Quark potentials on the color channels between two quarks in lattice calculations

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Contents

Color forces in color channels (1)

- Color singlet
  - Attraction, confinement, color-singlet hadrons
  - Linearly rising confining potential at larger distances
  - Many lattice calculations: ref. Bali, PR343(2001).
- Color anti-triplet
  - Attraction
Diquark; ref, Anselmino, et al., RMP65(1995)1199
  - Strongly interacting diquark in the hadron ?
（Jaffe and Wilczek, PRL95(2003)232003）

Color forces in color channels (2)

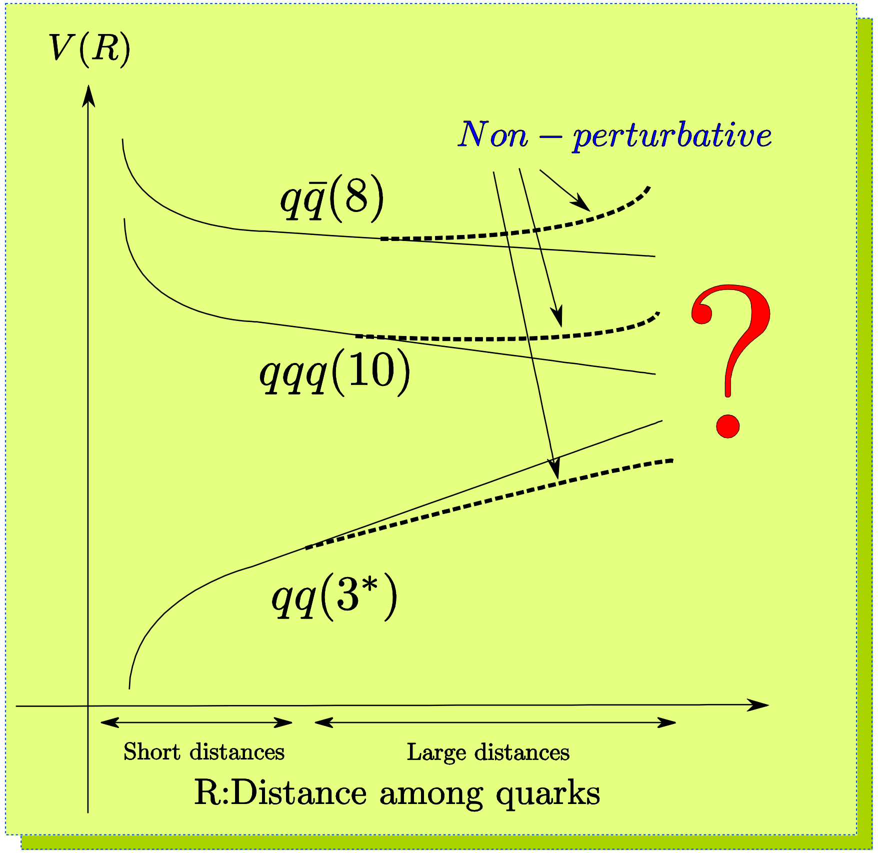
- Color octet
  - Repulsion
    it’s a weakest force in two quark state
  - Detail study of J/Ψ photo production：
    Color octet model
    Exp. ：CLEO Collab. PRD70,072001
    The. ：Cacciari and Kramer, PRL76,4128(1999), Bratten and Fleming, PRL74,3327(1995), etc.

Color forces in color channels (3)

Color Forces for 2 or 3 quarks

Quarks	Channel	Color Factor $C = \langle\lambda_a\lambda_b\rangle$ or $\langle\lambda_a\lambda_b\lambda_c\rangle$
$q\bar{q}$	1	-4/3
	8	1/6
qq	3^*	$-2/3$
	6	1/3
qqq	1	-2
	$8(8')$	$-1/2$
	10	1

$V(R) \sim C/R$ for short distances



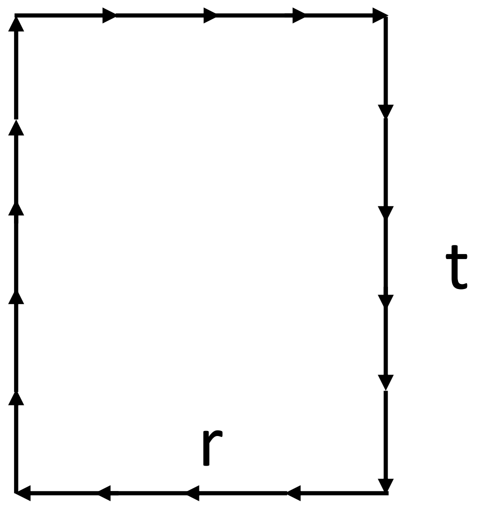
Potentials in Wilson loops

Definition due to the gauge-invariant manner

$$V(r) = -\lim_{t\to\infty}\frac{d}{dt}\ln\langle W(r,t)\rangle$$

$$\langle W(r,t)\rangle \sim c_s e^{-V_s(r)t} + c_o e^{-V_o(r)t}$$

$$V_s(r) \gg V_o(r)$$



$$W(C) \equiv Tr\left(\prod_{x,\mu} U_\mu(x)\right)$$

Page 34 of G.S.Bali, Phys. Rep. 343 (2001)

Potentials in Polyakov line

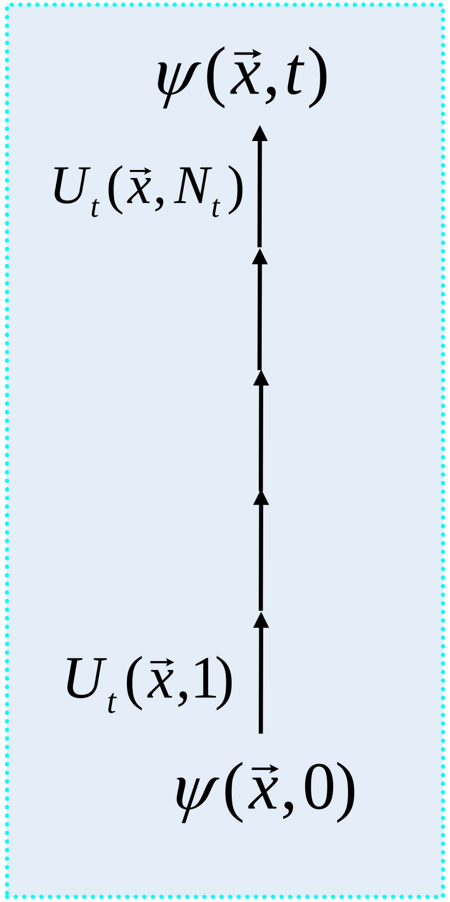
(McLerran, RMP58, 1021(1986))

$$\left(\frac{1}{i}\frac{\partial}{\partial t} - t^a A_0^a(\vec{x},t)\right)\psi(\vec{x},t) = 0$$

$$\psi(\vec{x},t) = T\exp\left(i\int_0^t dt' t^a A_0^a(\vec{x},t')\right)\psi(\vec{x},0)$$

$$\equiv L(\vec{x})\psi(\vec{x},0)$$

$$L(\vec{x}) \equiv U_0(\vec{x},1)\,U_0(\vec{x},2)\cdots\cdots\cdots U_0(\vec{x},N_t)$$



PLC in each color channel (1)

