The ϕ meson at finite density, revisited

Workshop on J-PARC hadron physics in 2014

@ Tokai, Japan

10.2.2014

Philipp Gubler (RIKEN, Nishina Center)

Collaborator: Keisuke Ohtani (Tokyo Tech)

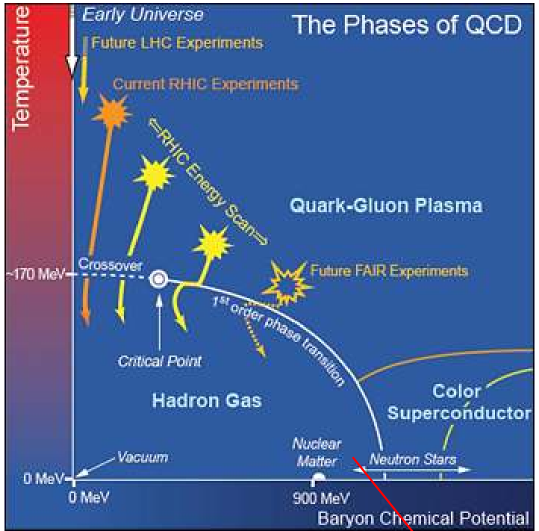
Contents

- Motivation
- Update on QCD sum rules of light vector mesons at finite density
- Results
- Discussion
- Conclusion

Introduction: Vector mesons at finite density

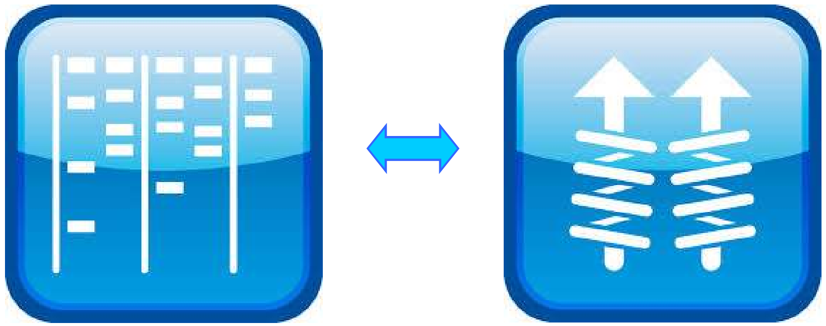
Basic Motivation:

Understanding the behavior of matter under extreme conditions



$$\boxed{\rho,\ \phi,\ \omega,\ D\ \ldots}$$

Understanding the origin of mass and its relation to chiral symmetry of QCD



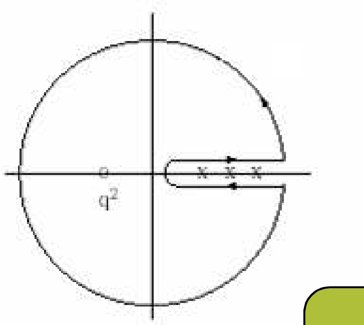
- Vector mesons: clean probe for experiment
- To be investigated at J-PARC
- Firm theoretical understanding is necessary for interpreting the experimental results!

QCD sum rules

M.A. Shifman, A.I. Vainshtein and V.I. Zakharov,
Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

In this method the properties of the two point correlation function is fully exploited:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T\{\chi(x)\overline{\chi}(0)\} | 0 \rangle$$



$$\rightarrow \Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

is calculated “perturbatively”, using OPE

spectral function of the operator χ

After the Borel transformation:

$$G_{OPE}(M) = \frac{1}{\pi} \int_0^\infty ds \frac{1}{M^2} e^{-\frac{s}{M^2}} \mathrm{Im}\Pi(s)$$

