Heavy quarkonium potential from lattice QCD

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T. Kawanai, SS, PRL 107 (2011) 091601
T. Kawanai, SS, PRD85 (2012) 091503(R)
Why back to quark potential models?

Why back to quark potential models?

Charmonium-like XYZ mesons are discovered

“Exotic” = “Non-standard”?

XYZ mesons could not be simply explained by a constituent quark description as quark and antiquark bound states

“Standard” states can be defined in potential models

→ Does it sound reliable?

Meson *local* operator

\[ \bar{q}(x) \Gamma q(x) \]

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( ^{2S+1}L_J )</th>
<th>( J^{PC} )</th>
<th>Meson</th>
<th>Charmonium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_5 )</td>
<td>( ^1S_0 )</td>
<td>0−+</td>
<td>( \pi )</td>
<td>( \eta_c )</td>
</tr>
<tr>
<td>( \gamma_i )</td>
<td>( ^3S_1 )</td>
<td>1−−</td>
<td>( \rho, \omega )</td>
<td>( J/\psi )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( ^3P_0 )</td>
<td>0++</td>
<td>( \sigma, a_0, f_0 )</td>
<td>( \chi_0(1P) )</td>
</tr>
<tr>
<td>( \gamma_5 \gamma_i )</td>
<td>( ^3P_1 )</td>
<td>1++</td>
<td>( a_1 )</td>
<td>( \chi_1(1P) )</td>
</tr>
<tr>
<td>( \gamma_i \gamma_j )</td>
<td>( ^1P_1 )</td>
<td>1+-</td>
<td>( b_1 )</td>
<td>( h_c(1P) )</td>
</tr>
</tbody>
</table>

\[ \Delta M_{\text{hyp}} = 114(1) \text{MeV} \]

Charmonia

DDbar threshold
Status of lattice QCD spectroscopy

\[ \begin{align*}
1S_0 & \quad 3S_1 & \quad 1P_1 & \quad 3P_0 & \quad 3P_1 & \quad 1D_2 & \quad 3D_2 & \quad 3D_3 & \quad 1F_3 & \quad 3F_3 \\
& & & & & & & & & \\
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\end{align*} \]

Nf=2 Clover

Pion mass \( m_\pi \approx 1.0 \) GeV
lattice cut off \( 1/a=2.6 \) GeV

Higher spin and exotic states:
non-local operator
\[
\bar{q}(x) \Gamma U(x, y) q(y)
\]
gauge links

G. Bali, S. Collins, C. Ehmann, PRD84 (2011) 094506
Why back to quark potential models?

∗ Interquark potential in non-relativistic quark potential models

T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

\[
V_{cc} = -\frac{4 \alpha_s}{3} \frac{1}{r} + \sigma r + \frac{32\pi\alpha_s}{9m_q^2} \delta(r) \mathbf{S}_q \cdot \mathbf{S}_\bar{q} + \frac{1}{m_q^2} \left[ \left( \frac{2\alpha_s}{r^3} - \frac{b}{2r} \right) \mathbf{L} \cdot \mathbf{S} + \frac{4\alpha_s}{r^3} T \right]
\]

- Cornell potential
- Spin-dependent potential

• Spin-spin, tensor, LS terms appear as corrections in powers of \(1/m_q\)
• Spin-dependent potentials determined by one-gluon exchange at tree level

→ There are large theoretical ambiguities for higher-mass charmonia

The reliable interquark potential derived from lattice QCD is hence desired at the charm quark mass
Static heavy quark potential from Wilson loops

\[ r_0 \approx 0.5 \text{ fm} \]

Lattice QCD exhibits the "Cornell-type potential" at the leading order in \( 1/m_Q \) expansion (pNRQCD)

\[ V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0 \]
Static heavy quark potential from Wilson loops

spin-dependent potentials at $O(1/m_Q^2)$ in pNRQCD


Koma-san’s talk
Static heavy quark potential from Wilson loops

But, the result is **not satisfactory**:

- applicability of pNRQCD is **doubtful at the charm mass**
- **quench approximation** (not applicable in full QCD)
- spin-spin potential seems to be “**attractive**”
New approach
Potential from BS amplitude

