Global Analysis of Fragmentation Functions

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"Hadron-structure physics at J-PARC and related topics"

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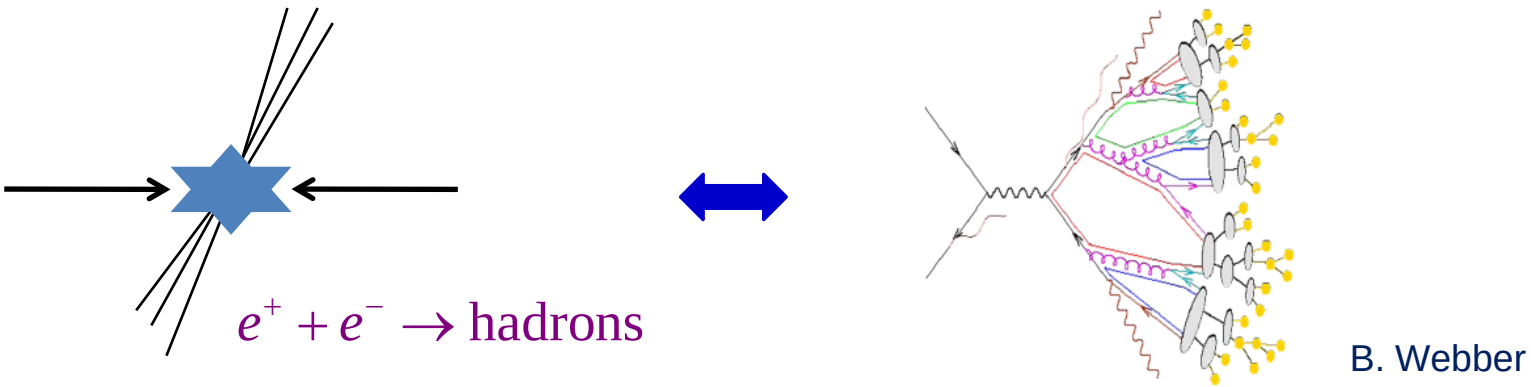
Introduction

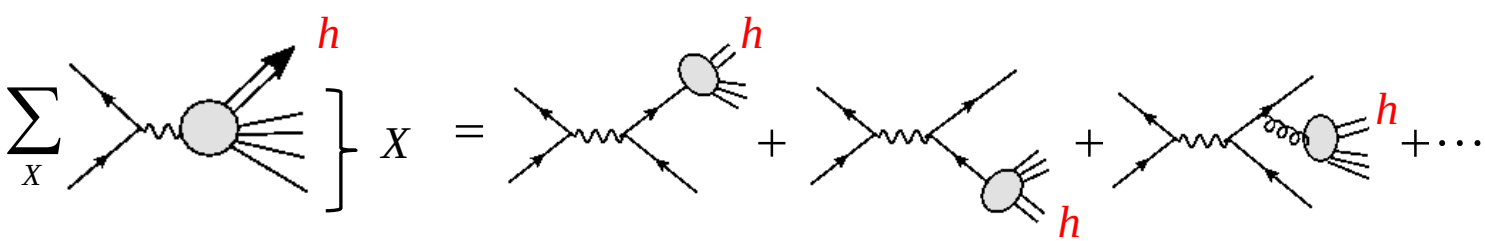
Hadron production in hard processes

- Hadron production in hard scattering processes involves many steps.

parton production, branching, color neutralization, hadronization, etc.



- However, if we consider inclusive single hadron production, the situation is highly simplified .

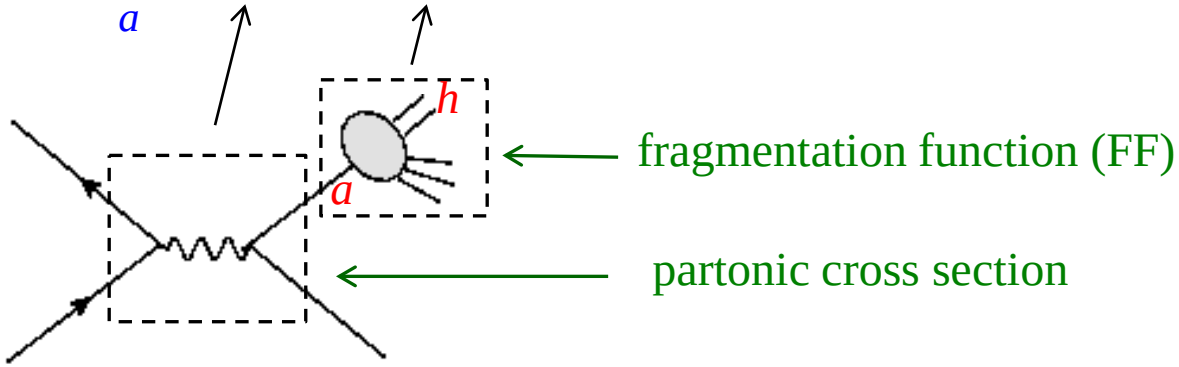


Factorization formula

- Cross Section $(e^+ + e^- \to h + \text{anything})$

$$d\sigma^h = \sum_a d\hat{\sigma}^a \otimes D_a^h(z)$$

$z = P_h / P_a$



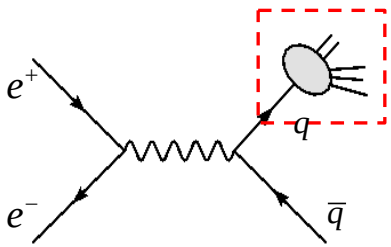
- Fragmentation Functions (FFs)
  - key ingredients for calculating the inclusive production of a hadron.

- FFs are non-perturbative and have to be determined from experiments.

