格子QCD計算によるheavy Qを含む中間子間相互作用の研究

-- T_{cc}およびT_{cs}の探索 --

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(Hadrons to Atomic nuclei from Lattice QCD)

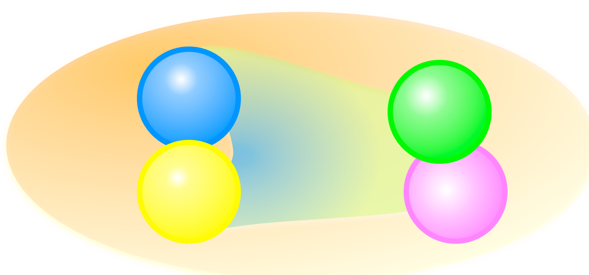
原子核媒質中のハドロン研究=魅力と課題= @J-PARC, Aug. 5--6, 2013

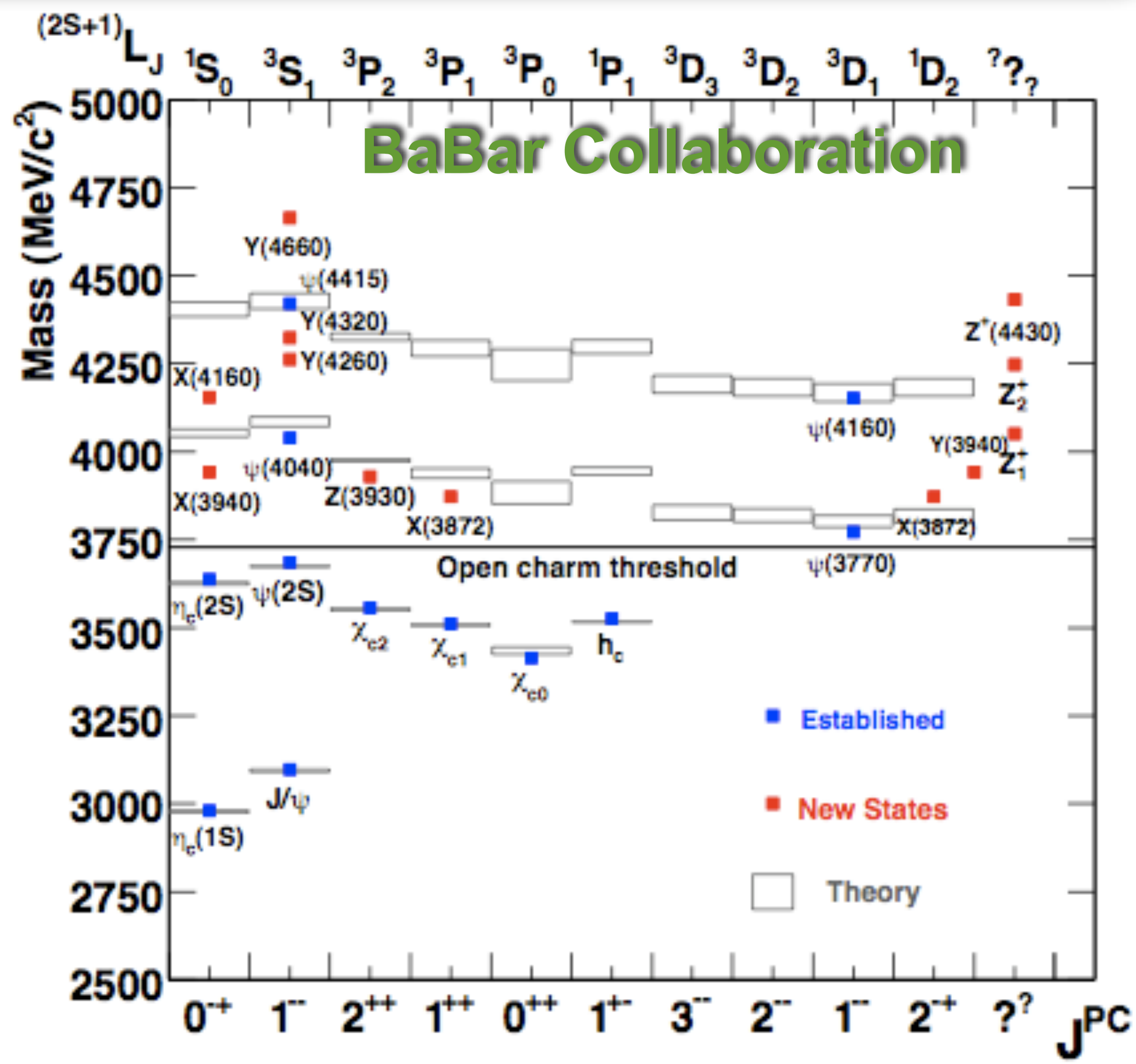
Charming systems…?

- **Constituent quark models:**
  mass spectra below open charm threshold

  Godfrey, Isgur, PRD 32 (1985).
  Barnes, Godfrey, Swanson, PRD 72 (2005).

- **Charmonium-like (X, Y, Z) states:**
  candidates of exotic hadrons ($c c^{bar}$ +…)





- **Tetraquarks ($T_{QQ'} = QQ'q^{bar}q^{bar}/Q^{bar}Q'^{bar}qq$):**
  $Q^{(')}$ can be strange-, charm- or bottom-quark

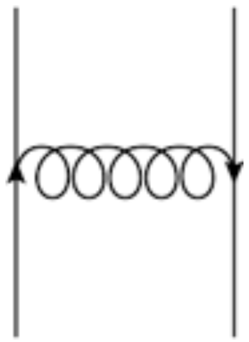
Possible candidates of exotic hadrons
➡ Tetraquarks have not been experimentally discovered yet

Bound tetraquarks $T_{QQ'}$?

Why can we expect possible bound T_{QQ}'s?

H. J. Lipkin, PLB172, 242 (1986).

Constituent quark models suggest bound $T_{QQ'}$
because of strongly attractive color magnetic interactions



- Color magnetic interaction (CMI) : hadron mass splitting

$$V_{\mathrm{CMI}} = -C \cdot \alpha_s \sum_{i<j} \frac{(\vec{\lambda}(i) \cdot \vec{\lambda}(j))(\vec{\sigma}(i) \cdot \vec{\sigma}(j))}{M_i M_j} \delta^3(\vec{r}_i - \vec{r}_j)$$

- Color-spin matrix elements : $\langle v_{ij} \rangle = -\langle (\vec{\lambda}(i) \cdot \vec{\lambda}(j))(\vec{\sigma}(i) \cdot \vec{\sigma}(j)) \rangle$

$<v_{ij}>$	C=1	C=8	C=3	C=6^{bar}
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

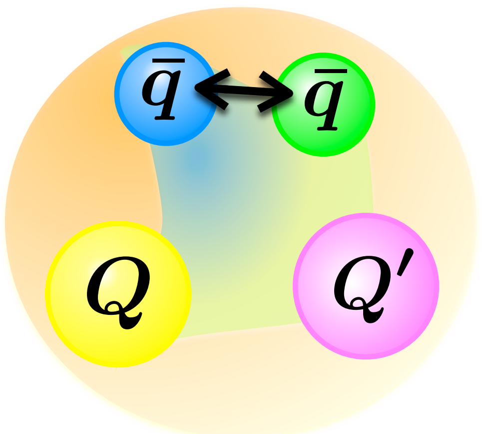
- C=3, S=0 (I=0) : -8
- C=6^{bar}, S=1 (I=0) : -4/3
- C=3, S=1 (I=1) : 8/3
- C=6^{bar}, S=0 (I=1) : 4

attractive ↕ repulsive

- CMI proportional to $1/M_i$: strongly attractive $u^{bar}d^{bar}$-diquark pair

➡ B.E. (T_{cc} ($J^P=1^+$, I=0)) ~ 70 MeV

J. Vijande, A. Valcarce, PRC80, 035204 (2009)



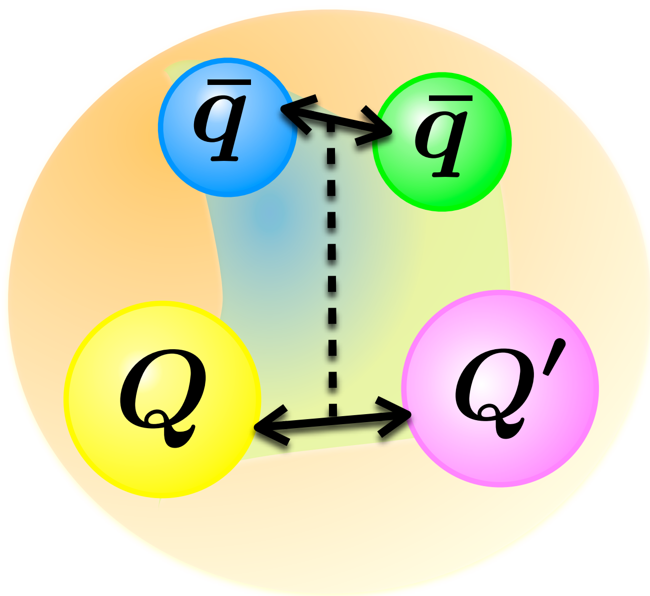
If bound $T_{QQ'}$ is found...

