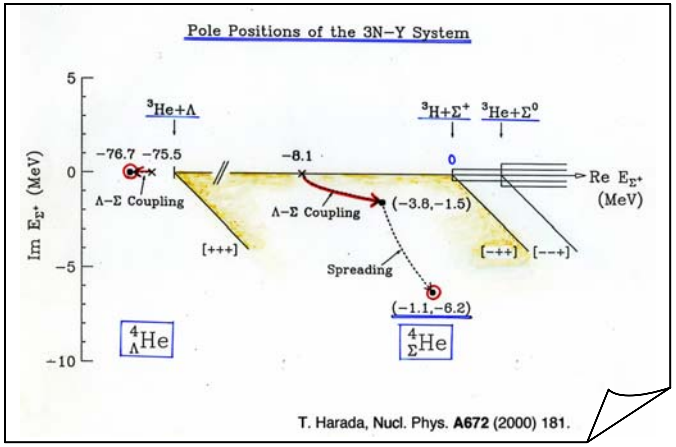

${}^{3}_{\Lambda}$H
${}^{4}_{\Lambda}$H
${}^{5}_{\Lambda}$He
-0.10 [0.15%]
-0.13
-0.92 [0.98%]
-0.99
-2.04
-2.06 [1.49%]
SC97e(S)
NN:G3RS
p_{Σ}
-2.75 [1.55%]
-3.12 (MeV)



Theory:

T. Harada,
Phys. Rev. Lett. 81 (1998) 5287



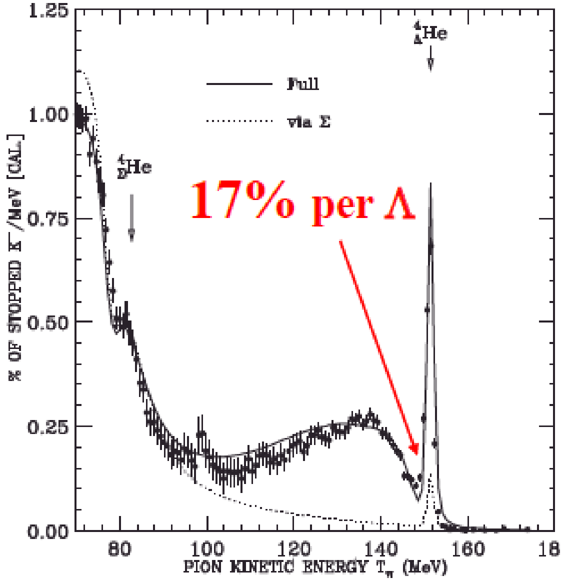


T. Harada, Nucl. Phys. **A672** (2000) 181.

BNL : (1998)

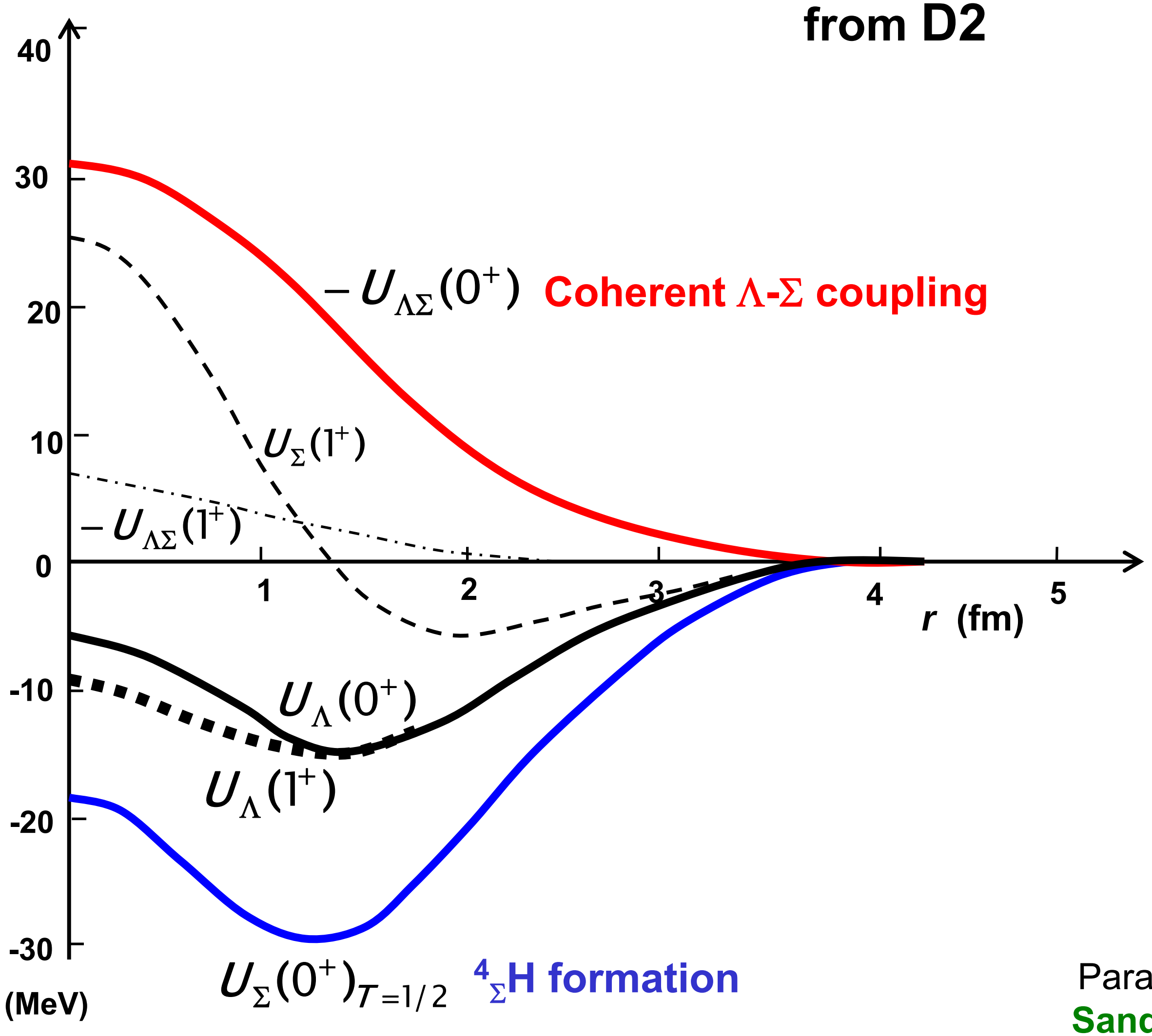
T. Nagae, R.E. Chrien et al.,
Phys. Rev. Lett. **80** (1998) 1605.

