Hidden-Strangeness Partners of $X(3872)$

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§1. Introduction

Short review on tetra-quark mesons

- **Flavor wavefunctions :**

$$\{qq\bar{q}\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus \{[qq](\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\} \oplus (qq)(\bar{q}\bar{q})$$

$\Updownarrow$

No $(K\pi)_{I=3/2}$ scalar resonance at $E \lesssim 1.8$ GeV

- **Color :** $\begin{cases} \cancel{6_c \times \bar{6}_c} \\ \bar{3}_c \times 3_c \end{cases}$ $\begin{cases} \cancel{6_c \times \bar{6}_c} \\ \bar{3}_c \times 3_c \end{cases}$ (**Probably small mixing in heavy mesons**)

- J^P **:** $\begin{cases} 0^+ \\ 0^+, 1^+, 2^+ \end{cases}$ $\begin{matrix} 1^+ \\ (0^+, 1^+, 2^+) \end{matrix}$ $\begin{matrix} \text{Flavor symmetry} \\ \text{Broken flavor symmetry} \end{matrix}$

* **The diquarkonium model with a large $SU_f(4)$ symmetry breaking,**
  Maiani et al., PRD 71, 014028 (2005)
* **A simple tetra-quark model as the 0th order approximation of $SU_f(4)$ symmetry breaking in (spatial) wave functions**
  K. T., PTP 118, 821 (2007); arXiv:0706.3944 [hep-ph];
  PRD68, 011501(R) (2003)

Flavor symmetry breaking in (spatial) wave functions
$\sim$ **Deviation of form factor of vector current from unity (*)**

Phenomenological and measured (extrapolated) form factors:

$$\begin{cases} SU_f(3): & \begin{cases} f_+^{(\pi K)}(0) = & 0.961 \pm 0.008 & \text{Leutwyler \& Roos} \\ \dfrac{f_+^{(\pi D)}(0)}{f_+^{(\bar{K} D)}(0)} = & \begin{cases} 1.00 \pm 0.11 \pm 0.02 & \text{FNAL E687} \\ 0.99 \pm 0.08 & \text{CLEO} \end{cases} \end{cases} \\ SU_f(4): & f_+^{(\bar{K} D)}(0) = \quad 0.74 \pm 0.03 \quad \text{PDG'96} \end{cases}$$

(*) $\begin{cases} Q = \int d^3\underline{x}\{V^0(\underline{x}, t)\} \\ \text{Nornalization: } f_+(0) = 1 \text{ in the symmetry limit.} \end{cases}$

§2. Tetra-Quark States

* **Diquarkonium model**: $[qq]_S$, $(S = 0, 1)$

  $\Uparrow$

  flavor symmetry breking

  - **Scalar:** $[qq]_0[\bar{q}\bar{q}]_0$, $[qq]_1[\bar{q}\bar{q}]_1$
  - **Axial-vector:** $[qq]_0[\bar{q}\bar{q}]_1 \oplus [qq]_1[\bar{q}\bar{q}]_0$, $[qq]_1[\bar{q}\bar{q}]_1$
  - **Tensor:** $[qq]_1[\bar{q}\bar{q}]_1$

* **Simple tetra-quark model**: $[qq]_0$, $(qq)_1$

  $\Uparrow \quad \Uparrow$

  survive in the flavor symmetry limit

  - **Scalar:** $[qq]_0[\bar{q}\bar{q}]_0$
  - **Axial-vector:** $[qq]_0(\bar{q}\bar{q})_1 \oplus (qq)_1[\bar{q}\bar{q}]_0$
  - **Tensor:** $\left[(qq)_1(\bar{q}\bar{q})_1\right]$

