

# **Improved constraints on chiral SU(3) dynamics from kaonic hydrogen**

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**with**

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**References:**

**Y.I., Hyodo, Weise, PLB 706 (2011) 63.**

**Y.I., Hyodo, Weise, arXiv:1201.6549 [nucl-th].**

**Workshop on `Future Prospects of Hadron Physics at J-PARC  
and Large Scale Computational Physics', Feb. 9-11. 2012**

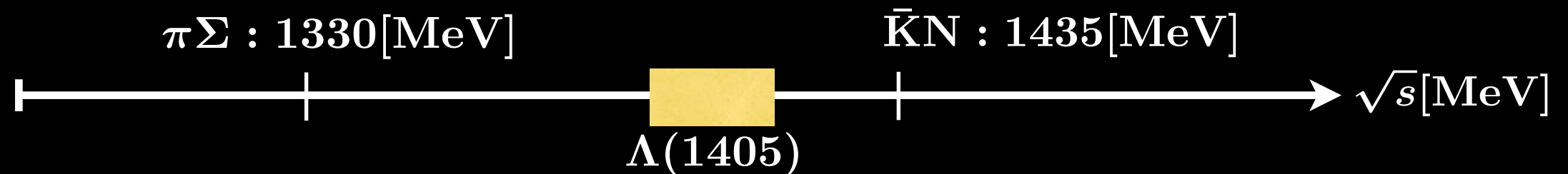
# **Contents**

- ▶ **Introduction to low energy anti-K -- nucleon interactions**
- ▶ **Chiral SU(3) effective field theory and coupled-channel dynamics**
- ▶ **Improved constraints on chiral SU(3) dynamics by SIDDHARTA kaonic hydrogen results**
- ▶ **Fit results**
- ▶ **Uncertainty analysis from SIDDHARTA results**
- ▶ **Predictions from “improved” chiral SU(3) dynamics**
- ▶ **Summary and future plans**

# Anti-kaon -- nucleon interactions

**Anti-K -- nucleon ( $K^{\text{bar}}N$ ) interaction in  $l=0$  is strongly attractive**

- ✓ Presence of  $\Lambda(1405)$  :  $K^{\text{bar}}N$  quasi-bound state
- ✓ Repulsive shift of kaonic hydrogen, scattering data, ...



**$K^{\text{bar}}N$  system strongly couples to  $\pi\Sigma$  continuum**

- ✓ Width of  $\Lambda(1405)$ ,  $K^{\text{bar}}$  atoms

**$K^{\text{bar}}N$  interaction is basic ingredient for application to**

- ✓  $K^{\text{bar}}$ -nuclei,  $K^{\text{bar}}$  in medium,  $K^{\text{bar}}$  atoms, ...

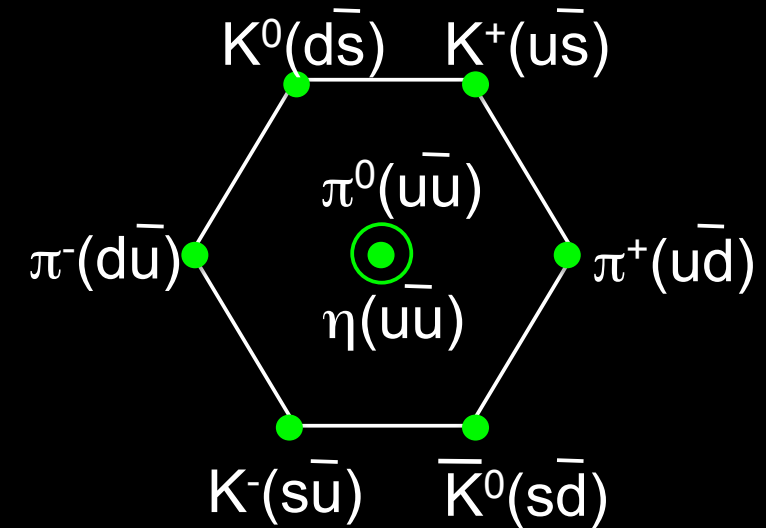
**In order to discuss many application,  
evidently determination of  $K^{\text{bar}}N$  interaction is important issue**

# Anti-kaon -- nucleon interactions

**Kaon : Nambu-Goldstone (NG) boson [ chiral SU(3) ]**

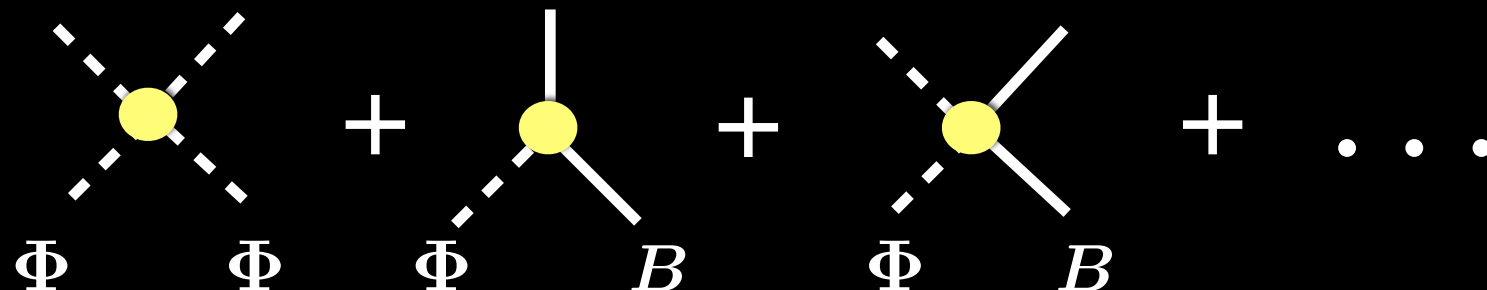
**Chiral Effective Field Theory :**

**interactions between NG bosons and baryons**



$$\mathcal{L}_{\text{eff}}(\Phi, B) = \mathcal{L}_M(\Phi) + \mathcal{L}_{\Phi B}(\Phi, B)$$

**well organized by systematic low-energy expansion in  $p/\Lambda_\chi$ ,  $m_\pi/\Lambda_\chi$**



$\Phi$ : Meson field,  $B$ : Baryon field

**Chiral perturbation theory works for low-energy  $\pi\pi$  and  $\pi N$  systems,  
but not for  $S=-1$  meson-baryon sector ( $K^{\text{bar}}N$ ,  $\pi\Sigma$ , ...)**