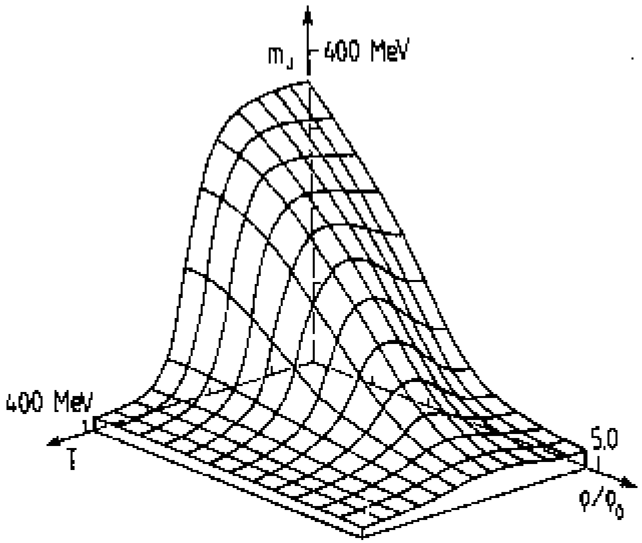
More on the OPE in matter

perturbative Wilson coefficients

non-perturbative condensates

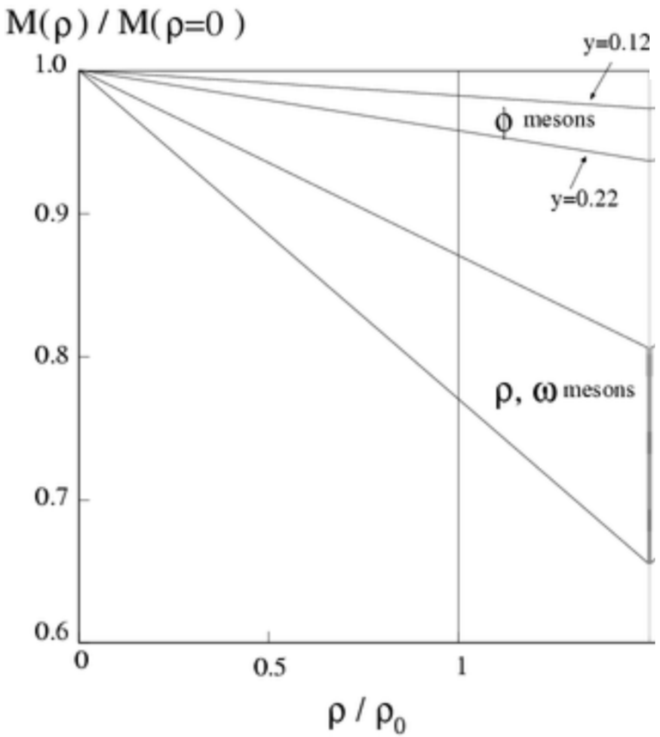
$$i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle = C_I(q^2)I+\sum_n C_n(q^2)\langle 0|O_n|0\rangle$$

$$\underbrace{\begin{aligned}\langle 0|O_n|0\rangle = &\langle 0|\overline{q}q|0\rangle, \\ &\langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle, \\ &\langle 0|\overline{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q|0\rangle, \\ &\langle 0|\overline{q}q\overline{q}q|0\rangle, \ldots\end{aligned}}_{\text{Change in hot or dense matter!}}$$



Important early study

T. Hatsuda and S.H. Lee, Phys. Rev. C **46**, R34 (1992).



Vector meson masses mainly drop due to changes of the quark condensates.

The most important condensates are:

$\langle \bar{q}q\bar{q}q \rangle_\rho$ for $\rho,\ \omega$

$m_s \langle \bar{s}s \rangle_\rho$ for ϕ

Important assumption:

$$\langle \bar{q}q\bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_\rho^2,\ \langle \bar{s}s\bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_\rho^2$$

Might be wrong!

Structure of QCD sum rules for the light vector mesons

$$\frac{1}{M^2}\int_0^\infty ds e^{-\frac{s}{M^2}}\rho(s) = c_0 + \frac{c_1}{M^2} + \frac{c_2}{M^4} + \frac{c_3}{M^6} + \dots$$

<u>Vacuum</u>

$\boxed{\rho,\ \omega}$

$$c_0 = 1 + \frac{\alpha_s}{\pi}$$

$$c_1 = -6m_q^2 \sim 0$$

$$c_2 = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle + 8\pi^2 m_q\langle\overline{q}q\rangle$$

$$c_3 = -\frac{896}{81}\kappa\pi^3\alpha_s\langle\overline{q}q\rangle^2$$

$\boxed{\phi}$

$$c_0 = 1 + \frac{\alpha_s}{\pi}$$

$$c_1 = -6m_s^2$$

$$c_2 = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle + 8\pi^2 m_s\langle\overline{s}s\rangle$$

$$c_3 = -\frac{896}{81}\kappa\pi^3\alpha_s\langle\overline{s}s\rangle^2$$

Finite density effects

Modification of condensates

$$\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_0 + \langle N|\bar{q}q|N\rangle \rho = \langle \bar{q}q \rangle_0 + \frac{\sigma_{\pi N}}{2m_q}\rho$$

$$\langle \bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_0 + \langle N|\bar{s}s|N\rangle \rho = \langle \bar{s}s \rangle_0 + \frac{\sigma_{sN}}{m_s}\rho$$

$$= \langle \bar{s}s \rangle_0 + y\frac{\sigma_{\pi N}}{2m_q}$$

$$\frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{q}q|N\rangle}$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_\rho = \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 - \frac{8}{9}(M_N - \sigma_{\pi N} - \sigma_{sN})\rho$$

