Global Analysis of Fragmentation Functions

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Introduction
Hadron production in hard processes

- Hadron production in hard scattering processes involves many steps.
  
  parton production, branching, color neutralization, hadronization, etc.

\[ e^+ + e^- \rightarrow \text{hadrons} \]

- However, if we consider **inclusive** single hadron production, the situation is highly simplified.

\[ \sum_x X = \quad + \quad + \quad + \cdots \]
• Cross Section  \((e^+ + e^- \rightarrow h + \text{anything})\)

\[
d\sigma^h = \sum_a d\hat{\sigma}^a \otimes D_a^h(z)
\]

\[z = \frac{P_h}{P_a}\]

• Fragmentation Functions (FFs)
  — key ingredients for calculating the inclusive production of a hadron.

• FFs are non-perturbative and have to be determined from experiments.
  “global analysis”
• FFs are universal.

- $e^+e^-$ annihilation
- Semi-inclusive DIS
- Hadroproduction

• Precise knowledge of FFs is useful & necessary for various phenomenology.
  ex.
  — Determination of parton distribution from semi-inclusive DIS.
  — Investigation of dense media through jet quenching in heavy-ion collisions.
  — Study of semi-hard QCD dynamics (multiplicity, hadronization, etc.).
Scale dependence

- DGLAP eq.

\[
\frac{\partial}{\partial \ln Q^2} D_j^h(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ij}(\frac{z}{\xi}, \alpha_s) D_i^h(\xi, Q^2) \equiv \frac{\alpha_s(Q^2)}{2\pi} \sum_i P_{ij} \otimes D_i(z, Q^2)
\]

\(\text{(timelike) splitting functions}\)

\(P_{ji}(z, \alpha_s) = P_{ji}^{(0)}(z) + \alpha_s P_{ji}^{(1)}(z) + \cdots\)

\(\text{LO}\)

\(\text{NLO}\)

- Multiplicity in e+e- annihilation

\[
F^h(z, Q^2) \equiv \frac{1}{\sigma_{tot}} \frac{d\sigma^h}{dz} = \sum_i \int_z^1 \frac{d\xi}{\xi} C_i(\frac{z}{\xi}, Q^2) D_i^h(\xi, Q^2) \quad Q = \sqrt{s}, \ z = 2E_h / \sqrt{s}
\]

\(C_q = \text{LO} \quad + \quad \text{NLO} \quad + \quad \text{LO} \quad + \quad \text{NLO} \quad + \cdots\)

\(C_g = \text{LO} \quad + \quad \text{NLO} \quad + \quad \text{LO} \quad + \quad \text{NLO} \quad + \cdots\)

Corresponding to each accuracy, we define \(\text{LO} \ F^h, \text{NLO} \ F^h\).
Global $\chi^2$ analysis of FFs

- How to determine FFs from the experimental data.

FFs at the initial scale:

$$D_i^h(x, Q_0^2) = N_i^h x^{\alpha_i^h} (1 - x)^{\beta_i^h}$$

$Q_0 \sim 1$GeV

DGLAP evolution:

$$D_i^h(x, Q^2) \rightarrow D_i^h(x, Q^2)$$

$$Q \sim \sqrt{s}$$

Observables

$$\frac{1}{\sigma_{tot}} \cdot \frac{d\sigma}{dz} (e^+ e^- \rightarrow hX) = \sum C_i \otimes D_i^h(z, Q^2)$$

$$z = 2E_h / Q$$

$$\chi^2 = \frac{(d\sigma^{data} - d\sigma^{theo})^2}{(\delta d\sigma^{data})^2}$$

$\{N_i^h, \alpha_i^h, \beta_i^h\}$

exp. data
HKNS07 FFs

A global analysis for the pion, kaon & proton FFs.

(with uncertainties for the first time.)

HKNS07 uses e+e- data only, in which the LEP/SLC data are dominant.

→ the gluon FFs are poorly determined.

DSS07 (de Florian, Sassot, Stratmann) and AKK08 (Albino, Kniehl, Kramer)
uses the e+e-, ep, pp(ppbar) and e+e-, pp(ppbar) data, respectively.
What’s new? — New data from Belle.

Large-z, small-Q region is covered with very high precision.

→ Constraints on Gluon FF & q-qbar decomposition.

“Preprint data” (arXiv:1301.6183) was released recently. We are currently working on it. Normalization problem?
We employ the same parametrization as in HKNS07.