**Non-perturbative coupled-channels dynamics needed**

# Theoretical framework for MB scattering

**Non-perturbative dynamics : coupled-channels Bethe-Salpeter equation**

**10 channels :  $K^{\text{bar}}N=(K^{\text{bar}}p, K^{\text{bar}}n)$ ,  $\pi\Lambda$ ,  $\pi\Sigma=(\pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+)$ ,  $\eta\Lambda$ ,  $\eta\Sigma^0$ ,  $K\Xi=(K^+\Xi^-, K^0\Xi^0)$**

$$T_{ij}(\sqrt{s}) = V_{ij}(\sqrt{s}) + \sum_n V_{in}(\sqrt{s}) G_n(\sqrt{s}) T_{nj}(\sqrt{s})$$

**Interactions determined by matching with chiral EFT order by order**

$$V^{(1)} = T^{(1)}(\sqrt{s}), \quad V^{(2)} = T^{(2)}(\sqrt{s}), \quad V^{(3)} = T^{(3)}(\sqrt{s}) - T^{(1)}(\sqrt{s})G(\sqrt{s})T^{(1)}(\sqrt{s}), \quad \dots$$

**Loop integral : finite parts include subtraction constant  $a(\mu)$**

$$G(\sqrt{s}) = \underbrace{a(\mu)}_{\text{subtraction constant}} + \frac{1}{32\pi^2} \left[ \log\left(\frac{m_\phi^2 M_B^2}{\mu^4}\right) + \frac{m_\phi^2 - M_B^2}{s} \log\left(\frac{m_\phi^2}{M_B^2}\right) \right] \\ - \frac{1}{16\pi^2} \left[ 1 + \frac{4|q_{\text{c.m.}}|}{\sqrt{s}} \text{artanh}\left(\frac{2\sqrt{s}|q_{\text{c.m.}}|}{(m_\phi + M_B)^2 - s}\right) \right]$$

**Scattering amplitude T is consistent with**

**non-perturbative coupled-channels dynamics + chiral SU(3) symmetry**

**(Coupled-channels chiral SU(3) dynamics)**

Kaiser, Siegel, Weise, NPA 594 (1995) 325.

Oset, Ramos, NPA 635 (1998) 99.; Oller, Meissner, PLB 500 (2001) 263. + many others...

Recent review : Hyodo, Jido, PPNP 67 (2012) 55.

# Coupled-channels chiral SU(3) dynamics

## Leading order contributions from chiral EFT

### Leading order (Tomozawa-Weinberg) term : O(p)

$$\mathcal{L}^{\text{TW}} = \frac{i}{8f^2} \text{tr}[\bar{B} \gamma^\mu [v_\mu, B]]$$

Weinberg, PRL 17, 616 (1966).

Tomozawa, Nuov. Cim. 46A, 707 (1966).

$$v_\mu = \Phi \boxed{(\partial_\mu \Phi)} - \boxed{(\partial_\mu \Phi)} \Phi + \dots \quad \Phi: \text{Meson field}, B: \text{Baryon field}$$

↓

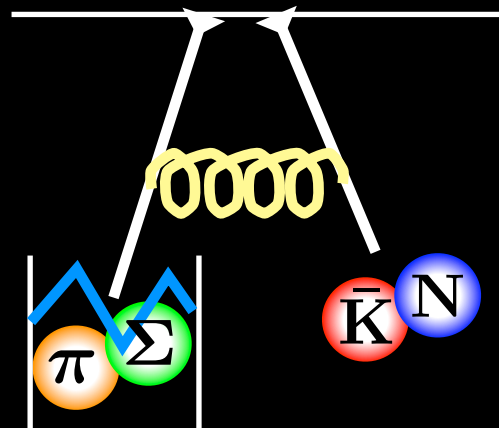
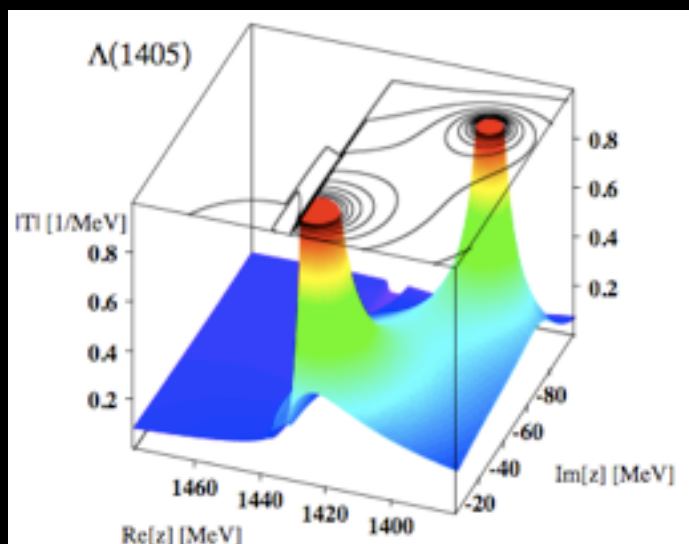
$$V_{ij}^{\text{TW}} = -\frac{C_{ij}}{8f_i f_j} \mathcal{N}_i \mathcal{N}_j \boxed{(\omega_i + \omega_j)}$$

( $\mathcal{N}_i = \sqrt{E_i + M_i}$ )

- ✓ **Strength : (physical) meson decay constant**
- ✓ **Coupling : chiral SU(3) symmetry**
- ✓ **Derivative coupling : (meson) energy dependence**

► With TW term alone, low-energy K-p scattering data are described fairly well

►  $\Lambda(1405)$  : two poles (K<sup>bar</sup>N bound,  $\pi\Sigma$  resonance)



Jido et al., NPA723, 205 (2003).

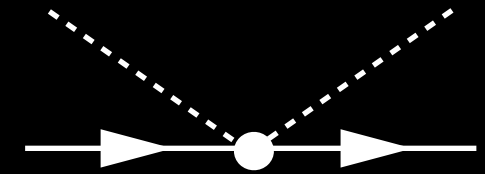
Hyodo, Weise, PRC77, 035204 (2008).