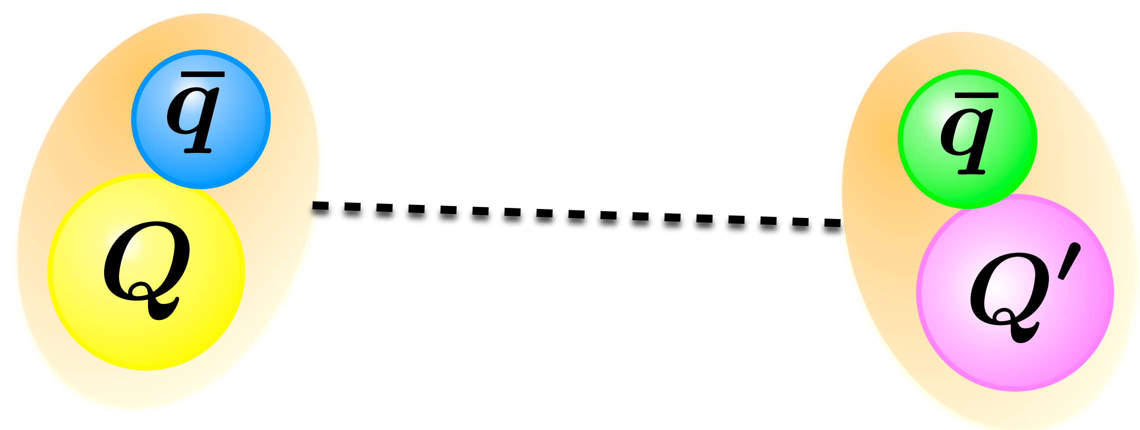
What is structure of bound $T_{QQ'}$?

➡ Genuine tetraquark?
(colorful CMI?)

➡ Meson-meson molecule?
(importance of pion exchange?)

S. Ohkoda et al., PRD86, 034019 (2012).





➡ Admixture of both?

If $T_{QQ'}$ is bound...,
model independent analysis for structures of $T_{QQ'}$ is necessary

Motivation:
1) Possible bound $T_{QQ'}$ search from lattice QCD simulation
2) If exists, structure studies

Outline

1) Introduction to tetraquarks

2) Scattering on the lattice -- a brief introduction --

3) Multi-hadrons on the lattice

4) Lattice potentials -- HAL QCD method --

5) Results of LQCD simulations

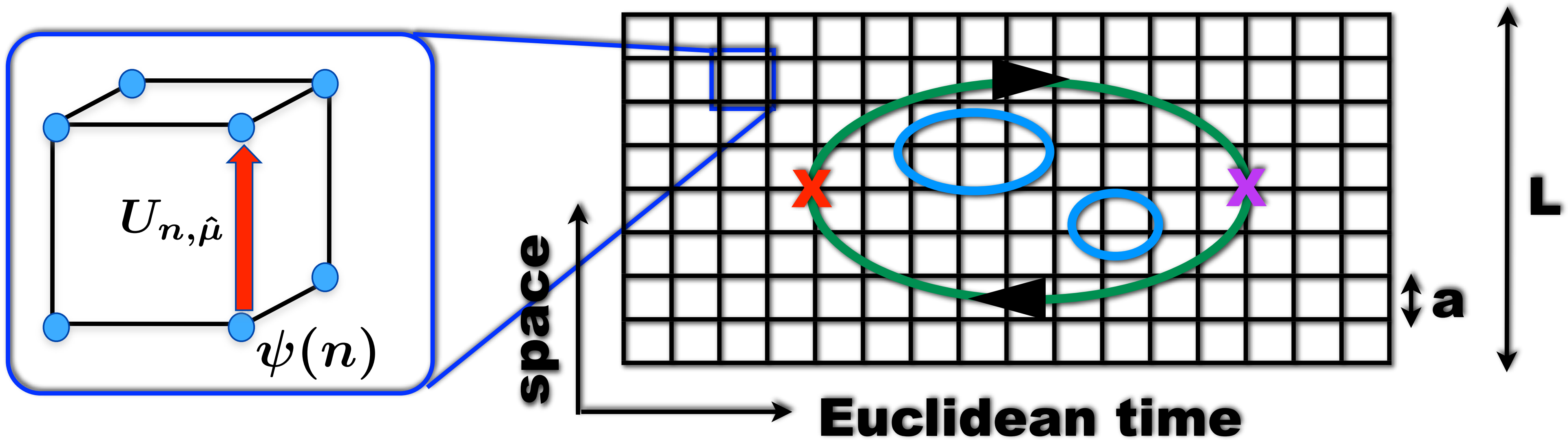
6) Summary

Lattice QCD -- a brief introduction --

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2} tr F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi$$

➡ **Path integral formalism in Euclidean space-time**

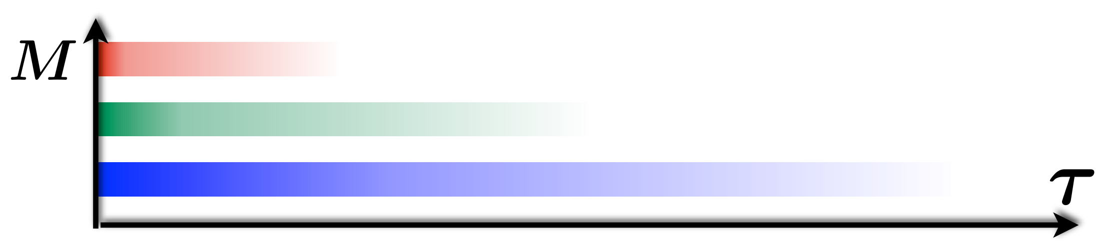
- **Quarks : $\psi(n)$**
- **Gluons : $U_{n,\mu}$**



e.g.) meson masses

$$\langle O(\tau)\rangle = A_0 e^{-M_0\tau} + A_1 e^{-M_1\tau} + A_2 e^{-M_2\tau} + \cdots$$

$$\langle O(\tau)\rangle = \sum_{\vec{x}} \langle 0|\bar{q}(\vec{x},\tau)\Gamma q(\vec{x},\tau)\left(\bar{q}(\vec{x},0)\Gamma q(\vec{x},0)\right)^{\dagger}|0\rangle$$



At large τ region, ground states dominate correlation functions (Ground state saturation)

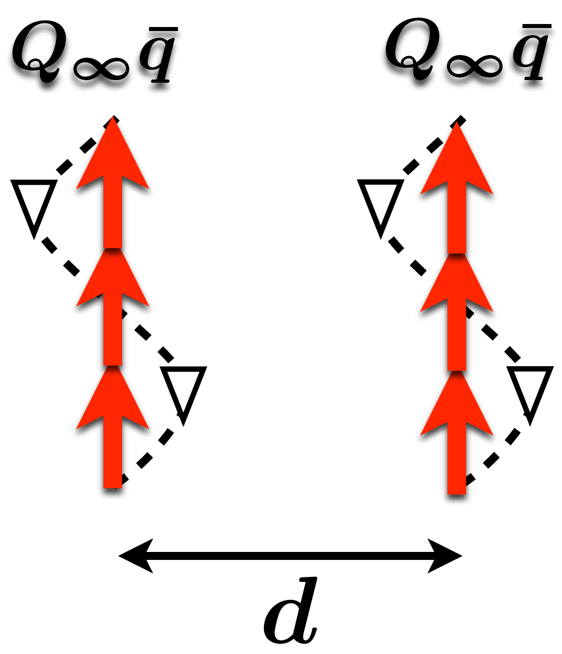
- **Well defined statistical field theory**
- **Gauge invariant formalism**
- **Non-perturbative method**