- **Favored FFs:**
  \[
  D_{u}^{\pi^{+}}(z,Q_0^2) = D_{d}^{\pi^{+}}(z,Q_0^2) = N_u^{\pi^{+}}z^{\alpha_u^{\pi^{+}}}(1-z)^{\beta_u^{\pi^{+}}}
  \]

- **Disfavored FFs:**
  \[
  D_{u}^{\pi^{+}}(z,Q_0^2) = D_{d}^{\pi^{+}}(z,Q_0^2) = D_{s}^{\pi^{+}}(z,Q_0^2) = D_{\bar{s}}^{\pi^{+}}(z,Q_0^2)
  = N_{\bar{u}}^{\pi^{+}}z^{\alpha_{\bar{u}}^{\pi^{+}}}(1-z)^{\beta_{\bar{u}}^{\pi^{+}}}
  \]

- **Gluon FF:**
  \[
  D_{g}^{\pi^{+}}(z,Q_0^2) = N_{g}^{\pi^{+}}z^{\alpha_{g}^{\pi^{+}}}(1-z)^{\beta_{g}^{\pi^{+}}}
  \]

- **Charm FF:**
  \[
  D_{c}^{\pi^{+}}(z,m_c^2) = D_{\bar{c}}^{\pi^{+}}(z,m_c^2) = N_{c}^{\pi^{+}}z^{\alpha_{c}^{\pi^{+}}}(1-z)^{\beta_{c}^{\pi^{+}}}
  \]

- **Bottom FF:**
  \[
  D_{b}^{\pi^{+}}(z,m_b^2) = D_{\bar{b}}^{\pi^{+}}(z,m_b^2) = N_{b}^{\pi^{+}}z^{\alpha_{b}^{\pi^{+}}}(1-z)^{\beta_{b}^{\pi^{+}}}
  \]

**Initial scale(s):**

- \(3, Q_0^2 < Q^2 < m_c^2\)
- \(4, m_c^2 < Q^2 < m_b^2\)
- \(5, m_b^2 < Q^2 < m_t^2,\)
- \(6, m_t^2 < Q^2\)

\(Q_0^2 = 1 \text{ GeV}^2, \quad m_c = 1.43 \text{ GeV}, \quad m_b = 4.3 \text{ GeV} \)
Parameterization for initial kaon FFs

K^+

6 independent FFs.

\begin{align*}
\text{favored FFs: } & \quad D_u^K(z, Q_0^2) = N_u^{K^+} z^{\alpha_u^{K^+}} (1 - z)^{\beta_u^{K^+}} & m_s \neq m_u \\
\text{disfavored FFs: } & \quad D_s^K(z, Q_0^2) = N_s^{K^+} z^{\alpha_s^{K^+}} (1 - z)^{\beta_s^{K^+}} \\
\text{disfavored FFs: } & \quad D_{u\bar{u}}^K(z, Q_0^2) = D_{d\bar{d}}^K(z, Q_0^2) = D_{c\bar{c}}^K(z, Q_0^2) = D_{s\bar{s}}^K(z, Q_0^2) = N_{u\bar{u}}^{K^+} z^{\alpha_{u\bar{u}}^{K^+}} (1 - z)^{\beta_{u\bar{u}}^{K^+}} \\
\text{gluon FF: } & \quad D_g^K(z, Q_0^2) = N_g^{K^+} z^{\alpha_g^{K^+}} (1 - z)^{\beta_g^{K^+}} \\
\text{charm FF: } & \quad D_c^K(z, m_c^2) = D_{\bar{c}}^K(z, m_c^2) = N_c^{K^+} z^{\alpha_c^{K^+}} (1 - z)^{\beta_c^{K^+}} \\
\text{bottom FF: } & \quad D_b^K(z, m_b^2) = D_{\bar{b}}^K(z, m_b^2) = N_b^{K^+} z^{\alpha_b^{K^+}} (1 - z)^{\beta_b^{K^+}} \\

\end{align*}

- Momentum sum rule

\[
\sum_h M_i^h \equiv \sum_h \int_0^1 z D_i^h(z, Q_0^2) dz = \sum_h N_i^h B(\alpha_i^h + 2, \beta_i^h + 1) \quad \Rightarrow \quad 0 < M_i^h < 1
\]

Actually, we take \( M_i^h \) instead of \( N_i^h \) as the fitting parameters.
Impact of the Belle data