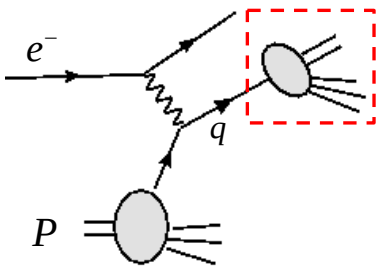
"global analysis"

Universality

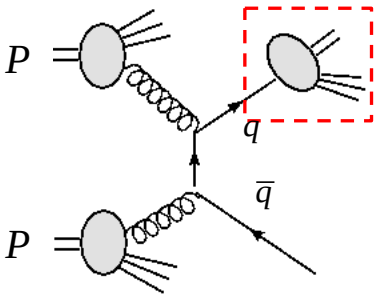
- FFs are universal.



e^+e^- annihilation



semi-inclusive DIS



hadroproduction

- Precise knowledge of FFs is useful & necessary for various phenomenology.

ex.

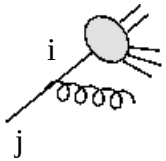
— Determination of parton distribution from semi-inclusive DIS.
— Investigation of dense media through jet quenching in heavy-ion collisions.
— Study of semi-hard QCD dynamics (multiplicity, hadronization, etc.) .

Scale dependence

- DGLAP eq.

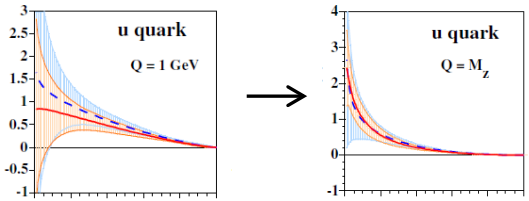
$$\frac{\partial}{\partial \ln Q^2} D_j^h(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{z}{\xi},\alpha_s\right) D_i^h\left(\xi,Q^2\right) \equiv \frac{\alpha_s(Q^2)}{2\pi} \sum_i P_{ij} \otimes D_i(z,Q^2)$$

↔ multiple emission of partons



(timelike) splitting functions

$$P_{ji}\left(z,\alpha_s\right) = \underbrace{\underbrace{P_{ji}^{(0)}(z)}_{LO} + \alpha_s P_{ji}^{(1)}(z)}_{NLO} + \cdots$$



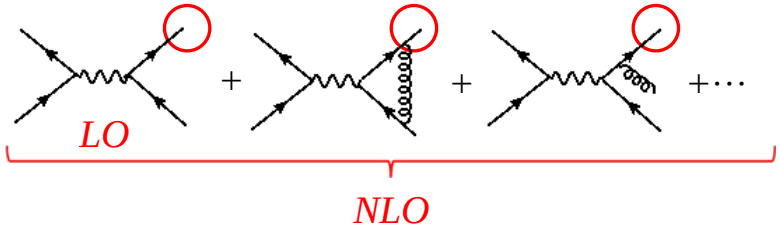
HKNS07

$D_u^{\pi^+}$ at $Q = 1\text{GeV}$ $\quad$ $D_u^{\pi^+}$ at $Q = M_Z$

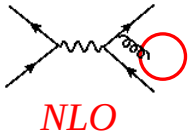
- Multiplicity in e+e- annihilation

$$F^h(z,Q^2) \equiv \frac{1}{\sigma_{tot}} \frac{d\sigma^h}{dz} = \sum_i \int_z^1 \frac{d\xi}{\xi} C_i\left(\frac{z}{\xi},Q^2\right) D_i^h\left(\xi,Q^2\right) \qquad Q = \sqrt{s},\ z = 2E_h / \sqrt{s}$$

$C_q =$



$C_g =$



Corresponding to each accuracy, we define LO F^h, NLO F^h.

Global χ^2 analysis of FFs

- How to determine FFs from the experimental data.

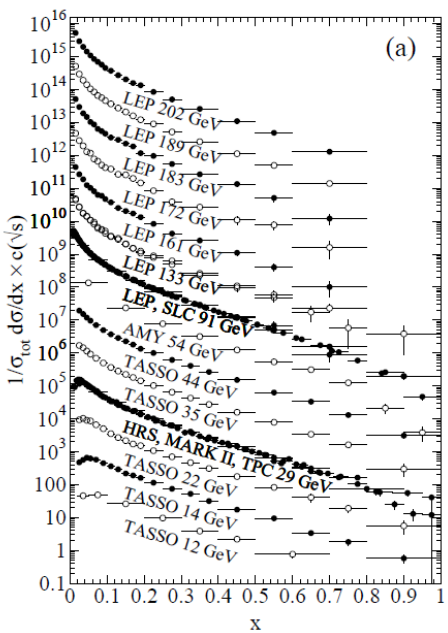
FFs at the initial scale : $D_i^h\left(x,Q_0^2\right) = N_i^h x^{\alpha_i^h}(1-x)^{\beta_i^h}$ $\quad Q_0 \sim 1\text{GeV}$

DGLAP evolution : $D_i^h\left(x,Q_0^2\right) \rightarrow D_i^h\left(x,Q^2\right)$ $\quad Q \sim \sqrt{s}$

Observables

$$\frac{1}{\sigma_{tot}} \cdot \frac{d\sigma}{dz}(e^+e^- \rightarrow hX) = \sum C_i \otimes D_i^h(z,Q^2)$$

$$z = 2E_h / Q$$



exp. data

$$\chi^2 = \frac{(d\sigma^{data} - d\sigma^{theo})^2}{(\delta d\sigma^{data})^2}$$

$$\{N_i^h, \alpha_i^h, \beta_i^h\}$$

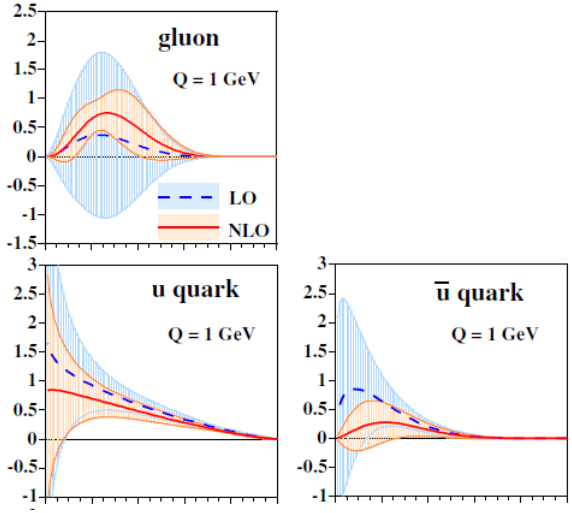
HKNS07 FFs

Hirai, Kumano, Nagai, Sudoh, PRD75(2007) 094009.