Nadkarni, PRD33,3738

- Color decomposition in quark-antiquark for SU(N)

$$\overline{N} \otimes N = 1 \oplus (N^2 - 1)$$

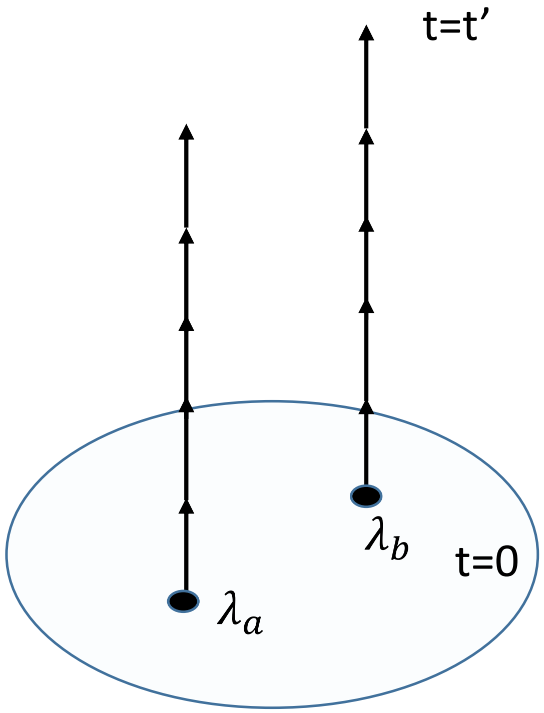
- Quark-antiquark correlator is made by

$$G_{q\bar{q}} = e^{-V_1} P_1 + e^{-V_8} P_8$$

- Singlet and octet potentials with the PLC for SU(3)

$$\mathrm{e}^{-V_1(R)} = \frac{1}{3}\left\langle \mathrm{TrL(R)L^{\dagger}(0)} \right\rangle$$

$$\mathrm{e}^{-V_8(R)} = \frac{8}{9}\left\langle \mathrm{TrL(R)TrL^{+}(0)} \right\rangle - \frac{3}{8}\left\langle \mathrm{TrL(R)L^{+}(0)} \right\rangle$$



PLC in each color channel (2)

Nadkarni, PRD33,3738

- Color decomposition in quark-quark potential.

$$N \otimes N = \frac{1}{2} N(N+1) \oplus \frac{1}{2} N(N-1)$$

- qq correlator is made by the following two parts

$$G_{qq} = e^{-V_{sym}} P_{sym} + e^{-V_{antisym}} P_{antisym}$$

- Symmetric and antisymmetric potentials are defined as

$$\mathrm{e}^{-V_{sym}(R)} = \frac{3}{4} \langle \mathrm{TrL(R)TrL(0)} \rangle + \frac{3}{4} \langle \mathrm{TrL(R)L(0)} \rangle$$

$$\mathrm{e}^{-V_{anti-sym}(R)} = \frac{3}{2} \langle \mathrm{TrL(R)TrL(0)} \rangle - \frac{3}{2} \langle \mathrm{TrL(R)L(0)} \rangle$$

Coulomb gauge QCD

Simple and obvious !

1. Coulomb gauge is a physical gauge:
   - Hamiltonian formulation with positive-definite Fock space, suited for variational analyses, etc.
   - Transverse gluons are clearly defined.
   - Instantaneous forces carry color interactions among color charges (in analogy of QED)
2. Confinement Mechanism
   - ***Instantaneous term*** produces a linearly rising potential: It's necessary condition for confinement.
   - Strong confining forces are in the proximity of Gribov regions; namely, from the lowest-mode of FP eigenvalues.
   - ***The retarded term*** including vacuum effect approaches a phenomenological Cornel potential in value.

Brief history of
Coulomb gauge confinement

Gribov, 1978,NPB139
-- Study of confinement --

Zwangier, 1998, NPB518,237
-- Color-Coulomb inst. pot. --

Greensite et.al, 2003, PRD67,094503
-- Inst. pot. is a linearly riging potential by su(2) computation --

A.Nakamura, TS, 2006, PTP115,189
-- Inst. pot. is a linearly riging potential by su(3) computation --

Hamiltonian

- Hamiltonian in the Coulomb gauge QCD

$$H = \frac{1}{2}\int d^3x\left(E_i^2 + B_i^2\right) + \frac{1}{2}\int d^3x d^3y\left(\rho(x)D(x,y)\rho(y)\right)$$

- Faddeev-Popov term in the Coulomb gauge QCD

$$D(\vec{x},\vec{y}) = \int d^3z\left[\frac{1}{M(\vec{x},\vec{y})}(-\vec{\partial}_{\vec{z}}^2)\frac{1}{M(\vec{x},\vec{y})}\right] \quad M = -(\vec{\partial}^2 + g\vec{A}\times\vec{\partial})$$

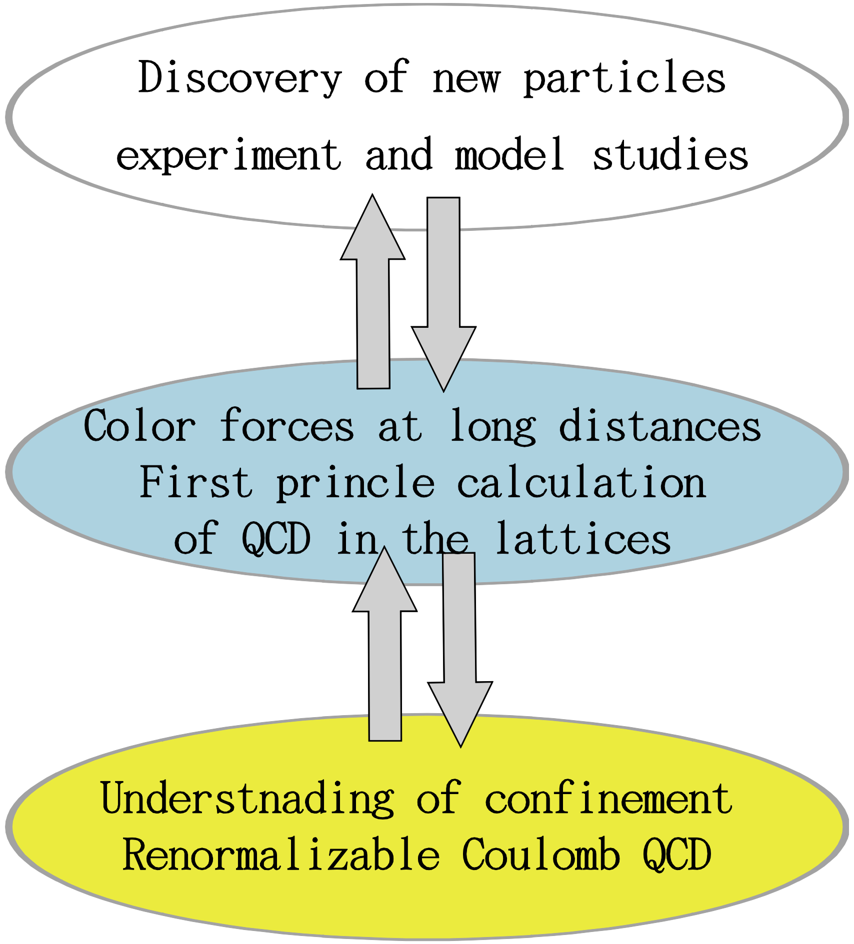
- Time-time component of the gluon propagators.