T. Harada



Y- $(NNN)_{T=1/2}$: interactions

from D2



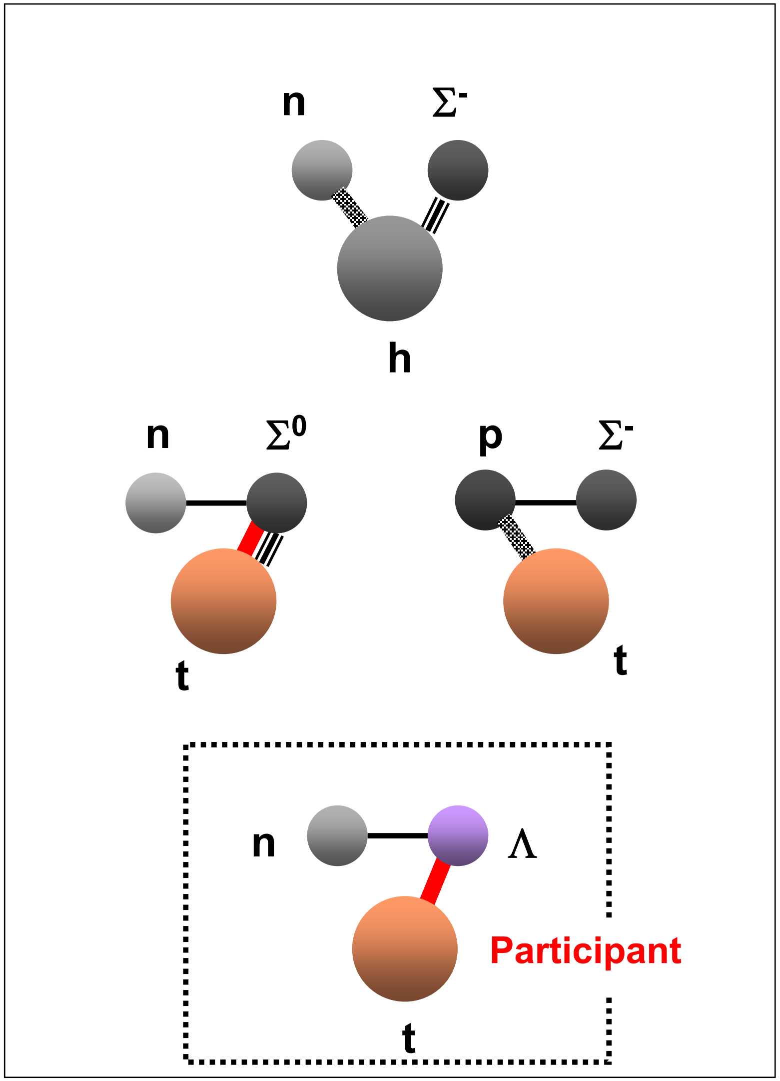
Parametrized by
Sander Myint Oo

Coupling scheme

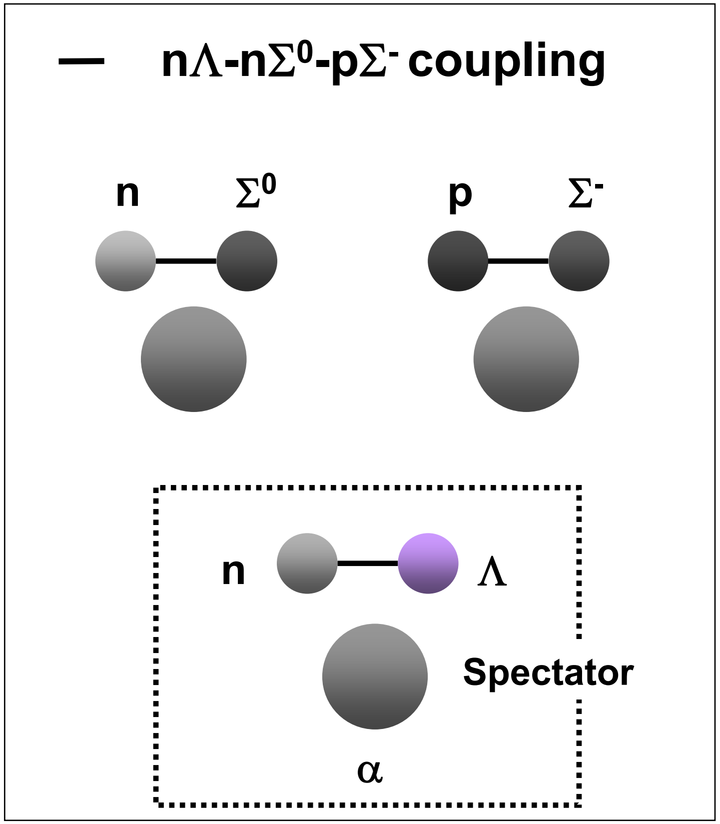
🟥 Coherent Λ-Σ coupling

☰ $^4_\Sigma$H formation

▩ α formation



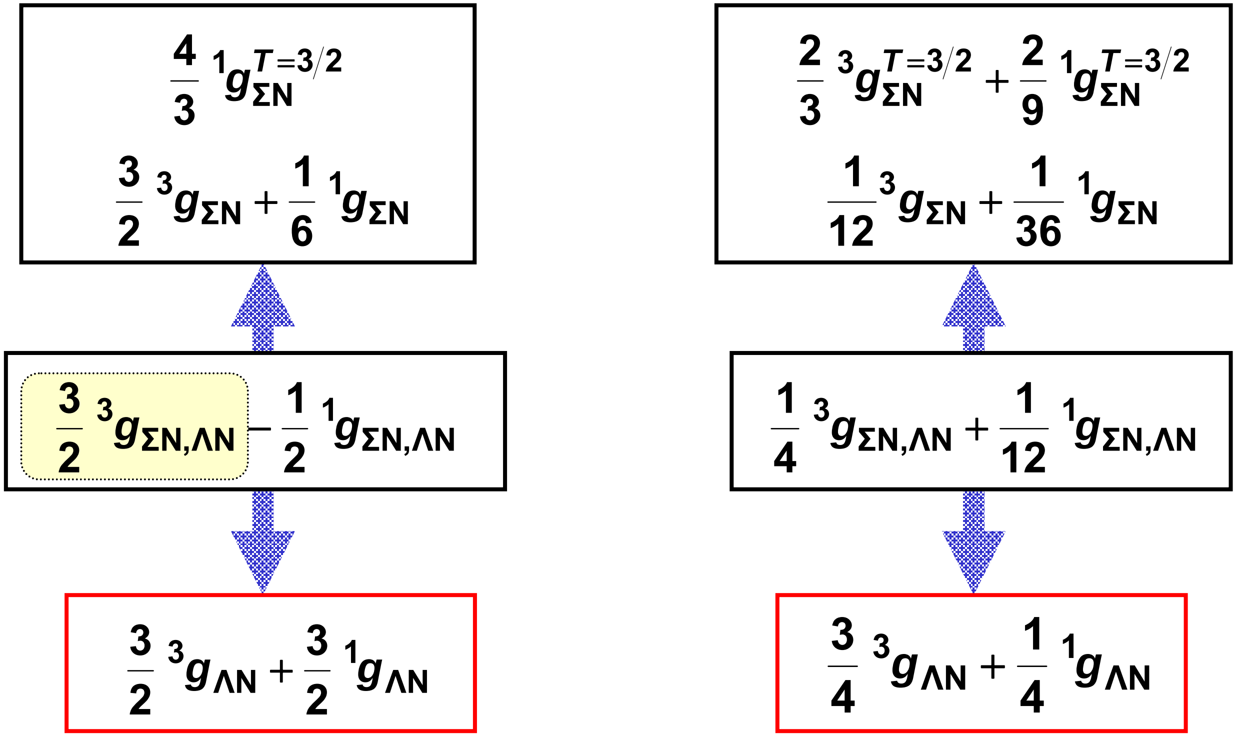
$^6_\Lambda$H - n



$^7_\Lambda$He - n

YN interaction weights in ${}^{6}_{\Lambda}$H(0$^+$)

g for p-shell N is the sum of
even & odd state effective interactions.



$$\frac{4}{3}\,{}^{1}g_{\Sigma N}^{T=3/2}$$

$$\frac{3}{2}\,{}^{3}g_{\Sigma N}+\frac{1}{6}\,{}^{1}g_{\Sigma N}$$

$$\frac{3}{2}\,{}^{3}g_{\Sigma N,\Lambda N}-\frac{1}{2}\,{}^{1}g_{\Sigma N,\Lambda N}$$

$$\frac{3}{2}\,{}^{3}g_{\Lambda N}+\frac{3}{2}\,{}^{1}g_{\Lambda N}$$

from *s*-shell nucleons

$$\frac{2}{3}\,{}^{3}g_{\Sigma N}^{T=3/2}+\frac{2}{9}\,{}^{1}g_{\Sigma N}^{T=3/2}$$

$$\frac{1}{12}\,{}^{3}g_{\Sigma N}+\frac{1}{36}\,{}^{1}g_{\Sigma N}$$

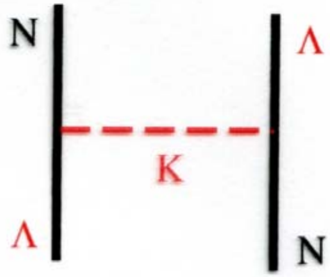
$$\frac{1}{4}\,{}^{3}g_{\Sigma N,\Lambda N}+\frac{1}{12}\,{}^{1}g_{\Sigma N,\Lambda N}$$

$$\frac{3}{4}\,{}^{3}g_{\Lambda N}+\frac{1}{4}\,{}^{1}g_{\Lambda N}$$

from *p*-shell neutrons

Program code
swe3/LH6.f

Brueckner-Hartree-Fock Calculation on Gaussian Basis



Hyperon-nucleus potential

$$U^{\mathrm{eq}}_{\mu}(\vec{r}_1)\varphi_{\mu}(\vec{r}_1) \equiv U_{\mathrm{H}}(\vec{r}_1)\varphi_{\mu}(\vec{r}_1) + \int d\vec{r}_2 U_{\mathrm{F}}(\vec{r}_1,\vec{r}_2)\varphi_{\mu}(\vec{r}_2)$$

$$U_{\mathrm{H}}(\vec{r}_1) = \int d\vec{r}_2 \sum_{\nu} \varphi^{*}_{\nu}(\vec{r}_2)\overset{\text{g-matrix}}{\boxed{g(\vec{r}_1,\vec{r}_2)}}\varphi_{\nu}(\vec{r}_2)$$

$$U_{\mathrm{F}}(\vec{r}_1,\vec{r}_2) = -\sum_{\nu} \varphi^{*}_{\nu}(\vec{r}_2) g(\vec{r}_1,\vec{r}_2)\varphi_{\nu}(\vec{r}_1)$$

$$g(\vec{r}_1-\vec{r}_2) = \sum_{j} \boxed{\gamma^{\mu}_{j}} \exp\left\{-\left((\vec{r}_1-\vec{r}_2)/c_j\right)^2\right\}, \quad c_1,\ldots.,c_{20} \text{ fixed}$$

$$U^{\mathrm{eq}}_{\mu}(\vec{r}_1) = \sum_{j} \boxed{\alpha^{\mu}_{j}} \exp\left\{-\left(r_1/a_j\right)^2\right\}, \quad a_1,\ldots.,a_{10} \text{ fixed}$$

$$\varphi_{\mu}(\vec{r}_1) = \sum_{j} \boxed{\beta^{\mu}_{j}} \exp\left\{-\left(r_1/b_j\right)^2\right\} r_1^{\ell} y_{\ell_1 s_1 j_1 m_1}(\hat{r}_1), \quad b_1,\ldots.,b_{20} \text{ fixed}$$

Self-consistently determined

Matrix elements for s-shell hyperon:

$$\boxed{\sum_{\phi_2}\sum_{\mu_2}\left\langle (a_1{''}\, j_1\mu_1\phi_1)(a_2{''}\, j_2\mu_2\phi_2)\right| g \left|(a_1 j_1\mu_1\phi_1)(a_2 j_2\mu_2\phi_2) - \text{exch.}\right\rangle}$$

$$= (2j_2+1)(2\ell_2+1)^2 \sum_{\ell'=0}^{\ell_2}\sum_{\ell'''=0}^{\ell_2}\left(\frac{M_1}{M_1+M_2}\right)^{\ell'+\ell'''}\sqrt{\binom{2\ell_2}{2\ell'}\binom{2\ell_2}{2\ell'''}}\sum_n\sum_{n''}(2n+1)(2n''+1)$$

$$\times\sum_\kappa \boxed{\begin{gathered}\int_0^\infty dr\, r^{\ell'+\ell''+2}\exp\left(-\left(A_{12}{''}+A_{12}+c_\kappa^2\right)r^2\right)\int_0^\infty dR\, R^{2\ell_2-\ell'-\ell'''+2}\exp\left(-(a_1{''}+a_2{''}+a_1+a_2)R^2\right)\\ \times i^{n''}\, j_{n''}(ia_{12}{''}\, rR)\, i^n\, j_n(ia_{12}rR)\end{gathered}}$$

$$\times\sum_{\tilde\ell}\left(\ell' n 0 0|\tilde\ell 0\right)\left(\ell''' n'' 0 0|\tilde\ell 0\right)\sum_{\tilde L}\left(\ell_2-\ell' n 0 0|\tilde L 0\right)\left(\ell_2-\ell''' n'' 0 0|\tilde L 0\right)\begin{Bmatrix} n & \ell' & \tilde\ell \\ \ell_2 & \tilde L & \ell_2-\ell\end{Bmatrix}\begin{Bmatrix} n'' & \ell''' & \tilde\ell \\ \ell_2 & \tilde L & \ell_2-\ell'''\end{Bmatrix}$$

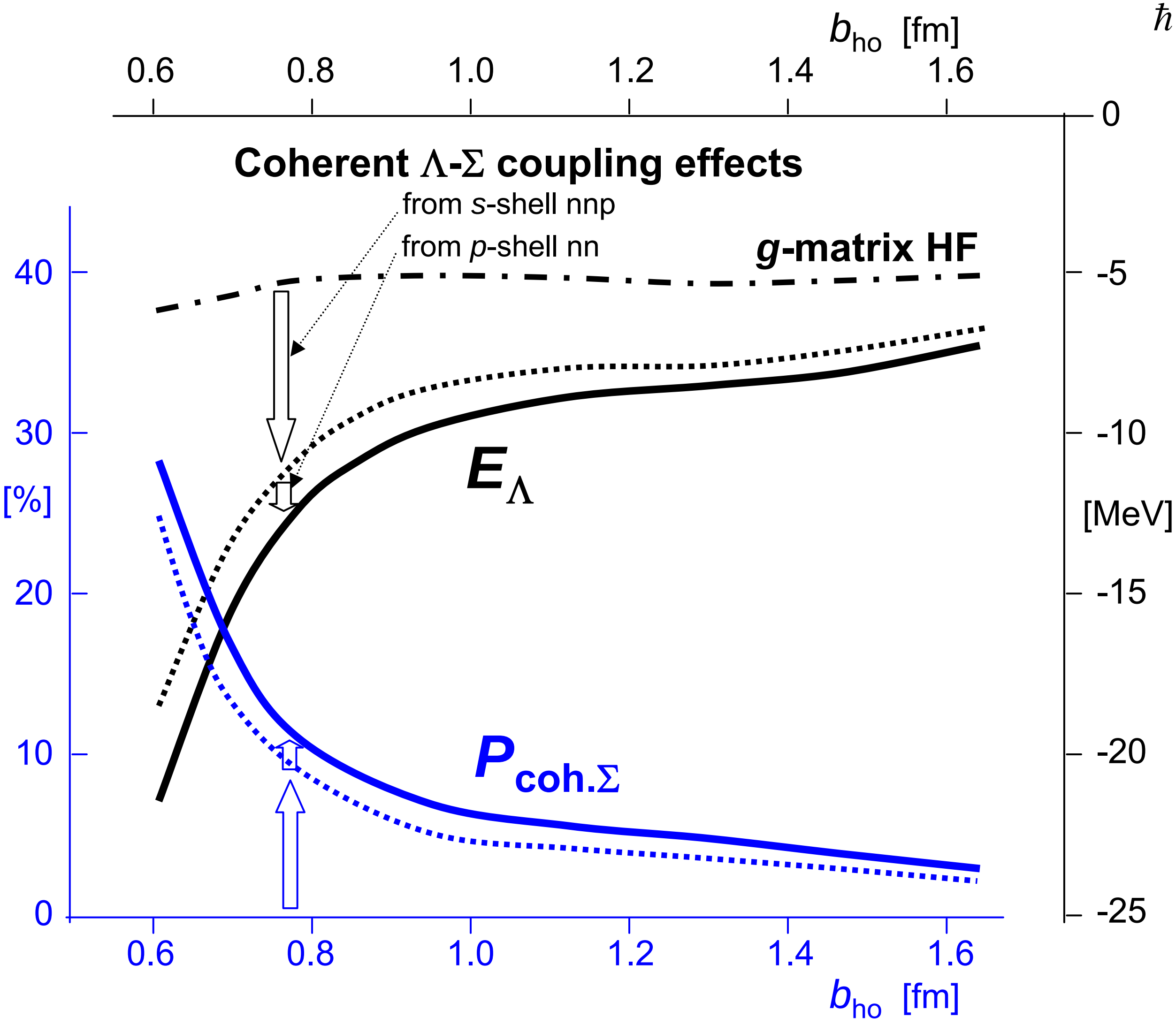
$$\times\left[\sum_{K=\ell_2}\frac{2K+1}{2}\begin{Bmatrix} j_2 & K & \frac{1}{2} \\ 0 & \frac{1}{2} & \ell_2\end{Bmatrix}^2\sum_{J=\tilde\ell}(2J+1)\begin{Bmatrix}\tilde\ell & 0 & J \\ K & \tilde L & \ell_2\end{Bmatrix}^2\frac{1-(-)^{n-\ell'}}{2}\left\{\boxed{V_{\tilde\ell,S=0}^{J,F=0}(\kappa)}\right\}_{\text{Yn and Yp}}\right.$$