- A global analysis for the pion, kaon & proton FFs.
  (with uncertainties for the first time.)

- HKNS07 uses e+e- data only, in which the LEP/SLC data are dominant.
  → the gluon FFs are poorly determined.

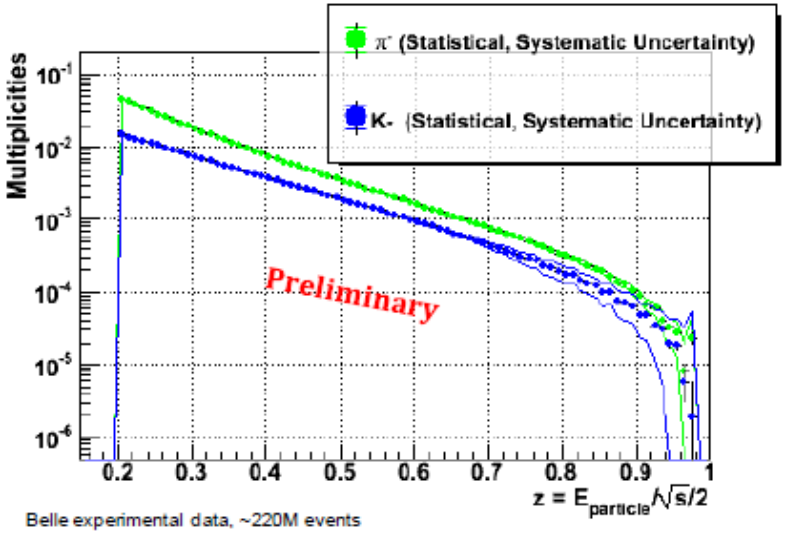
FFs of π^+



♣ DSS07 (de Florian, Sassot, Stratmann) and AKK08 (Albino, Kniehl, Kramer) uses the e+e-, ep, pp(ppbar) and e+e-, pp(ppbar) data, respectively.

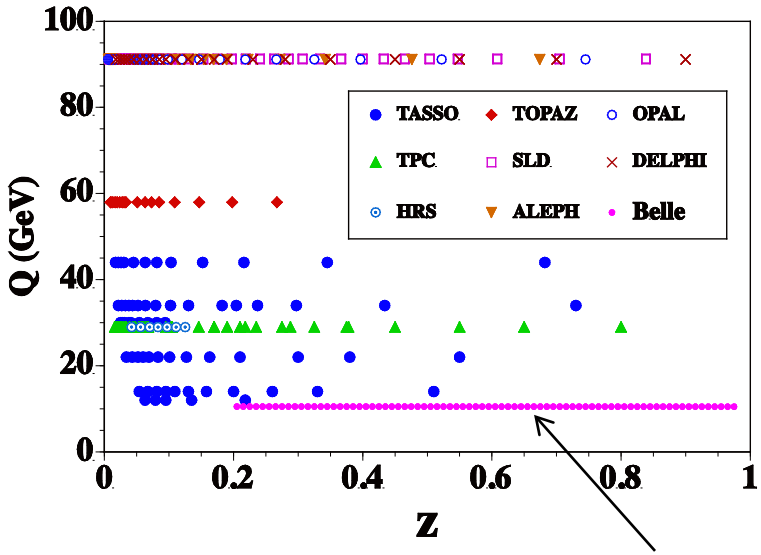
Update from HKNS07

What's new ? — New data from Belle.



Leitgab DIS2012 (preliminary data)

Kinematical coverage (pion)



Belle data

— Large-z, small-Q region is covered with very high precision.
→ Constraints on Gluon FF & q-qbar decomposition.

— "Preprint data" (arXiv:1301.6183) was released recently.
We are currently working on it. Normalization problem?

Parametrization for initial pion FFs

- We employ the same parametrization as in HKNS07.

π^+ 5 independent FFs.

u & $\bar{d}$ favored FFs: $D_u^{\pi^+}(z,Q_0^2) = D_{\bar{d}}^{\pi^+}(z,Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}}$

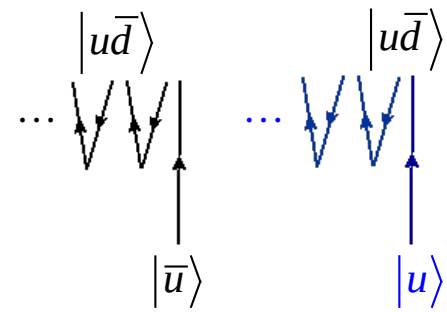
$\bar{u}$ & d & s & $\bar{s}$ disfavored FFs: $D_{\bar{u}}^{\pi^+}(z,Q_0^2) = D_d^{\pi^+}(z,Q_0^2) = D_s^{\pi^+}(z,Q_0^2)$

$$= D_{\bar{s}}^{\pi^+}(z,Q_0^2) = N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1-z)^{\beta_{\bar{u}}^{\pi^+}}$$