• At the Z-pole (LEP/SLC)

\[ F^h(z, M^2_Z) \approx (c^u_v + c^u_A) \left[ D^h_{u^+}(z, M^2_Z) \right] + (c^d_v + c^d_A) \left[ D^h_{d^+}(z, M^2_Z) + D^h_{s^+}(z, M^2_Z) \right] \approx 0.33 \sum_q D^h_{q^+}(z, M^2_Z) \]

flavor singlet component

• Far from the Z-pole (Belle, TASSO/TPC/HRC/TOPAZ)

\[ F^h(z, Q^2) \approx \frac{4}{9} \left[ D^h_{u^+}(z, Q^2) \right] + \frac{1}{9} \left[ D^h_{d^+}(z, Q^2) + D^h_{s^+}(z, Q^2) \right] \]

— c-quark, b-quark FFs are well determined from the flavor tagged data.
— Suppose we have very precise data at and far from the Z-pole, we can determine 2 independent components of the quark FFs.
— Remaining flavor decomposition & determination of the gluon FF come from the mixing through scale evolution.

pion FFs: \[ D^\pi^+_{\text{fav.}} (z, Q^2_0) & D^\pi^+_{\text{dis.}} (z, Q^2_0) + D^\pi^+_g (z, Q^2_0) \]

kaon FFs: \[ D^{K^+}_{\text{fav.}} (z, Q^2_0) & D^{K^+}_{\text{fav.}} (z, Q^2_0) & D^{K^+}_{\text{dis.}} (z, Q^2_0) + D^{K^+}_g (z, Q^2_0) \]
DGLAP evolution

• NS & S evolution eqs.

\[ a = \alpha_s / (2\pi), \quad t = \ln Q^2 \]

\[ q^\pm = q \pm \bar{q}, \quad \Sigma = \sum_q q^+ \]

Non-singlet

\[ \frac{\partial}{\partial t} q^- = a \cdot P_{NS}^- \otimes q^- \]

\[ \frac{\partial}{\partial t} (u^+ - d^+) = a \cdot P_{NS}^+ \otimes (u^+ - d^+), \text{ etc.} \]

Singlet

\[ \frac{\partial}{\partial t} \Sigma = a \cdot P_{qq} \otimes \Sigma + a \cdot n_f P_{qg} \otimes g, \]

\[ \frac{\partial}{\partial t} g = a \cdot P_{qq} \otimes \Sigma + a \cdot P_{gg} \otimes g \]

\[ \begin{cases} P_{q_i q_j} = \delta_{ij} P_{qq}^V + P_{qq}^S, \\ P_{q_i \bar{q}_j} = \delta_{ij} P_{q\bar{q}}^V + P_{q\bar{q}}^S \end{cases} \quad \rightarrow \quad P_{NS}^\pm = P_{qq}^V \pm P_{q\bar{q}}^V, \quad P_{PS} = P_{qq}^S + P_{q\bar{q}}^S, \quad P_{qq} = P_{NS}^+ + P_{PS}^S \]
Going back to multiplicity,,,

- The multiplicity at a higher scale $Q^2 \ll M_Z^2$

\[
N_{\pi^+}(z,Q^2) = C_u(z/y) \otimes \left[u^+(y,Q^2) + \frac{1}{4}d^+(y,Q^2) + \frac{1}{4}s^+(y,Q^2)\right] + C_g(z/y) \otimes g(y,Q^2)
\]

\[
= C_u(z/y) \otimes \left[\left(\frac{1}{2}Q^+(y,Q^2) - \frac{1}{4}d^+(y,Q^2) - \frac{1}{4}s^+(y,Q^2)\right) + \frac{1}{2} \right] + C_g(z/y) \otimes g(y,Q^2)
\]

\[
= C_u(z/y) \otimes \left[U_{NS}^+(y/x;Q^2,Q_0^2) \otimes \left(u^+(x,Q_0^2) - \frac{1}{4}d^+(y,Q_0^2) - \frac{1}{4}s^+(y,Q_0^2)\right)\right]
\]

\[
\quad + \frac{1}{2} \left(U_{\Sigma\Sigma}(y/x;Q^2,Q_0^2) \otimes \Sigma(x,Q_0^2) + U_{g\Sigma}(y/x;Q^2,Q_0^2) \otimes g(x,Q_0^2)\right)\]

\[
\quad + C_g(z/y) \otimes \left[U_{\Sigma g}(y/x;Q^2,Q_0^2) \otimes \Sigma(x,Q_0^2) + U_{gg}(y/x;Q^2,Q_0^2) \otimes g(x,Q_0^2)\right]
\]