vacuum polarization (retarded) part

$$g^2\left\langle A_0(x)A_0(y)\right\rangle = V(x-y) + P(x-y)$$

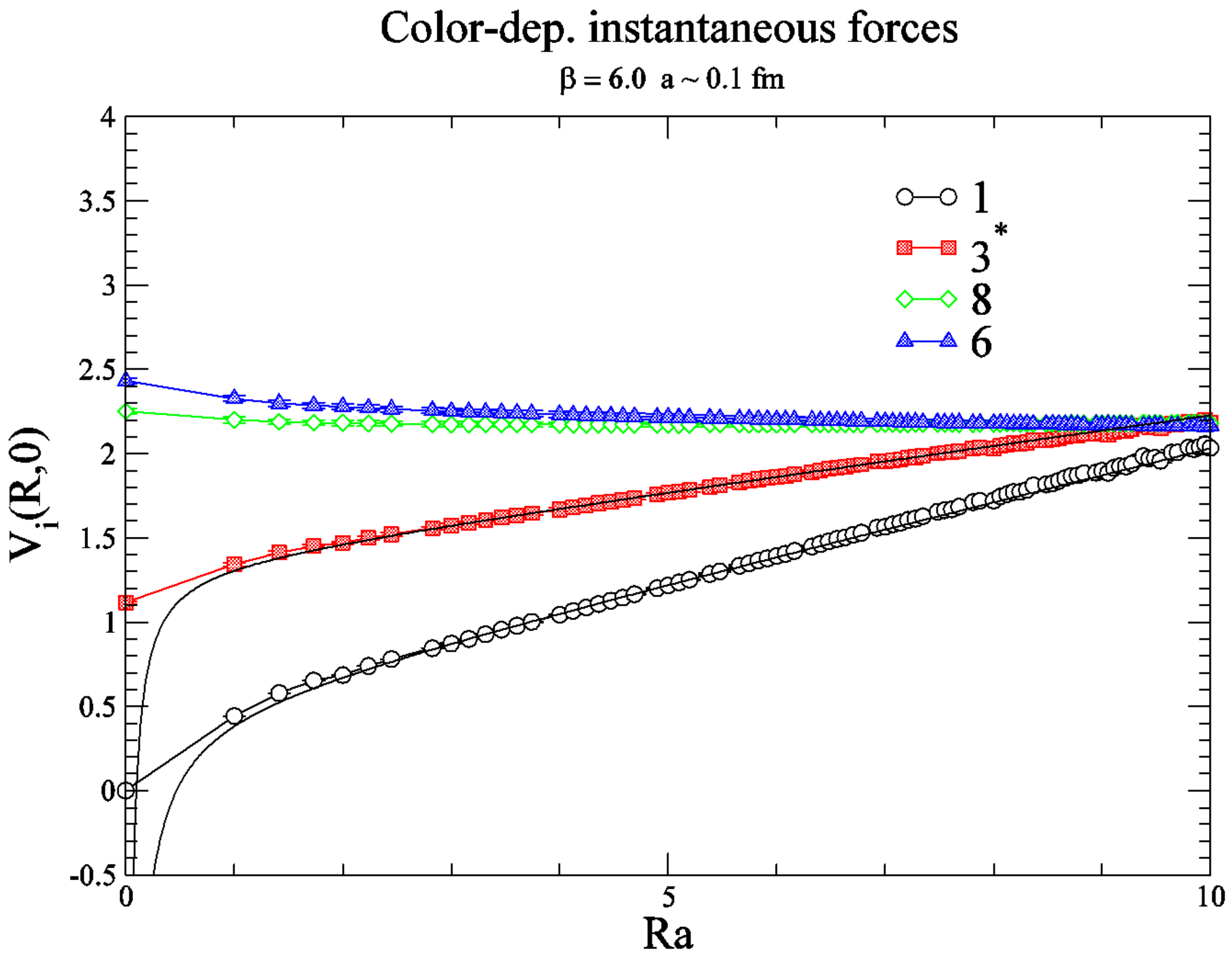
Instantaneous part $V(x-y) = g^2\left\langle D(\vec{x},\vec{y})\right\rangle \delta(x_4 - y_4)$

Plan of this study



This gives a great insight into confinement and chiral physics of QCD.