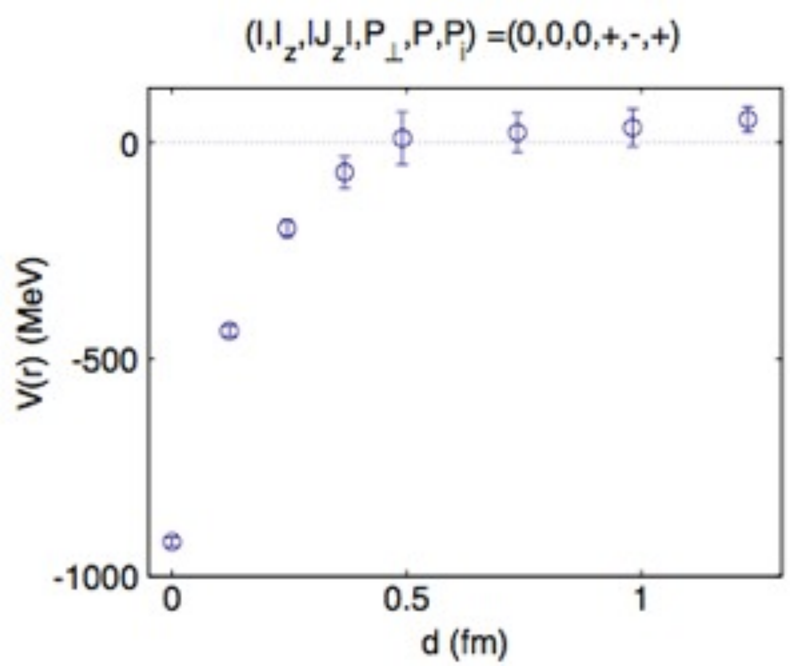
➡ **Monte Calro calculation**

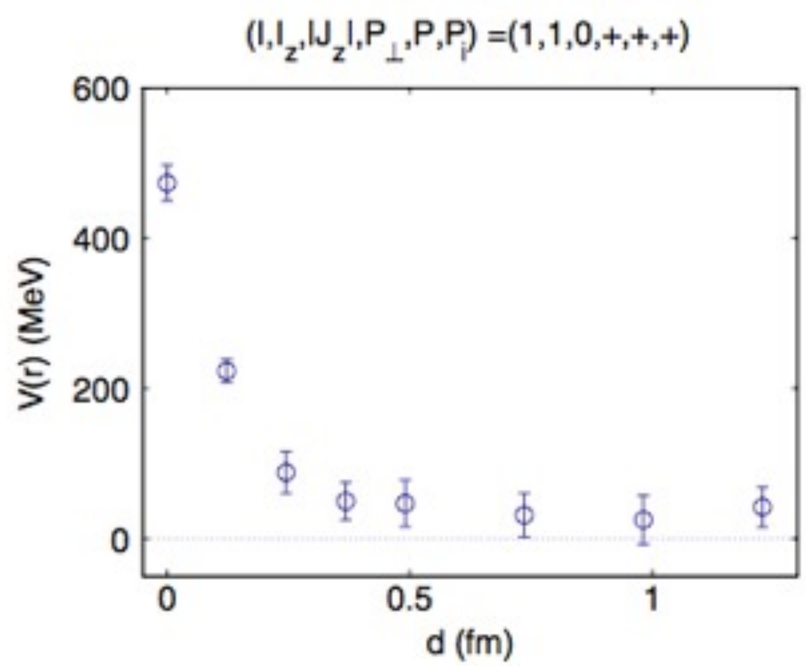
Lattice QCD studies of T_{bb}

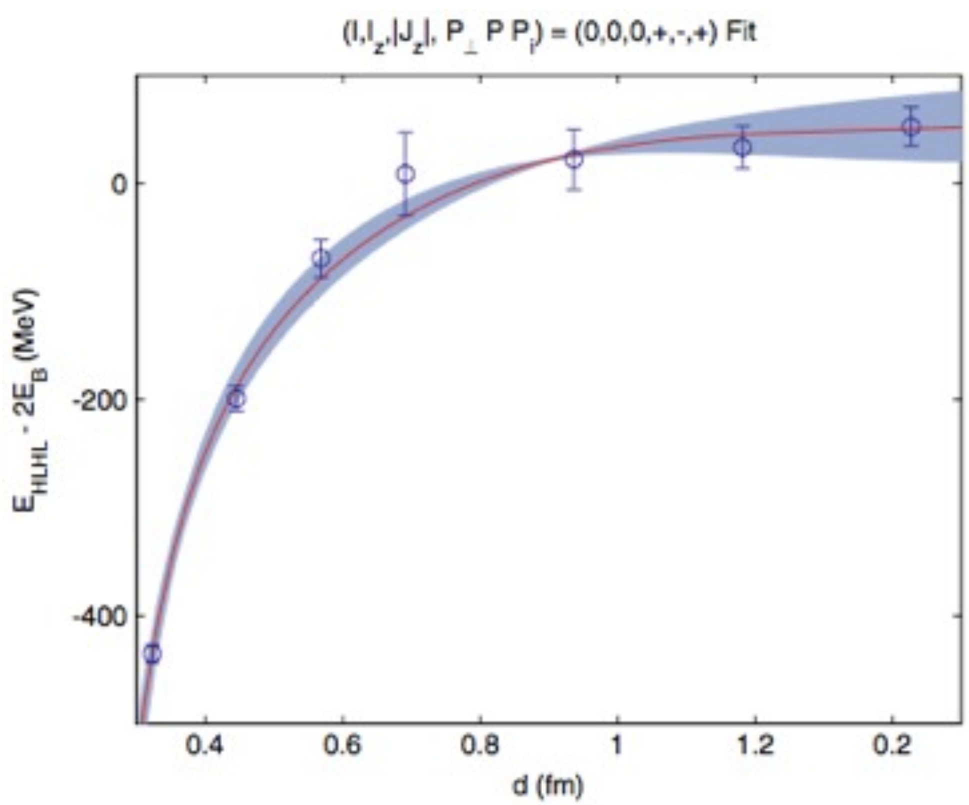
✓ **Interaction energies for meson-meson in static limit ($Q\bar{q}$) -- ($Q\bar{q}$)**

Z. Brown, K Orginos, PRD86, 114506 (2012); P. Bicudo, M. Wagner, PRD87, 114511 (2013).









➡ **used for T_{bb} search**

Solve Schrodinger equation with input :

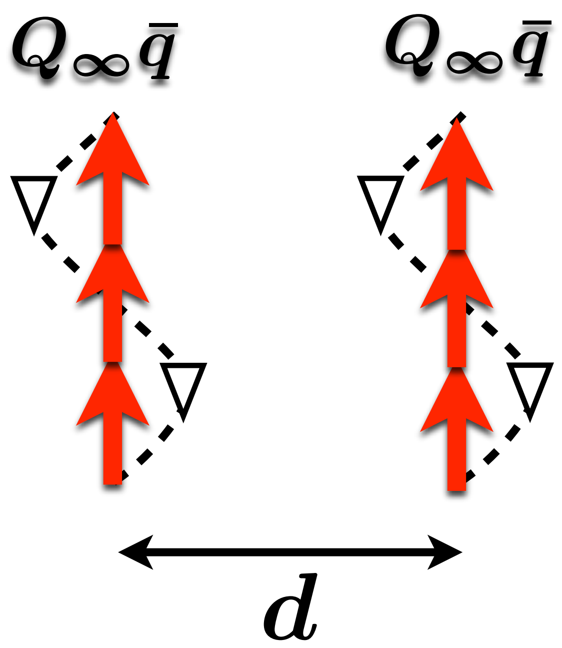
1) static limit ($Q\bar{q}$) -- ($Q\bar{q}$) "potential"

2) physical B meson mass

$$\mathrm{B.E.} = 50(5)\mathrm{MeV}$$

$$\sqrt{\langle r^2 \rangle} = 0.383(6)\mathrm{fm}$$

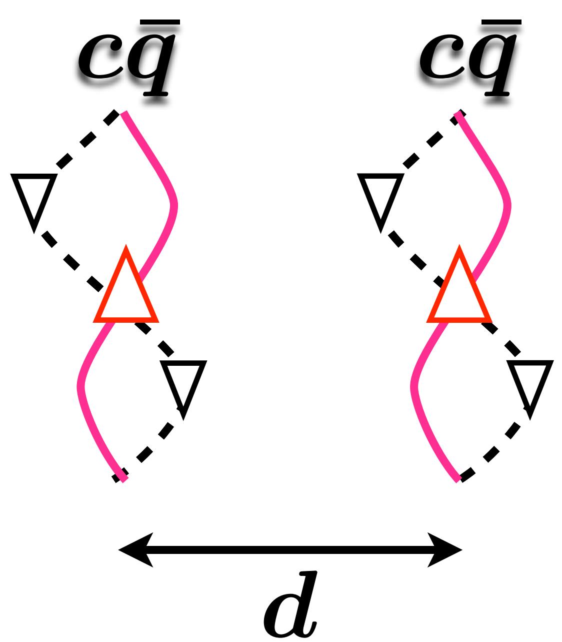
Search for T_{cc} & T_{cs} on the lattice



➡ Dynamics of charm quarks should be appropriately taken into account, since charm quarks are relatively "light"

- Our approach: HAL QCD method to search for bound Tcc & Tcs

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007); Aoki, Hatsuda, Ishii, PTP123, 89 (2010).



Advantages

➡ dynamics of charm quarks

➡ size measurement, if bound states exist

➡ tetraquark-like or molecular-like?

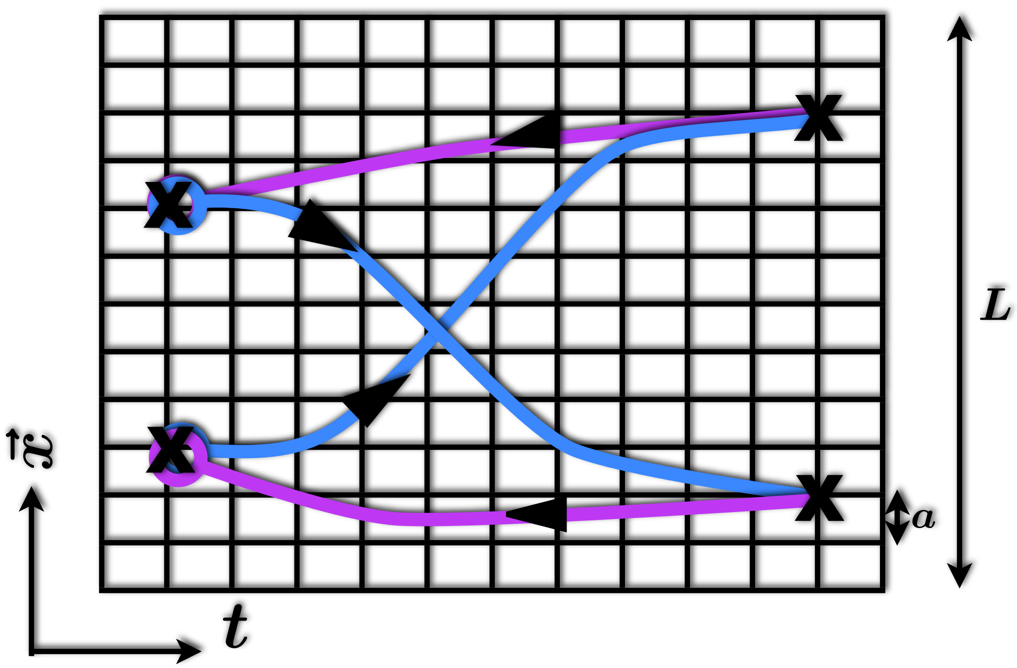
Scattering on the lattice

Key quantity : Equal-time Nambu-Bethe-Salpeter amplitude

$$\psi(\vec{r},t) = \sum_{\vec{x},\vec{X},\vec{Y}} \langle 0|\phi_1(\vec{x}+\vec{r},t)\phi_2(\vec{x},t)\phi_1(\vec{X},t=0)^\dagger\phi_2(\vec{Y},t=0)^\dagger|0\rangle$$

$$= \sum_{\vec{k}} A_{\vec{k}} \exp\left[-W(\vec{k})t\right]\psi_{\vec{k}}(\vec{r})$$

$$\psi^{(l)}_{\vec{k}}(r) \sim \frac{e^{i\delta_l(k)}}{kr}\sin(kr+\delta_l(k)-l\pi/2)$$