$$\left.+\cdots\left\{\boxed{V_{\tilde\ell,S=0}^{J,F=1}(\kappa)}\right\}_{\text{Yn and Yp}}+\cdots\left\{\boxed{V_{\tilde\ell,S=1}^{J,F=0}(\kappa)}\right\}_{\text{Yn and Yp}}+\cdots\left\{\boxed{V_{\tilde\ell,S=1}^{J,F=1}(\kappa)}\right\}_{\text{Yn and Yp}}\right]$$

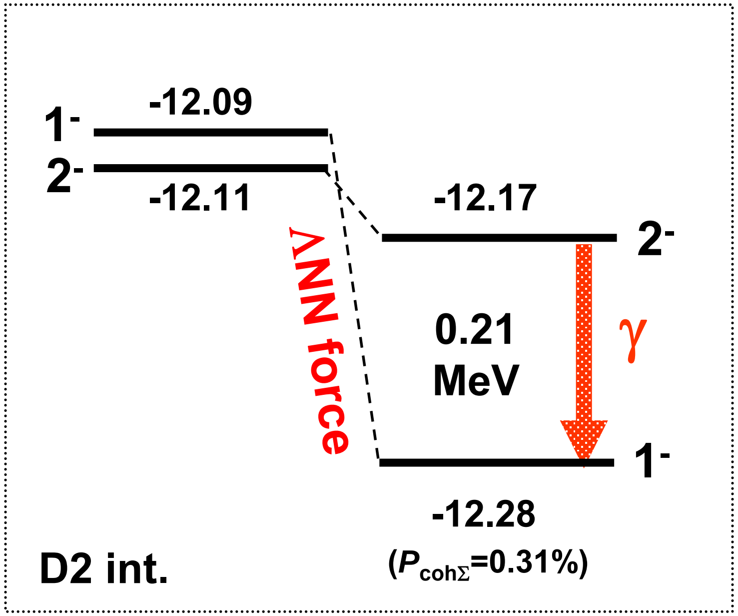
NSC97f is used.

Dependence of coherent Λ-Σ coupling on $^{6}_{\Lambda}$H size

$$\hbar\omega = \frac{\hbar^2}{M}\frac{1}{b_{\mathrm{ho}}^2}$$



$\boxed{^{10}_{\Lambda}\mathrm{Li}} = {}^{6}_{\Lambda}\mathrm{H} + \alpha$



1^- -12.09

2^- -12.11

-12.17 2^-

ΛNN force

0.21 MeV

γ

1^- -12.28

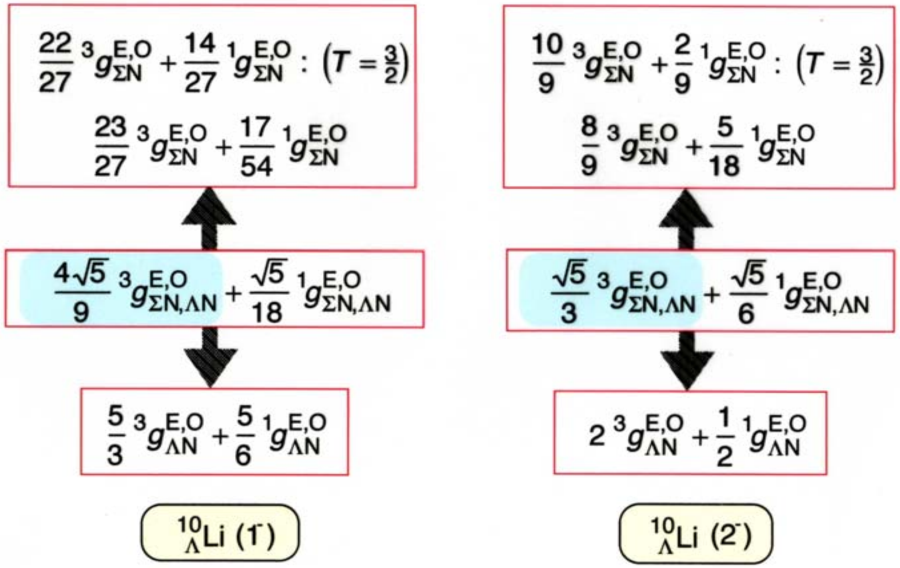
($P_{\mathrm{coh}\Sigma}$=0.31%)

D2 int.

BHF cal.

Y. Akaishi & Khin Swe Myint

YN interaction from p-shell nucleons



$\frac{22}{27}\,{}^3g^{E,O}_{\Sigma N} + \frac{14}{27}\,{}^1g^{E,O}_{\Sigma N} : (T = \frac{3}{2})$

$\frac{23}{27}\,{}^3g^{E,O}_{\Sigma N} + \frac{17}{54}\,{}^1g^{E,O}_{\Sigma N}$

$\frac{4\sqrt{5}}{9}\,{}^3g^{E,O}_{\Sigma N,\Lambda N} + \frac{\sqrt{5}}{18}\,{}^1g^{E,O}_{\Sigma N,\Lambda N}$

$\frac{5}{3}\,{}^3g^{E,O}_{\Lambda N} + \frac{5}{6}\,{}^1g^{E,O}_{\Lambda N}$

$^{10}_{\Lambda}\mathrm{Li}\ (1^-)$

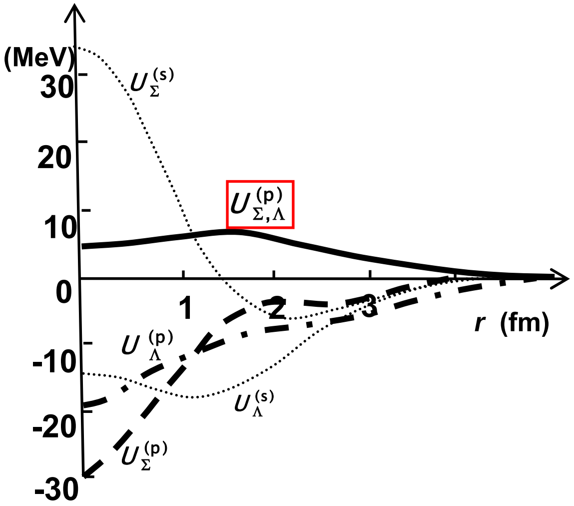
$\frac{10}{9}\,{}^3g^{E,O}_{\Sigma N} + \frac{2}{9}\,{}^1g^{E,O}_{\Sigma N} : (T = \frac{3}{2})$

$\frac{8}{9}\,{}^3g^{E,O}_{\Sigma N} + \frac{5}{18}\,{}^1g^{E,O}_{\Sigma N}$

$\frac{\sqrt{5}}{3}\,{}^3g^{E,O}_{\Sigma N,\Lambda N} + \frac{\sqrt{5}}{6}\,{}^1g^{E,O}_{\Sigma N,\Lambda N}$

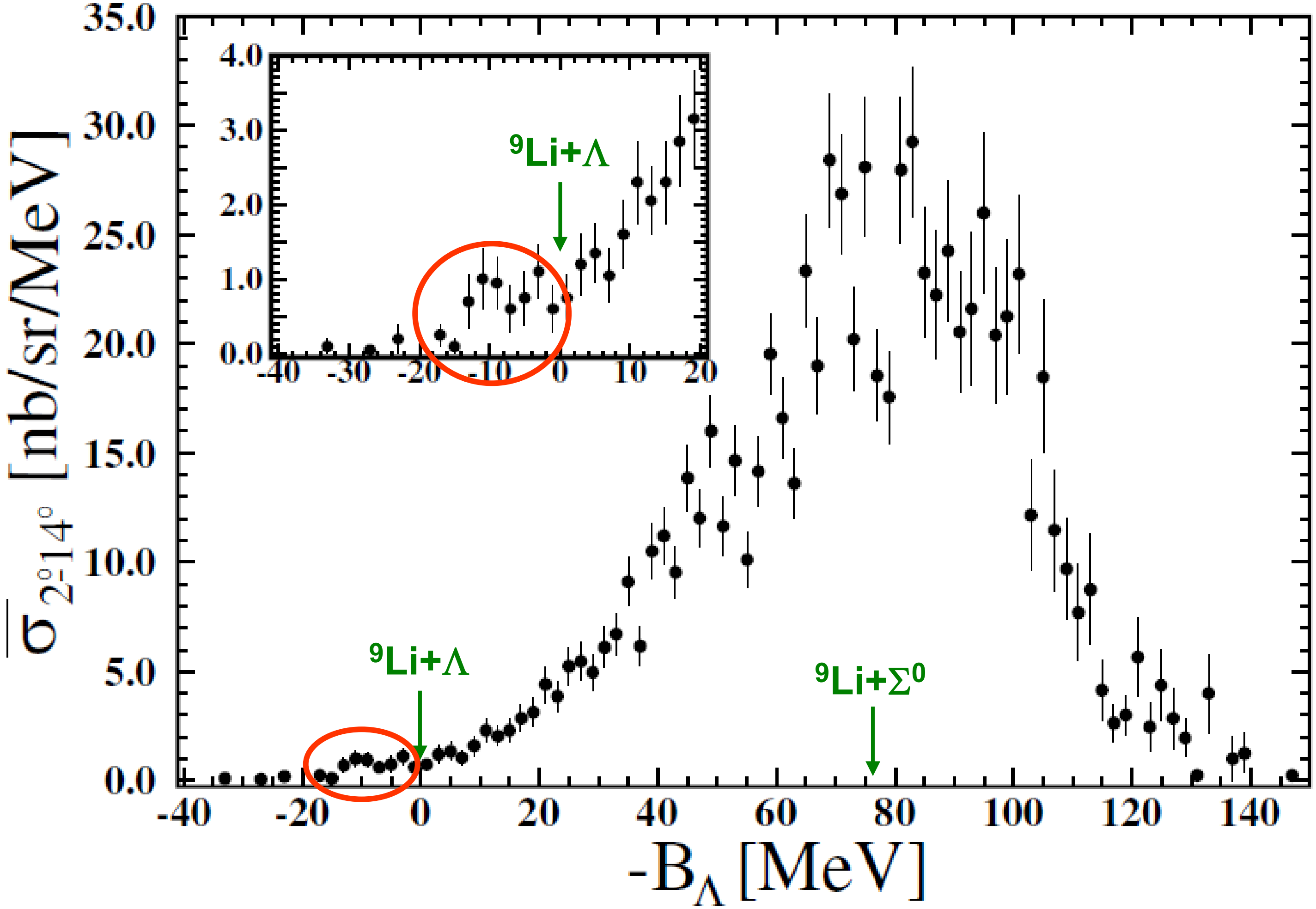
$2\,{}^3g^{E,O}_{\Lambda N} + \frac{1}{2}\,{}^1g^{E,O}_{\Lambda N}$