Color-dependent instantaneous forces



$$C_1 = -\frac{4}{3}, \text{ singlet}$$

$$C_{3^*} = -\frac{2}{3}, \text{ anti-triplet}$$

$$C_8 = \frac{1}{6}, \text{ octet}$$

$$C_6 = \frac{1}{3}, \text{ sextet}$$

$$V(R) = c_0 + KR - \frac{A}{R}$$

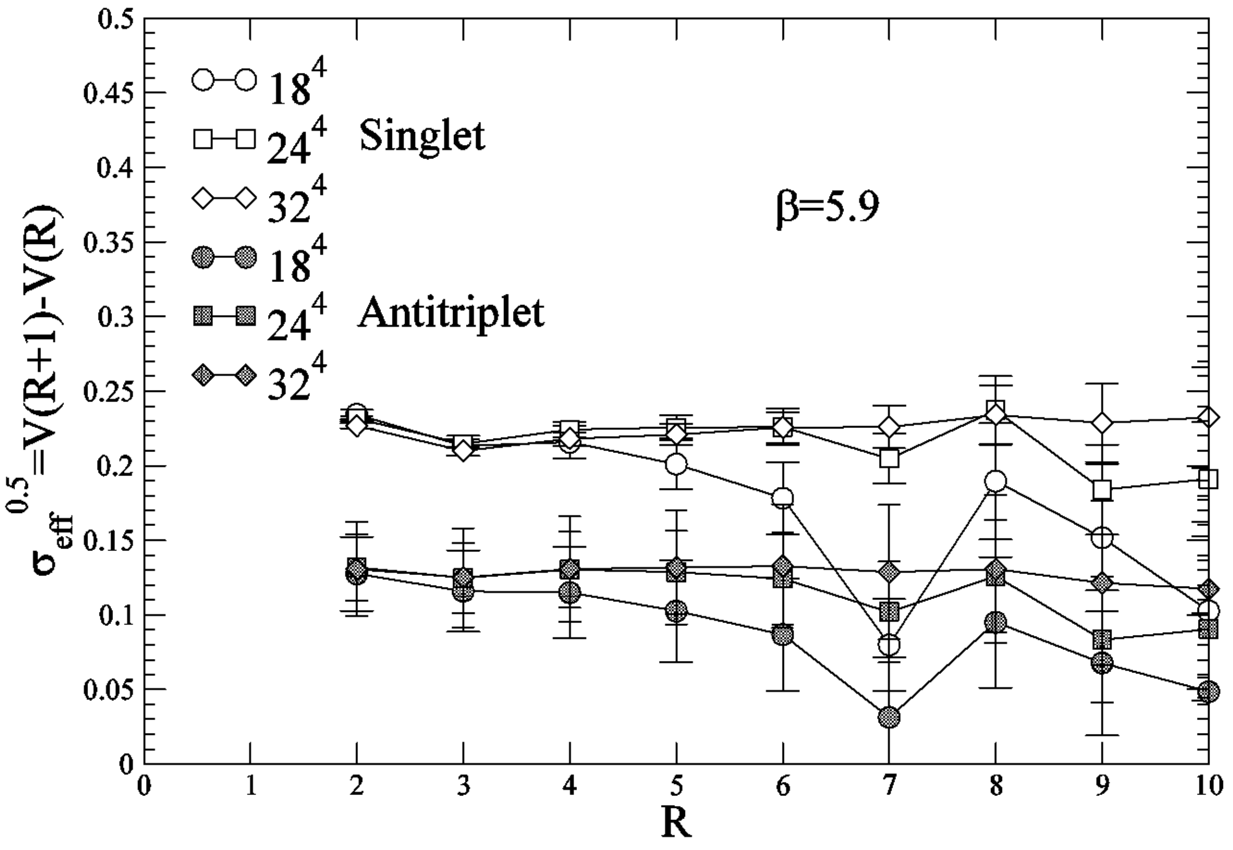
$$C_1/C_{3^*} = (-4/3)/(-2/3) = 2$$

$$\sqrt{\sigma}_1 a \sim 0.170 - 0.175$$

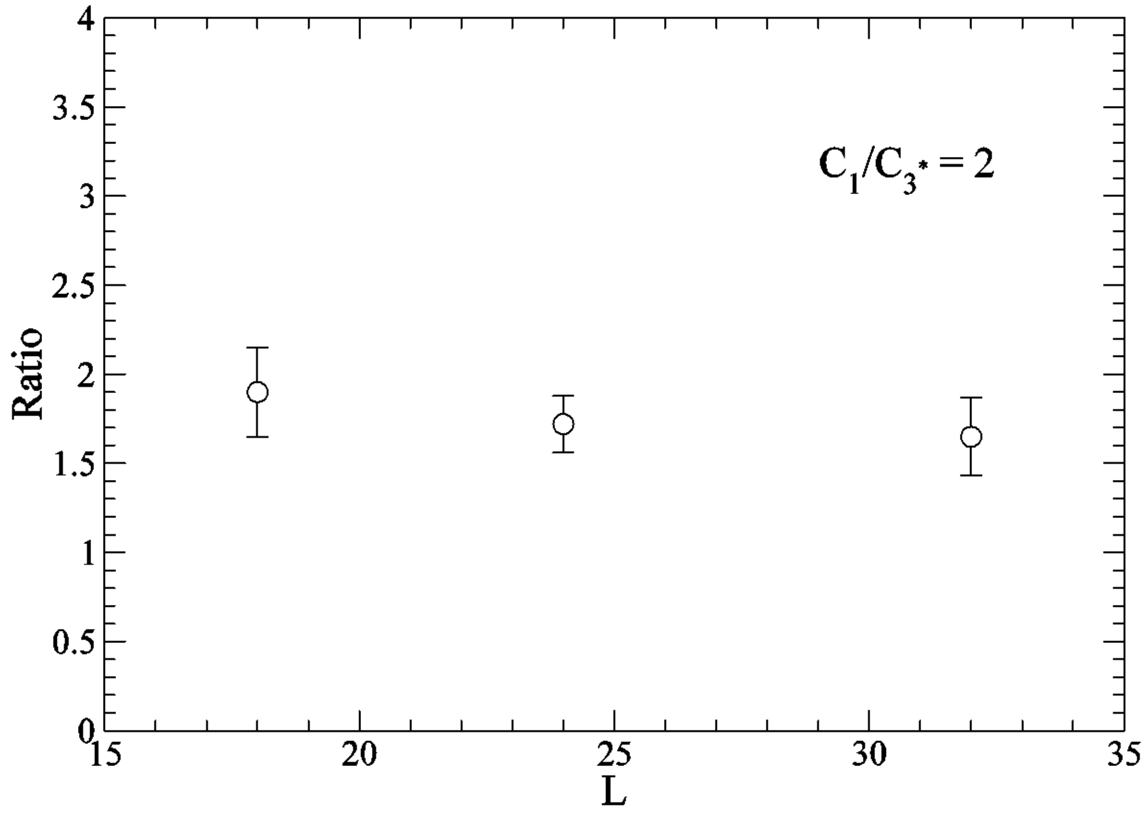
$$\sqrt{\sigma}_{3^*} a \sim 0.092 - 0.097$$

Nakagawa, et al. PRD77,034015,2008;PRD77,034015,2008

Effective string tension for attractive channels



Ratios of the color forces between singlet and anti-triplet channels



$$V(R) = c_0 + KR - \frac{A}{R}$$

$$K = V(R+1) - V(R)$$

$$\boxed{\mathrm{Ratio} = K_1 / K_{3^*}}$$

- Ratio of the effective string tensions between singlet and ant-triplet.
- Volume dependence seems to be small.

Divergence part of color-Coulomb instantaneous potential

- Infrared divergence parts

$$V_c^{IS} = 4\pi(T_1^a T_2^b)\int_0^\infty dp\frac{1}{p^2} \quad \text{(from terms depending on R)}$$

$$\Sigma_c^{IS} = 4\pi(T_i^a)^2\int_0^\infty dp\frac{1}{p^2} \quad \text{(from terms not depending on R)}$$

$$(T_1^a T_2^b)+(T_i^a)^2 = (-4/3)+(4/3) = 0 \text{ for } 1$$

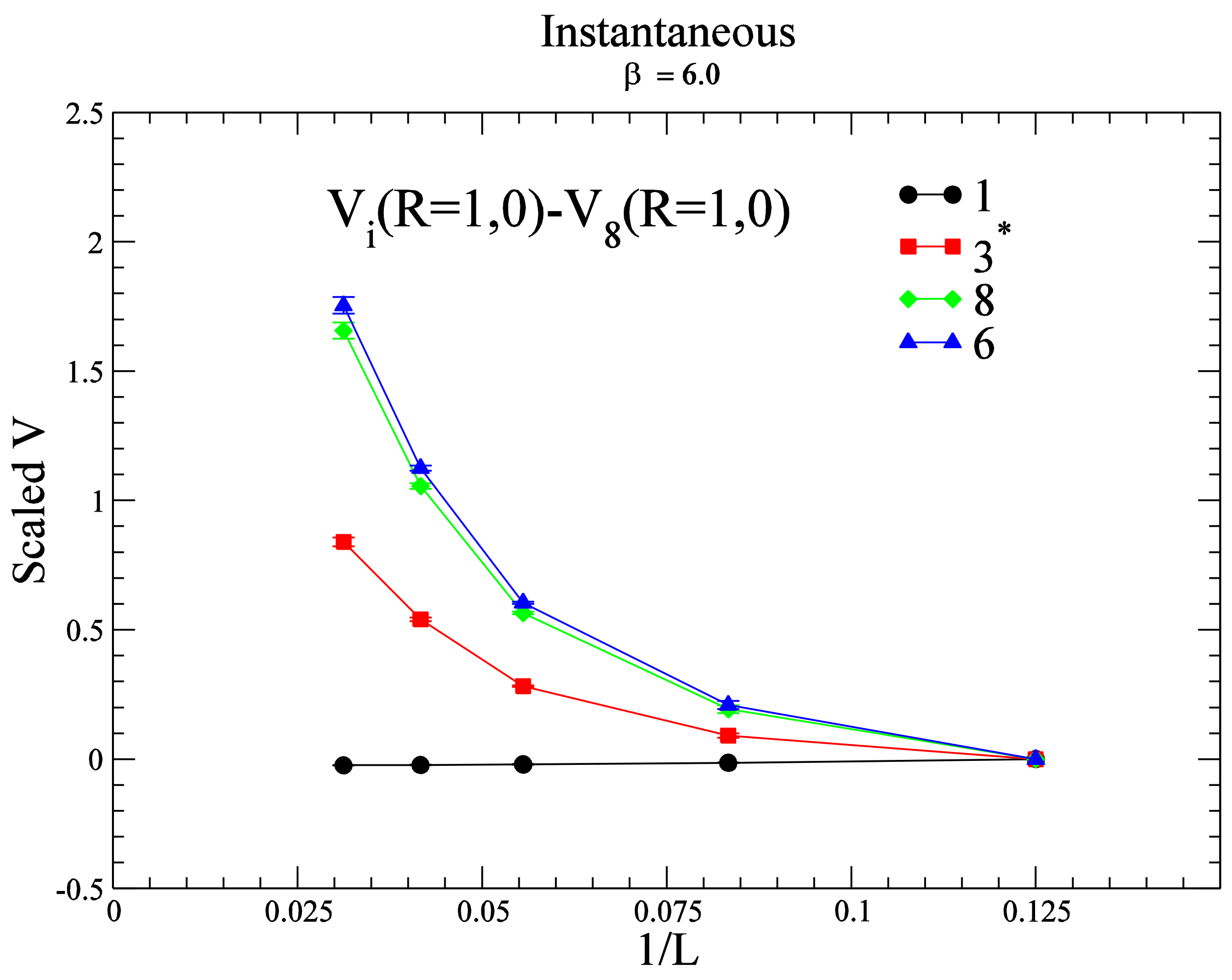
$$(T_1^a T_2^b)+(T_i^a)^2 = (\ \ 1/6)+(4/3) = 3/2 \text{ for } 8$$