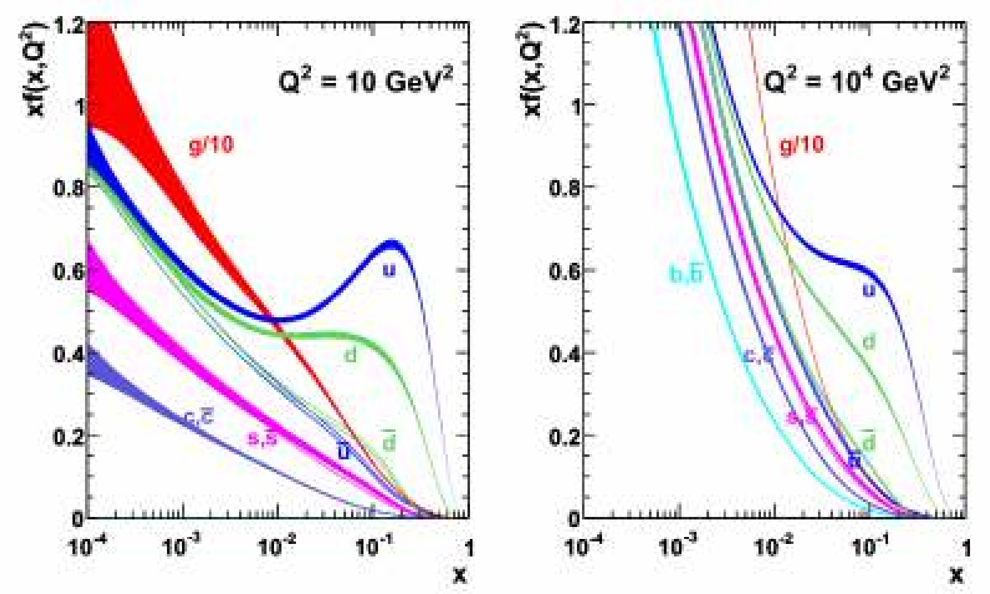
Higher twist terms

$$c_2(\rho) = c_2(0) + 4\pi^2 A_1^{q,s} M_N \rho - \frac{23}{9}\pi\alpha_s A_2^g M_N \rho$$

$$c_3(\rho) = c_3(0) - \frac{20}{3}\pi^2 A_3^{q,s} M_N^3 \rho$$

What has changed since 1992?

$$A_1^q = 2\int_0^1 dx x\left[q(x) + \bar{q}(x)\right]$$
$$A_3^q = 2\int_0^1 dx x^3\left[q(x) + \bar{q}(x)\right]$$
$$A_1^s = 2\int_0^1 dx x\left[s(x) + \bar{s}(x)\right]$$
$$A_3^s = 2\int_0^1 dx x^3\left[s(x) + \bar{s}(x)\right]$$
$$A_2^g = \int_0^1 dx x g(x)$$



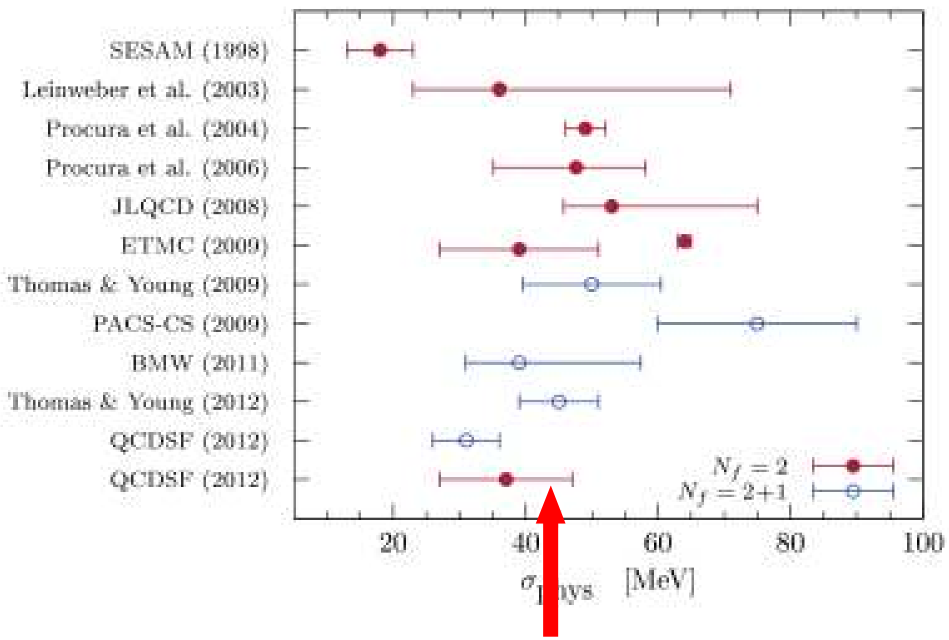
A.D. Martin, W.J. Stirling, R.S. Thorne and G. Watt, Eur. Phys. J. C **63**, 189 (2009).

1992		2013	
$A_1^q = 0.45$		$A_1^q = 0.61$	Some changes, but no big effect.
$A_3^q = 0.06$		$A_3^q = 0.065$	
$A_1^s = 0.05$		$A_1^s = 0.044$	
$A_3^s = 0.002$		$A_3^s = 0.0011$	
A_2^g: ignored		$A_2^g = 0.359$	non-negligible

What has changed since 1992?

The nuclear sigma term: $\sigma_{\pi N}$

Taken from G.S. Bali et al., Nucl. Phys. B866, 1 (2013).