- **Axial-vector mesons with $C = 2$ (?)**

$\Rightarrow$

- $*$ **Diquarkonium model :**
  - $X(3872) \sim [cn]_0[\bar{c}\bar{n}]_1 + [cn]_1[\bar{c}\bar{n}]_0$, $X_0 \sim [cn]_0[\bar{c}\bar{n}]_0 + [cn]_0[\bar{c}\bar{n}]_0$
  - $\Rightarrow m_{X(3872)} \sim m_{X_0}$ $\quad (n = u,\ d)$
- $*$ **Simple tetra-quark model:**
  - $X(3872) \sim [cn]_0(\bar{c}\bar{n})_1 + (cn)_1[\bar{c}\bar{n}]_0$, $\quad \hat{\delta}^c \sim \{[cn]_0[\bar{c}\bar{n}]_0\}_{I=0}$
  - $\Updownarrow$
  - $\Rightarrow m_{X(3872)} \not\simeq m_{\hat{\delta}^c}$ $\quad (D_{s0}^+(2317) \sim \{[cn]_0[\bar{s}\bar{n}]_0\}_{I=1})$

- **Predicted mass values of the lowest hidden-charm scalar meson:**

  - $*$ **Unitarized chiral model:**
    Gamerman et al., PRD 76, 074016 (2007)
  - $*$ **Diquarkonium model:**
    Maiani et al., PRD 71, 014028 (2005)

  $\Rightarrow m_{X_0} \simeq 3.7$ GeV

  $\Updownarrow$

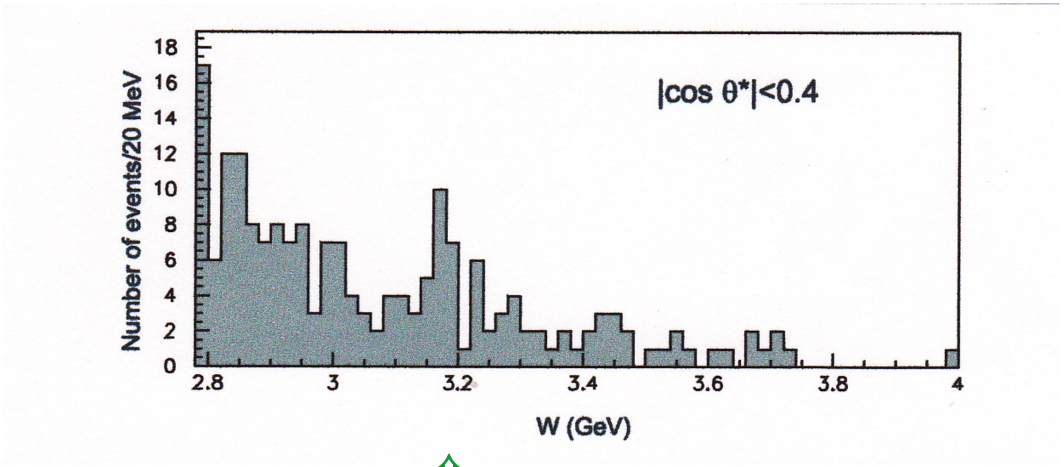
  - $*$ **Simple tetra-quark model:**
    K.T., PTP121, 211 (2009);
    arXiv:0805.4460 [hep-ph]

  $\Rightarrow m_{\hat{\delta}^c} \simeq 3.3$ GeV

  (close to $(m_{\hat{\delta}^c(3.2)})_{\exp} \simeq 3.2$ GeV)

$\hat{\delta}^c(3.2)$: Indication of an $\eta\pi$ peak around 3.2 GeV in $\gamma\gamma$ collision

Belle, PRD80, 032001 (2009)



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$\hat{\delta}^c(3.2)$

$$\left\{ \begin{array}{lll} * \text{ Charmonium-like:} & & m_{J/\psi} < (m_{\hat{\delta}^c(3.2)})_{\exp} \simeq 3.2 \text{ GeV} < m_{\chi_{c0}} \\ * \ \{c\not{\bar{c}}\} & : & I = 1 \end{array} \right.$$

$\Rightarrow$ **Hidden-charm (probably scalar) multi-quark system**

Confirmation of the $\eta\pi$ peak is awaited.