# Coupled-channels chiral SU(3) dynamics

**Interaction kernels : chiral EFT up to next-to-leading order**

**Leading order (Tomozawa-Weinberg) term :  $\mathcal{O}(p)$**

$$\mathcal{L}^{\text{TW}} = \frac{i}{8f^2} \text{tr}[\bar{B} \gamma^\mu [v_\mu, B]]$$



**input : meson decay constant**

**Direct and crossed Born terms :  $\mathcal{O}(p)$**

$$\mathcal{L}^{\text{Born}} = \text{tr} \left( \frac{D}{2} (\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}) + \frac{F}{2} (\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]) \right)$$



**input : axial-vector constants  
from hyperon beta decays**

$D=0.80, F=0.46 \rightarrow g_A=D+F=1.26$

**Next-to-leading order (NLO) terms :  $\mathcal{O}(p^2)$**

$$\begin{aligned} \mathcal{L}^{\text{NLO}} = & b_D \text{tr}(\bar{B} \{\chi_+, B\}) + b_F \text{tr}(\bar{B} [\chi_+, B]) + b_0 \text{tr}(\bar{B} B) \text{tr}(\chi_+) \\ & + d_1 \text{tr}(\bar{B} \{u^\mu, [u_\mu, B]\}) + d_2 \text{tr}(\bar{B} [u^\mu, [u_\mu, B]]) \\ & + d_3 \text{tr}(\bar{B} u^\mu) \text{tr}(u_\mu B) + d_4 \text{tr}(\bar{B} B) \text{tr}(u^\mu u_\mu) \end{aligned}$$



**input : 7 low energy constants  
( $b_D, b_F, b_0, d_1, d_2, d_3, d_4$ )**

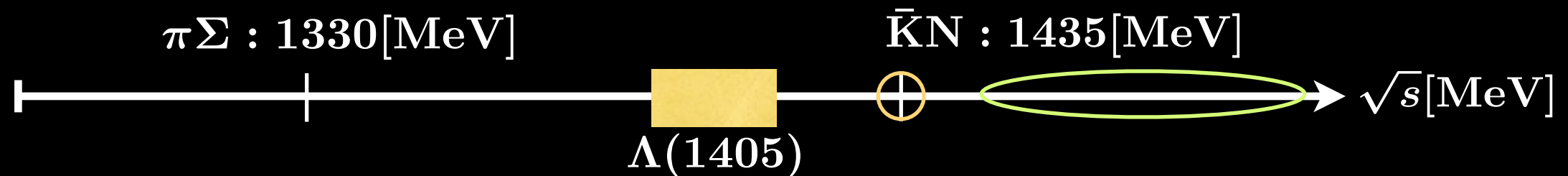
# Constraints on chiral SU(3) dynamics

## Parameters of coupled-channels chiral SU(3) dynamics

10 channels :  $K^{\text{bar}}N=(K^-p, K^{\text{bar}0}n)$ ,  $\pi\Lambda$ ,  $\pi\Sigma=(\pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+)$ ,  $\eta\Lambda$ ,  $\eta\Sigma^0$ ,  $K\Xi=(K^+\Xi^-, K^0\Xi^0)$

$a(\mu)$  : 6 isospin symmetric subtraction constants

NLO terms : 7 low energy constants



$K^-p$  cross sections : wide energy range above  $K^{\text{bar}}N$  threshold

Threshold branching ratios : accurate

$$\frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-)}{\Gamma(K^-p \rightarrow \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

$$\frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-, \pi^-\Sigma^+)}{\Gamma(K^-p \rightarrow \text{all inelastic channels})} = 0.66 \pm 0.01$$

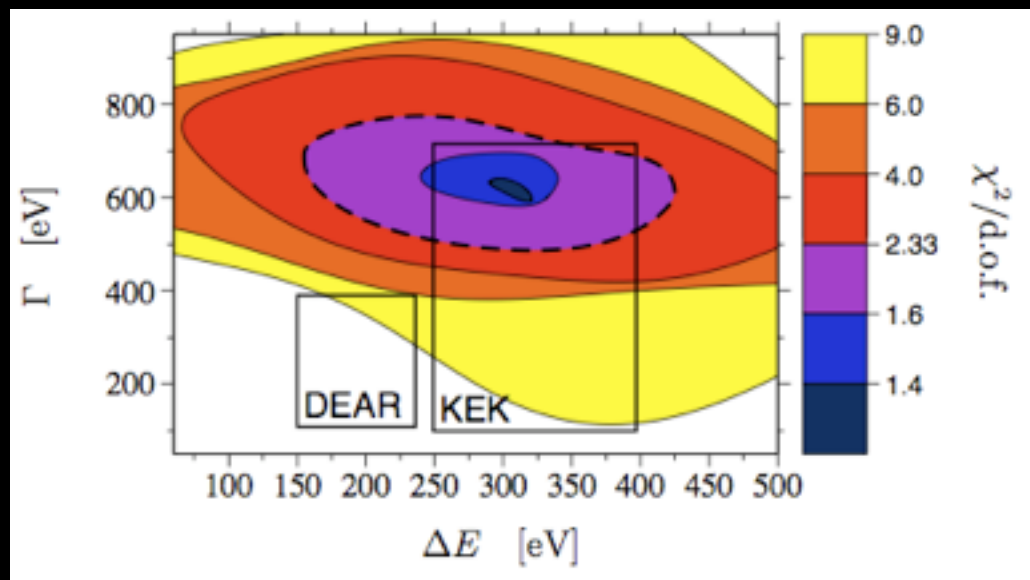
$$\frac{\Gamma(K^-p \rightarrow \pi^0\Lambda^0)}{\Gamma(K^-p \rightarrow \text{neutral channels})} = 0.19 \pm 0.02$$