Equal-time NBS amplitude used to determine

- **Scattering parameters:**

$$k\cot\delta(k) = \frac{1}{a} - \frac{1}{2}r_e k^2 + \cdots$$

- **NBS wave func. ~ wave func. in QM.**
  information on phase shift

- **Temporal correlation, W(k) : phase shift (Luscher's formula)** M. Lüscher, NPB354, 531 (1991).
- **Spacial correlation, ψ(r) : potential --> observable** CP-PACS Coll., PRD71, 094504(2005). Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Multi-hadron systems : progress & challenge

1) # of Wick contraction : $N_{\rm con} = \left(\frac{3}{2}A!\right)^2 \times 6^A \cdot 4^A$

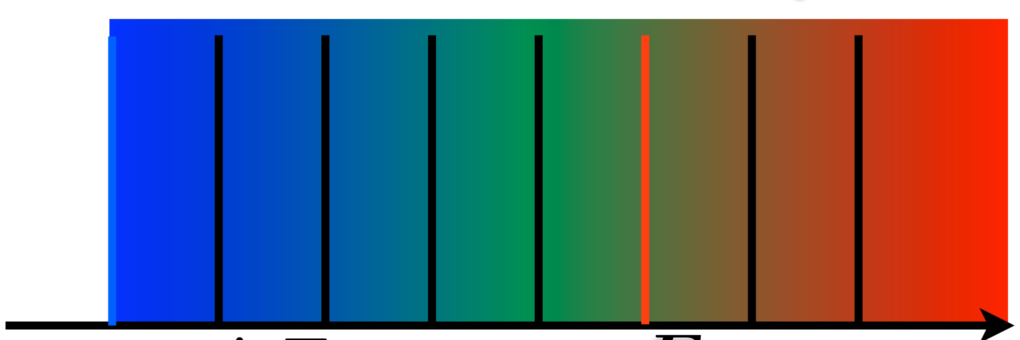
➡ Solution = Unified Contraction Algorithm

T. Doi, M. Endres, Comput. Phys. Commun. 184, 117 (2013).

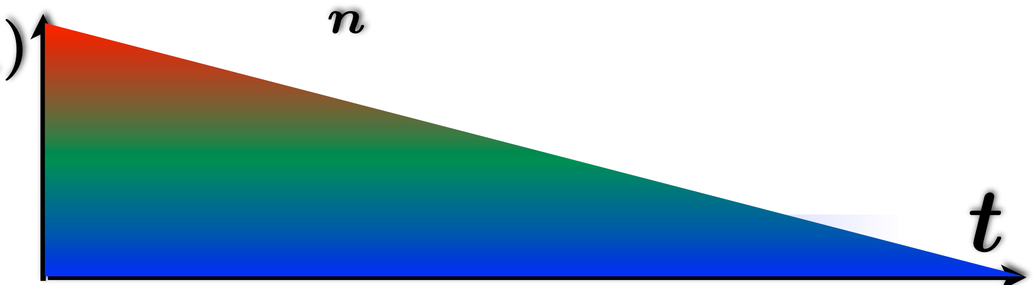
2) Ground/single state saturation :

- Excited state contamination : dense spectrum in large L limit

$$\vec{k}_n = \frac{2\pi\vec{n}}{L} \qquad \Delta E = \frac{\vec{k}^2}{2\mu} + \cdots$$



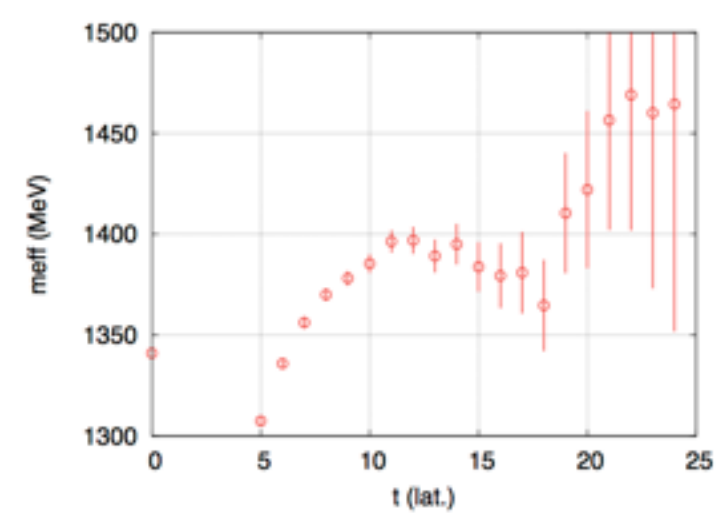
$$\langle O(t)\rangle = \sum_n A_n \exp(-W(\vec{k}_n)t)$$



- Signal-to-noise issue

✔ pion : $S/N \sim \text{const.}$

✔ nucleon : $S/N \sim \exp[-A(m_N - 3/2 m_\pi)t]$



Solution = HAL QCD method

Define energy-independent potential below inelastic threshold:

$$\psi_n(\vec{r}) = \langle 0|\phi_1(\vec{r}+\vec{x})\phi_2(\vec{x})|n\rangle$$

$$(\vec{k}_n^2 + \nabla^2)\psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r},\vec{r}')\psi_n(\vec{r}')$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Extract energy-independent potential through time-dependent Schrodinger eq.

Ishii et al.(HAL QCD Coll.), PLB712,437(2012).

$$R(\vec{r},t) = \sum_{n\leq n_{th}} \psi_n(\vec{r}) e^{-\Delta W(\vec{k}_n)t}$$

$$(-\partial_t - H_0 + \mathcal{O}(\partial_t^2))R(\vec{r},t) = \int d\vec{r}' U(\vec{r},\vec{r}')R(\vec{r}',t)$$

$$H_0 = -\frac{\nabla_r^2}{2\mu}$$

Velocity expansion: leading order central potential

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

Calculate observable: phase shift, binding energy, mean-square radius, ...

Advantage: We can obtain potentials w/o G.S. saturation

NN potential from HAL QCD method

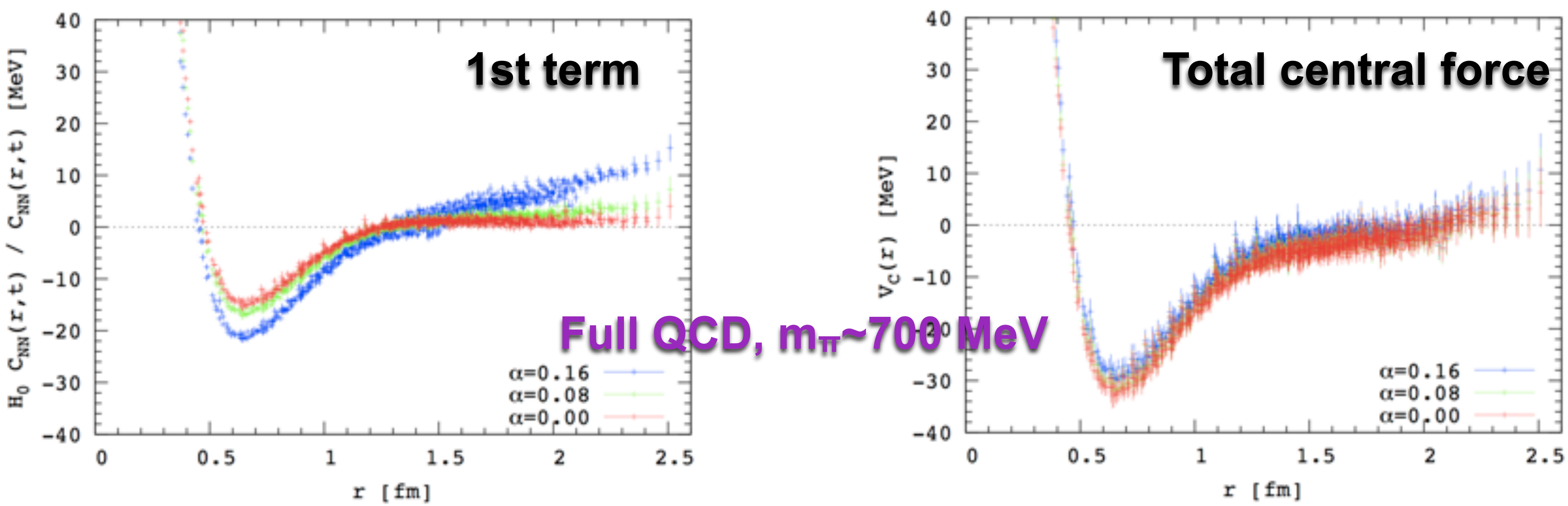
Ishii et al.(HAL QCD Coll.), PLB712,437(2012).

Central force of singlet NN system

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t) + \frac{1}{4M_N}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)}$$

Examine source operator dependence: possible contaminations are different

$$f(x,y,z) = 1 + \alpha\big(\cos(2\pi x/L) + \cos(2\pi y/L) + \cos(2\pi z/L)\big)$$



Operator dependence disappears

Ground (single) state saturation is not a question!!

Lattice QCD Setup : light quarks

N_f=2+1 full QCD configurations generated by PACS-CS Coll.