- Equal-time BS wave function
  \[ \phi_\Gamma (\mathbf{r}) = \sum_{\mathbf{x}} \langle 0 | Q(\mathbf{x}) \Gamma Q(\mathbf{x} + \mathbf{r}) | Q \bar{Q} \rangle \]

- Schrödinger eq. with non-local potential
  \[ -\frac{\nabla^2}{2\mu} \phi_\Gamma (\mathbf{r}) + \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \phi_\Gamma (\mathbf{r}') = E_\Gamma \phi_\Gamma (\mathbf{r}) \]

- Velocity expansion
  \[ v = |\nabla/m_Q| \]

- N-N potential

\[ U(\mathbf{r}', \mathbf{r}) = \{ V(\mathbf{r}) + V_S(\mathbf{r}) \mathbf{S}_Q \cdot \mathbf{S}_\bar{Q} + V_T(\mathbf{r}) S_{12} + V_{LS}(\mathbf{r}) \mathbf{L} \cdot \mathbf{S} + \mathcal{O}(\nabla^2) \} \delta(\mathbf{r}' - \mathbf{r}) \]

\( \bar{Q}Q \) potential from BS wave func.

- Ikeda-lida, arXiv:1011.2866

\[
\nabla^2 \phi_{\bar{Q}Q}(r) = \frac{\phi_{\bar{Q}Q}(r)}{m_Q} \left[ V(r) - E \right]
\]

The quark mass dependence is automatically encoded in the definition of the potential.

There are two key issues:

- Determination of the quark mass \( m_Q \)
- Behavior in the \( m_Q \to \infty \) limit
Novel determination of quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

\[
\left\{ -\frac{\nabla^2}{m_Q} + V_{Q\bar{Q}}(r) + S_Q \cdot S_Q V_{\text{spin}}(r) \right\} \phi_{\Gamma}(r) = E_{\Gamma} \phi_{\Gamma}(r) \quad \text{for} \quad \Gamma = \text{PS, V}
\]

Q. How can we determine a quark mass in the Schrödinger equation?

A. Look into asymptotic behavior of wave functions at long distances

\[
V_{\text{spin}}(r) - \Delta E_{\text{hyp}} = \frac{1}{m_Q} \left( \frac{\nabla^2 \phi_V(r)}{\phi_V(r)} - \frac{\nabla^2 \phi_{\text{PS}}(r)}{\phi_{\text{PS}}(r)} \right)
\]

Under a simple, but reasonable assumption of \( \lim_{r \to \infty} V_{\text{spin}}(r) = 0 \)

\[
m_Q = \lim_{r \to \infty} \frac{1}{\Delta E_{\text{hyp}}} \left( \frac{\nabla^2 \phi_{\text{PS}}(r)}{\phi_{\text{PS}}(r)} - \frac{\nabla^2 \phi_V(r)}{\phi_V(r)} \right)
\]
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

\[
m_Q = \lim_{r \to \infty} \frac{1}{\Delta E_{\text{hyp}}} \left( \frac{\nabla^2 \phi_{\text{PS}}(r)}{\phi_{\text{PS}}(r)} - \frac{\nabla^2 \phi_V(r)}{\phi_V(r)} \right)
\]

\[
\lim_{r \to \infty} V_{\text{spin}}(r) = 0
\]

\[
- m_Q \Delta E_{\text{hyp}}
\]
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

\[ V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0 \]

Consistent with the Wilson loops in the \( m_q \to \infty \) limit
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

The BS amplitude method can provide

- quark kinetic mass $m_Q$
- $m_Q$ dependence of the interquark potential
- spin-dependent potential (spin-spin)

in a self-consistent manner.
Several systematic tests within quenched QCD
Scaling test

at charm mass

central potential

$V(r) \ [\text{GeV}]$

$r \ [\text{fm}]$

$a=0.0931\text{fm} \ \text{coarse lattice}$

$a=0.0677\text{fm} \ \text{fine lattice}$

$L^3 \times T = 24^3 \times 48 \ \text{and} \ 32^3 \times 64$

Discretization error is well under control
Scaling test

at charm mass

spin-spin potential

\( \beta = 6.0, \ 6.2 \)

\( V(r) \) [GeV]

\( r \) [fm]

\( a=0.0931\text{fm} \) coarse lattice

\( a=0.0677\text{fm} \) fine lattice

\( L^3 \times T = 24^3 \times 48 \) and \( 32^3 \times 64 \)