# Constraints on chiral SU(3) dynamics

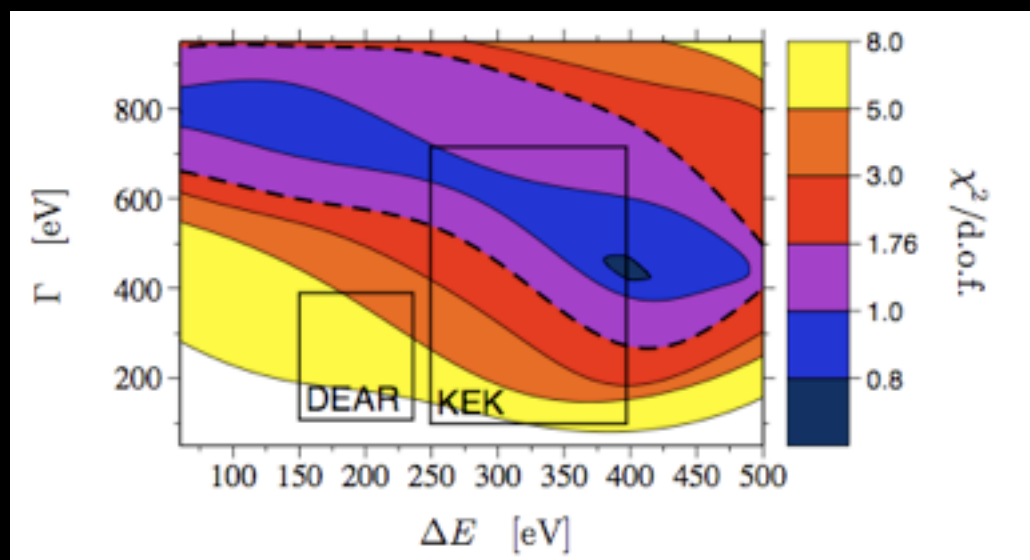
$\chi^2$  analysis w/ cross section data and branching ratios

## 1. predicted kaonic hydrogen shift & width

Leading order (TW)



TW+Born+NLO

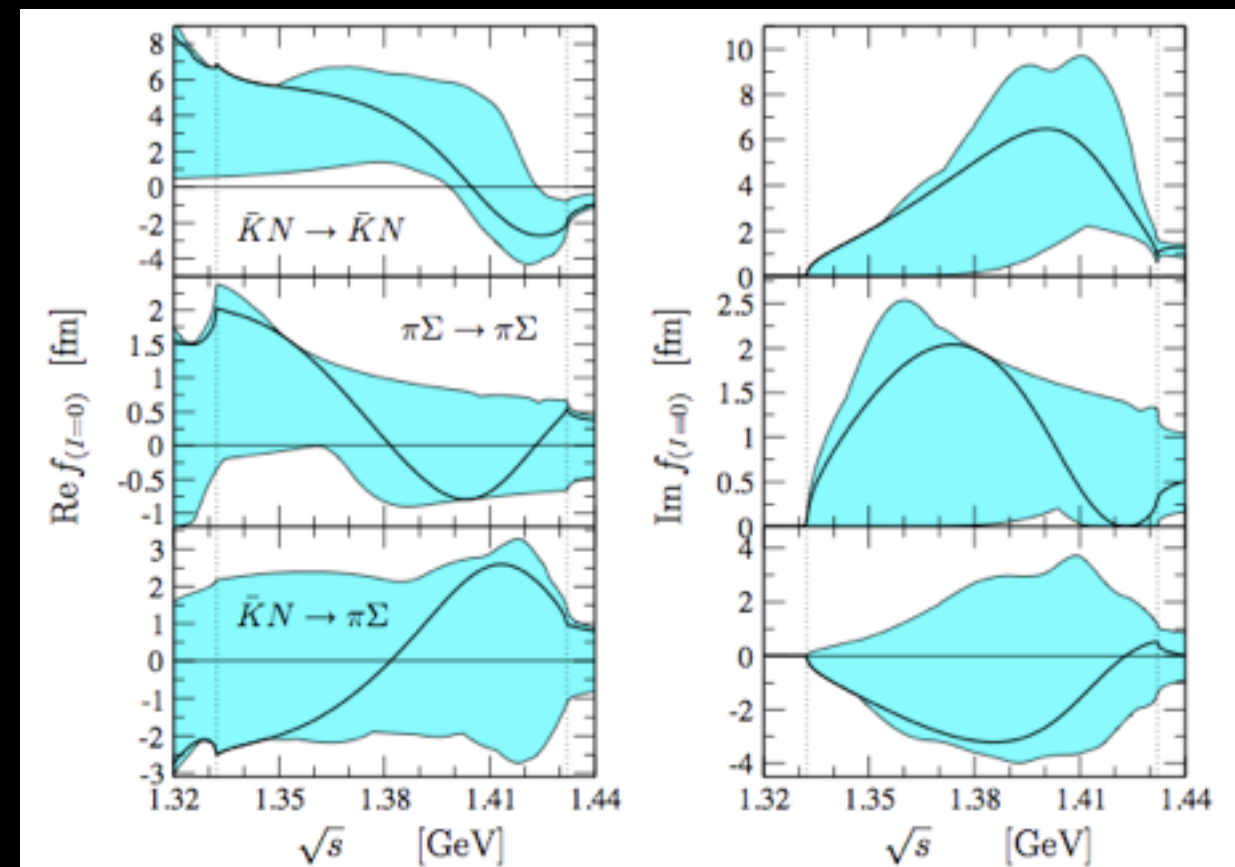


Nissler, PhD thesis (2008)

Borasoy, Nissler, Weise, EPJA25(2005).

Brasoy, Meissner, Nissler, PRC74(2006).

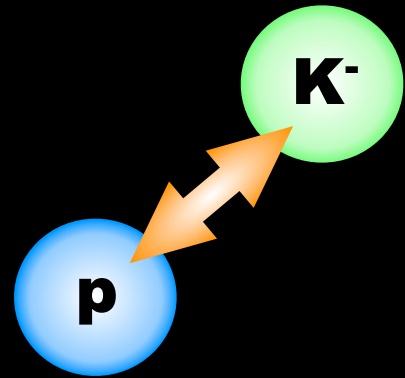
## 2. amplitudes (NLO), $K^{\text{bar}}N$ & $\pi\Sigma$