§3. Decay Properties of Hidden-Strangeness Partners of $X(3872)$

Hidden-Strangeness Partners of $X(3872)$

- **Diquarkonium model**
  - $X(3872) \sim \{[cn]_0[\bar{c}\bar{n}]_1 + [cn]_1[\bar{c}\bar{n}]_0\}_{I=0}, \ (n = u, \ d)$

    (with the isospin symmetry against the original model)
  - Its hidden-strangeness partners:

$$\begin{cases} \tilde{X}^s \sim \{[cs]_0[\bar{c}\bar{s}]_1 \pm [cs]_1[\bar{c}\bar{s}]_0\}, \\ \tilde{X'}^s \sim \{[cs]_1[\bar{c}\bar{s}]_1 - [cs]_1[\bar{c}\bar{s}]_1\} \end{cases}$$

- **Simple tetra-quark model**
  - $X(3872) \sim \{[cn]_0(\bar{c}\bar{n})_1 + (cn)_1[\bar{c}\bar{n}]_0\}_{I=0}, \ (n = u, \ d)$,
  - Its hidden-strangeness partners:

$$X^s \sim \{[cs]_0(\bar{c}\bar{s})_1 \pm (cs)_1[\bar{c}\bar{s}]_0\},$$

Coupling to ordinary mesons

Decompose each of tetra-quark states into a sum of products of $\{q\bar{q}\}$ pairs and then replace $\{q\bar{q}\}_{1_c}$ pairs by ordinary mesons,

- Simple tetra-quark model ($X(+) = X(3872)$):

$$X(+) = \frac{1}{4}\sqrt{\frac{1}{3}}\Big\{2[J/\psi\omega - \omega J/\psi] + [D^+ D^{*-} + D^{*+} D^-] - [D^- D^{*+} + D^{*-} D^+] + [D^0 \bar{D}^{*0} + D^{*0}\bar{D}^0] - [\bar{D}^0 D^{*0} + \bar{D}^{*0} D^0]\Big\} + \cdots,$$

$$X(-) = \frac{1}{2}\sqrt{\frac{1}{6}}\Big\{[\eta_c\omega - \omega\eta_c] - [\eta_0 J/\psi - J/\psi\eta_0] + [D^{*0}\bar{D}^{*0} - \bar{D}^{*0} D^{*0}] + [D^{*+} D^{*-} - D^{*-} D^{*+}]\Big\} + \cdots$$

$$X^s(+) = \frac{1}{2}\sqrt{\frac{1}{6}}\Big\{\sqrt{2}[J/\psi\phi - \phi J/\psi] + [D_s^+ D_s^{*-} - D_s^- D_s^{*+} + D_s^{*+} D_s^- - D_s^{*-} D_s^+]\Big\} + \cdots,$$

$$X^s(-) = \frac{1}{2}\sqrt{\frac{1}{6}}\Big\{[\eta_c\phi - \phi\eta_c] + [J/\psi\eta_s - \eta_s J/\psi] + \sqrt{2}[D_s^{*+} D_s^{*-} - D_s^{*-} D_s^{*+}]\Big\} + \cdots$$

where $\cdots$ includes contributions of excited mesons and colored $\{q\bar{q}\}$ pairs.

- **Diquarkonium model** $(X(3872) = \tilde{X}(+))$:

$$\tilde{X}(+) = \frac{1}{4}\sqrt{\frac{1}{3}}\Big\{2[J/\psi\omega + \omega J/\psi] - \sqrt{2}[D^{*0}\bar{D}^{*0} + \bar{D}^{*0}D^{*0} + D^{*+}D^{*-} + D^{*-}D^{*+}]\Big\} + \cdots$$

$$\tilde{X}(-) = \frac{1}{4}\sqrt{\frac{1}{3}}\Big\{\sqrt{2}[\eta_c\omega + \omega\eta_c + J/\psi\eta_0 + \eta_0 J/\psi] - [D^0\bar{D}^{*0} + \bar{D}^0 D^{*0} + D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0 + D^+D^{*-} + D^-D^{*+} + D^{*+}D^- + D^{*-}D^+]\Big\} + \cdots$$