Discretization error is well under control
Scaling test

spin-spin potential

$\beta = 6.0, 6.2, 6.47$

$V(r) [\text{GeV}]$

$r [\text{fm}]$

$a=0.0931\text{fm}$ coarse lattice

$a=0.0677\text{fm}$ fine lattice

$a=0.0469\text{fm}$ hyper fine lattice

fixed spacial size: $L \sim 2.2 \text{ fm}$

$L^3 \times T = 24^3 \times 48, 32^3 \times 64 \text{ and } 48^3 \times 96$
Lattice QCD simulations

$V(r) = V_{cc}(r) + S_Q \cdot S_{\bar{Q}} V_{\text{spin}}(r)$

$V_{\text{spin}}(r) \propto \nabla^2 V_{cc}(r)$

Our approach

Wilson loop approach

at charm mass

repulsive

Note: $M(0^-) < M(1^-)$

Y. Koma and M. Koma, NPB769 (2007) 79
Test of finite spacial size effect

$L^3 \times T = 24^3 \times 48 \text{ and } 32^3 \times 48$

$V(r) \text{ [GeV]}$

$r \text{ [fm]}$

$\beta = 6.0 \ (a = 0.093\text{fm})$

$L = 2.2\text{fm small lattice}$

$L = 3.0\text{fm large lattice}$
Full QCD at the physical point
Tuning RHQ parameters for **full QCD**

- RHQ action (Tsukuba-type) with 5 parameters
  - PACS-CS configurations at $m_\pi=156$ MeV
  - Relativistic Heavy Quark (RHQ) action for charm
  - $32^3 \times 64$ lattice
  - $a = 0.0907(13)$ fm
  - $L_a \sim 2.9$ fm
  - 198 configs

\[ \frac{1}{4} (M_{\eta_c} + 3M_{J/\psi}) = 3.070(1) \text{ GeV} \]

\[ \Delta M_{\text{hyp}} = 114(1) \text{ MeV} \]

- $c_{\text{eff}}^2 = 1.04(5)$

Namekawa et al., (PACS-CS), arXiv:1104.4600
How to treat heavy quarks

- Heavy quark mass introduces discretization errors of $O((ma)^n)$

- At charm quark, it becomes severe:
  
  $m_c \sim 1.5$ GeV and $1/a \sim 2$ GeV, then $m_c a \sim O(1)$

- Relativistic heavy quark (RHQ) approach:
  

- All $O((ma)^n)$ and $O(a\Lambda)$ errors are removed by the appropriate choice of six canonical parameters $\{m_0, \zeta, r_t, r_s, C_B, C_E\}$

  $$S_{\text{lat}} = \sum_{n,n'} \bar{\psi}_n K_{n,n'} \psi_{n'}$$

  explicit breaking of axis-interchange symmetry

  $$K = m_0 + \gamma_0 D_0 + \zeta \gamma_i D_i - \frac{r_t}{2} D_0^2 - \frac{r_s}{2} D_i^2 + C_B \frac{i}{4} \sigma_{ij} F_{ij} + C_E \frac{i}{2} \sigma_{0i} F_{0i}$$

- We follow the Tsukuba procedure to determine parameters

  S. Aoki, Y. Kuramashi, S.-I. Tominaga (1999)
Charmonium potential from full QCD

- Kawanai-Sasaki, PRD85 (2012) 091503(R)

* PACS-CS configurations at $m_\pi=156$ MeV

\[ V(r) = -\frac{A}{r} + \sigma r + V_0 \]

- $A_{c\bar{c}} = 0.813(22)$
- $\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV
- $A_\infty = 0.403(10)$
- $\sqrt{\sigma_\infty} = 0.462(2)$ GeV

Polyakov line correlator (off-axis)
Polyakov line correlator (on-axis)
BS wave function (off-axis)
BS wave function (on-axis)
Charmonium potential from full QCD

**Spin-independent $c\bar{c}$ potential**

- Lattice data
- NRp model

```
V(r) [GeV]
0.5
0
-0.5
-1
-1.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 r [fm]
```

- lattice results
  - $A_{c\bar{c}} = 0.813(22)$
  - $\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV