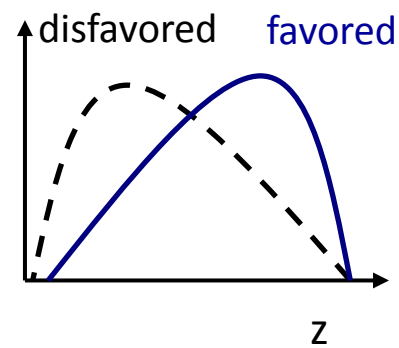
gluon FF: $D_g^{\pi^+}(z,Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1-z)^{\beta_g^{\pi^+}}$

charm FF: $D_c^{\pi^+}(z,m_c^2) = D_{\bar{c}}^{\pi^+}(z,m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1-z)^{\beta_c^{\pi^+}}$

bottom FF: $D_b^{\pi^+}(z,m_b^2) = D_{\bar{b}}^{\pi^+}(z,m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1-z)^{\beta_b^{\pi^+}}$



Feynman & Field picture



Initial scale(s):

$$n_f = \begin{cases} 3, & Q_0^2 < Q^2 < m_c^2 \\ 4, & m_c^2 < Q^2 < m_b^2 \\ 5, & m_b^2 < Q^2 < m_t^2 \\ 6, & m_t^2 < Q^2 \end{cases}, \quad \begin{pmatrix} Q_0^2 = 1\ \mathbf{GeV}^2 \\ m_c = 1.43\ \mathbf{GeV} \\ m_b = 4.3\ \mathbf{GeV} \end{pmatrix}$$

Parameterization for initial kaon FFs

K^+ 6 independent FFs.

u favored FFs: $D_u^{K^+}(z,Q_0^2) = N_u^{K^+} z^{\alpha_u^{K^+}} (1-z)^{\beta_u^{K^+}}$ $m_s \neq m_u$

$\bar{s}$ favored FFs: $D_{\bar{s}}^{K^+}(z,Q_0^2) = N_{\bar{s}}^{K^+} z^{\alpha_{\bar{s}}^{K^+}} (1-z)^{\beta_{\bar{s}}^{K^+}}$

$\bar{u}$ & d & $\bar{d}$ & s disfavored FFs: $D_{\bar{u}}^{K^+}(z,Q_0^2) = D_d^{K^+}(z,Q_0^2) = D_{\bar{d}}^{K^+}(z,Q_0^2) = D_s^{K^+}(z,Q_0^2) = N_{\bar{u}}^{K^+} z^{\alpha_{\bar{u}}^{K^+}} (1-z)^{\beta_{\bar{u}}^{K^+}}$

gluon FF: $D_g^{K^+}(z,Q_0^2) = N_g^{K^+} z^{\alpha_g^{K^+}} (1-z)^{\beta_g^{K^+}}$

charm FF: $D_c^{K^+}(z,m_c^2) = D_{\bar{c}}^{K^+}(z,m_c^2) = N_c^{K^+} z^{\alpha_c^{K^+}} (1-z)^{\beta_c^{K^+}}$

bottom FF: $D_b^{K^+}(z,m_b^2) = D_{\bar{b}}^{K^+}(z,m_b^2) = N_b^{K^+} z^{\alpha_b^{K^+}} (1-z)^{\beta_c^{K^+}}$

- Momentum sum rule

$$\sum_h M_i^h \equiv \sum_h \int_0^1 z D_i^h(z,Q_0^2)dz = \sum_h N_i^h B(\alpha_i^h + 2, \beta_i^h + 1) \Rightarrow \boxed{0 < M_i^h < 1}$$

Actually, we take M_i^h instead of N_i^h as the fitting parameters.

Impact of the Belle data

LO

$$D^h_{q^+} \equiv D^h_q + D^h_{\bar{q}}$$

- At the Z-pole (LEP/SLC)

$$F^h(z, M_Z^2) \approx (c_V^{u\,2} + c_A^{u\,2})\left[D^h_{u^+}(z, M_Z^2) \right.$$
$$\left. + (c_V^{d\,2} + c_A^{d\,2})\left[D^h_{d^+}(z, M_Z^2) + D^h_{s^+}(z, M_Z^2) \right] \right] \approx 0.33 \sum_q D^h_{q^+}(z, M_Z^2)$$

flavor singlet component

- Far from the Z-pole (Belle, TASSO/TPC/HRC/TOPAZ)

$$F^h(z, Q^2) \approx \frac{4}{9}\left[D^h_{u^+}(z, Q^2) \right] + \frac{1}{9}\left[D^h_{d^+}(z, Q^2) + D^h_{s^+}(z, Q^2) \right]$$

  - c-quark, b-quark FFs are well determined from the flavor tagged data.
  - Suppose we have very precise data at and far from the Z-pole, we can determine 2 independent components of the quark FFs.
  - Remaining flavor decomposition & determination of the gluon FF come from the mixing through scale evolution.

pion FFs: $D^{\pi^+}_{fav.}(z, Q_0^2)$ & $D^{\pi^+}_{dis.}(z, Q_0^2)$ + $D^{\pi^+}_g(z, Q_0^2)$

kaon FFs : $D^{K^+}_{fav.}(z, Q_0^2)$ & $D^{K^+\prime}_{fav.}(z, Q_0^2)$ & $D^{K^+}_{dis.}(z, Q_0^2)$ + $D^{K^+}_g(z, Q_0^2)$

DGLAP evolution

- NS & S evolution eqs.

$$a = \alpha_s / (2\pi), \quad t = \ln Q^2$$

$$q^{\pm} = q \pm \bar{q}, \quad \Sigma = \sum_q q^+$$

(1) $$\frac{\partial}{\partial t} q^- = a \cdot P_{NS}^- \otimes q^-$$

Non-singlet

$$\frac{\partial}{\partial t}(u^+ - d^+) = a \cdot P_{NS}^+ \otimes (u^+ - d^+), \text{ etc.}$$

(2) $$\begin{cases} \frac{\partial}{\partial t}\Sigma = a \cdot P_{qq} \otimes \Sigma + a \cdot n_f P_{gq} \otimes g \ , \\ \frac{\partial}{\partial t} g = a \cdot P_{qg} \otimes \Sigma + a \cdot P_{gg} \otimes g \end{cases}$$