$$(T_1^a T_2^b)+(T_i^a)^2 = (-2/3)+(4/3) = 2/3 \text{ for } 3^*$$

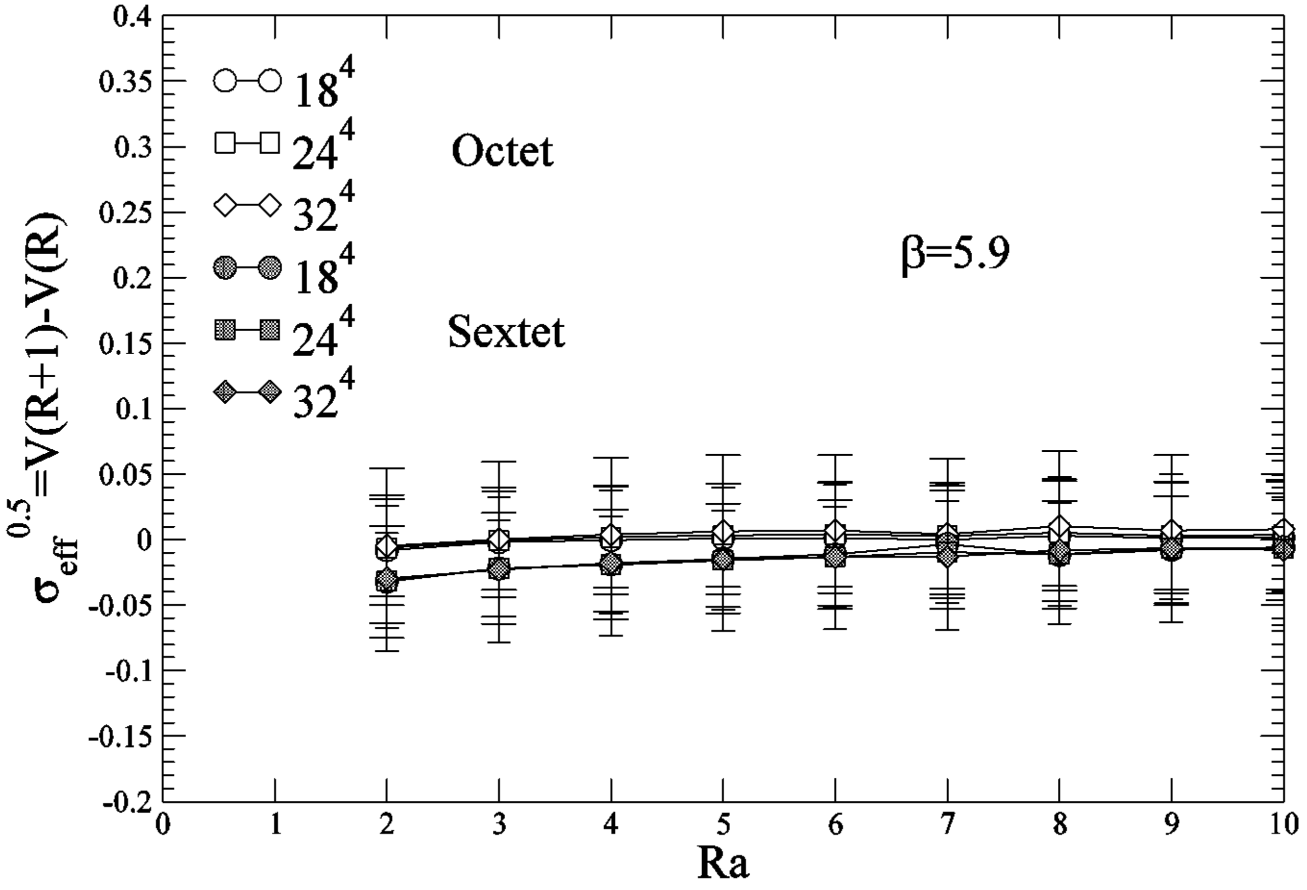
$$(T_1^a T_2^b)+(T_i^a)^2 = (\ \ 1/3)+(4/3) = 5/3 \text{ for } 6$$