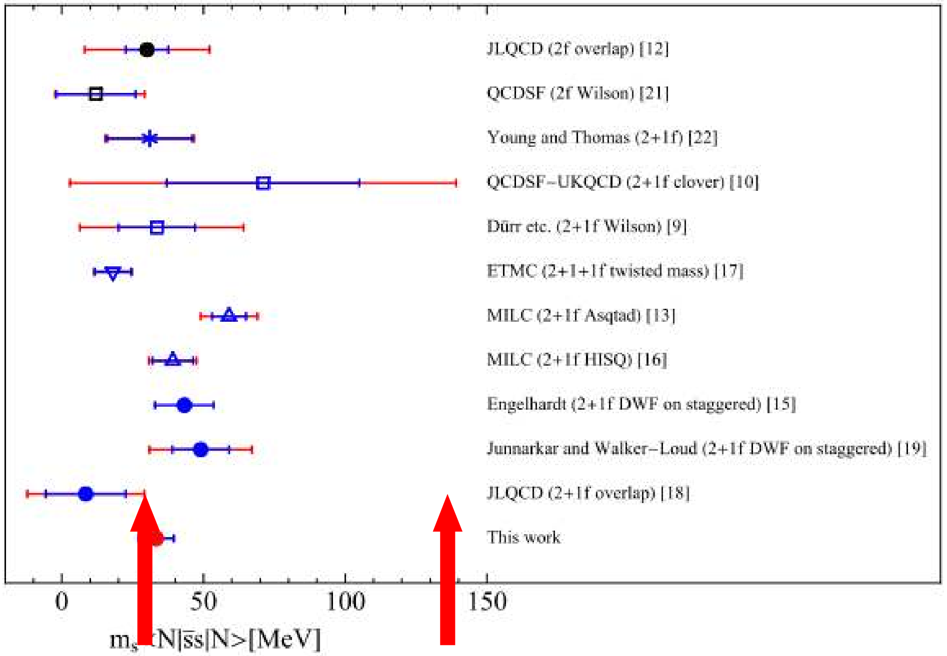
Value used by Hatsuda and Lee: 45 MeV

Recent lattice results are mostly consistent with the old values.
The latest trend might point to a somewhat smaller value.

What has changed since 1992?

The strangeness content of the nucleon: $$y = \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{q}q|N\rangle}$$

Taken from M. Gong et al. (χQCD Collaboration), arXiv:1304.1194 [hep-ph].



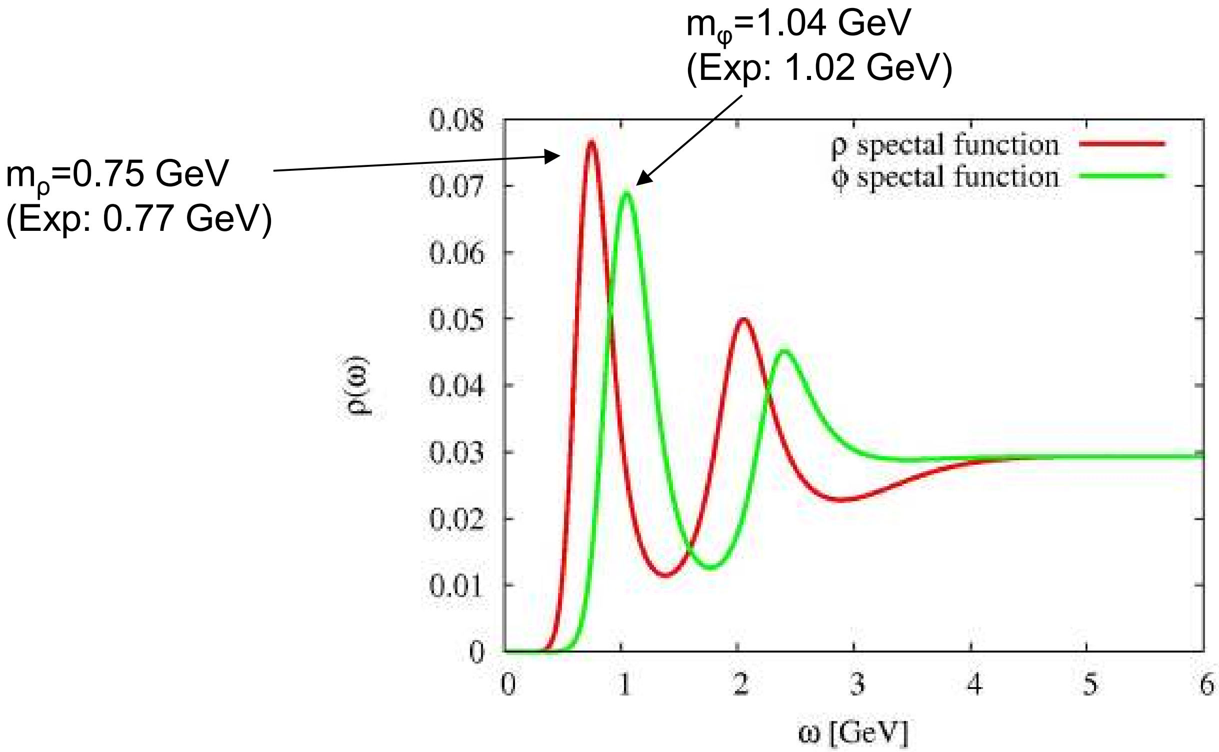
y ~ 0.04

Value used by Hatsuda and Lee: y=0.22
Too big!!