Singlet

$$\begin{cases} P_{q_i q_j} = \delta_{ij} P_{qq}^V + P_{qq}^S, \\ P_{q_i \bar{q}_j} = \delta_{ij} P_{q\bar{q}}^V + P_{q\bar{q}}^S \end{cases} \longrightarrow \quad \begin{array}{l} P_{NS}^{\pm} = P_{qq}^V \pm P_{q\bar{q}}^V, \quad P_{PS} = P_{qq}^S + P_{q\bar{q}}^S, \\ P_{qq} = P_{NS}^+ + P_{PS}^S \end{array}$$

Going back to multiplicity,,,

- The multiplicity at a higher scale $Q^2 \ll M_Z^2$

$$N^{\pi^+}(z,Q^2) = C_u(z/y)\otimes\left[u^+(y,Q^2)+\frac{1}{4}d^+(y,Q^2)+\frac{1}{4}s^+(y,Q^2)\right]+C_g(z/y)\otimes g(y,Q^2)$$

$$= C_u(z/y)\otimes\left[\left(\frac{1}{2}u^+(y,Q^2)-\frac{1}{4}d^+(y,Q^2)-\frac{1}{4}s^+(y,Q^2)\right)+\frac{1}{2}\quad{}^2\right]+C_g\ z\quad y)\otimes g(y,Q^2)$$

$$= C_u(z/y)\otimes\left[U_{NS}^+\left(y/x;Q^2,Q_0^2\right)\otimes\left(\frac{1}{2}u^+(x,Q_0^2)-\frac{1}{4}d^+(y,Q_0^2)-\frac{1}{4}s^+(y,Q_0^2)\right)\right.$$

$$\left.+\ \frac{1}{2}\left(U_{\Sigma\Sigma}\left(y/x;Q^2,Q_0^2\right)\otimes\Sigma(x,Q_0^2)+U_{g\Sigma}\left(y/x;Q^2,Q_0^2\right)\otimes g(x,Q_0^2)\right)\right]$$

$$+C_g(z/y)\otimes\left[U_{\Sigma g}\left(y/x;Q^2,Q_0^2\right)\otimes\Sigma(x,Q_0^2)+U_{gg}\left(y/x;Q^2,Q_0^2\right)\otimes g(x,Q_0^2)\right]$$

Evolution operators

$$U_{NS}^+\left(y/x;Q^2,Q_0^2\right),\ U_{\Sigma\Sigma}\left(y/x;Q^2,Q_0^2\right),\ U_{gg}\left(y/x;Q^2,Q_0^2\right)\ \sim\ 1+*\ \alpha_s\log\left(Q^2/Q_0^2\right)+\cdots$$

$$U_{\Sigma g}\left(y/x;Q^2,Q_0^2\right),\quad U_{g\Sigma}\left(y/x;Q^2,Q_0^2\right)\ \sim\quad *\ \alpha_s\log\left(Q^2/Q_0^2\right)+\cdots$$

$$N^{\pi^+}(z,Q^2) = C_u(z/y) \otimes \Big[U^+_{NS}\left(y/x;Q^2,Q_0^2\right) \otimes \left(\frac{1}{4} D^{fav.}(y,Q_0^2) - \frac{1}{4} D^{unfav.}(y,Q_0^2) \right)$$

$$+ U_{\Sigma\Sigma}\left(y/x;Q^2,Q_0^2\right) \otimes \left(D^{fav.}(x,Q_0^2) + 2D^{unfav.}(y,Q_0^2) \right) + U_{\Sigma g}\left(y/x;Q^2,Q_0^2\right) \otimes \frac{1}{2} g(x,Q_0^2) \Big]$$

$$+ C_g(z/y) \otimes \Big[U_{g\Sigma}\left(y/x;Q^2,Q_0^2\right) \otimes \left(2D^{fav.}(x,Q_0^2) + 4D^{unfav.}(y,Q_0^2) \right) + U_{gg}\left(y/x;Q^2,Q_0^2\right) \otimes g(x,Q_0^2) \Big]$$

—— dominant components

$$U^+_{NS}\left(y/x;Q^2,Q_0^2\right) \simeq U_{\Sigma\Sigma}\left(y/x;Q^2,Q_0^2\right)$$

⇨ Scale evolution can be used mainly to constrain gluon FF.

Results with the Belle's preliminary data

(1) Pion Fragmentation Functions

Results for π^+ fit

- Total number of data = 342 (Belle 78)

$\Lambda_{QCD}^{n_f=4} = \mathbf{0.220}$ (LO), $\mathbf{0.323}$ (NLO)

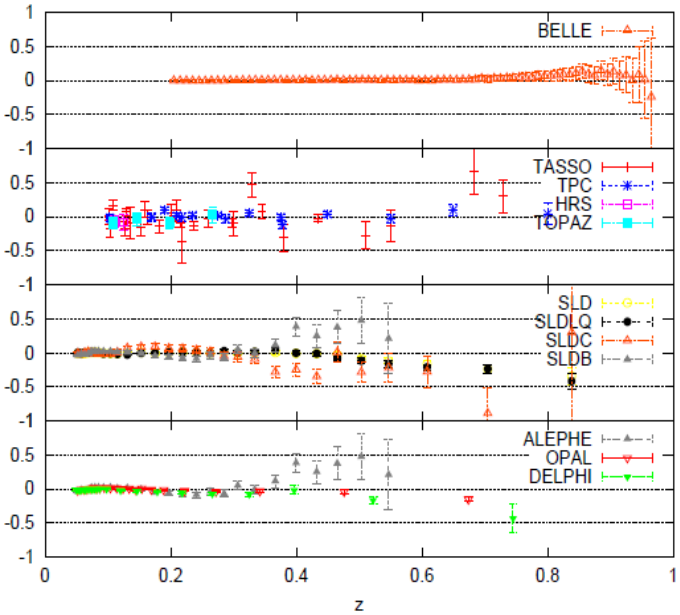
— To meet the momentum sum rule, we fixed a parameter for the gluon FF $\beta_g^{\pi^+} = 8$.