To determine NLO parameters, we need precise kaonic hydrogen data

# SIDDHARTA kaonic hydrogen measurement

## Kaonic hydrogen energy shift and width by strong interaction

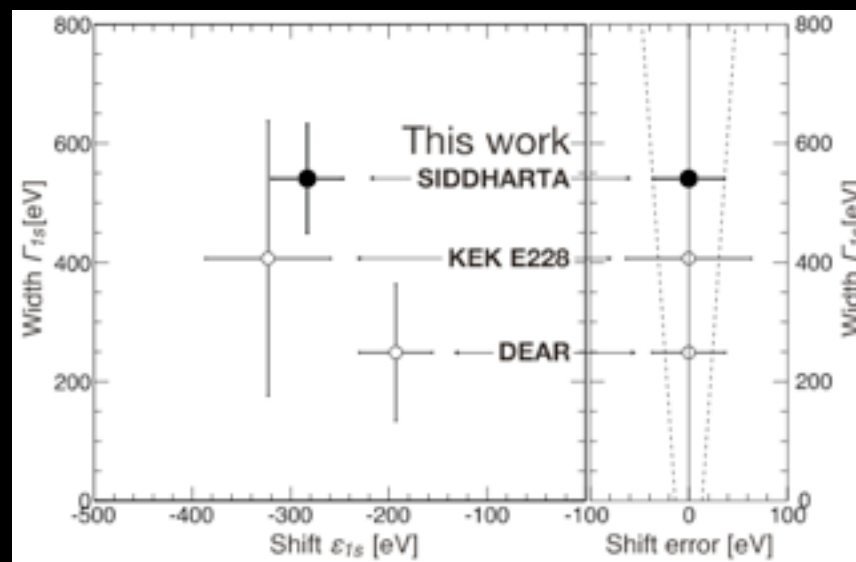
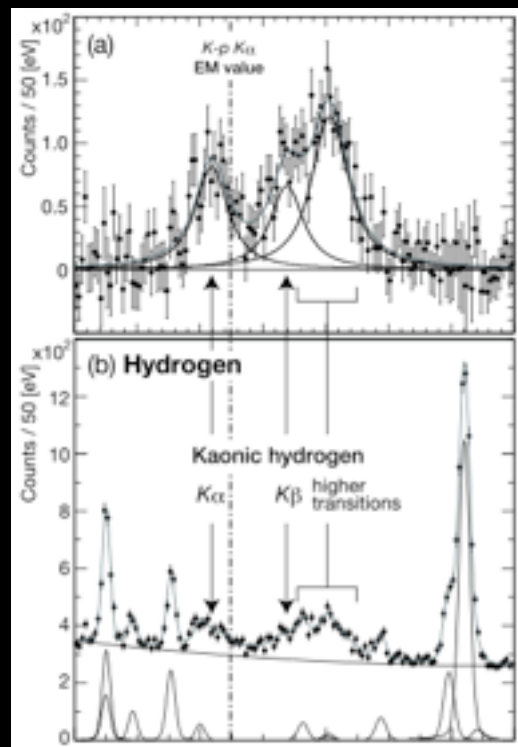


**Kaonic hydrogen : mainly EM binding system**  
**strong interaction -> shift & width -> scattering length**

$$\Delta E - \frac{i}{2}\Gamma = -2\alpha^3 \mu_c^3 a_{K-p} [1 - 2\alpha \mu_c (\ln \alpha - 1) a_{K-p}]$$

Meissner, Raha, Rusestky, EPJ C35 (2004).

## New precision data of kaonic hydrogen by SIDDHARTA Coll.



Bazzi et al., PLB 704 (2011).

Bazzi et al., arXiv:1201.4635[nucl-ex].

## SIDDHARTA shift & width

$$\Delta E = -283 \pm 36 \text{ (stat.)} \pm 6 \text{ (syst.) eV}$$

$$\Gamma = 541 \pm 89 \text{ (stat.)} \pm 22 \text{ (syst.) eV}$$



$$\text{Re } a_{K-p} = -0.65 \pm 0.10 \text{ [fm]}$$

$$\text{Im } a_{K-p} = 0.81 \pm 0.15 \text{ [fm]}$$

**This work :**

**Systematic  $\chi^2$  analysis w/**

**SIDDHARTA results + cross sections + threshold branching ratios**

# Fit results

**We employ physical meson decay constants**

**(  $f_\pi=92.4$ ,  $f_K=119\pm1$ ,  $f_\eta=130\pm5$  [MeV] )**

| <b>kaonic hydrogen shift &amp; width</b>                                                                                   | <b>TW</b>                    | <b>TW+Born</b>                | <b>NLO (full)</b>            | <b>EXP.</b>                       |
|----------------------------------------------------------------------------------------------------------------------------|------------------------------|-------------------------------|------------------------------|-----------------------------------|
| <b><math>-\Delta E</math> (eV)</b>                                                                                         | <b>373</b>                   | <b>377</b>                    | <b>306</b>                   | <b><math>283\pm36\pm6</math></b>  |
| <b><math>\Gamma</math> (eV)</b>                                                                                            | <b>495</b>                   | <b>514</b>                    | <b>591</b>                   | <b><math>541\pm89\pm22</math></b> |
| <b>Threshold branching ratios</b>                                                                                          |                              |                               |                              |                                   |
| $\frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-)}{\Gamma(K^- p \rightarrow \pi^- \Sigma^+)}$                                | <b>2.36</b>                  | <b>2.36</b>                   | <b>2.37</b>                  | <b><math>2.36\pm0.04</math></b>   |
| $\frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-, \pi^- \Sigma^+)}{\Gamma(K^- p \rightarrow \text{all inelastic channels})}$ | <b>0.66</b>                  | <b>0.66</b>                   | <b>0.66</b>                  | <b><math>0.66\pm0.01</math></b>   |
| $\frac{\Gamma(K^- p \rightarrow \pi^0 \Lambda^0)}{\Gamma(K^- p \rightarrow \text{neutral channels})}$                      | <b>0.20</b>                  | <b>0.19</b>                   | <b>0.19</b>                  | <b><math>0.19\pm0.02</math></b>   |
| <b>Pole position of <math>\Lambda(1405)</math><br/>(MeV)</b>                                                               | <b>1422-i16<br/>1384-i90</b> | <b>1421-i17<br/>1385-i105</b> | <b>1424-i26<br/>1381-i81</b> |                                   |
| <b><math>\chi^2/\text{d.o.f.}</math></b>                                                                                   | <b>1.12</b>                  | <b>1.15</b>                   | <b>0.96</b>                  |                                   |