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & Wilson clover
- Gauge coupling : β=1.90
- Lattice spacing : a=0.0907(13) (fm) ($\Lambda_{lat.}$=2176(MeV))
- Box size : 32^3x64 -> L~2.9 (fm)
- Hopping parameters :
  - set1 : (κ_{ud}, κ_s)=(0.13700, 0.13640)
  - set2 : (κ_{ud}, κ_s)=(0.13727, 0.13640)
  - set3 : (κ_{ud}, κ_s)=(0.13754, 0.13640)
- Conf. # : [set1]:399, [set2]:400, [set 3]:450
- Wall source

Light meson mass [set1, set2, set3] (MeV)
M_π=699(1), 572(2), 411(2) [PDG:135 (π^0)]
M_K=787(1), 714(1), 635(2) [PDG:498 (K^0)]

Lattice QCD Setup : charm quarks

✤ Tsukuba-type Relativistic Heavy Quark (RHQ) action

Aoki et al., PTP109, 383 (2003)

Cutoff errors, $O((ma)^n)$ and $O(a\Lambda_{QCD})$, are removed by adjusting RHQ parameters, $\{m_0, \nu, r_s, C_E, C_B\}$.

$$S^{\mathrm{RHQ}} = \sum_{x,y} \bar{q} D_{x,y} q(y)$$

$$D_{x,y} = m_0 + \gamma_0 D_0 + \nu \gamma_i D_i - a r_t D_0^2 - a r_s D_i^2 - a C_E \sigma_{0i} F_{0i} - a C_B \sigma_{ij} F_{ij}$$

- We are allowed to choose r_t=1
- We are left with $O((a\Lambda_{QCD})^2)$ error (~ a few %)

We use RHQ parameters tuned by Namekawa et al.

Y. Namekawa et al., PRD84, 074505 (2011)

Charmed meson mass [set1, set2, set3] (MeV)

$M_{\eta c}$=3024(1), 3005(1), 2988(2) [PDG:2981]

$M_{J/\psi}$=3142(1), 3118(1), 3097(2) [PDG:3097]

M_D=1999(1), 1946(1), 1912(1) [PDG:1865 (D^0)]

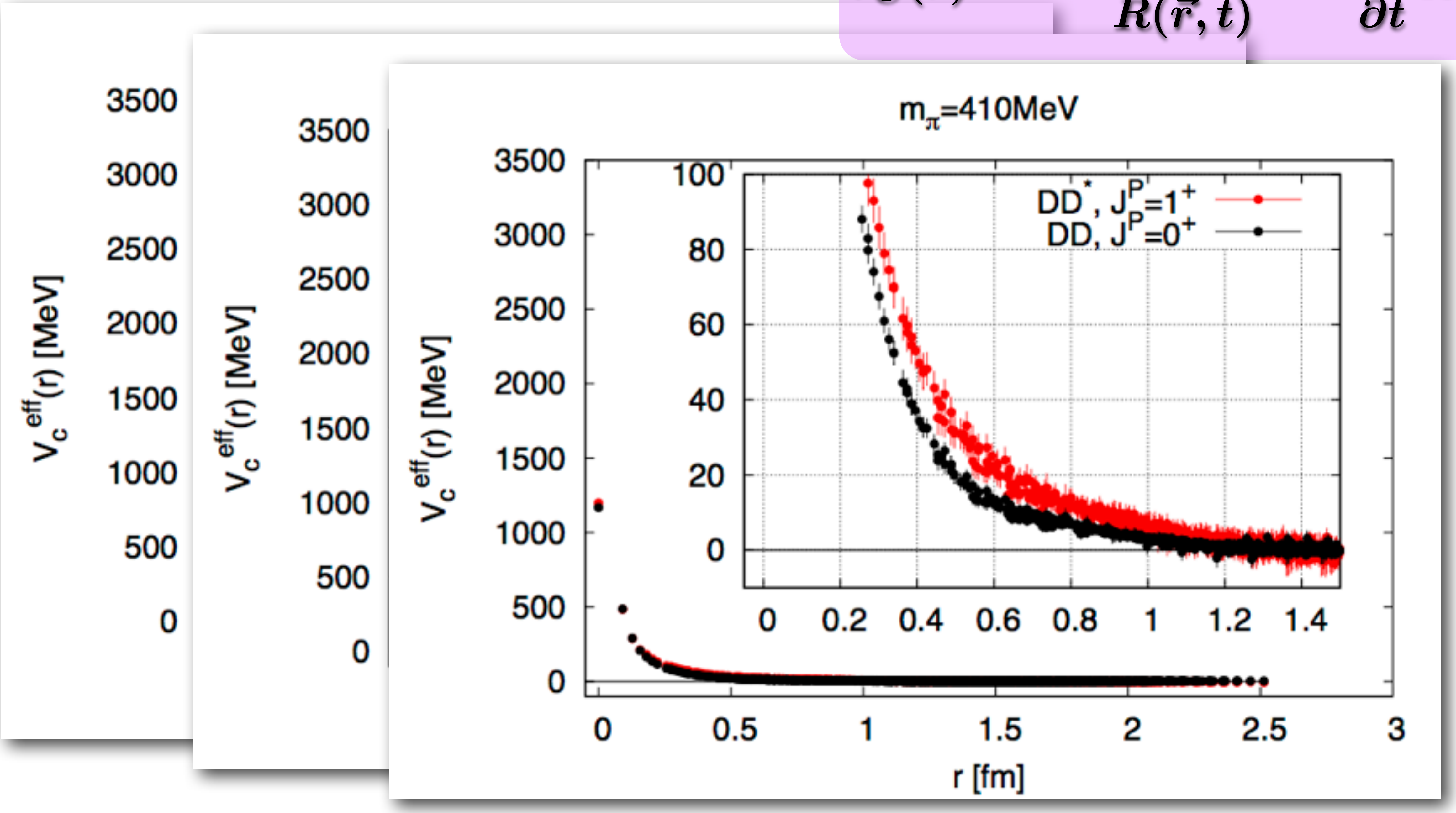
M_{D^*}=2159(4), 2099(6), 2059(8) [PDG:2007 (D^{*0})]

Results : isospin 1 channels

S-wave DD$^{(*)}$ potentials: $T_{cc}(0^+, 1^+(1))$

Y. Ikeda et al. [HAL QCD Coll.], in preparation.

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

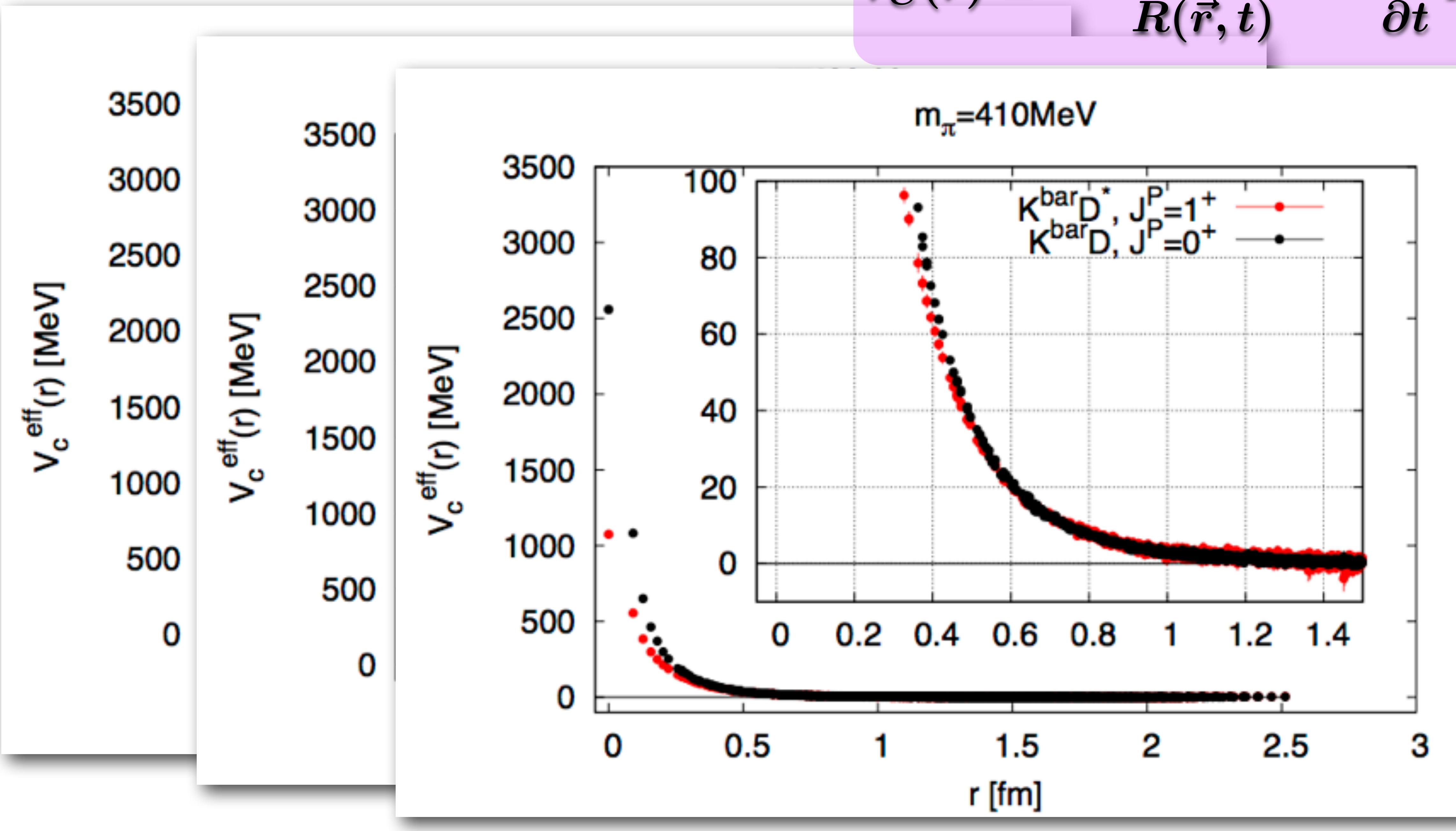


- Repulsive DD and DD* potentials
- Weak quark mass dependence
- It is unlikely to form bound state even at physical point