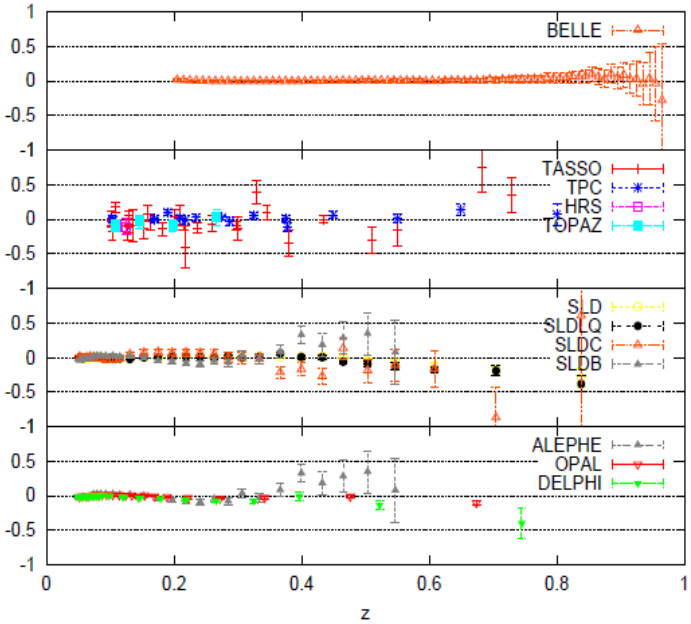
— Good convergences for both of the LO & NLO FFs.

$[D(z,Q_0^2) = Nz^{\alpha}(1-z)^{\beta}]$

LO $\chi^2 = 612.9, \quad \chi^2/(d.o.f) = 1.87$

NLO $\chi^2 = 498.2, \quad \chi^2/(d.o.f) = 1.52$





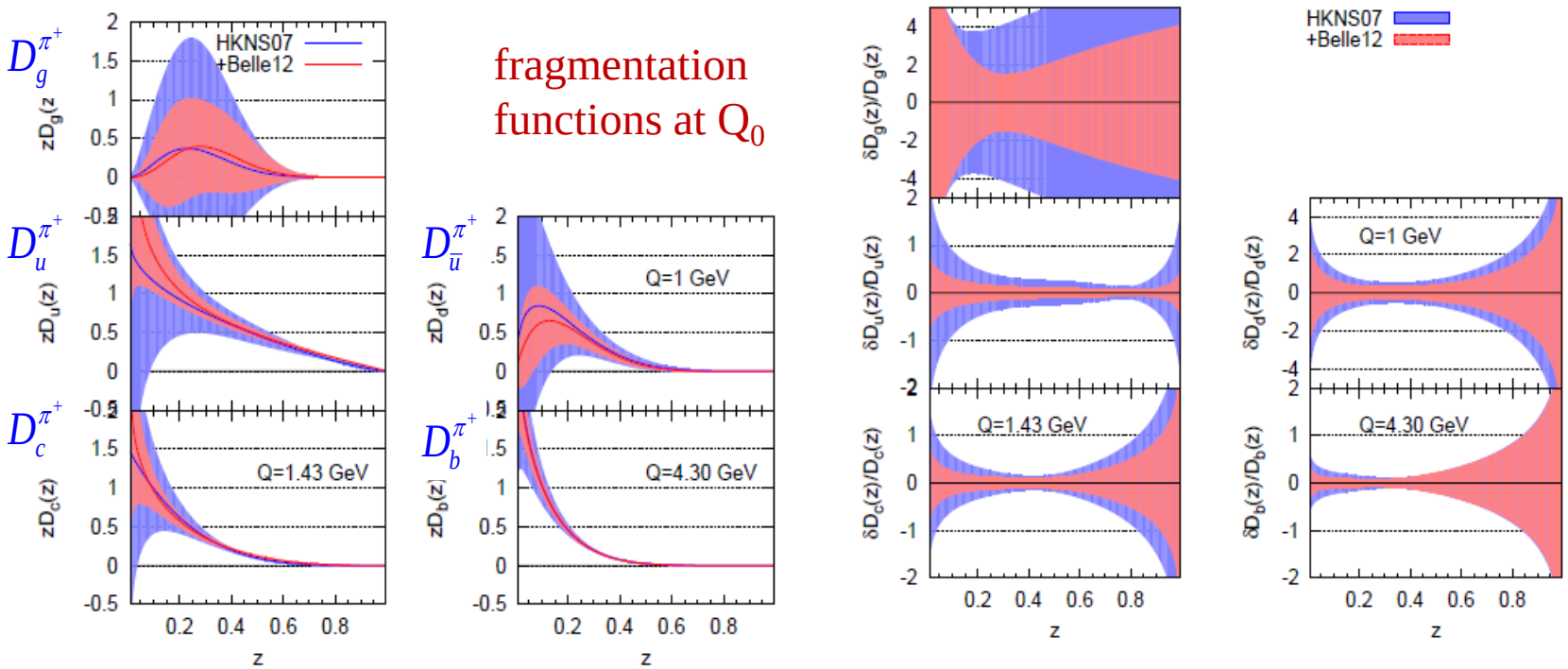
(Data-Theory)/Theory

Results for π^+ FFs (LO)