$^{10}_{\Lambda}\mathrm{Li}\ (2^-)$

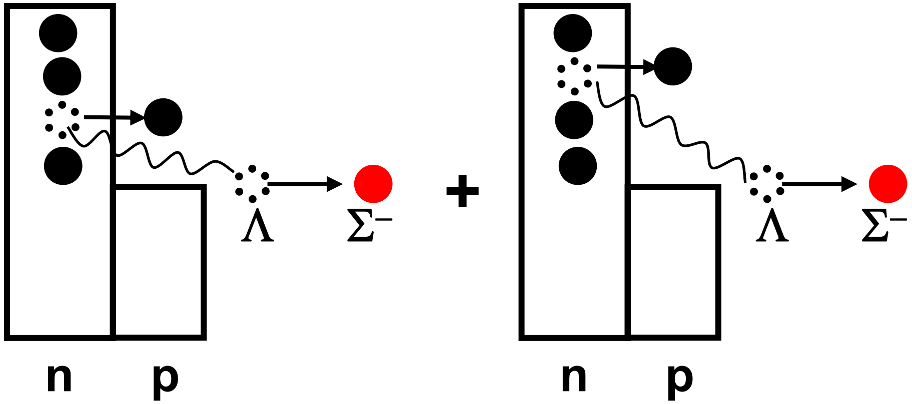


$^{10}B\ (\pi^-, K^+)^{10}_{\Lambda}Li$ spectrum

P.K. Saha et al. (T. Fukuda), Phys. Rev. Lett. 94 (2005) 052502

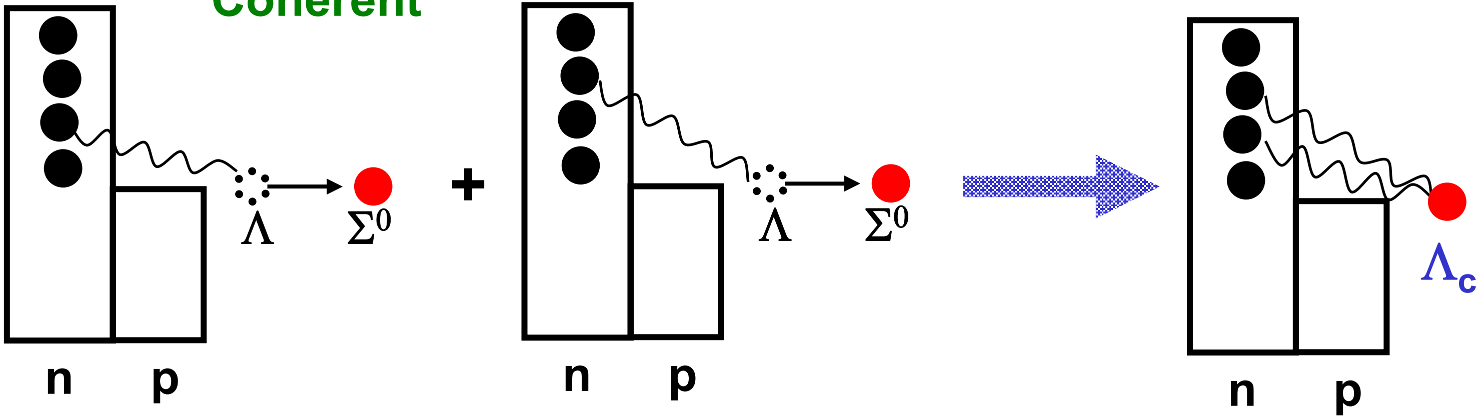


Incoherent

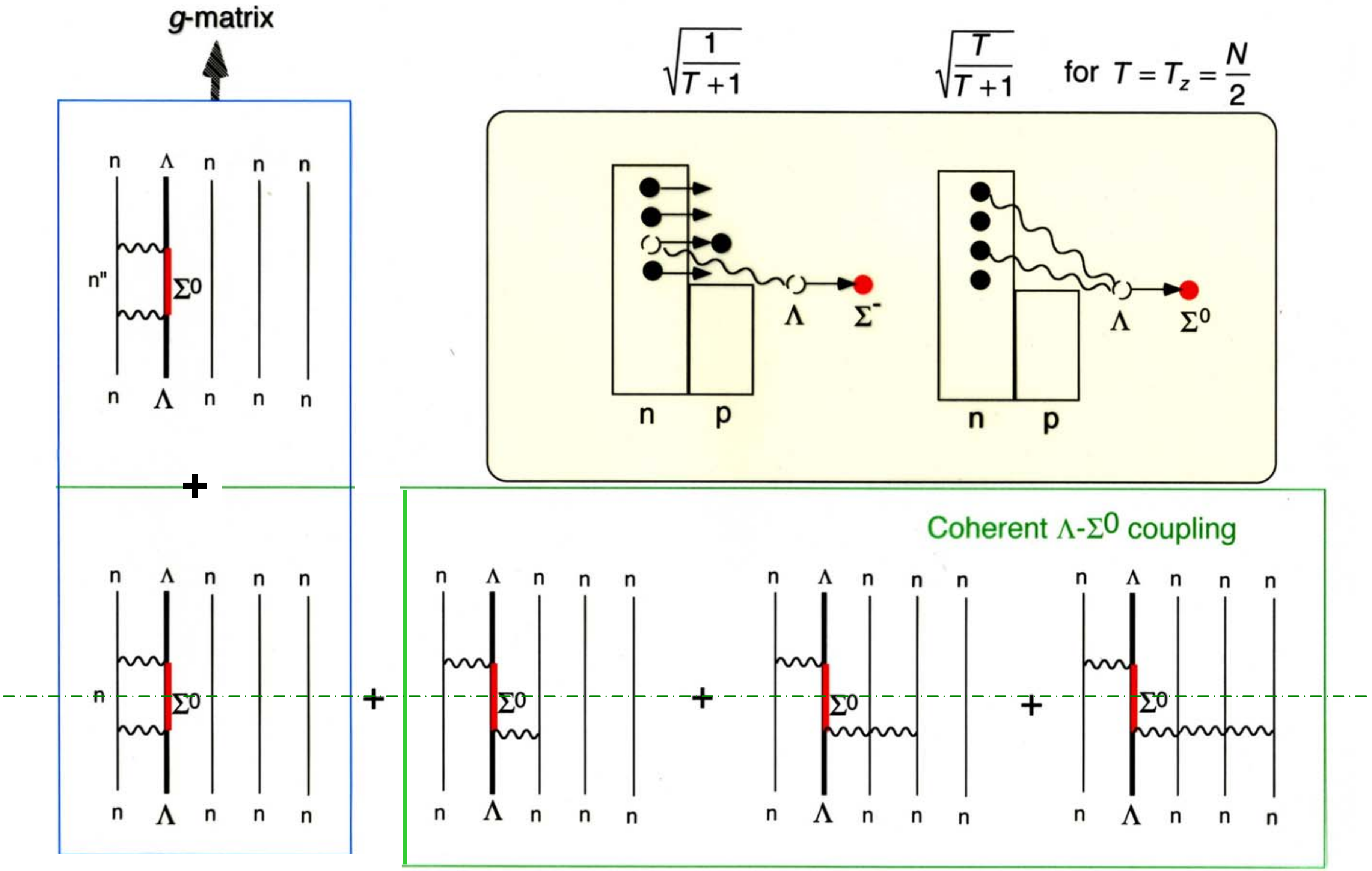


Coherent

$\sqrt{\frac{T}{T+1}}$ for $T = T_z = \frac{N}{2}$



$\Lambda_{coh} = \Lambda/\Sigma^0$



$\sqrt{\frac{1}{T+1}}$ $\sqrt{\frac{T}{T+1}}$ for $T = T_z = \frac{N}{2}$

Coherent Λ-Σ^0 coupling

Relativistic mean field model

Baryons: n, p, Λ, Σ

Mesons: σ, ρ, ω

For Λ and Σ^0

$$\left(\not{p}-\gamma^0 g_{\Lambda\Lambda\omega}\omega_0-M_\Lambda+g_{\Lambda\Lambda\sigma}\sigma\right)\Lambda-\xcancel{\boxed{\gamma^0 g_{\Lambda\Sigma\rho}\rho_0}}\Sigma^0=0$$

$$\left(\not{p}-\gamma^0 g_{\Sigma\Sigma\omega}\omega_0-M_\Sigma+g_{\Sigma\Sigma\sigma}\sigma\right)\Sigma^0-\xcancel{\boxed{\gamma^0 g_{\Sigma\Lambda\rho}\rho_0}}\Lambda=0$$

For mesons

$$m_\sigma{}^2\sigma=\sum g_{BB\sigma}\langle\bar{B}B\rangle$$

$$m_\omega{}^2\omega^0=\sum g_{BB\omega}\langle\bar{B}\gamma^0 B\rangle$$

$$m_\rho{}^2\rho^0=\sum g_{BB\rho}\langle\bar{B}\gamma^0 B\rangle+\xcancel{\boxed{g_{\Lambda\Sigma\rho}\left(\langle\bar{\Lambda}\gamma^0\Sigma\rangle+\langle\bar{\Sigma}\gamma^0\Lambda\rangle\right)}}$$

"Normal state of infinite matter"

Baryons in the medium carry the same quantum numbers in vacuum.

N.K. Glendenning, Astrophys. J. 293 (1985) 470.