S-wave $D^{(*)}K^{bar}$ potential : $T_{cs}(0^+, 1^+(1))$

Y. Ikeda et al. [HAL QCD Coll.], in preparation.

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$



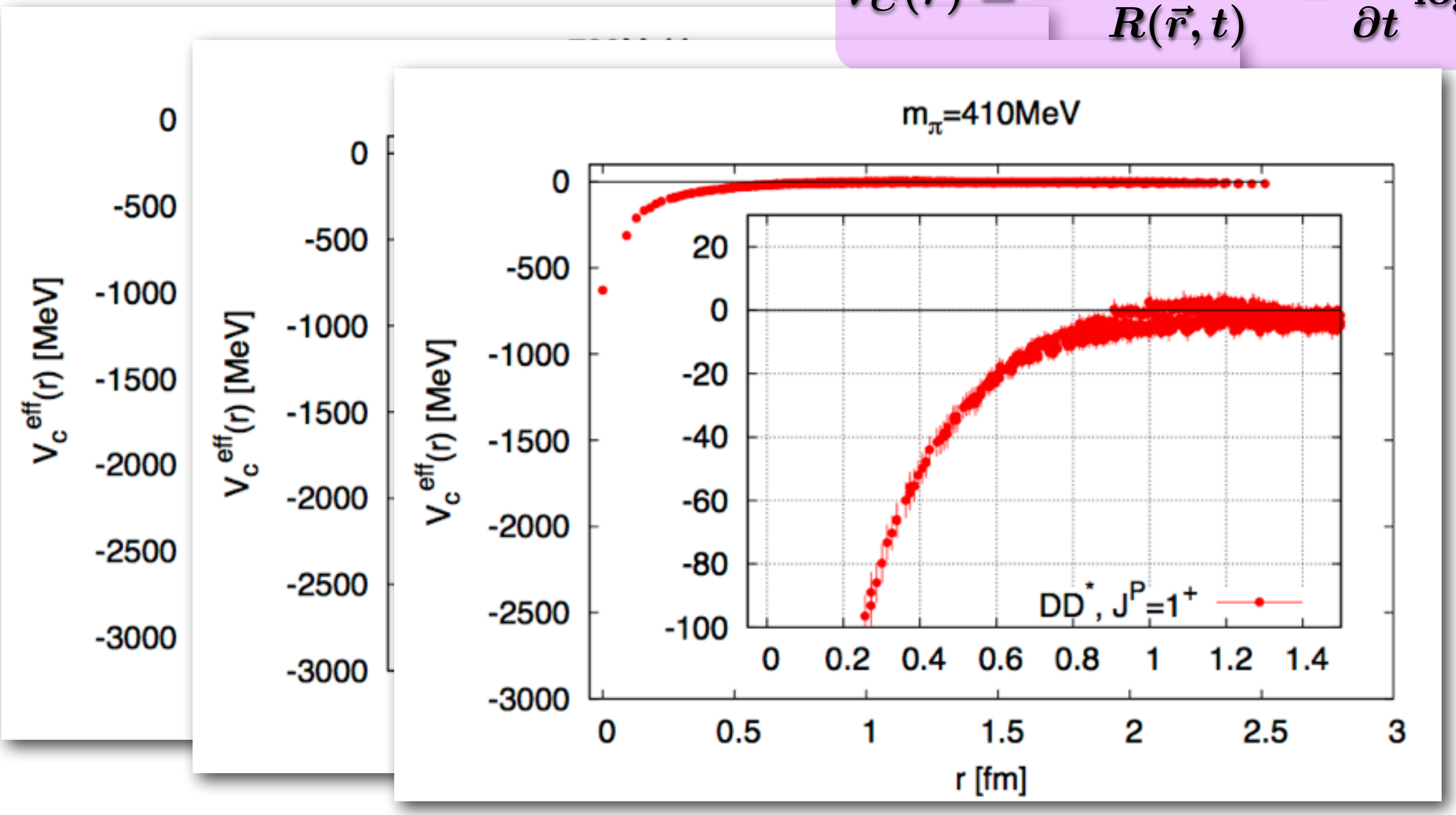
- Repulsive $K^{bar}D$ and $K^{bar}D^*$ potentials
- Weak quark mass dependence
- It is unlikely to form bound state even at physical point

Results : isospin 0 channel

S-wave DD* potential : $T_{cc}(1^+(0))$

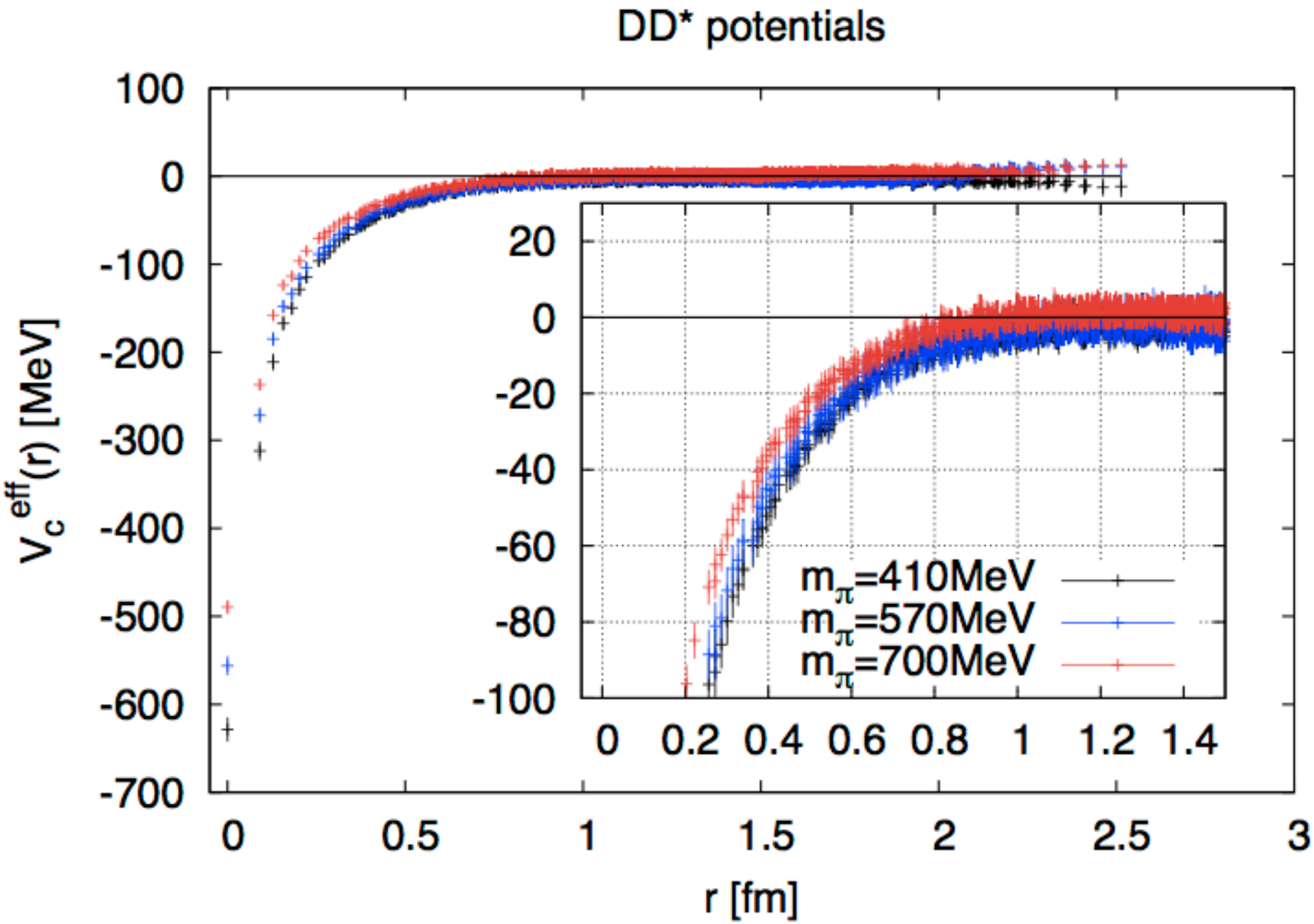
Y. Ikeda et al. [HAL QCD Coll.], in preparation.