**Non-trivial observations (NLO):**

**1. subtraction constants are all natural size,  $a(\mu=1\text{GeV}) \sim 10^{-2}$**

**2. NLO parameters are all small (hierarchy maintains)**

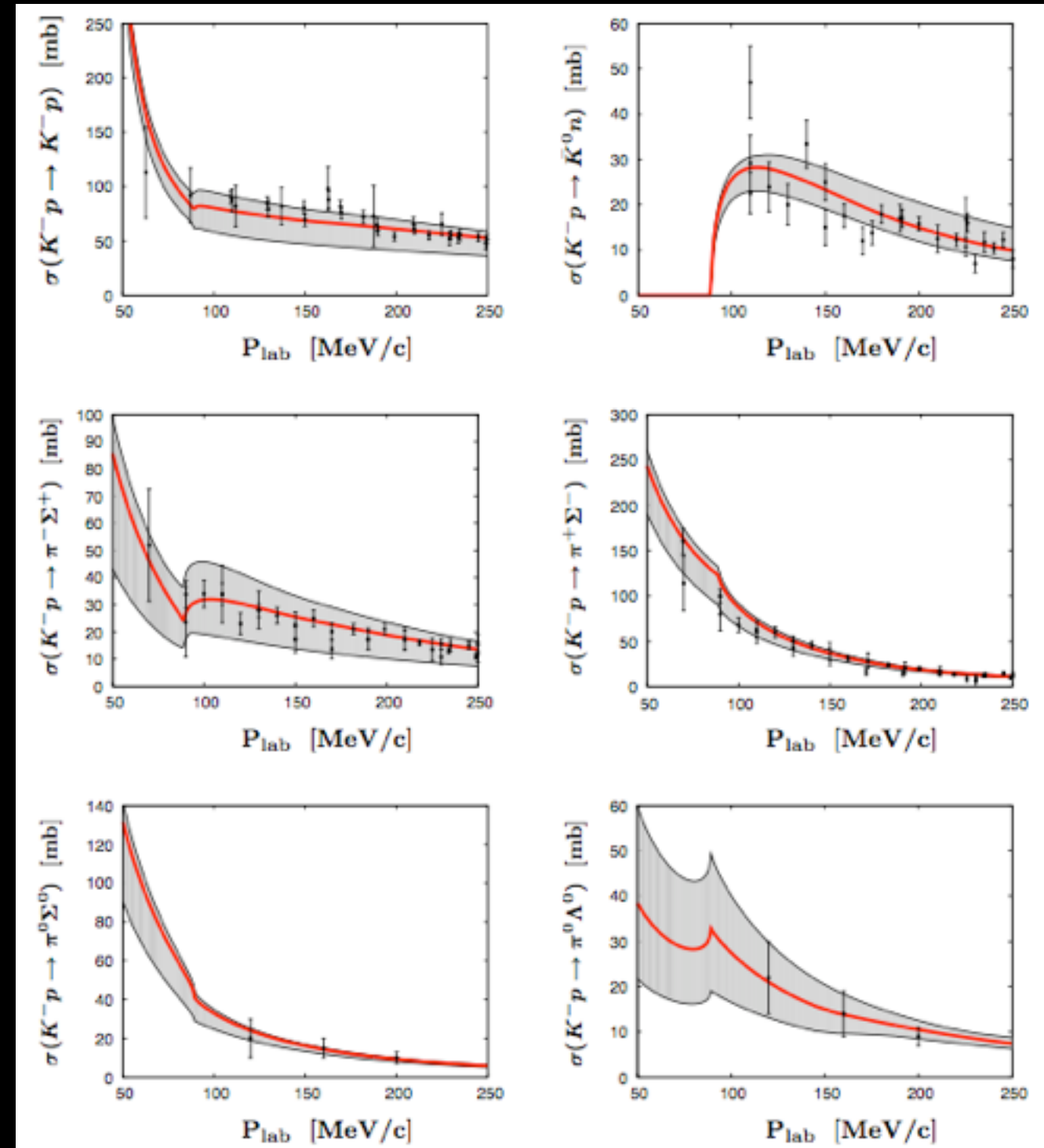
# Uncertainty analysis

## Uncertainty from SIDDHARTA error & $\pi\Lambda$ cross section ( $I=1$ )

**Total cross sections of  $K^-p$  reaction**

$$\sigma_{ij}(\sqrt{s}) = \frac{q_i}{q_j} \frac{|T_{ij}(\sqrt{s})|^2}{16\pi s}$$

**w/ Coulomb interaction for elastic scattering**



**SIDDHARTA data :**

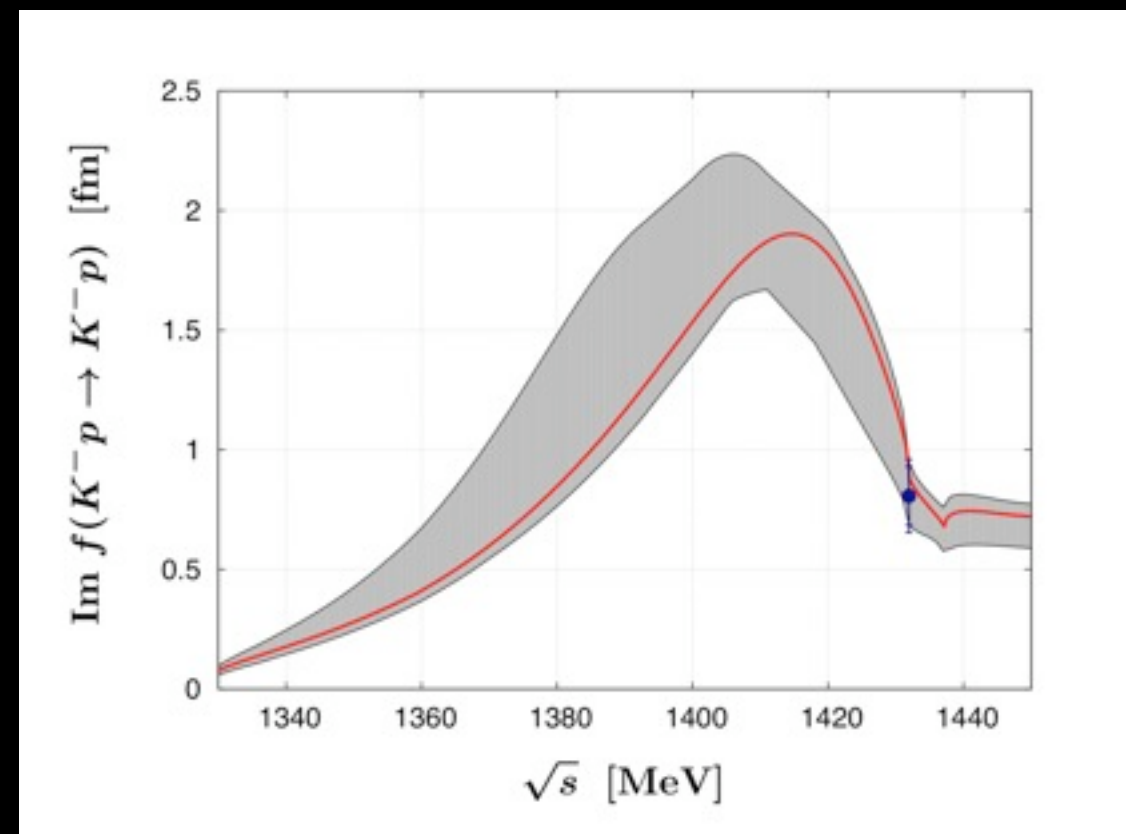