Effective $\Lambda\Lambda$ interaction in neutron-rich hypernuclei

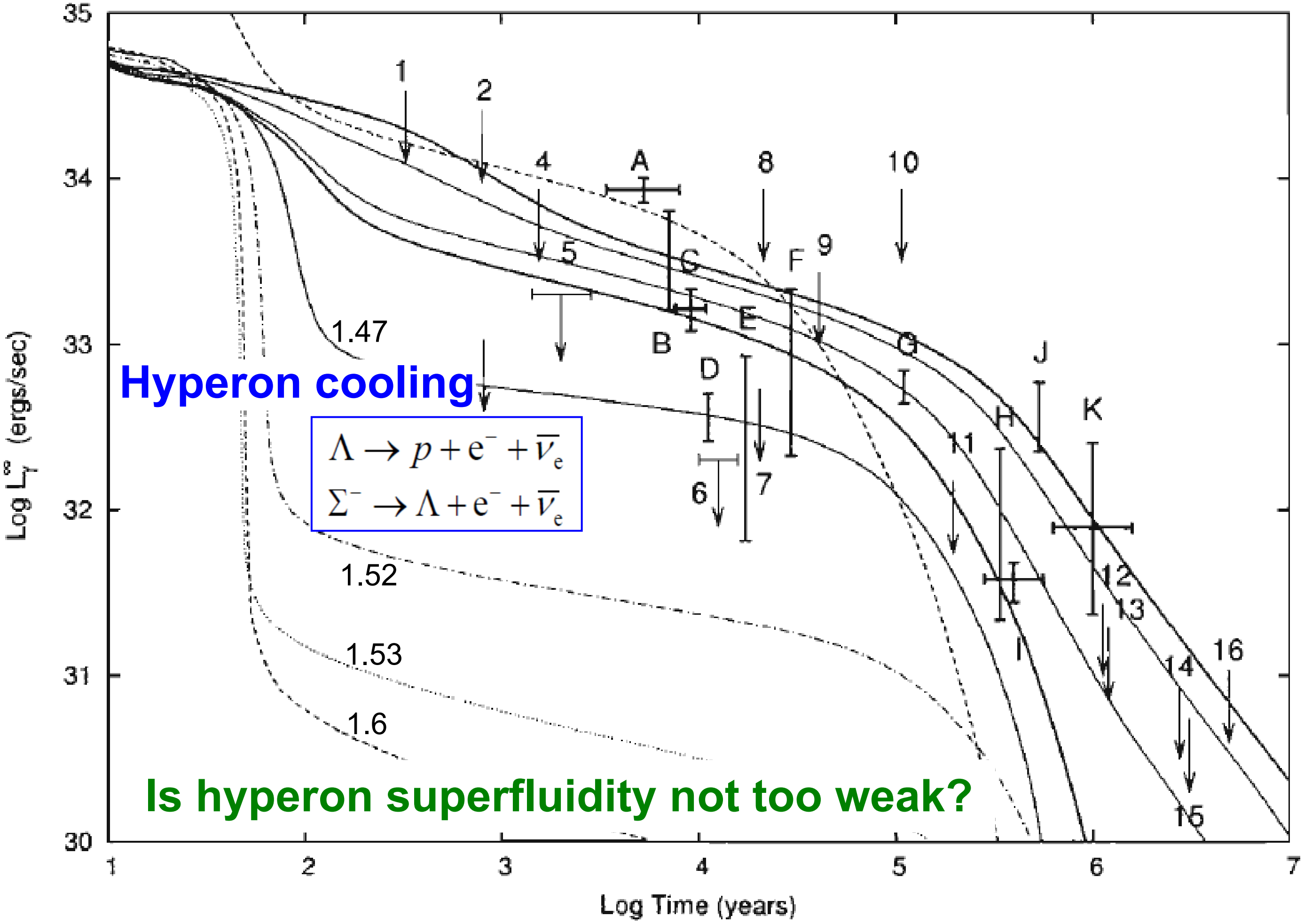
in collaboration with

Aye Aye Min & Khin Swe Myint

二重ハイパー核 ${}^{5}_{\Lambda\Lambda}\mathrm{H}$ を取り上げる。ここでのテーマは「$\Lambda\Lambda$有効相互作用は、中性子物質中とN=Z核物質中で同じか？」である。中性子星中での YY 相互作用を知るために、${}^{5}_{\Lambda\Lambda}\mathrm{H}$ の実験データが欠かせないことを示す。

Thermal evolution of hyperon-mixed neutron stars

S. Tsuruta, J. Sadino, A. Kobelski, M.A. Teter, A.C. Liebmann,
T. Takatsuka, K. Nomoto & H. Umeda,
Astrophys. J. 691 (2009) 621



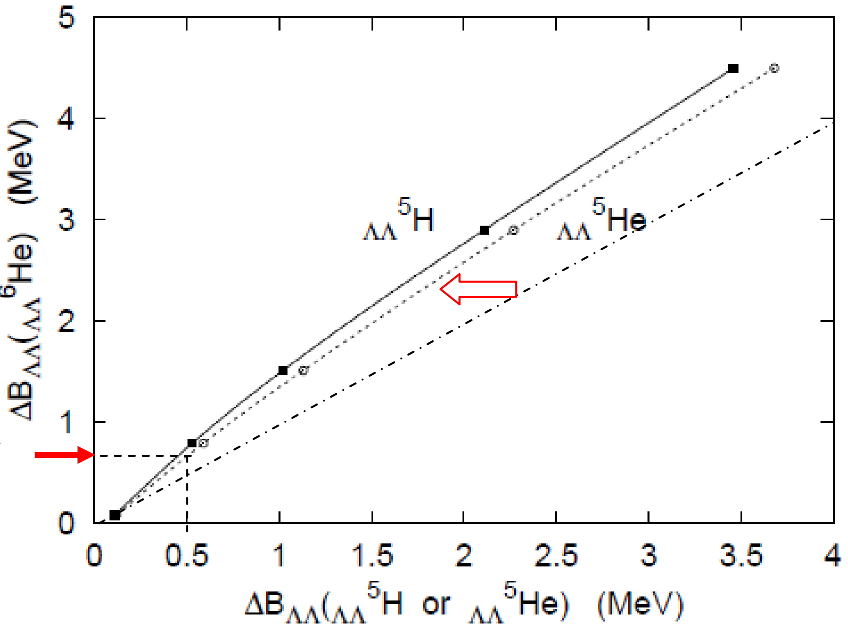
Hyperon cooling

$$\Lambda \rightarrow p + e^- + \overline{\nu}_e$$
$$\Sigma^- \rightarrow \Lambda + e^- + \overline{\nu}_e$$

Is hyperon superfluidity not too weak?

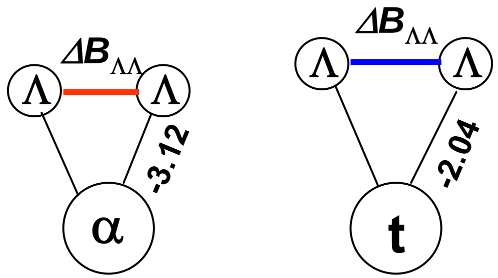
A. Gal, Invited Lecture at J-PARC, Tokai, on Feb. 9, 2012,

(neglecting K.S. Myint et al., Eur. Phys. J. A 16)



NAGARA
0.67+-0.17 MeV

← **Simple physics!**



I.N. Filikhin, A. Gal, Nucl. Phys. A **707** (2002) 491

s-wave Faddeev calculations of $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ vs. $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H},\ {}^{5}_{\Lambda\Lambda}\mathrm{He})$

$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) \equiv \mathrm{B}_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) - 2\mathrm{B}_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$

$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) \approx 1$ MeV implies that ${}^{5}_{\Lambda\Lambda}\mathrm{H}$ & ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ are also bound

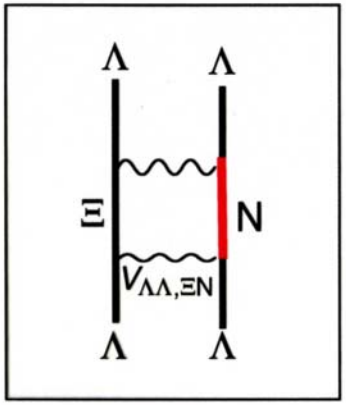
${}^{5}_{\Lambda\Lambda}\mathrm{H}$ & ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ may mark the onset of ΛΛ binding

Pauli Suppression Effect in ${}^{\;6}_{\Lambda\Lambda}\mathrm{He}$

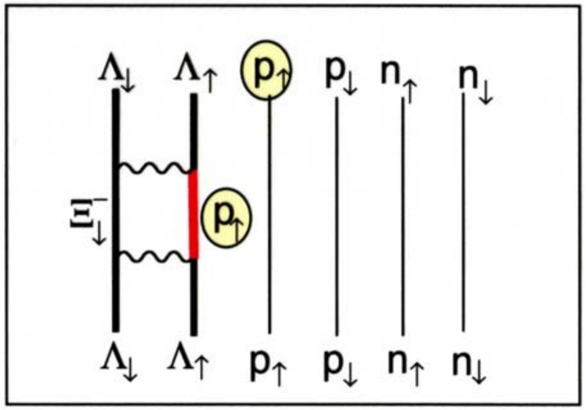
$$\Delta V_{\mathrm{Pauli}} = V_{\Lambda\Lambda,\Xi^-\mathrm{p}} \frac{P_\alpha}{\Delta M} V_{\Xi^-\mathrm{p},\Lambda\Lambda} + V_{\Lambda\Lambda,\Xi^0\mathrm{n}} \frac{P_\alpha}{\Delta M} V_{\Xi^0\mathrm{n},\Lambda\Lambda}$$

$$\Delta M = M_\Xi + M_\mathrm{N} - 2M_\Lambda + 2B_\Lambda\left({}^{5}_{\Lambda}\mathrm{He}\right) = 32.0\,\mathrm{MeV}$$

$$P_\alpha = \left|\mathrm{N(0s)}\right\rangle_{\alpha\,\alpha}\left\langle\mathrm{N(0s)}\right|$$



Free space



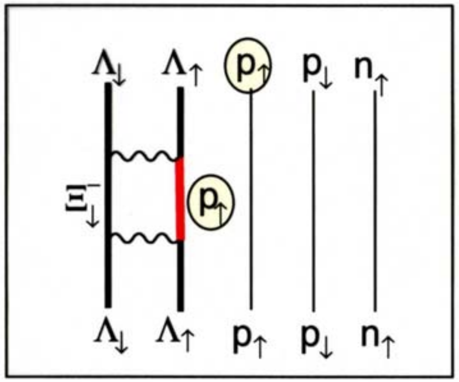
${}^{\;6}_{\Lambda\Lambda}\mathrm{He}$

The Pauli suppression effect must be included in fitting the empirical value of $\Delta B_{\Lambda\Lambda}\left({}^{\;6}_{\Lambda\Lambda}\mathrm{He}\right)$.

Khin Swe Myint, S. Shinmura & Y. Akaishi, Eur. Phys. J. A 16 (2003) 21

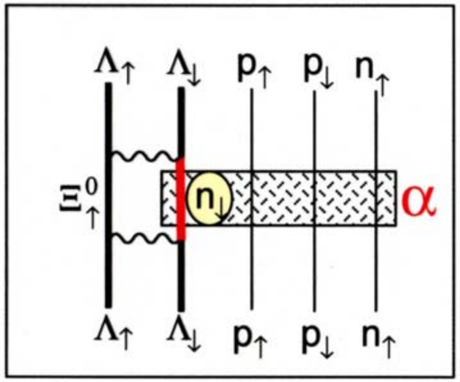
Five-Body Double-Λ Hypernuclei

Pauli suppression effect



$^{5}_{\Lambda\Lambda}$He

Alpha formation effect



$^{5}_{\Lambda\Lambda}$He

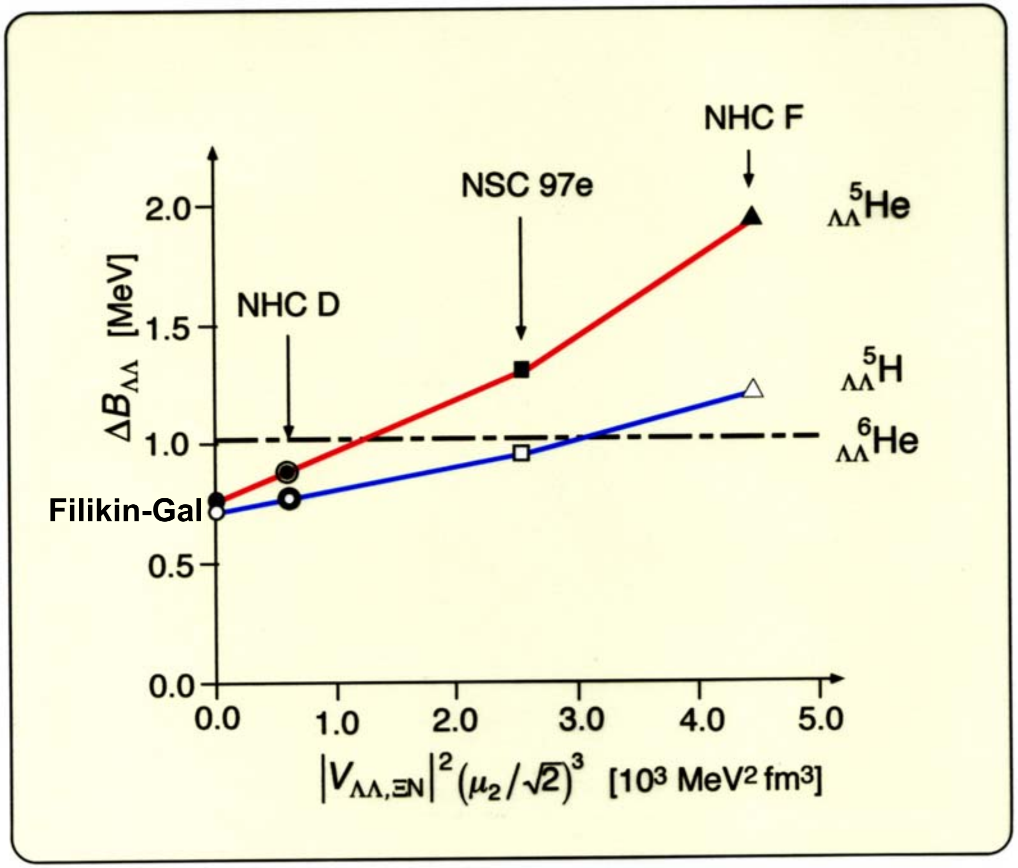
$$|T=0\rangle = \sqrt{\tfrac{1}{2}}\left(-|\Xi^- p\rangle + |\Xi^0 n\rangle\right)$$

$$\boxed{\Delta V_{\text{alpha}} = \frac{1}{2}\left(V_{\Lambda\Lambda,\Xi^0 n}\frac{P_h}{\Delta M}V_{\Xi^0 n,\Lambda\Lambda}\right) - \frac{1}{2}\left(V_{\Lambda\Lambda,\Xi^0 n}\frac{P_\alpha}{\Delta M_\alpha}V_{\Xi^0 n,\Lambda\Lambda}\right)}$$

$$\Delta M_\alpha = M_{\Xi^0} + M_\alpha - M_h - 2M_\Lambda + 2\bar{B}_\Lambda\left({}^{4}_{\Lambda}\text{He}\right) = 5.69\ \text{MeV}$$

$$\Delta M = 28.83\ \text{MeV}$$

$\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H})$ vs. $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$