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$



- Attractive DD* potential is observed

S-wave DD* potential

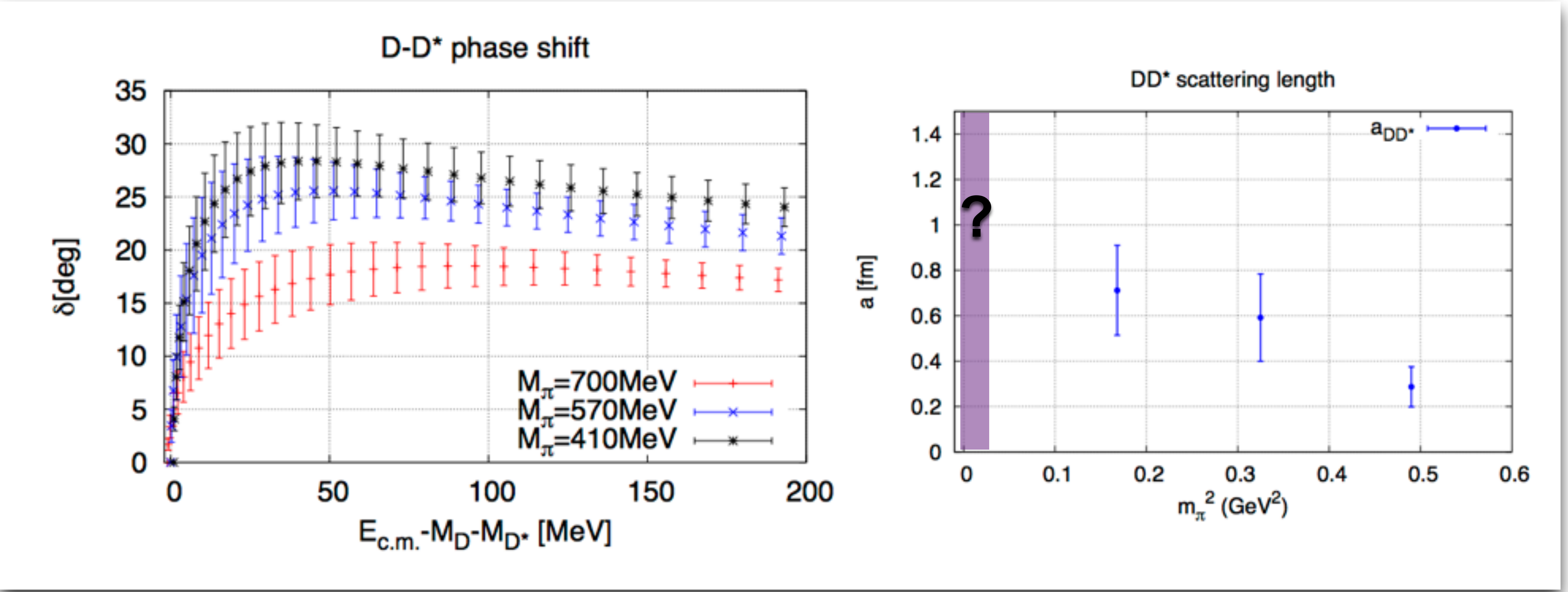


- Relatively weak quark mass dependence
- Check whether bound Tcc exist or not --> phase shift analysis

S-wave phase shift : $T_{cc}(1^+(0))$

- fit multi-range gaussian: $f(r) = \sum_i a_i e^{-\nu_i r^2}$

- solve Schrodinger equation in an infinite volume

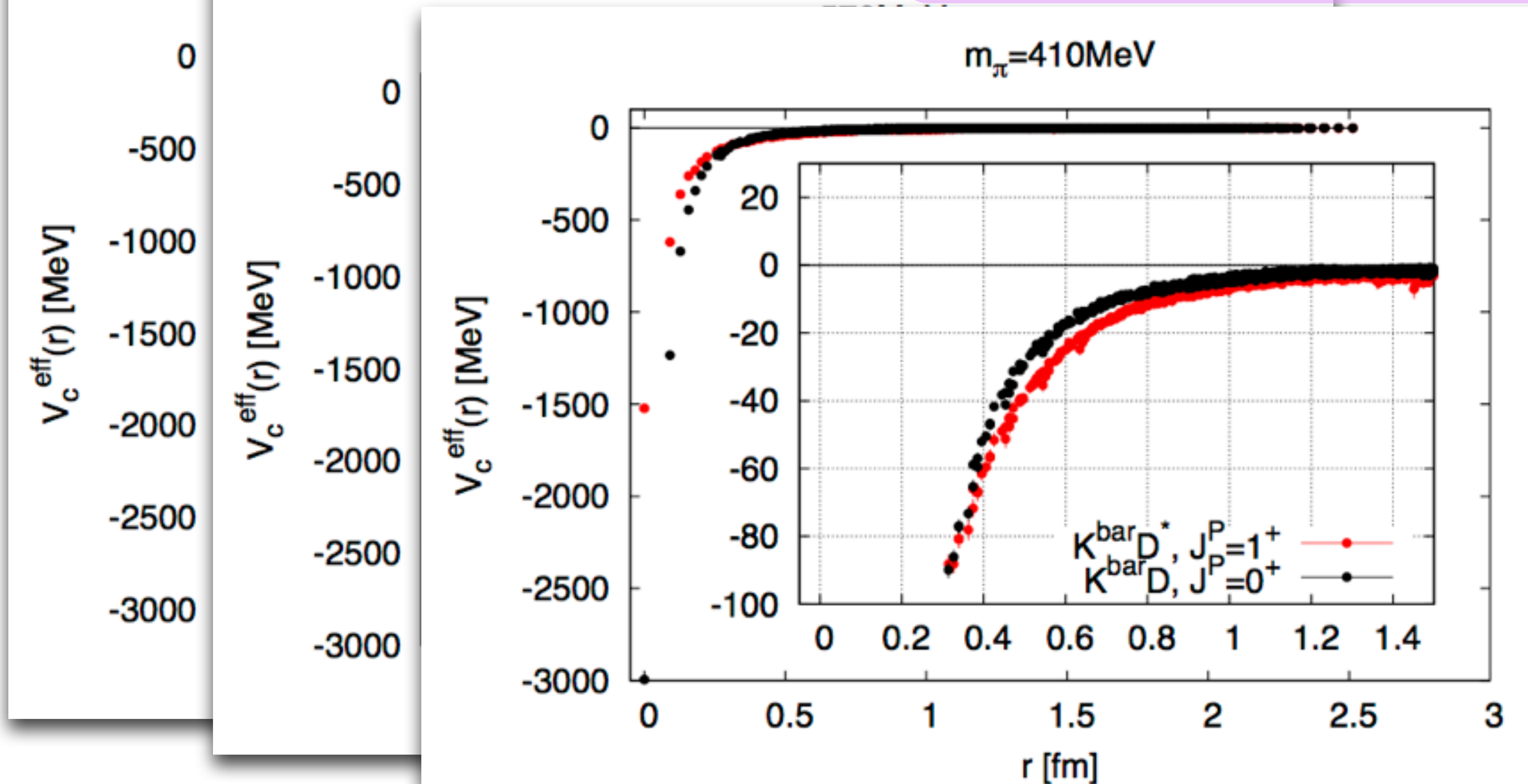


- Attraction is not enough strong to generate bound state
- Attraction gets stronger as decreasing quark mass
- For definite conclusion, physical point simulations are necessary

S-wave $D^{(*)}K^{bar}$ potential : $T_{cs}(0^+, 1^+(0))$

Y. Ikeda et al. [HAL QCD Coll.], in preparation.

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t} \log R(\vec{r},t)$$

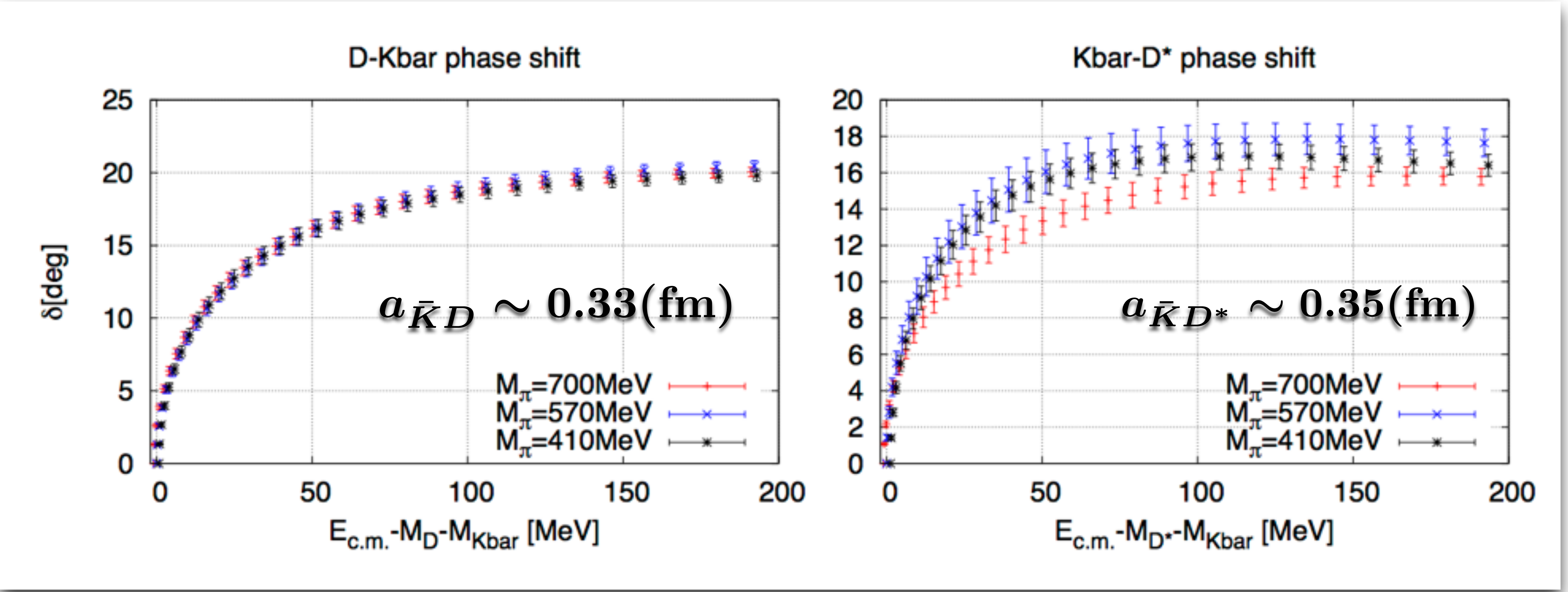


- Attractive $K^{bar}D$ and $K^{bar}D^*$ potentials
- Weak quark mass dependence
- Check whether bound Tcs exist or not --> phase shift analysis