The value of y has shrinked by a factor of about 5: a new analysis is necessary!

Results for the vacuum case

Analysis of the sum rule is done using the maximum entropy method (MEM), which allows to extract the spectral function from the sum rules without any phenomenological ansatz.



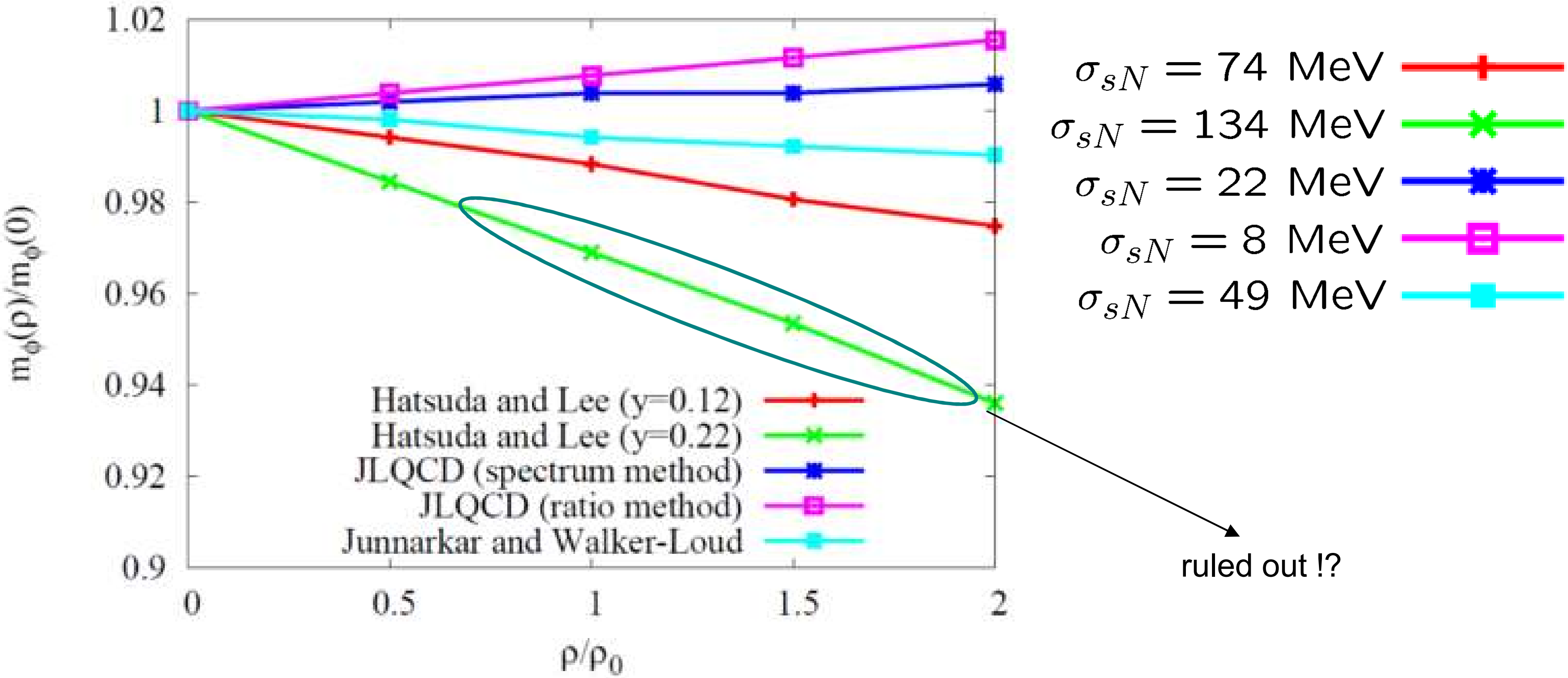
Used input parameters:

$$m_q = 4.7^{+0.9}_{-0.3}[\text{MeV}] \qquad \langle \overline{q}q \rangle_0 = -(0.232 \pm 0.06)^3[\text{GeV}^3]$$

$$m_s = 128 \pm 7[\text{MeV}] \qquad \langle \overline{s}s \rangle_0 = (0.6 \pm 0.1)\langle \overline{q}q \rangle_0$$

$$\alpha_s = 0.5 \qquad \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 = 0.012 \pm 0.0036[\text{GeV}^4]$$

φ meson at finite density



The φ meson mass shift strongly depends on the strange sigma term.