$\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He}) > \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$

contradicts with Filikhin & Gal's remark:
Nucl. Phys. A**707** (2002) 491.

Observations of $^{5}_{\Lambda\Lambda}\mathrm{He}$ and $^{5}_{\Lambda\Lambda}\mathrm{H}$
are significant to deduce the coupling strength.

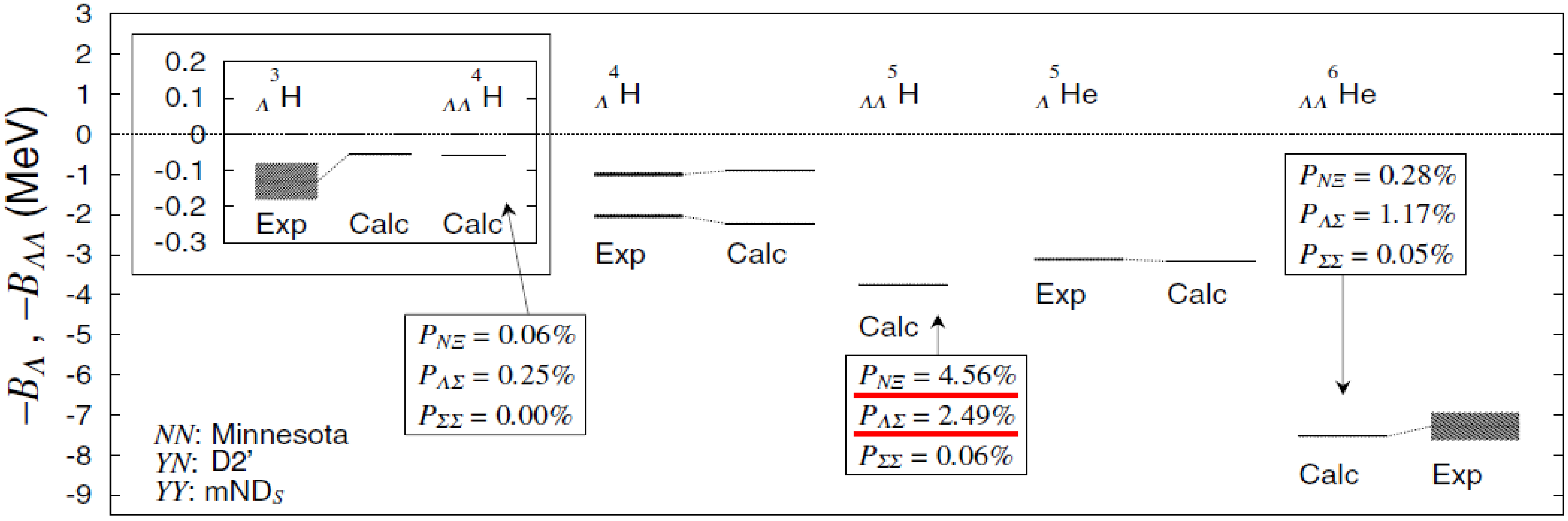
PRL 94, 202502 (2005) PHYSICAL REVIEW LETTERS week ending 27 MAY 2005

Fully Coupled Channel Approach to Doubly Strange *s*-Shell Hypernuclei

H. Nemura,[1,*] S. Shinmura,[2] Y. Akaishi,[3] and Khin Swe Myint[4]

$\overset{T=0}{\Lambda\Lambda}$ - $p\Xi^-$ - $n\Xi^0$ - $\overset{T=1}{\Lambda\Sigma^0}$ - $\Sigma^+\Sigma^-$ - $\Sigma^0\Sigma^0$ couplings

YY	$B_{\Lambda\Lambda}({}^{4}_{\Lambda\Lambda}\mathrm{H})$	$B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H})$	$B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He})$	$B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$
ND_S	0.107	4.05	3.96	7.94
mND_S	0.058	3.75	3.66	7.54
NF_S	0.128	3.84	3.77	7.53
Expt.				$7.25 \pm 0.19^{+0.18}_{-0.11}$



$\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H})$ vs. $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2\overline{B}_{\Lambda}^{\mathrm{cal}}$$

	$\overline{B}_{\Lambda}^{\mathrm{cal}}$
${}^{4}_{\Lambda}\mathrm{H}$	1.24
${}^{4}_{\Lambda}\mathrm{He}$	1.21
${}^{5}_{\Lambda}\mathrm{He}$	3.18

$\overline{B}_{\Lambda} = \dfrac{3B_{\Lambda}(1^{+}) + B_{\Lambda}(0^{+})}{4}$ for ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$

mNDs	$B_{\Lambda\Lambda}$	$\Delta B_{\Lambda\Lambda}$
${}^{5}_{\Lambda\Lambda}\mathrm{H}$	3.75	1.27
${}^{5}_{\Lambda\Lambda}\mathrm{He}$	3.66	1.24
${}^{6}_{\Lambda\Lambda}\mathrm{He}$	7.54	1.18

(unit in MeV)

1.27/1.18=1.08

$$\approx \langle \phi_{\Lambda\Lambda}^{(t)} | V_{\Lambda\Lambda}^{(t)} | \phi_{\Lambda\Lambda}^{(t)} \rangle / \langle \phi_{\Lambda\Lambda}^{(\alpha)} | V_{\Lambda\Lambda}^{(\alpha)} | \phi_{\Lambda\Lambda}^{(\alpha)} \rangle$$

This is not a fair comparison of effective interactions, because $\phi_{\Lambda\Lambda}^{(\alpha)}$ is more compact than $\phi_{\Lambda\Lambda}^{(t)}$.

NFs	$B_{\Lambda\Lambda}$	$\Delta B_{\Lambda\Lambda}$
${}^{5}_{\Lambda\Lambda}\mathrm{H}$	3.84	1.36
${}^{5}_{\Lambda\Lambda}\mathrm{He}$	3.77	1.35
${}^{6}_{\Lambda\Lambda}\mathrm{He}$	7.53	1.17

(unit in MeV)

1.36/1.17=1.16

$\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{H})$ is larger than $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$!

ATMS calculation of $^{5}{}_{\Lambda\Lambda}$H and $^{6}{}_{\Lambda\Lambda}$He

by Aye Aye Min

DATA for ATMS program

Double-Λ hypernuclei	Instead of 260.0 (for ZVCE)	Total energy		Note
5LLH	249.56	-0.38400D+01 (UPPER)	-0.38542D+01 (EIGEN)	NNN = 101 NT = 1000000 ICORE = 1

$$V_{\Lambda\Lambda}^{(t)}(r) = 5000.0 \exp(-(r/0.355)^2) - 249.6 \exp(-(r/0.855)^2) \quad \text{in } {}^{5}_{\Lambda\Lambda}\mathrm{H}$$

Double-Λ hypernuclei	Instead of 240.0 (for ZVCE)	Total energy		Note
6LLHe	225.61	-0.75301D+01 (UPPER)	-0.75408D+01 (EIGEN)	NNN = 101 NT = 1000000 ICORE = 2

$$V_{\Lambda\Lambda}^{(\alpha)}(r) = 5000.0 \exp(-(r/0.355)^2) - 225.6 \exp(-(r/0.855)^2) \quad \text{in } {}^{6}_{\Lambda\Lambda}\mathrm{He}$$

[in MeV, fm]

If $V^{(\alpha)}_{\Lambda\Lambda}$, instead of $V^{(t)}_{\Lambda\Lambda}$, is used for $^{5}{}_{\Lambda\Lambda}$H ,

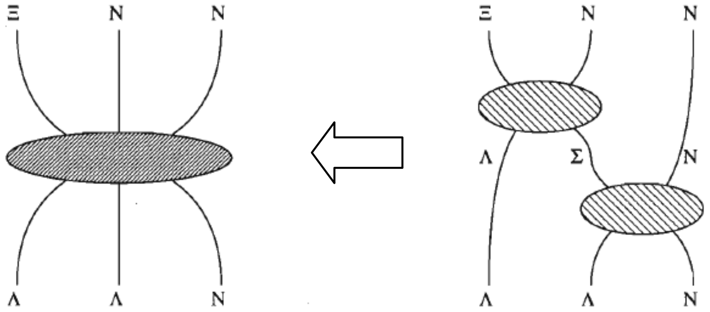
$B_{\Lambda\Lambda}$($^{5}{}_{\Lambda\Lambda}$H) is reduced to 3.45 MeV which gives $\Delta B_{\Lambda\Lambda}$= 0.97 MeV.

$$\left\langle \phi^{(t)}_{\Lambda\Lambda} \middle| V^{(t)}_{\Lambda\Lambda} \middle| \phi^{(t)}_{\Lambda\Lambda} \right\rangle \Big/ \left\langle \phi^{(t)}_{\Lambda\Lambda} \middle| V^{(\alpha)}_{\Lambda\Lambda} \middle| \phi^{(t)}_{\Lambda\Lambda} \right\rangle \approx$$