S-wave phase shift : $T_{cs}(0^+, 1^+(0))$

- fit multi-range gaussian: $f(\mathrm{r}) = \sum_i a_i e^{-\nu_i r^2}$

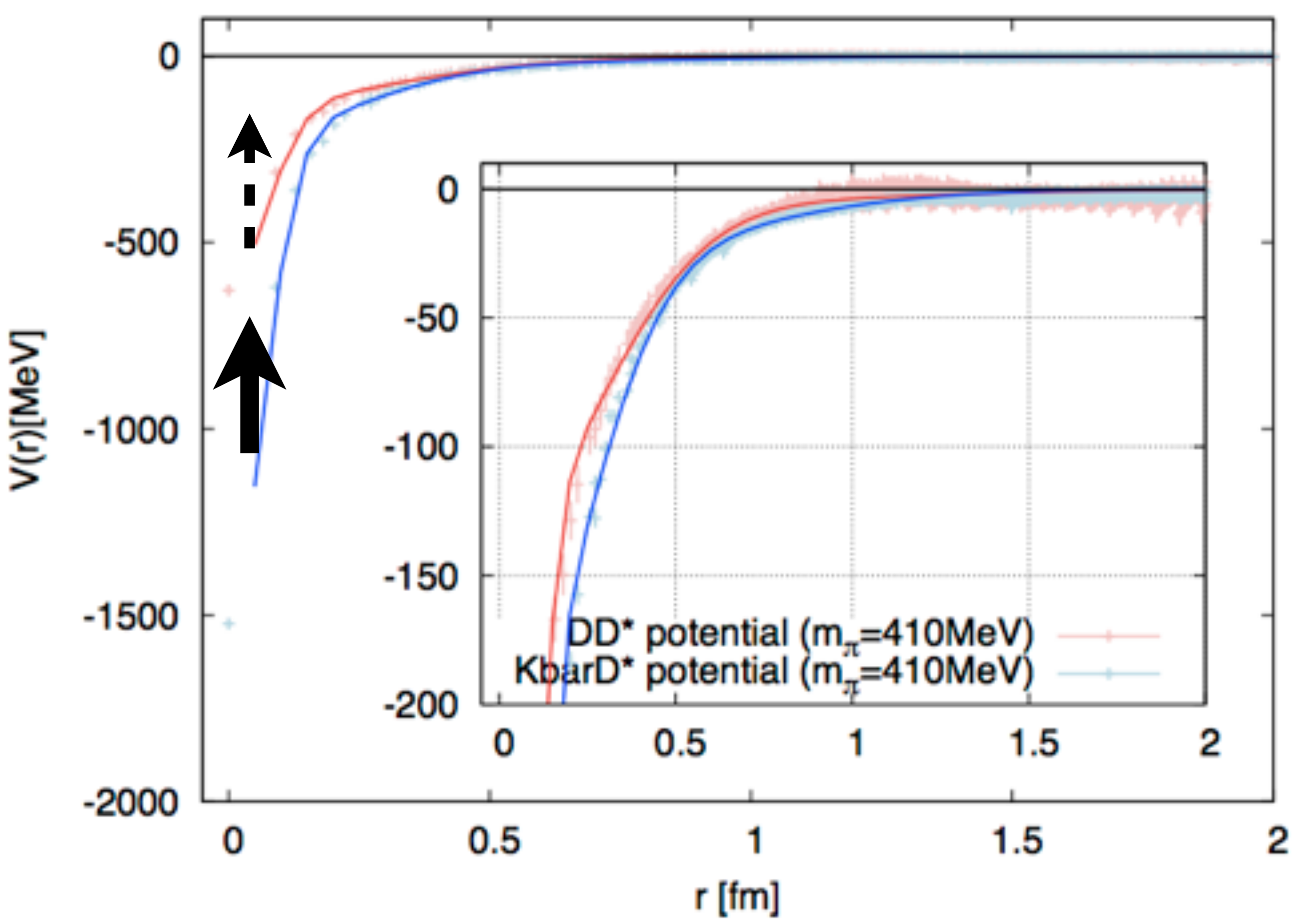
- solve Schrodinger equation in an infinite volume

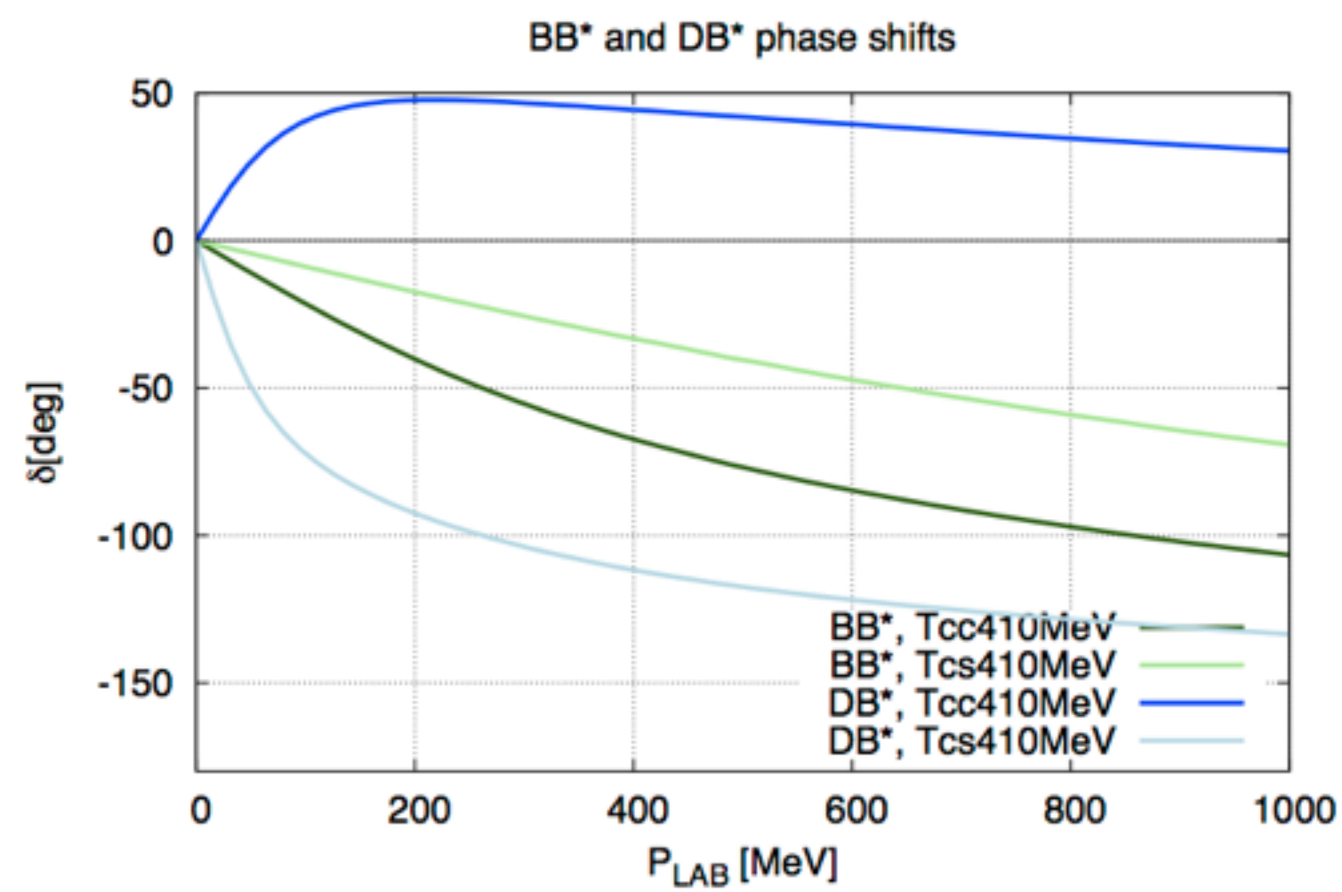


- Attractions are not enough strong to generate bound states
- Weak quark mass dependence of phase shifts

Just for fun

• Lattice DD* / K^{bar}D* potential + physical mass of BB*(T_{bb}), DB*(T_{bc})
--> aggressive setup (maybe too strong potentials even at physical point)





w/ LQCD DD* potential

• BB* system includes a bound state ~ T_{bb}
• DB* system is scattering state...

Summary

- Search for Tcc, Tcs on the lattice@m_π=410, 570, 700MeV

➡ N_f=2+1 full QCD simulation (PACS-CS configuration)
➡ Charm quarks: Relativistic Heavy Quark action

➡ Tcc, Tcs($J^P=0^+, 1^+$, I=1) : s-wave MM channels are repulsive
Bound states are unlikely...

➡ Tcc, Tcs($J^P=0^+, 1^+$, I=0) : s-wave MM channels are attractive,
but not strong enough to form bound states@m_π=410, 570, 700MeV
➡ $a_{DD^*} > a_{KbarD} \sim a_{KbarD^*}$ (attraction: Tcc(1^+) channel > Tcs($0^+, 1^+$) channel)
Large kinetic energy due to kaon in Tcs channels

- Future plan

➡ Physical point simulation
➡ Coupled-channel analysis (DD*-D*D*,...)