- NR quark model
  - $A_{NRp} = 0.7281$
  - $\sqrt{\sigma_{NRp}} = 0.3775$ GeV

**Spin-spin $c\bar{c}$ potential**

- Lattice data
- NRp model

```
V_s(r) [GeV]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.3 0.4 0.5 0.6 0.7 0.8 0.9 r [fm]
```

- good agreement
- large difference

**Refinement of spin-dependent potentials**

→ change the fine structure of charmonia

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Kawanai-Sasaki, PRD85 (2012) 091503(R)

<table>
<thead>
<tr>
<th>functional form</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa-type</td>
<td>0.287(8) GeV</td>
<td>0.894(32) GeV</td>
<td>7.28</td>
</tr>
<tr>
<td>Exponential-type</td>
<td>0.825(19) GeV</td>
<td>1.982(24) GeV</td>
<td>1.46</td>
</tr>
<tr>
<td>Gaussian-type</td>
<td>0.314(4) GeV</td>
<td>1.020(11) GeV^2</td>
<td>22.79</td>
</tr>
</tbody>
</table>

Non-relativistic potential model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026
Spin-spin potential from full QCD

finite-range repulsive potential

Non-relativistic potential model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026
\[ \Delta M_{\text{hyp}}(S-\text{wave}) = 47.3(3) \text{ MeV} \]

\[ \Delta M_{\text{hyp}}(\text{Exp.}) = 69.3(2.8) \text{ MeV} \]
spin-spin $b\bar{b}$ potential from full QCD

very preliminary

needs a confirmation through lattice cutoff dependence studies
Summary

- **New method to calculate QQ^{bar} potential at finite quark mass**
  - We propose a self-consistent determination of quark mass from the BS wave function
  - We confirm that spin-independent potential is consistent with the Wilson loop result in the m_Q → ∞ limit
- **Application to determine charmonium potential in full QCD**
  - Central potential resembles the NRp model
  - Spin-spin potential properly exhibits the short range repulsive interaction
  - Bottomonium potential (now under way)

→ Improves interquark potentials from lattice QCD
→ Refines a guideline of “exotic” quarkonia XYZ
Backup Slides
Rough estimates with lattice inputs

\[ V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0 \]

Coulomb part:

\[ E_C^n = 2m_c + V_0 - \frac{A^2}{2n^2m_c} \]

Linear part:

\[ E_L^n = 2m_c + V_0 + \lambda_n \left( \frac{\sigma^2}{m_c} \right)^{\frac{1}{3}} \]

\[ \lambda_1 = 2.388, \ \lambda_2 = 4.088 \quad \text{roots of Airy function} \]

Full QCD inputs

\[ \begin{align*}
A &= 0.861(17) \\
\sqrt{\sigma} &= 0.394(7) \text{ GeV} \\
V_0 &= -0.059(15) \text{ GeV} \\
m_c &= 1.74(3) \text{ GeV}
\end{align*} \]
Rough estimates with lattice inputs

\[ V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0 \]

Full QCD inputs

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\begin{align*}
A &= 0.861(17) \\
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V_0 &= -0.059(15) \text{ GeV} \\
m_c &= 1.74(3) \text{ GeV}
\end{align*}
\]

Coulomb part:

\[ E_{1S} = 2.78 \text{ GeV}, \quad E_{2S} = 3.26 \text{ GeV}, \quad \Delta E_{2S-1S} = 0.484 \text{ GeV} \]

Linear part:

\[ E_{1S} = 3.99 \text{ GeV}, \quad E_{2S} = 4.40 \text{ GeV}, \quad \Delta E_{2S-1S} = 0.408 \text{ GeV} \]

\[ E_{1S}^{\exp} = 3.07 \text{ GeV}, \quad E_{2S}^{\exp} = 3.67 \text{ GeV}, \quad \Delta E_{2S-1S}^{\exp} = 0.606 \text{ GeV} \]
Results from charmonium potential given by matching perturbative and lattice QCD


lattice QCD inputs

\[ V_c(r), \ V_S(r) \text{ for } r \geq 0.14 \text{ fm} \]
with \( m_c = 1.74(3) \text{ GeV} \)

pQCD inputs

\[ V_c(r), \ V_S(r) \text{ for } r \leq 0.14 \text{ fm} \]

\[ \overline{m_c^{MS}} (\mu = \overline{m_c^{MS}}) = 1.21(4) \text{ GeV} \]