1.27/0.97 = 1.31 for mNDs and 1.36/0.97 = 1.40 for NFs

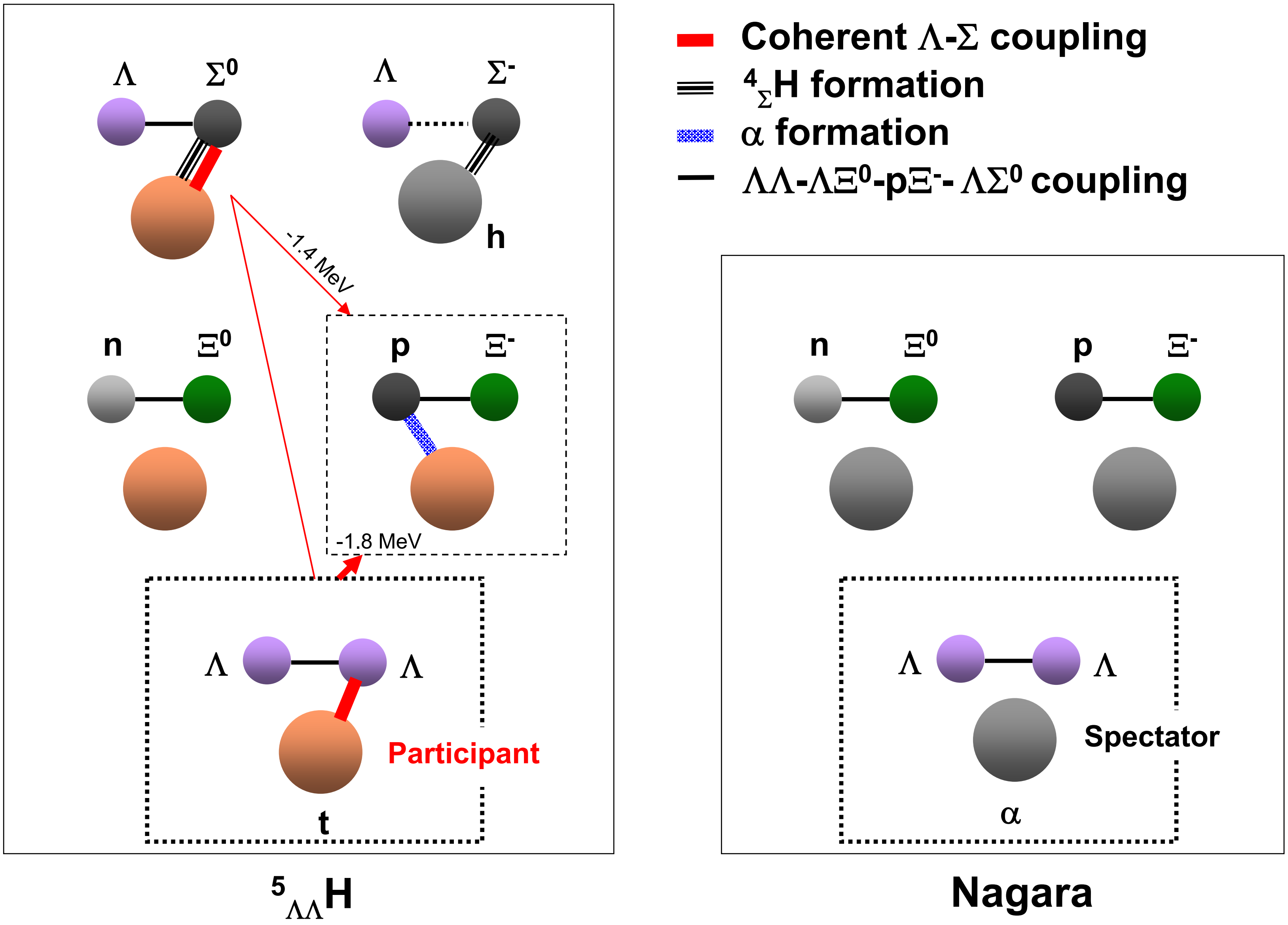
Effective ΛΛ interaction in $^{5}{}_{\Lambda\Lambda}$H is more attractive than that in $^{6}{}_{\Lambda\Lambda}$He by 30 ~ 40%.

Nemura's three-body force



H. Nemura, AIP Conf. Proc. 1011 (2008) 129

Coupling scheme



🟥 Coherent Λ-Σ coupling

≡ $^4_\Sigma$H formation

▨ α formation

— $\Lambda\Lambda$-$\Lambda\Xi^0$-pΞ^-- $\Lambda\Sigma^0$ coupling

$^5_{\Lambda\Lambda}$H

Nagara

Production 1

Stopped Ξ^- on ^{6}Li

M. May, Nouvo Cim. A 102 (1989) 401
D. Zhu, C.B. Dover, A. Gal & M. May, Phys. Rev. Lett. 67 (1991) 2268

⇩

~3% branching

Ξ^- ^{6}Li —>

$^{6}_{\Lambda\Lambda}$He + n + 31.88 MeV (1)

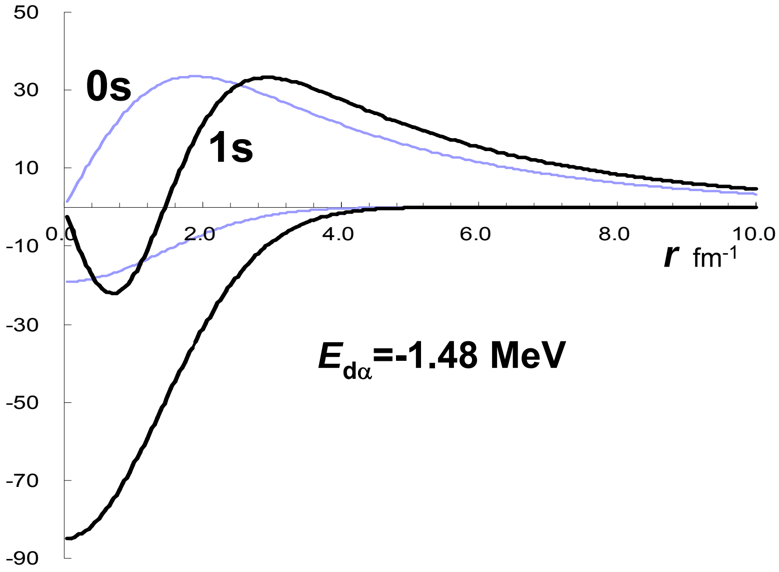
$^{5}_{\Lambda\Lambda}$H + d + 13..... MeV (2)

$^{5}_{\Lambda}$He + Λ + n + 27.75 MeV (3)

$^{4}_{\Lambda}$H + Λ + d + 9.08 MeV (4)

$^{4}_{\Lambda}$H + Λ + p + n + 6.86 MeV
$^{4}_{\Lambda}$He + Λ + n + n + ... MeV
^{4}He + Λ + Λ + n + 24.63 MeV
$^{3}_{\Lambda}$H + Λ + d + n + ... MeV
$^{3}_{\Lambda}$H + Λ + p + n + n + ... MeV
etc.

d-α relative wave function of ^{6}Li



Formation ratio of $^{6}_{\Lambda\Lambda}$He to $^{5}_{\Lambda\Lambda}$H from stopped Ξ^- on ^{6}Li

Aye Aye Min, Khin Swe Myint, J. Esmaili, Y. Akaishi,
Few-Body Syst. 54 (2013) 381

^{6}Li(α-d)	2S absorption	2P absorption	3D absorption
1s	**0.24**	**0.62**	**0.89**
0s	0.016	0.024	0.029
h.o.-1s	0.12	0.39	0.42
MC sampling	1,600,000,000	51,200,000,000	51,200,000,000

S 0.04% P 30.3% D 68.9%

$$^{5}_{\Lambda\Lambda}\mathrm{H} / ^{6}_{\Lambda\Lambda}\mathrm{He} \approx 1.24$$

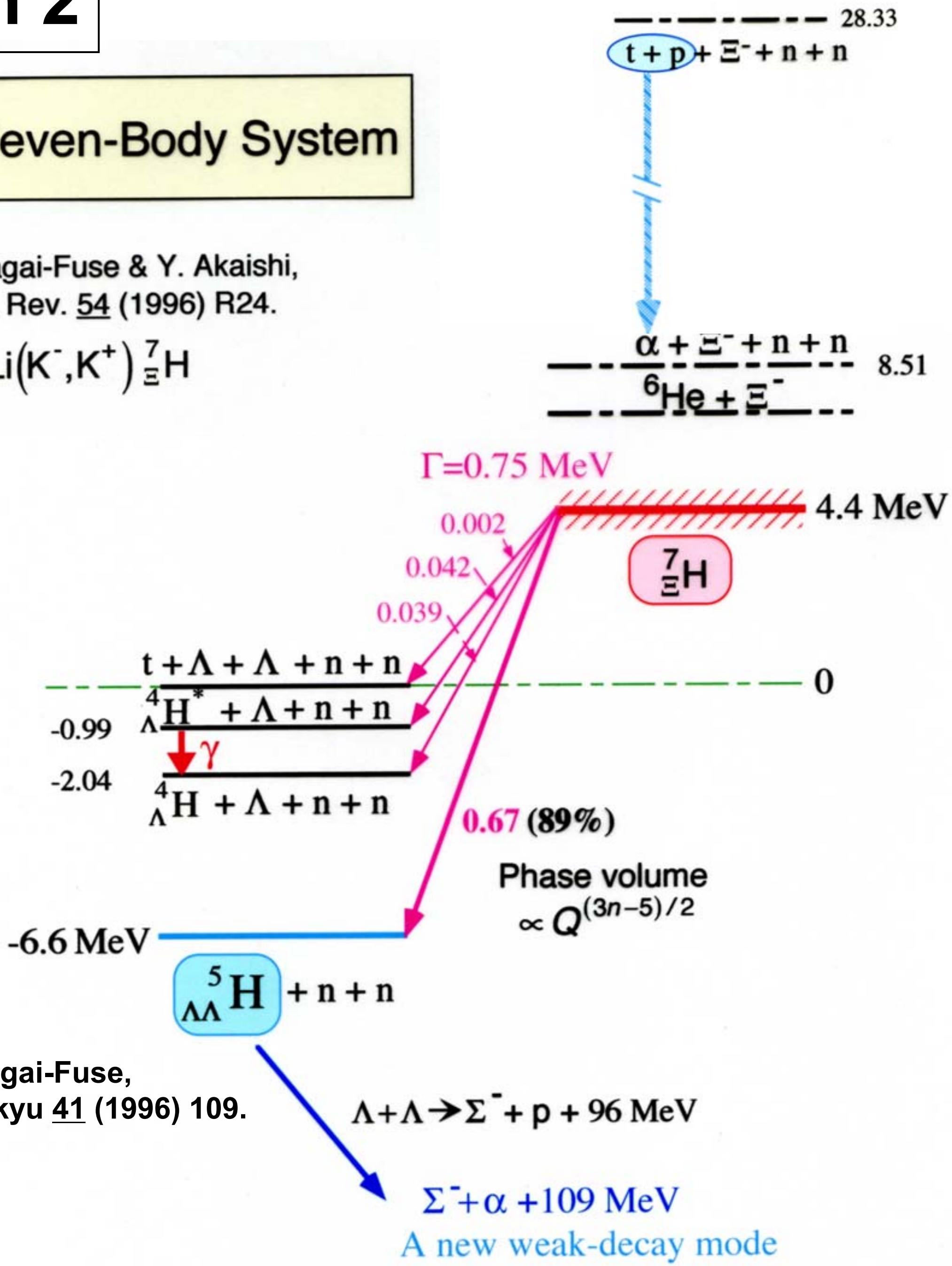
A large population of $^{5}_{\Lambda\Lambda}$H

Production 2

S=-2 Seven-Body System

I. Kumagai-Fuse & Y. Akaishi,
Phys. Rev. 54 (1996) R24.

$^{7}\mathrm{Li}(K^{-},K^{+})\,^{7}_{\Xi}\mathrm{H}$



Phase volume
$\propto Q^{(3n-5)/2}$

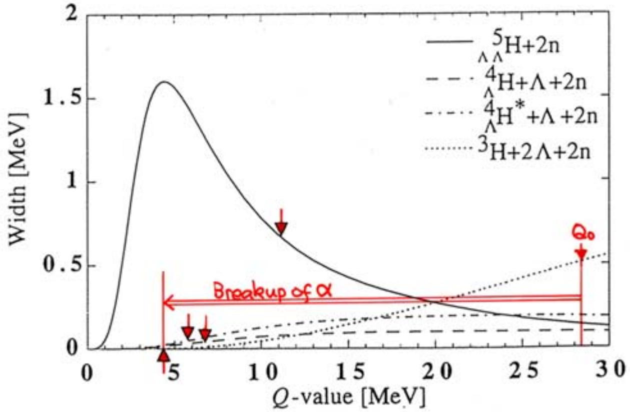
$\Lambda+\Lambda \rightarrow \Sigma^{-} + p + 96$ MeV

$\Sigma^{-}+\alpha$ +109 MeV
A new weak-decay mode

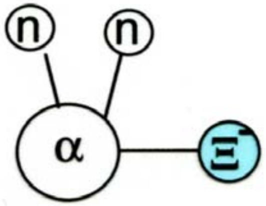
I. Kumagai-Fuse,
Gensikaku Kenkyu 41 (1996) 109.