- LO pion FFs with uncertainties

relative uncertainties $\delta D_i^{\pi^+} / D_i^{\pi^+}$

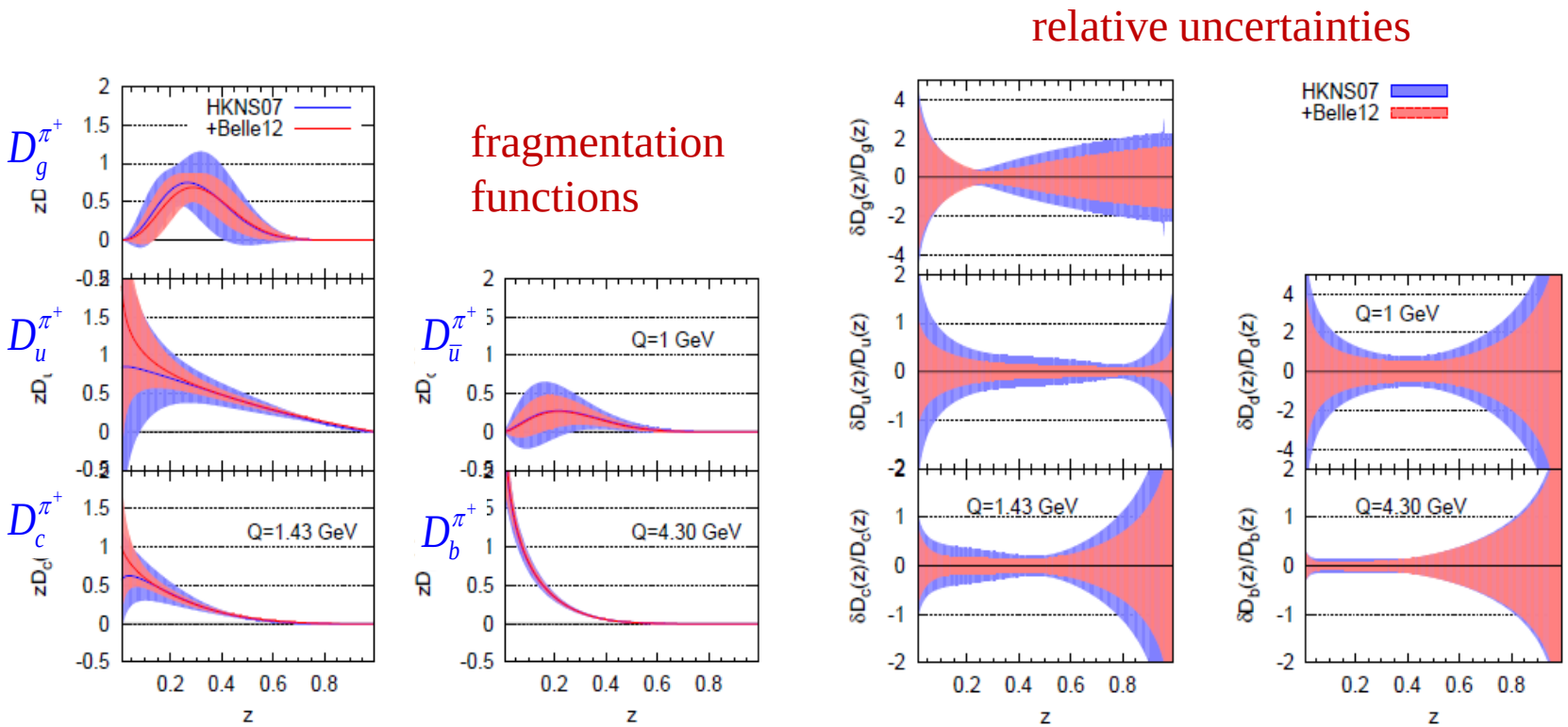
fragmentation functions at Q_0



- Uncertainty of the gluon FF and light-quark FFs are all reduced.

Results for π^+ FFs (NLO)

- NLO pion FFs with uncertainties



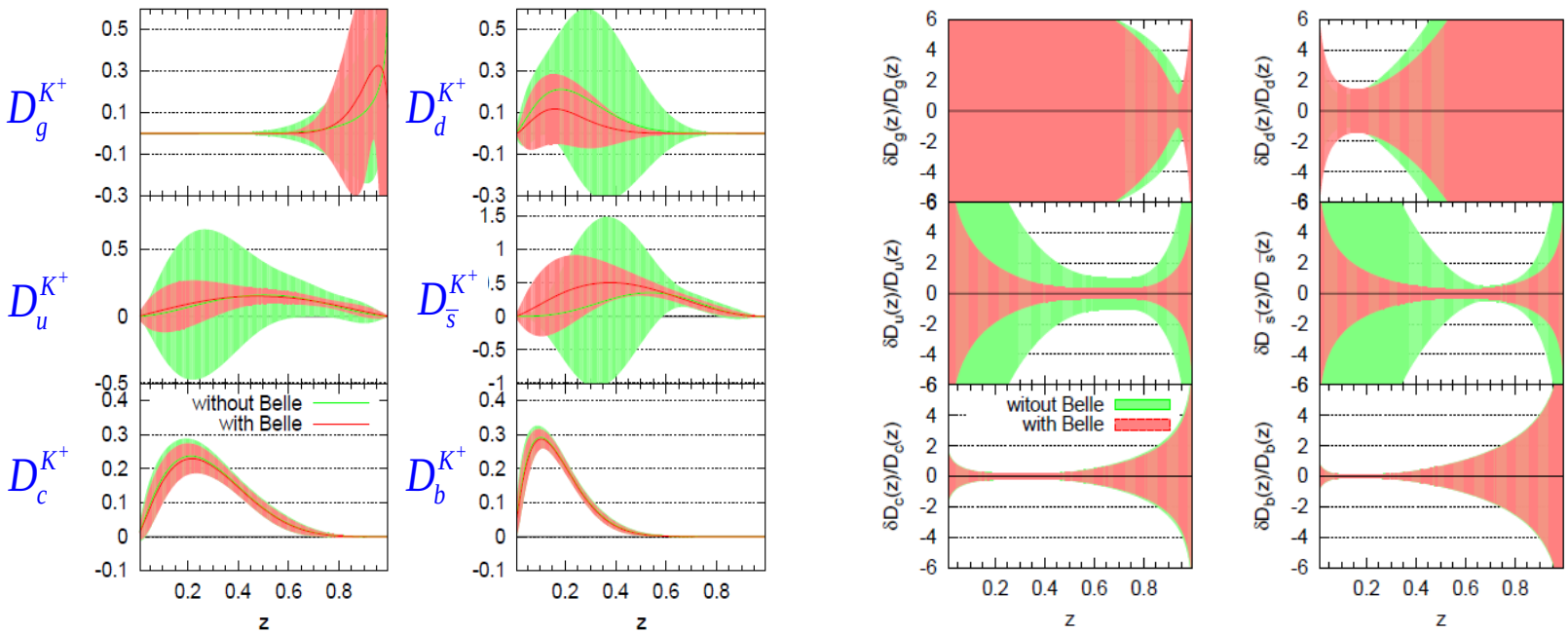
- Uncertainties of the gluon & light quark FFs are reduced.