$$4:9:10 \text{ for } 3^*, 8 \text{ and } 6$$

Volume effect



Effective string tension for repulsive channels



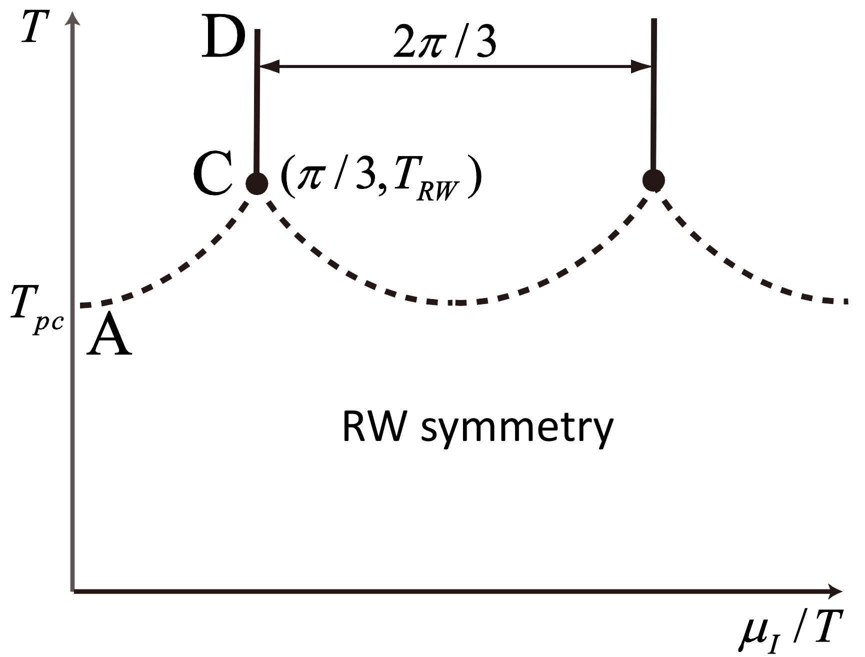
Imaginary Chemical Potentials

Sign problem for real chemical potential

$$\Delta(\mu)^{+} = \gamma_5 \Delta(-\mu^*)\gamma_5$$

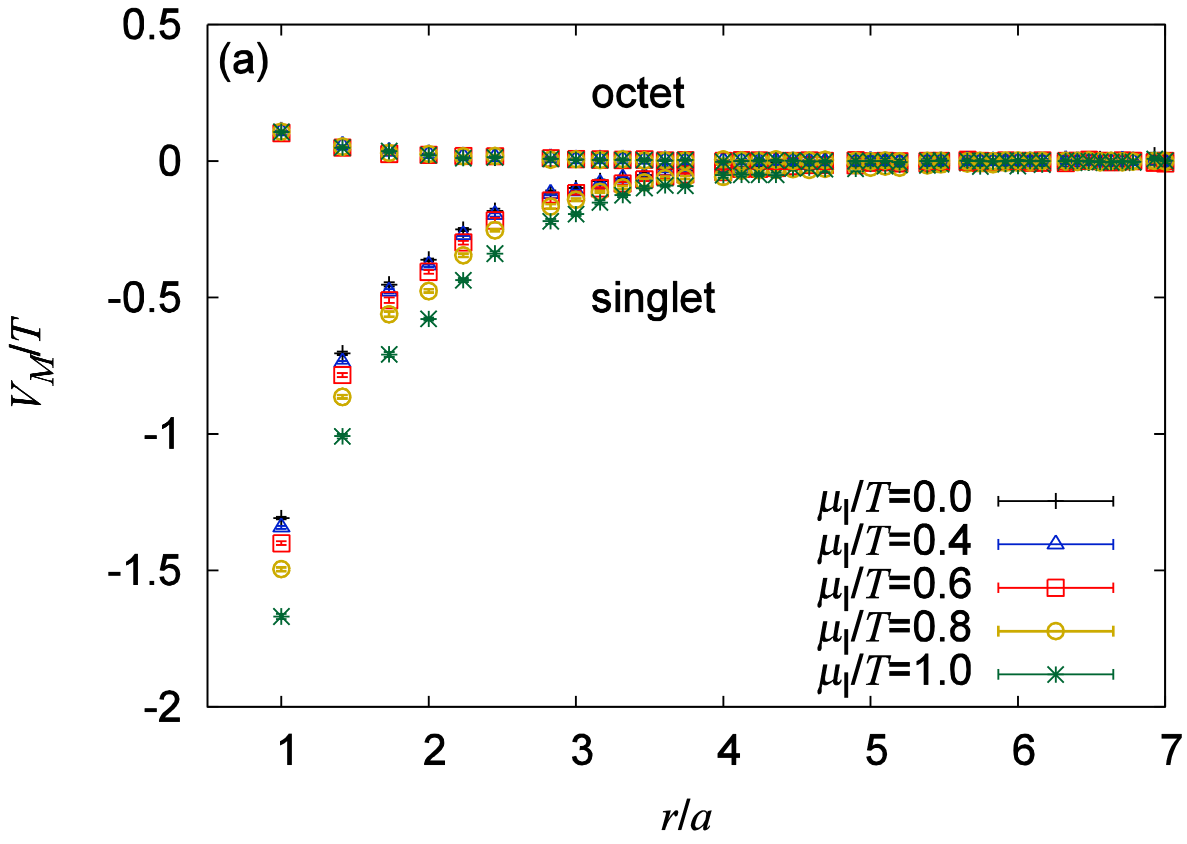
$$\mu \to i\mu_I$$

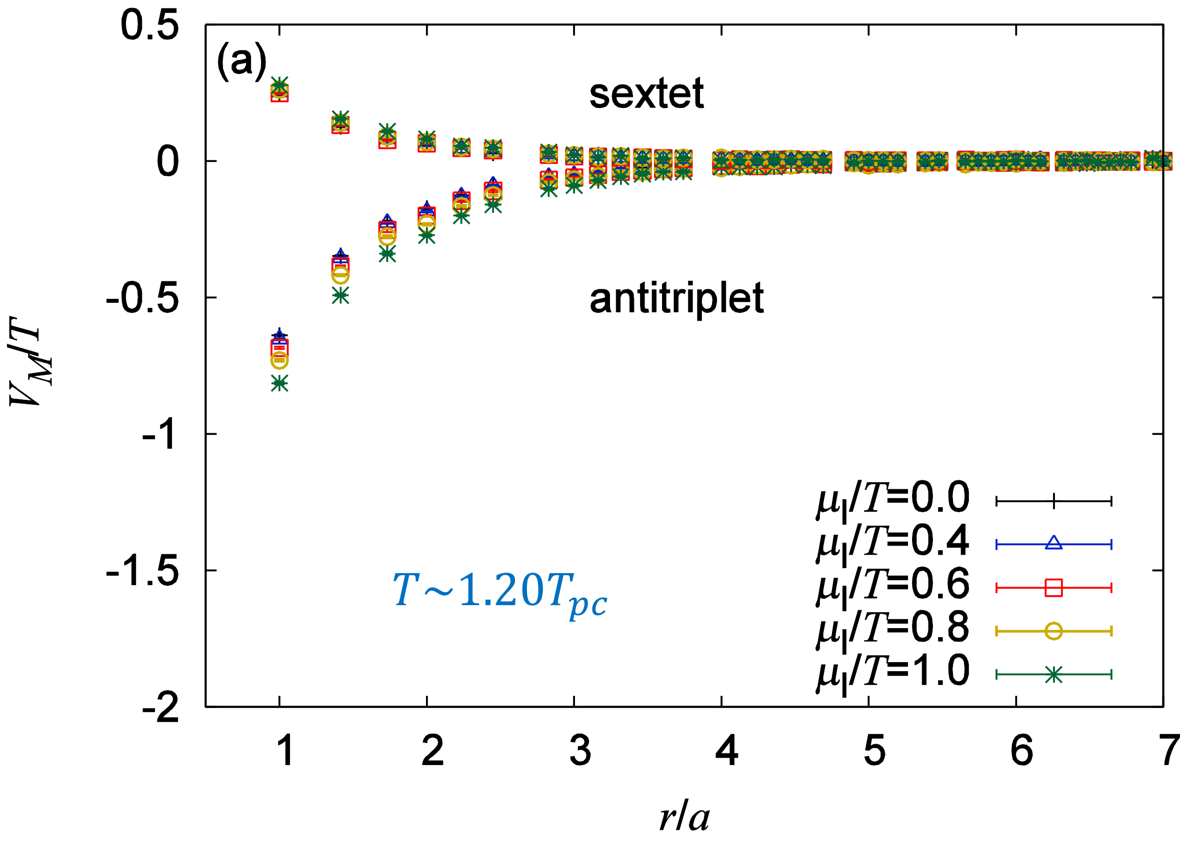
$$Z\left(\frac{\mu_I}{T}\right) = Z\left(\frac{\mu_I}{T} + \frac{2\pi k}{N_c}\right)$$