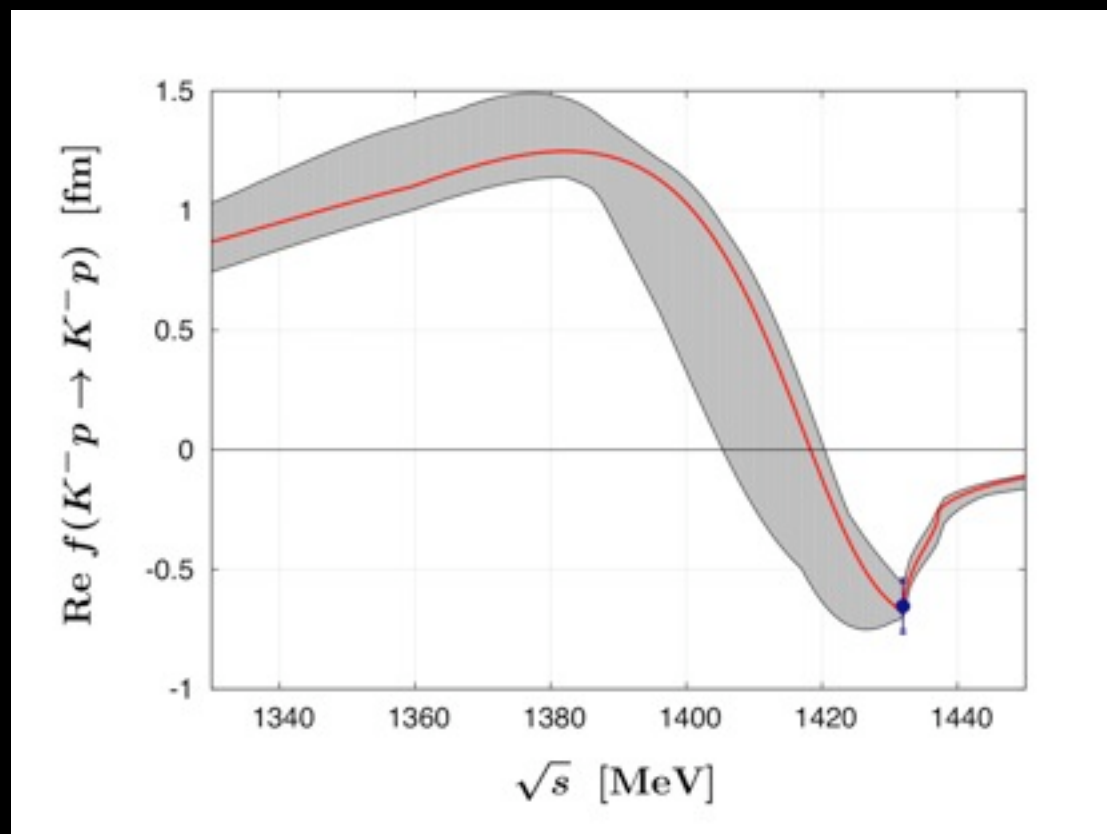
**consistent with all of scattering data**

**impact on  $K^-p \rightarrow \pi^0 \Sigma^0$  ( $I=0$  channel) cross section**

# Prediction I. $K^-p$ forward amplitudes

**$K^-p$  forward amplitudes near threshold and subthreshold extrapolation**

$$f_{ij}(\sqrt{s}) = \frac{1}{8\pi\sqrt{s}} T_{ij}(\sqrt{s})$$



**Uncertainties considerably reduced from previous studies**

Nissler, PhD thesis (2008)

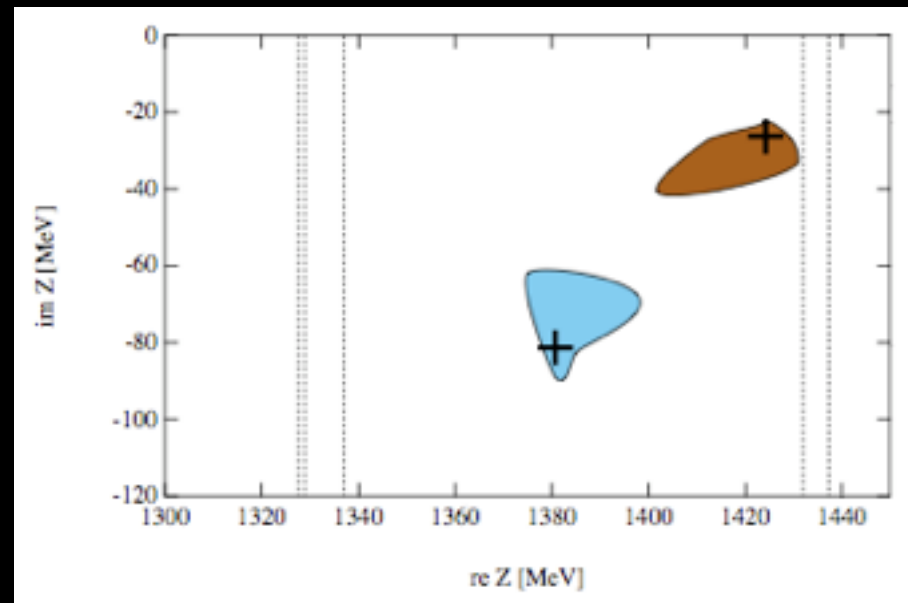
Borasoy, Nissler, Weise, EPJA25(2005).

Brasoy, Meissner, Nissler, PRC74(2006).