What happened?

Let us examine the OPE at finite density more closely:

$$c_2(\rho) =$$

$$c_2(0) + \rho\Big[-\frac{2}{27}M_N^0 + 2m_s\langle N|\bar{s}s|N\rangle + A_1^s M_N \; -\frac{23}{36}\frac{\alpha_s}{\pi}A_2^g M_N\Big]$$

-65 MeV

16 MeV

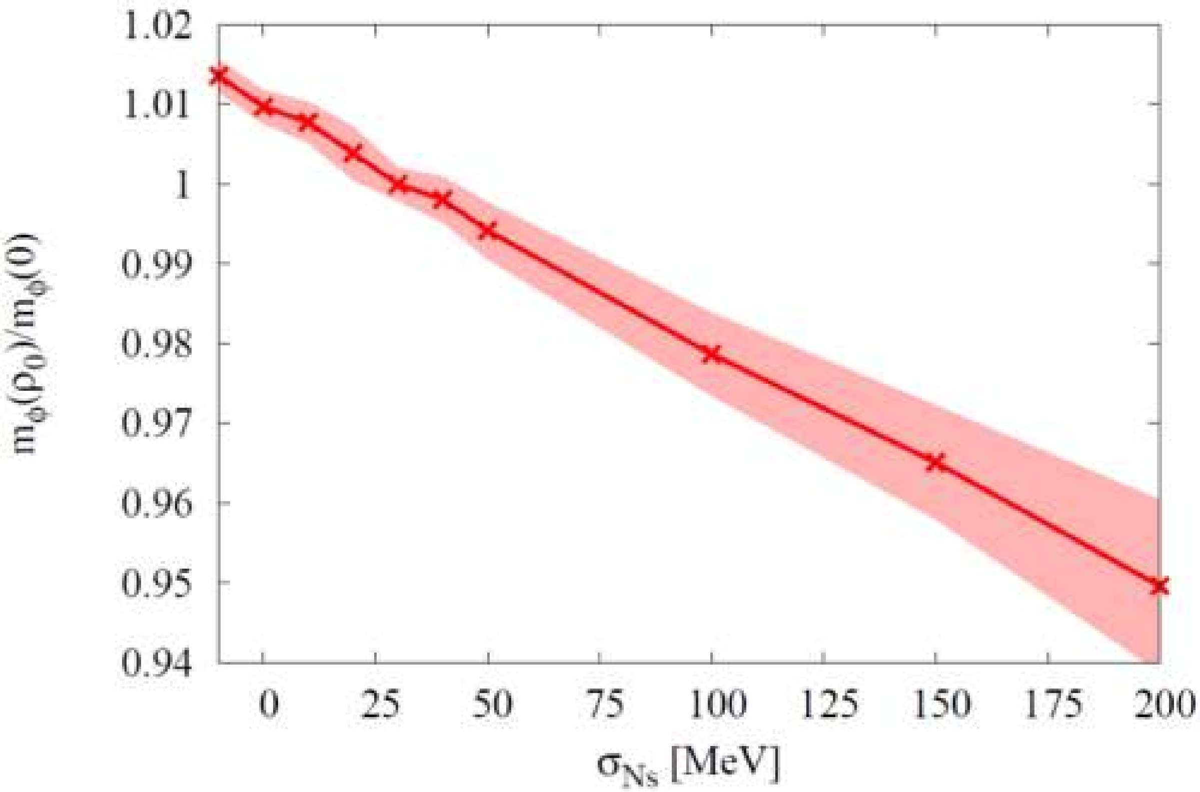
98 MeV

41 MeV

-34 MeV

Measuring the φ meson mass shift in nuclear matter provides a strong constraint to the strange sigma term!

Relation between the φ meson mass shift and the strange sigma term



$$\frac{m_\phi(\rho)}{m_\phi(0)} - 1 = \left[b_0 + b_1\left(\frac{\sigma_{sN}}{1\,\mathrm{MeV}}\right)\right]\frac{\rho}{\rho_0}$$