Roberge, Weiss, NPB275,734(1986)
Nagata, Nakamura, PRD83,2011,114507

Deconfinement phase

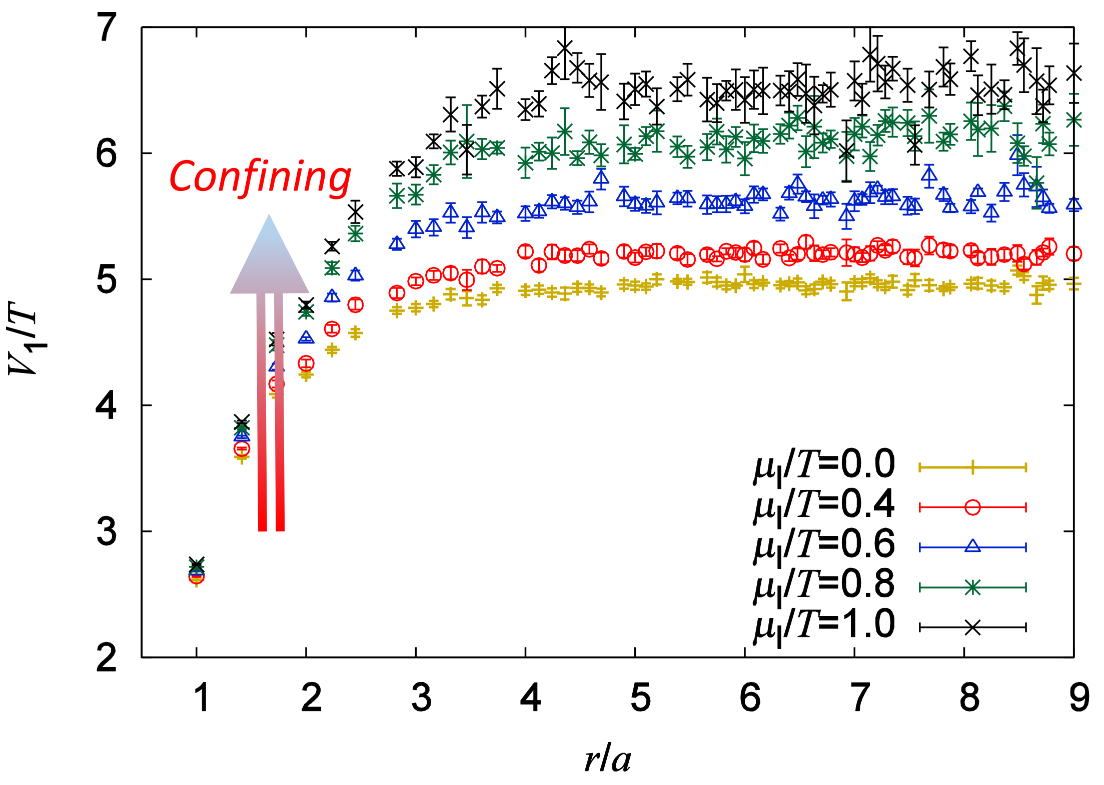


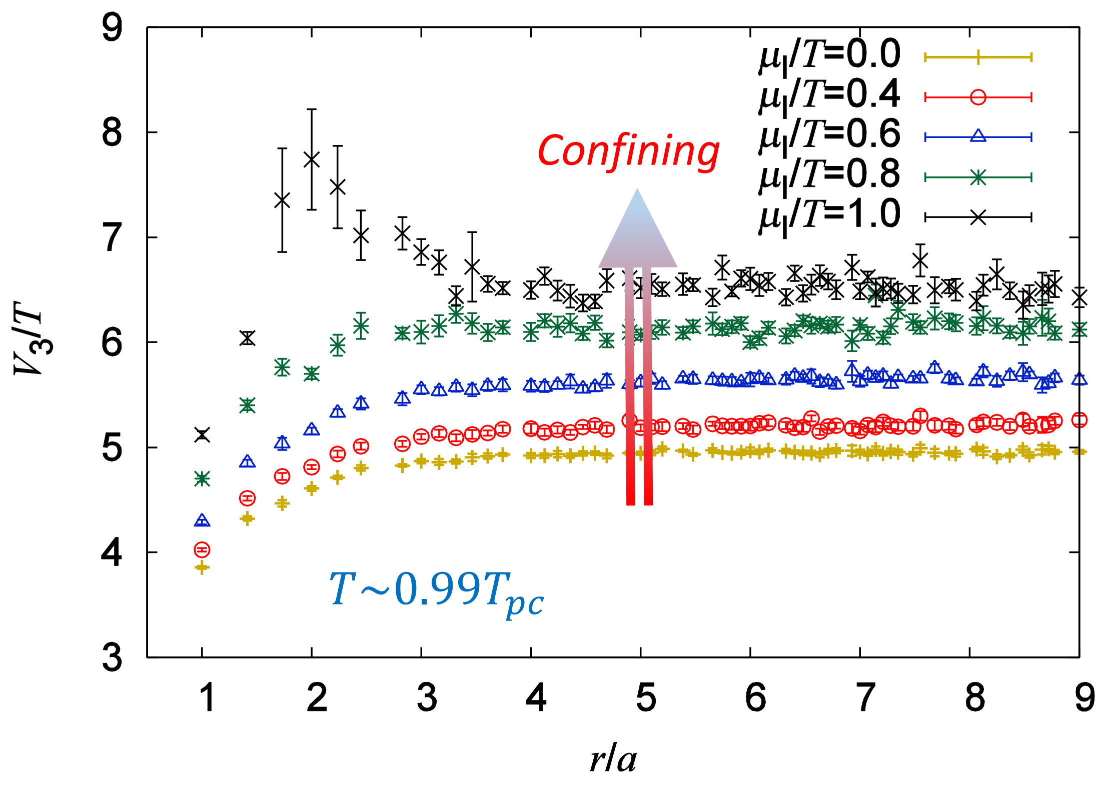


$T \sim 1.20 T_{pc}$

Takahashi J., et al. , hep-lat/1308.2489

Deconfinement → Confinement

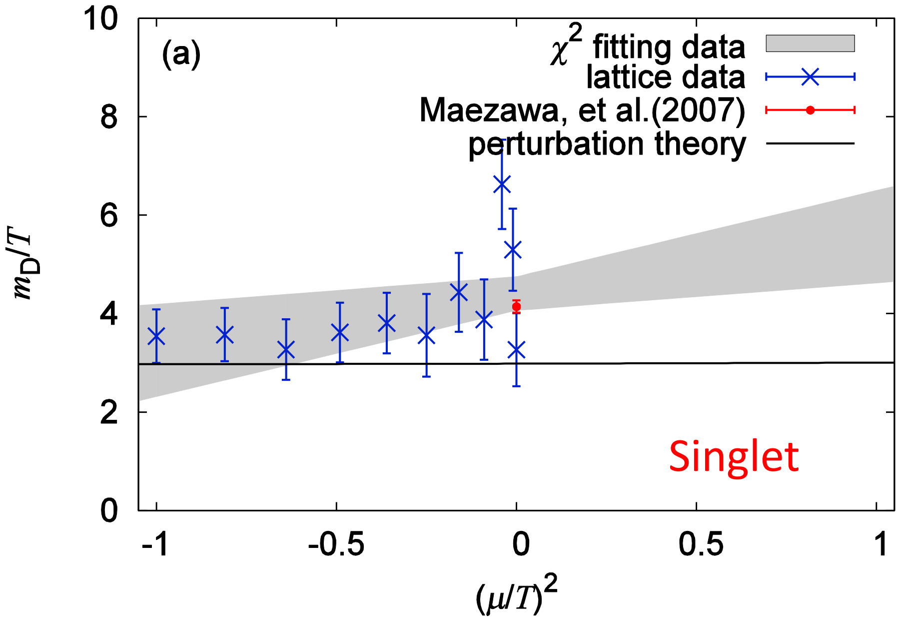


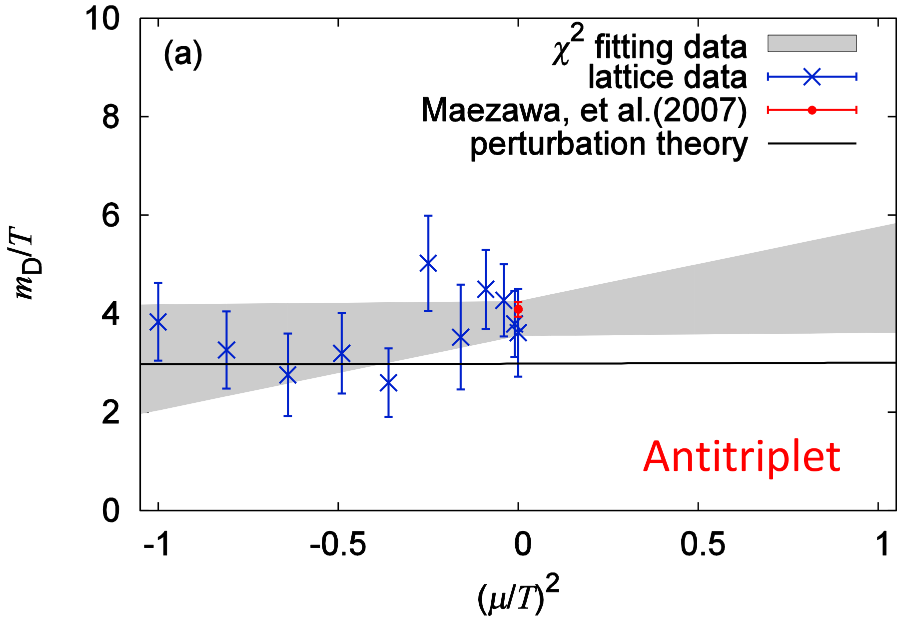


Preliminary

Color-Debye masses at finite density

$$V(R) \sim \frac{e^{-m_D R}}{R}$$

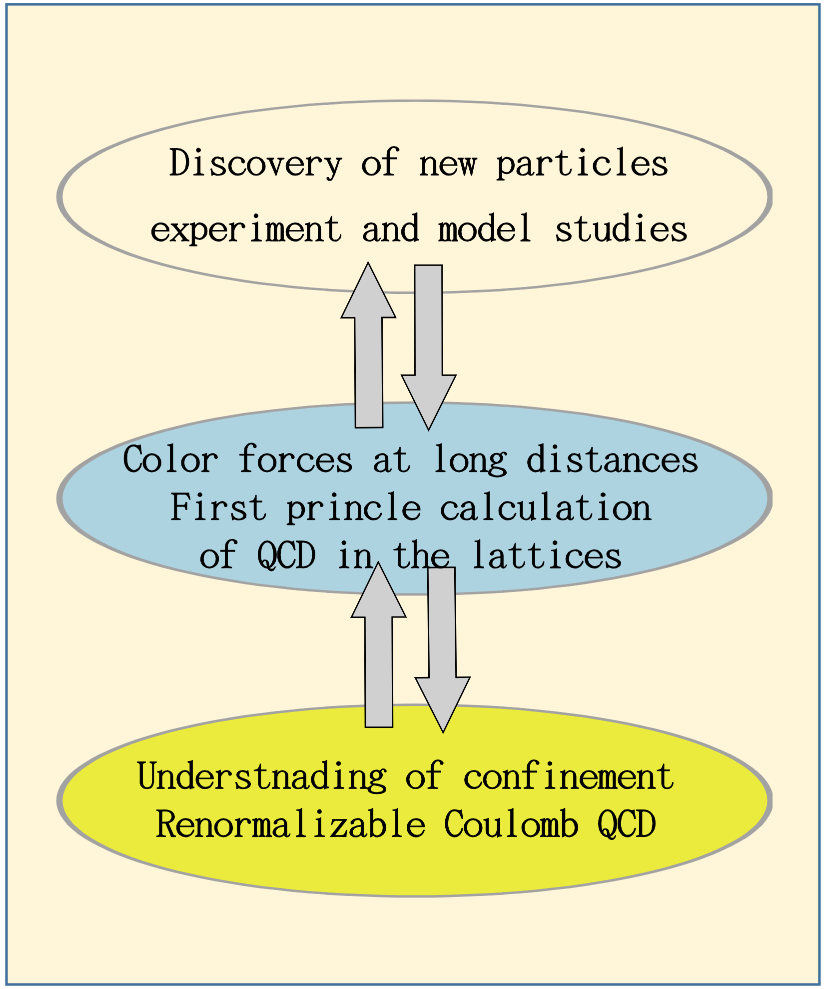




$T \sim 1.20 T_{pc}$

Summary

1. Coulomb gauge QCD theory enables us to compute color forces of SU(3) irreducible rep. in the confinement / deconfinement phases.
2. Those forces (instantaneous) depend basically on the quadratic Casimir factors, that is, $<\lambda\lambda>$.

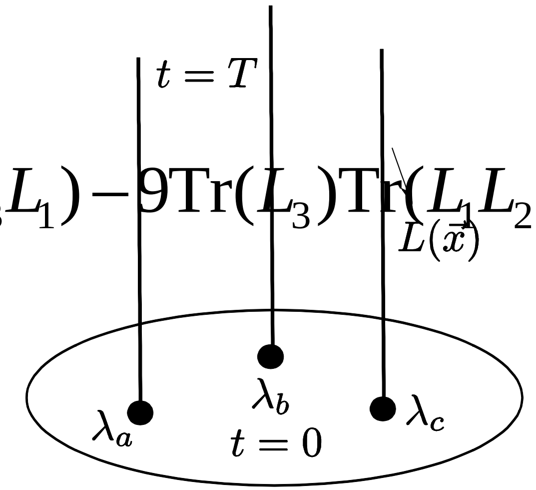


PLC in each color channel

Three-quark state

- Singlet channel

$$
\begin{aligned}
e^{-V^{1}_{qqq}(R)} = \Big[\ & 27\mathrm{Tr}(L_1)\mathrm{Tr}(L_2)\mathrm{Tr}(L_3) \\