**Evolution operators**

\[
U_{NS}^+(y/x;Q^2,Q_0^2), \quad U_{\Sigma\Sigma}(y/x;Q^2,Q_0^2), \quad U_{gg}(y/x;Q^2,Q_0^2) \sim 1 + * \alpha, \log(Q^2/Q_0^2) + * \alpha_s \log(Q^2/Q_0^2) + \ldots
\]

\[
U_{\Sigma g}(y/x;Q^2,Q_0^2), \quad U_{g\Sigma}(y/x;Q^2,Q_0^2) \sim * \alpha_s \log(Q^2/Q_0^2) + \ldots
\]
\[ N^{\pi^+}(z, Q^2) = C_u(z/y) \otimes \left[ U_{NS}^+(y/x; Q^2, Q_0^2) \otimes \left( \frac{1}{4} D_{\text{fav.}}(y, Q_0^2) - \frac{1}{4} D_{\text{unfav.}}(y, Q_0^2) \right) + U_{\Sigma} \left( y/x; Q^2, Q_0^2 \right) \otimes \left( D_{\text{fav.}}(x, Q_0^2) + 2D_{\text{unfav.}}(y, Q_0^2) \right) + U_{\Sigma g} \left( y/x; Q^2, Q_0^2 \right) \otimes \frac{1}{2} g(x, Q_0^2) \right] \\
+ C_g(z/y) \otimes \left[ U_{\Sigma S g} \left( y/x; Q^2, Q_0^2 \right) \otimes \left( 2D_{\text{fav.}}(x, Q_0^2) + 4D_{\text{unfav.}}(y, Q_0^2) \right) + U_{gg} \left( y/x; Q^2, Q_0^2 \right) \otimes g(x, Q_0^2) \right] \]

--- dominant components

\[ U_{NS}^+(y/x; Q^2, Q_0^2) \simeq U_{\Sigma} \left( y/x; Q^2, Q_0^2 \right) \]

\[ \Rightarrow \quad \text{Scale evolution can be used mainly to constrain gluon FF.} \]
Results with the Belle’s preliminary data

(1) Pion Fragmentation Functions
Results for $\pi^+$ fit

- Total number of data = 342 (Belle 78)
  
  - To meet the momentum sum rule, we fixed a parameter for the gluon FF $\beta_g^{\pi^+} = 8$.
  - Good convergences for both of the LO & NLO FFs.

\[
\Lambda_{QCD}^{nf=4} = 0.220 \text{ (LO), } 0.323 \text{ (NLO)}
\]

\[
\chi^2 = 612.9, \quad \chi^2 / (d.o.f) = 1.87
\]

\[
\chi^2 = 498.2, \quad \chi^2 / (d.o.f) = 1.52
\]
Results for $\pi^+$ FFs (LO)

- LO pion FFs with uncertainties

- Uncertainty of the gluon FF and light-quark FFs are all reduced.

fragmentation functions at $Q_0$

relative uncertainties $\Delta D_i^{\pi^+} / D_i^{\pi^+}$
Results for $\pi^+$ FFs (NLO)

- NLO pion FFs with uncertainties

Uncertainties of the gluon & light quark FFs are reduced.
Results with the Belle’s preliminary data

(2) Kaon Fragmentation Functions
As in the pion case, we take a fixed value for the gluon parameter: $\alpha_g^{\pi^+} = 10$ to ensure $\beta_g^{\pi^+} > 0$.

Even after including the Belle data, the gluon FF is poorly determined, while the uncertainties of the light-quark FFs are all reduced.
Results of $K^+$ FFs (NLO)

- Total number of data = 322 (Belle 77) \( \chi^2 / (d.o.f.) = 1.02 \)
  \[ D(z,Q_0^2) = Nz^\alpha (1-z)^\beta \]
- A fixed value for the gluon parameter: \( \alpha_g^{\pi^+} = 15 \) to ensure mom.sum rule & \( \beta_g^{\pi^+} > 0 \).

- Similar result as the LO case.

The Belle data is effective for the flavor decomposition.
Summary

• The new data from Belle covers a wide range of $z$ (0.8 > $z$ > 0.2) at $Q = 10.58$ GeV.
  → Highly precise data at a scale far from the Z-pole (LEP/SLC).

• We studied the impact of the new Belle (preliminary) data on the determination of FFs.

- **$\pi^+$** • Uncertainty of the gluon FF & light quark FFs are reduced significantly at LO ($\approx 50\%$) and moderately at NLO ($\approx 30\%$).
  • Some of the quark FFs change values within the uncertainties.

- **$K^+$** • Almost no impact on the gluon FF, while the uncertainties of the light-quark FFs are significantly reduced (LO & NLO)
  • Some of the quark FFs change within the previous uncertainties.

• An update of HKNS07 with the final Belle data is coming.