$$b_0 = (1.24 \pm 0.36)10^{-2}$$

$$b_1 = (-3.23 \pm 0.49)10^{-4}$$

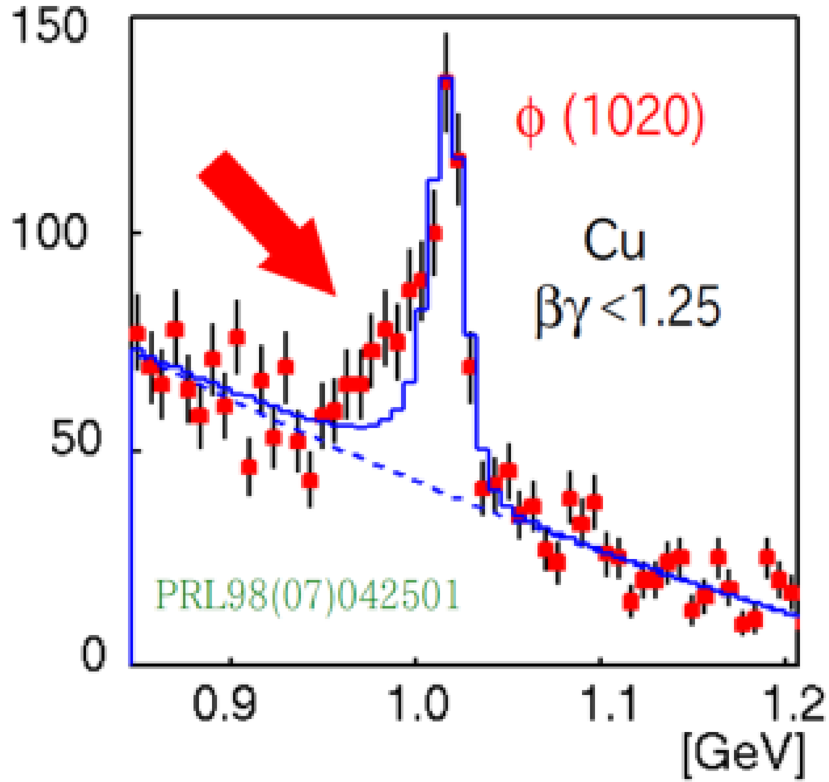
$$\frac{m_\phi(\rho)}{m_\phi(0)} - 1 = \left[b_0 + c_1 y\right]\frac{\rho}{\rho_0}$$

$$c_1 = (-1.97 \pm 0.30)10^{-1}$$

However…

Experiments seem to suggest something else:

Result of the E325 experiment at KEK



R. Muto et al, Phys. Rev. Lett. 98, 042501 (2007).

35 MeV mass reduction of the φ meson at nuclear matter density!

What could be wrong?

1. So far neglected condensates

Terms containing higher orders of m_s and other so far neglected terms could have a non-negligible effect.

$$m_s^3 \langle \bar{s}s \rangle, \; m_s^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle, \; m_s \langle \bar{s} g \sigma G s \rangle, \; \ldots$$

These terms do not have significant effects

2. α_s corrections

These corrections seem to be small

3. Underestimated density dependence of four-quark condensates

$$\langle \alpha_s (\bar{s}\gamma_\mu \gamma_5 \lambda^a s)^2 \rangle + \frac{2}{9} \langle \alpha_s (\bar{s}\gamma_\mu \lambda^a s) \sum_{q=u,d,s} (\bar{q}\gamma^\mu \lambda^a q) \rangle$$