& -9\mathrm{Tr}(L_1)\mathrm{Tr}(L_2L_3) - 9\mathrm{Tr}(L_2)\mathrm{Tr}(L_3L_1) - 9\mathrm{Tr}(L_3)\mathrm{Tr}(L_1L_2) \\
& +3\mathrm{Tr}(L_1L_2L_3) + 3\mathrm{Tr}(L_1L_3L_2)\ \Big]/6
\end{aligned}
$$

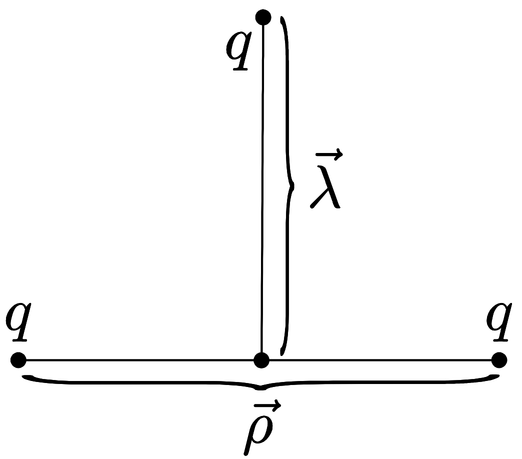


$$L(\vec{x}) = \Pi_{t=0}^{T} U_\mu(\vec{x}, t)$$

$Example:$

$$
\begin{aligned}
e^{-V^{8}_{q\bar{q}}(R)} &\sim a_1 TrLL^\dagger \\
&\quad - a_2 TrLTrL^\dagger \\
e^{-V^{10}_{qqq}(R)} &\sim b_1 TrLTrLTrL \\
&\quad + b_2 TrL(TrLL) \\
&\quad + \text{the other 4 terms}
\end{aligned}
$$

$V_{qqq}(\vec{\lambda}, \vec{\rho})$



Jacobi Coordinate

Divergence part of color-Coulomb instantaneous potential

- Hamiltonian

$$H = \frac{1}{2}\int d^3x\left(E_i^2 + B_i^2\right) + \frac{1}{2}\int d^3x d^3y\left(\rho(x) D(x,y) \rho(y)\right)$$

- Color-Coulomb instantaneous

$$V_{inst}(r) = \langle D \rangle = \left\langle \frac{1}{M}\left(-\partial_i^2\right)\frac{1}{M}\right\rangle$$

- Color charge density

$$\rho_a \sim T_1^a \delta(x - x_0) + T_2^a \delta(x - y_0)$$