Results with the Belle's preliminary data

(2) Kaon Fragmentation Functions

Results of K^+ FFs (LO)

- Total number of data = 322 (Belle 77) $\chi^2/(d.o.f) = 1.19$ $[D(z,Q_0^2) = Nz^{\alpha}(1-z)^{\beta}]$
  - As in the pion case, we take a fixed value for the gluon parameter: $\alpha_g^{\pi^+} = 10$ to ensure $\beta_g^{\pi^+} > 0$.

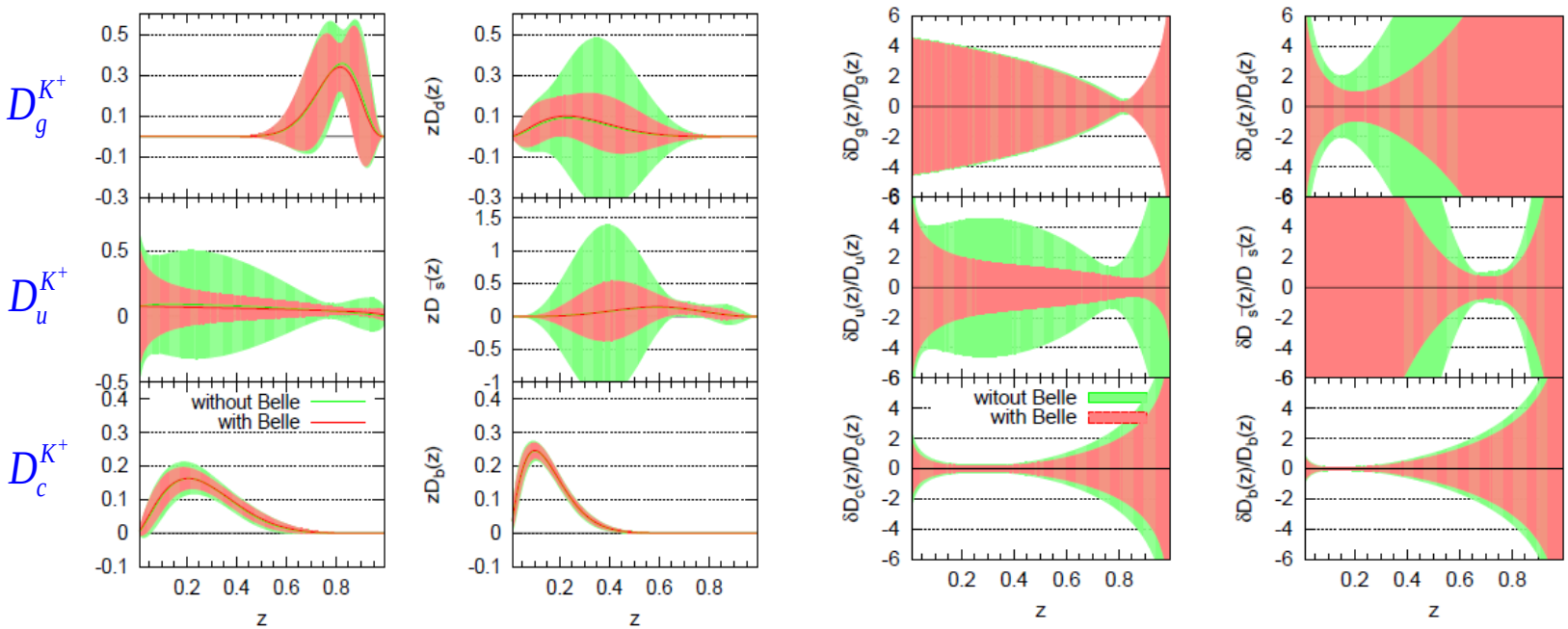


- Even after including the Belle data, the gluon FF is poorly determined, while the uncertainties of the light-quark FFs are all reduced.

Results of K+ FFs (NLO)

- Total number of data = 322 (Belle 77) $\chi^2/(d.o.f) = 1.02$ $[D(z,Q_0^2) = Nz^{\alpha}(1-z)^{\beta}]$

— A fixed value for the gluon parameter: $\alpha_g^{\pi^+} = 15$ to ensure mom.sum rule & $\beta_g^{\pi^+} > 0$.



- Similar result as the LO case.

The Belle data is effective for the flavor decomposition

Summary

- The new data from Belle covers a wide range of z (0.8>z>0.2) at Q = 10.58GeV.

  → Highly precise data at a scale far from the Z-pole (LEP/SLC).

- We studied the impact of the new Belle (preliminary) data on the determination of FFs.

π^+

- Uncertainty of the gluon FF & light quark FFs are reduced significantly at LO (~ 50%) and moderately at NLO (~ 30%).
- Some of the quark FFs change values within the uncertainties.

K^+

- Almost no impact on the gluon FF , while the uncertainties of the light-quark FFs are significantly reduced (LO & NLO)
- Some of the quark FFs change within the previous uncertainties.

- An update of HKNS07 with the final Belle data is coming.