This should be checked in the future

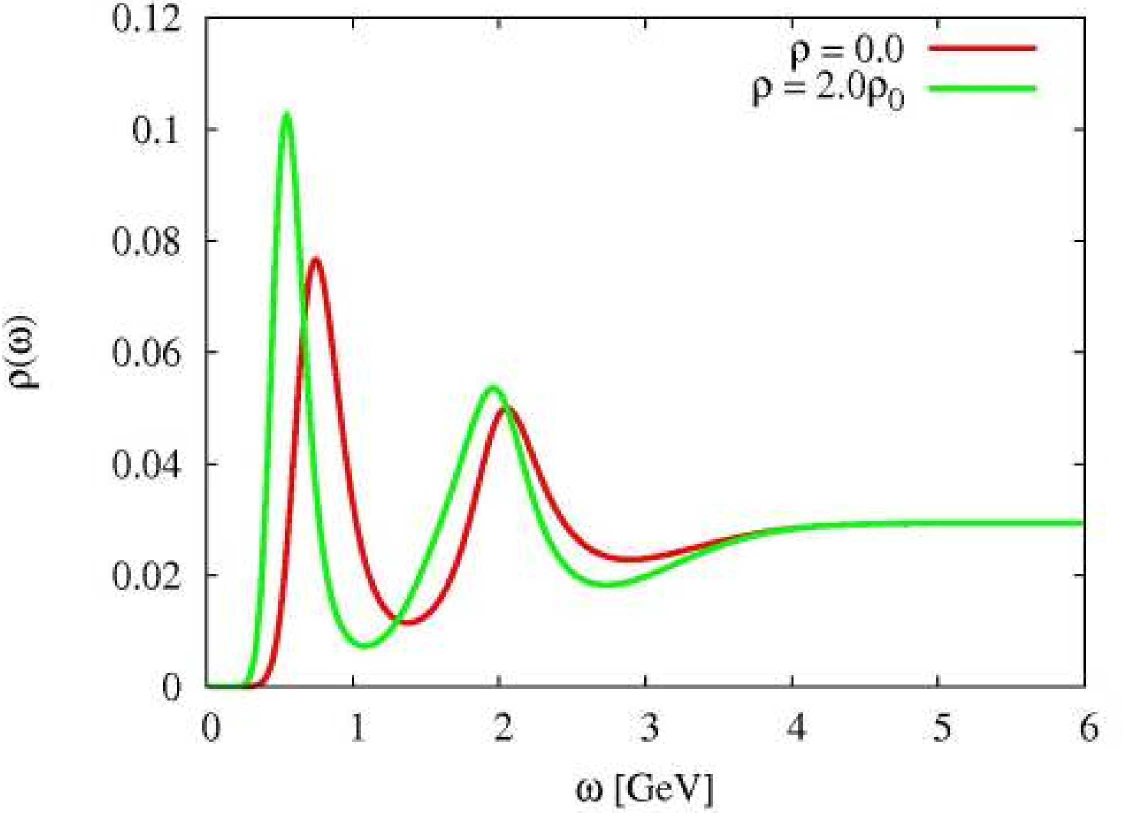
Conclusions

- We have reanalyzed the light vector meson sum rules at finite density using MEM and the newest sigma-term values
- For the φ-meson, due to the small strangeness content of the nucleon, the mass shift might be smaller than previously thought
- The φ-meson mass shift at finite density is found to be strongly correlated with the strangeness content of the nucleon
- Further study on the reliability of the obtained results are in progress

Backup slide

First results (ϱ meson at finite density)

200 MeV



$$\frac{m_\rho(\rho)}{m_\rho(0)} \simeq 1 - C\frac{\rho}{\rho_0}$$

0.12 ~ 0.19

Consistent with the result of Hatsuda-Lee.

Used input parameters: $\sigma_{\pi N} = 32 \pm 2[\mathrm{MeV}]$ A. Semke and M.F.M. Lutz, Phys. Lett. B **717**, 242 (2012).

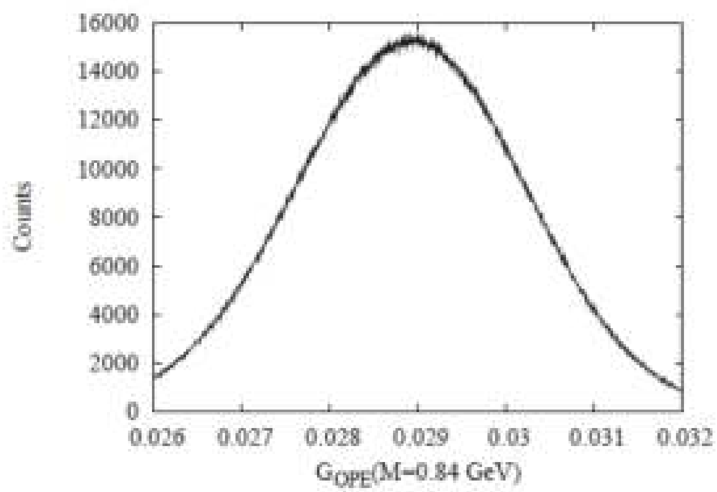
$M_N^0 = 880[\mathrm{MeV}]$ M. Procura, T.R. Hemmert and W. Weise, Phys. Rev. D **69**, 034505 (2004).

Estimation of the error of G(M)

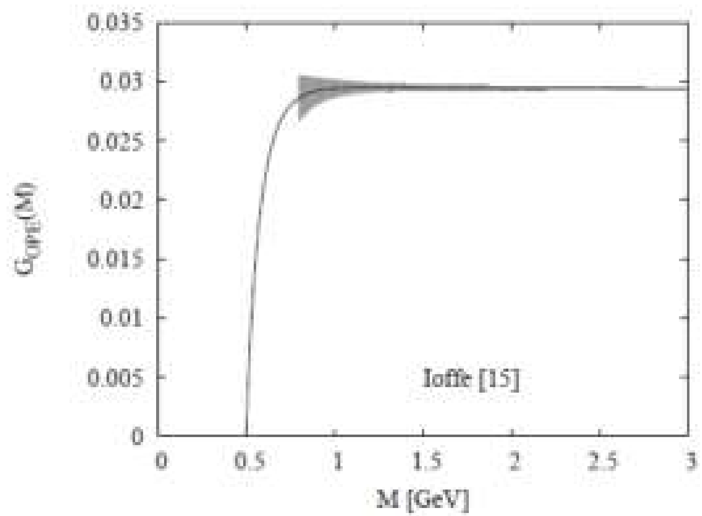
$$G_{OPE}(M) = \frac{1}{4\pi^2}(1+\frac{\alpha_s}{\pi})+\Big(2m\langle\bar{q}q\rangle+\frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\Big)\frac{1}{M^4} - \frac{112\pi}{81}\alpha_s\kappa\langle\bar{q}q\rangle^2\frac{1}{M^6}$$

Gaussianly distributed values for the various parameters are randomly generated. The error is extracted from the resulting distribution of $G_{OPE}(M)$.

D.B. Leinweber, Annals Phys. **322**, 1949 (1996).



$\longrightarrow$



PG, M. Oka, Prog. Theor. Phys. **124**, 995 (2010).