**--> basic input on  $K^{\text{bar}}$  bound states (Kaonic nuclei)**

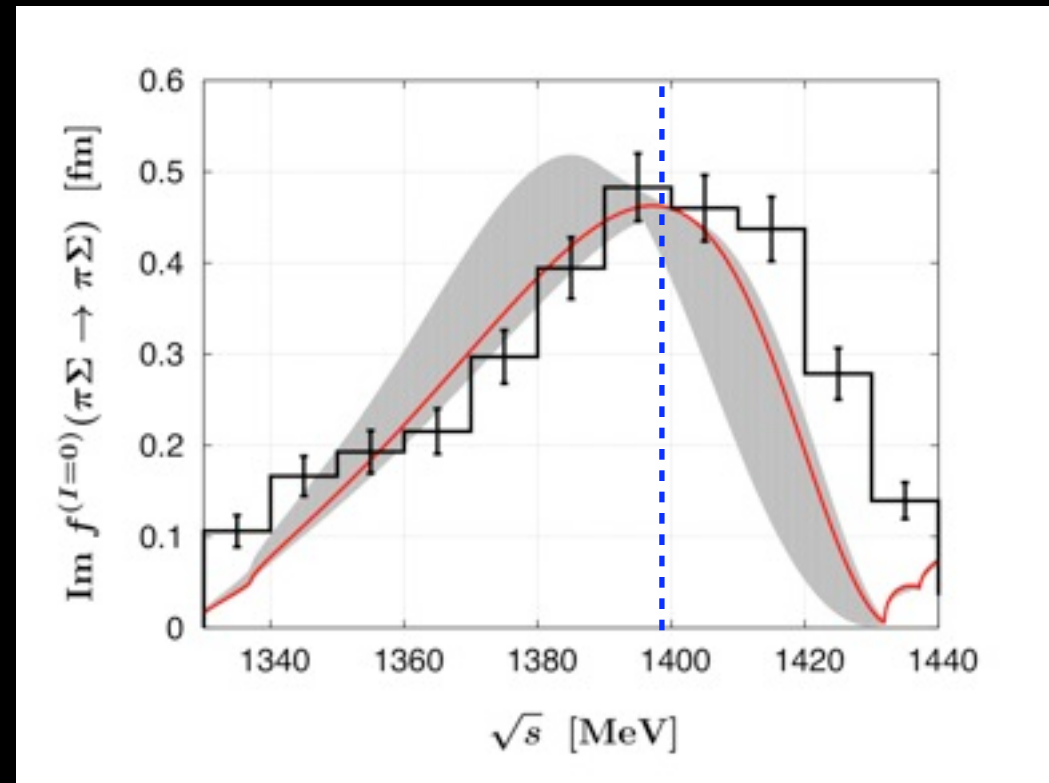
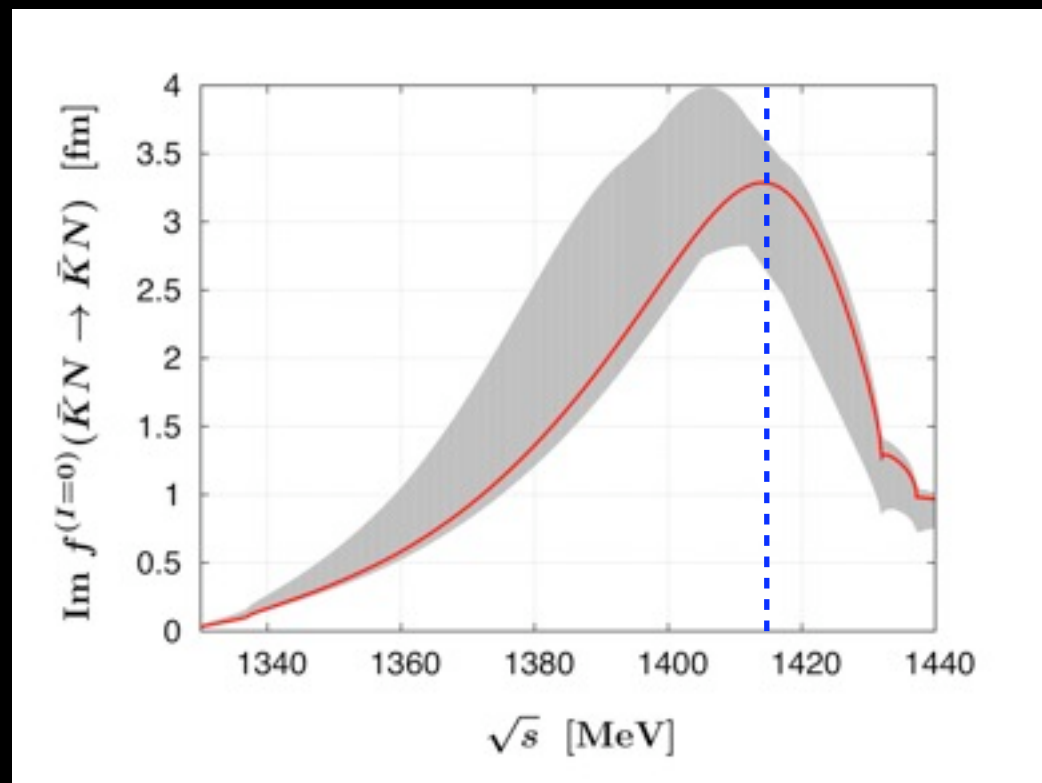
# Prediction II. $\Lambda(1405)$ pole positions

Pole of amplitudes --> resonance energy (mass, width)



Two-pole nature --> channel dependence of mass spectra

Jido et al., NPA723, 205 (2003).



Reaction models to compare with experiments (HADES, CLAS, J-PARC, ...)

# Summary

**Updated analysis of coupled-channels chiral SU(3) dynamics w/ SIDDHARTA result**

**Leading order (Tomozawa-Weinberg terms) : dominant contributions  
NLO : fine tuning for kaonic hydrogen (small NLO parameters)**

**Impact on physics involving  $K^{\text{bar}}$  meson**

**Update amplitudes : uncertainties considerably reduced**

**--> Useful for studies of  $K^{\text{bar}}$  nuclei**

**More improvements :**

**$\pi\Sigma$  scattering? (heavy baryon decay + LQCD input --> Hyodo's talk)**

**$K^-d$  atom experiment? ( $K^-p + K^-n \leftrightarrow I=0, 1$  amplitudes constructed)**

**Our on-going work**

**Construct effective potential models**

**Systematic studies of few-body systems**

**( energy of  $K$ -pp system,  $K$ -d scattering, ...)**