Selective population of ${}^{5}_{\Lambda\Lambda}\mathrm{H}$

I. Kumagai-Fuse & Y. Akaishi,
Phys. Rev. 54 (1996) R24.



$^{7}\mathrm{Li}\left(K^{-},K^{+}\right){}^{7}_{\Xi}\mathrm{H}$



$${}^{7}_{\Xi}\mathrm{H} \rightarrow \boxed{{}^{5}_{\Lambda\Lambda}\mathrm{H}} + n + n \quad \sim 11\ \mathrm{MeV}$$

$$\rightarrow {}^{4}_{\Lambda}\mathrm{H} + \Lambda + n + n \quad \sim 7\ \mathrm{MeV}$$

$$\rightarrow {}^{4}_{\Lambda}\mathrm{H}^{*} + \Lambda + n + n \quad \sim 6\ \mathrm{MeV}$$

$$\rightarrow {}^{3}\mathrm{H} + \Lambda + \Lambda + n + n \sim 5\ \mathrm{MeV}$$

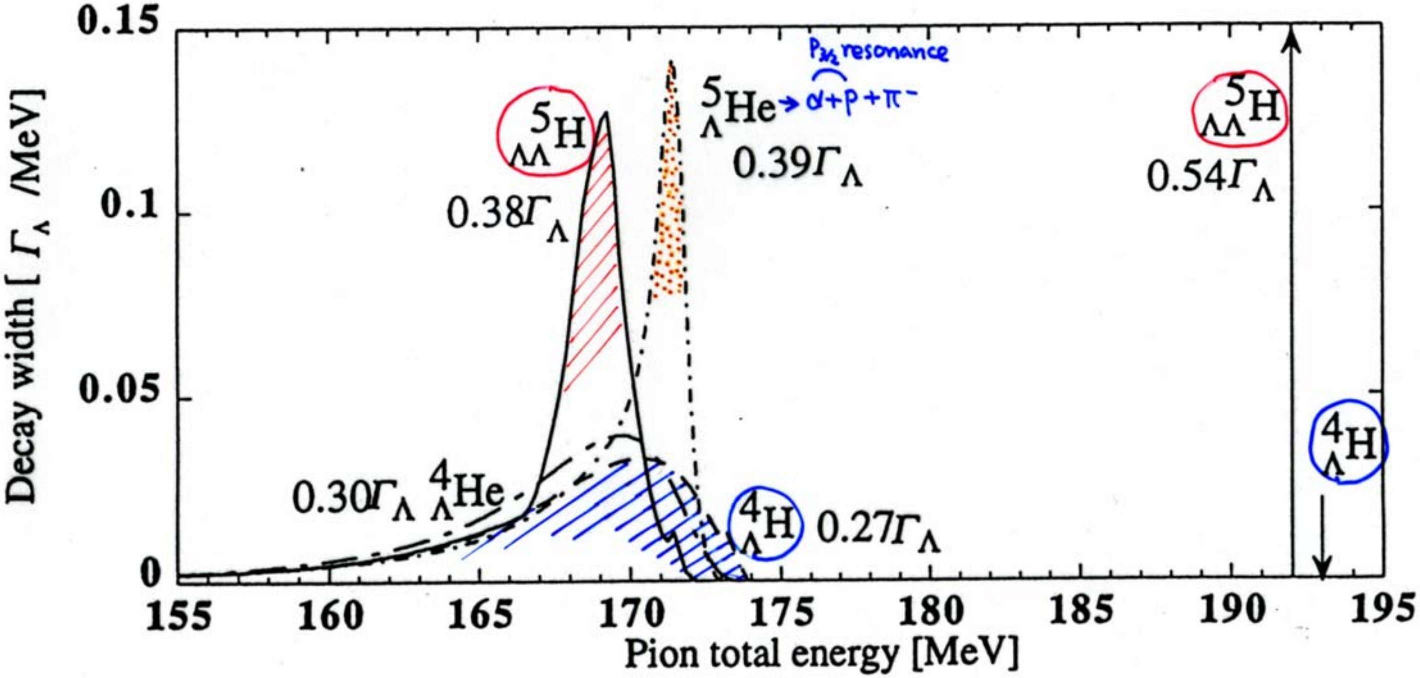
~90 %
branching

$$\Xi^{-} + p \rightarrow \Lambda + \Lambda + 28.33\ \mathrm{MeV} \longleftrightarrow B.E.(\alpha) = 28.30\ \mathrm{MeV}$$

Phase volume $\propto Q^{(3n-5)/2}$ for n-body breakup.

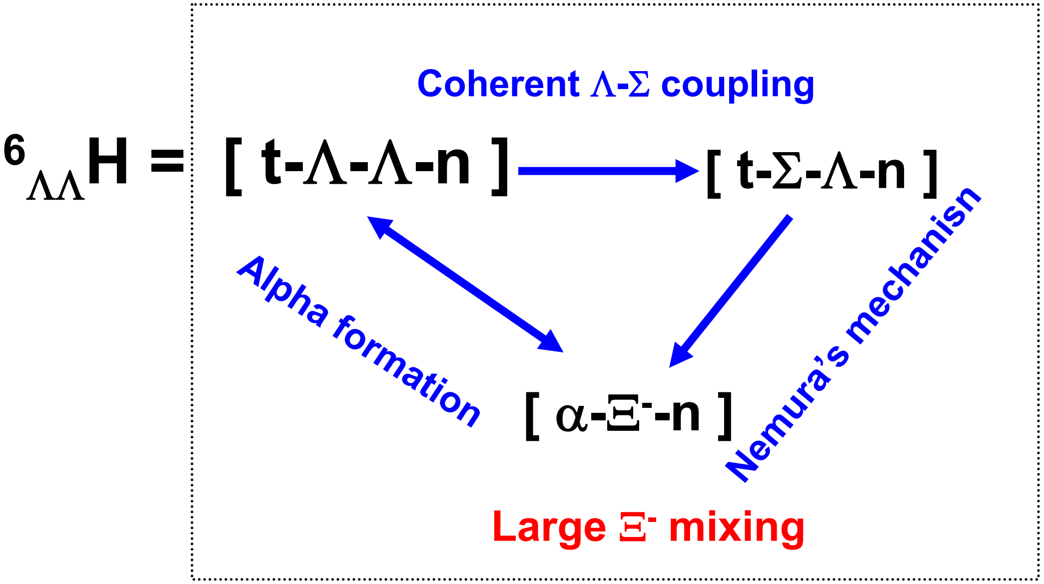
Weak-decay pion spectroscopy

I. Kumagai-Fuse et al., Genshikaku Kenkyu <u>41</u> (1996) 109



Production 3

^{6}Li (K^{-},K^{+}) $^{6}_{\Lambda\Lambda}$H



$^{6}_{\Lambda\Lambda}$H = [t-Λ-Λ-n]

Missing-mass spectroscopy in *S*=-2 sector
via one-step process !

Possible reactions

$^{6}\mathrm{Li}\left(K^{-}, K^{+}\right)\,^{6}_{\Lambda\Lambda}\mathrm{H}$

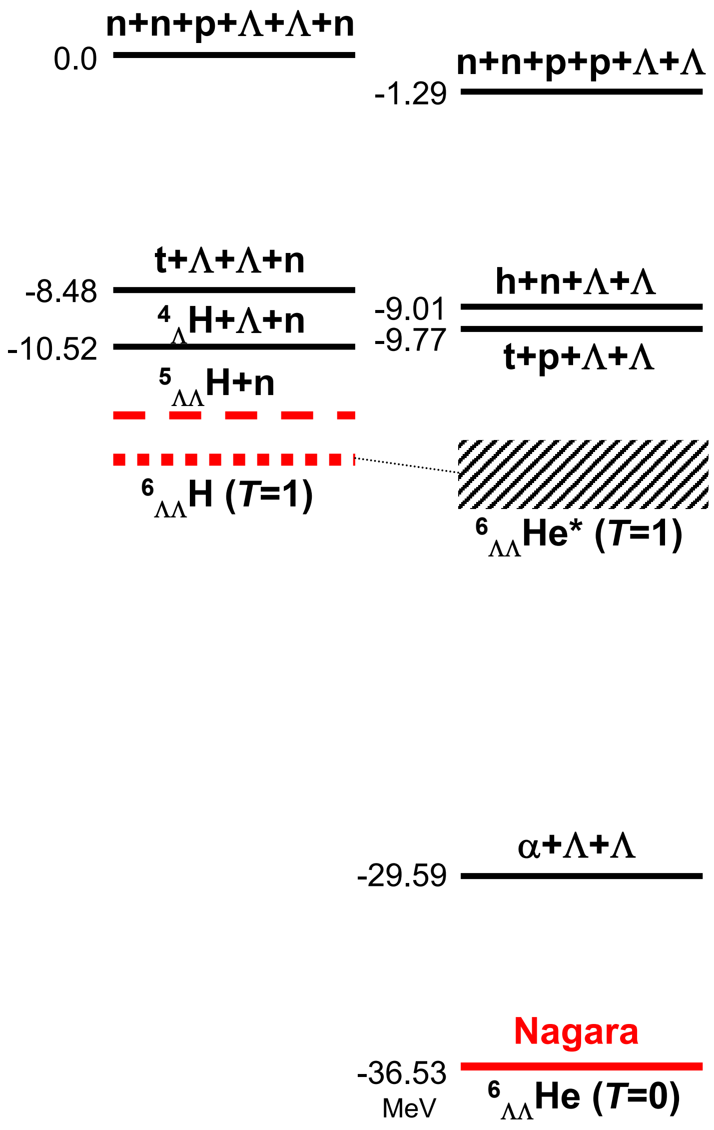
$^{6}\mathrm{Li}\left(K^{-}, K^{+}n\right)\,^{5}_{\Lambda\Lambda}\mathrm{H}$

$^{7}\underset{53d}{\mathrm{Be}}\left(K^{-}, K^{+}d\right)\,^{5}_{\Lambda\Lambda}\mathrm{H}$

etc.

Feasibility ?

Energy levels



0.0 **n+n+p+Λ+Λ+n**

-1.29 **n+n+p+p+Λ+Λ**

-8.48 **t+Λ+Λ+n**

-10.52 **$^{4}_{\Lambda}$H+Λ+n**

$^{5}_{\Lambda\Lambda}$H+n

$^{6}_{\Lambda\Lambda}$H (*T*=1)

-9.01 **h+n+Λ+Λ**

-9.77 **t+p+Λ+Λ**

$^{6}_{\Lambda\Lambda}$He* (*T*=1)

-29.59 **α+Λ+Λ**

Nagara

-36.53 MeV **$^{6}_{\Lambda\Lambda}$He (*T*=0)**

Concluding remarks

Study of few-body neutron-rich hypernuclei,

typically $^{6}_{\Lambda}\mathrm{H}$ and $^{5}_{\Lambda\Lambda}\mathrm{H}$,

is a doorway to neutron-star matter physics.

Key issues

Coherent Λ-Σ coupling

$\Lambda_{\mathrm{coh}}(\Lambda/\Sigma^{0})$

$\Lambda\Lambda$–$\Xi^{-}p$–$\Lambda\Sigma^{0}$ isospin-violating couplings

Superfluidity

"Transition" mean fields due to three-body forces

Thank you very much!