$$\tilde{X}^s(+) = \frac{1}{2}\sqrt{\frac{1}{3}}\Big\{(J/\psi\phi + \phi J/\psi) - (D_s^{*+}D_s^{*-} + D_s^{*-}D_s^{*+})\Big\} + \cdots$$

$$\tilde{X}^s(-) = \frac{1}{2}\sqrt{\frac{1}{6}}\Big\{(\eta_c\phi + \phi\eta_c) + (J/\psi\eta_s + \eta_s J/\psi) - (D_s^+D_s^{*-} + D_s^{*+}D_s^-) - (D_s^-D_s^{*+} + D_s^{*-}D_s^+)\Big\} + \cdots$$

$$\tilde{X}'^s(-) = \frac{1}{2}\sqrt{\frac{1}{6}}\Big\{[J/\psi\eta_s - \eta_s J/\psi] + [\phi\eta_c - \eta_c\phi] + [D_s^+D_s^{*-} + D_s^-D_s^{*+} - D_s^{*+}D_s^- - D_s^{*-}D_s^+]\Big\} + \cdots$$

Remarks:

- **Simple tetra-quark model:**
  - $X(+)$ **has all the necessary couplings to ordinary mesons which can reproduce the observed decays of** $X(3872)$.
    - $X(3872) \to D^0\bar{D}^{*0} + c.c. \to D^0\bar{D}\pi^0$ **and** $\to D^0\bar{D}\gamma$
      (Small phase space volume or radiative interactions $\Rightarrow$ **Small rates)**
    - $X(3872) \to (\omega J/\psi) \to \pi^+\pi^-\pi^0 J/\psi$
      (Small phase space volume $\Rightarrow$ **Small rates)**
    - $X(3872) \to (\omega J/\psi \to \rho^0 J/\psi) \to \pi^+\pi^- J/\psi$ **(through** $\omega\rho^0$ **mixing).**
      (Isospin non-conservation $\Rightarrow$ **Small rates)**
    - $X(3872) \to (\omega J/\psi) \to \gamma J/\psi$
      (Radiative interactions $\Rightarrow$ **Small rates)**
      $\Rightarrow$ **Narrow width of** $X(3872)$
  - $X(\pm)$ **have no unnatural coupling.**

- **Diquarkonium model:**
  - $\tilde{X}(+)$ **which should be assigned to** $X(3872)$ **does not couple to** $D\bar{D}^* + c.c.$ **within the framework of the present prescription.**

  $\Rightarrow$ **How to reproduce** $X(3872) \to D^0 \bar{D}^{*0}$ **in the diquarkonium model ?**
  - $\tilde{X}(-)$ **couples to** $D\bar{D}^* + c.c.$

    $X(3875)$ **(observed in the** $D^0\bar{D}^{*0} + c.c.$ **channel)** $\neq X(3872)$ **(?)**
  - **both of** $\tilde{X}^s(-)$ **and** $\tilde{X'}^s(-)$

    $\begin{cases} \text{do not couple to } D_s^{*+}D_s^{*-} \text{ which has } \mathcal{C} = - \text{ in } S\text{-wave (and } D\text{-wave),} \\ \text{and couple to } D_s^{+}D_s^{*-} \oplus D_s^{-}D_s^{*+} \end{cases}$
  - $\tilde{X}^s(+)$ **couples to** $D_s^{*+}D_s^{*-}$ **which has** $\mathcal{C} = -$ **in** S**-wave (and** D**-wave).**

Crude estimate of rates for two-body decays of partners of $X(3872)$:

Assume that the full width of $X(3872)$ is approximately saturated as

$$\Gamma_{X(3872)} \simeq \Gamma(X(3872) \to D^0\bar{D}^{*0} + c.c.) + \Gamma(X(3872) \to \pi^+\pi^-\pi^0\psi) + \Gamma(X(3872) \to \pi^+\pi^-\psi),$$

take the observed ratios of decay rates,

$$\begin{cases} \dfrac{\Gamma(X(3872) \to D^0\bar{D}^{*0} + c.c.)}{\Gamma(X(3872) \to \pi^+\pi^-\psi)} = 9.5 \pm 3.1, & \text{Belle} \\ \dfrac{\Gamma(X(3872) \to \pi^+\pi^-\pi^0\psi)}{\Gamma(X(3872) \to \pi^+\pi^-\psi)} = 0.8 \pm 0.3, & \text{Belle} \\ \dfrac{\Gamma(X(3872) \to \gamma\psi)}{\Gamma(X(3872) \to \pi^+\pi^-\psi)} = 0.22 \pm 0.09 & (*) \end{cases}$$

and the width $\Gamma_{X(3872)} = 3.9^{+2.8+0.2}_{-1.4-1.1}\text{MeV}$ Belle

(measured by using $X(3872) \to D^0\bar{D}^{*0} + c.c.$)

$$\Rightarrow \begin{cases} \Gamma(X(3872) \to D^0\bar{D}^{*0})_{\text{ph}} = 0.81\ ^{+0.72}_{-0.63}\ \text{MeV} \\ \Gamma(X(3872) \to \gamma\psi)_{\text{ph}} = 0.075^{+0.069}_{-0.061}\ \text{MeV} \end{cases}$$

(*) Obtained by compiling data on branching fractions from Belle

Masses of partners of $X(3872)$:

Quark counting with $\left\{ \begin{array}{l} m_s - m_n \simeq 100\ \mathrm{MeV}, \\ m_{X(3872)} \text{ and } m_{Z_c(3900)} \text{ as the input data} \end{array} \right.$

$$\Rightarrow \left\{ \begin{array}{l} m_{X(-)} \simeq m_{X_I(-)} = m_{Z_c(3900)}, \\ m_{X^s(+)} \simeq 4072\ \mathrm{MeV}, \\ m_{X^s(-)} \simeq 4100\ \mathrm{MeV} \end{array} \right.$$

Couplings to ordinary mesons:

- **(i) Flavor symmetry in couplings among tetra-quark and ordinary mesons,**
- **(ii) Same spatial wave function of tetra-quark mesons with opposite $\mathcal{C}$-parities,**
- **(iii) Vector meson dominance in radiative decays**

Two-body decays of partners of $X(3872)$, where $X_I^0(-) = Z_c^0(3900)$.

Input data:
$$\begin{cases} \Gamma(X(3872) \to D^0 \bar{D}^{*0})_{\rm ph} = 0.81\ ^{+0.72}_{-0.63}\ \text{MeV}, \\ \Gamma(X(3872) \to \gamma\psi)_{\rm ph} = 0.075^{+0.069}_{-0.061}\ \text{MeV} \end{cases}$$

Decay	Rate (MeV)	Decay	Rate (MeV)
$X(-) \to \eta_c \omega$	38^{+34}_{-30}	$X(-) \to \eta\psi$	24^{+22}_{-19} (∗)
$X_I^0(-) \to \eta_c \rho^0$	38^{+34}_{-30}	$X_I^0(-) \to \pi^0 \psi$	58^{+51}_{-45}
$X^s(-) \to \eta\psi$	20^{+18}_{-15} (∗)	$X^s(-) \to \eta_c \phi$	33^{+29}_{-25}
$X^s(+) \to \gamma\psi$	$0.12^{+0.11}_{-0.10}$	$X^s(+) \to \gamma\phi$	$0.74^{+0.68}_{-0.62}$

(∗) $\eta\eta'$ mixing with $\theta_P = -20^\circ$

K.T., arXiv:1304.7080, 1312.5791 [hep-ph]

$$\Gamma_{X_I^0(-)} \sim (20 - 200)\ \text{MeV} \Leftrightarrow \Gamma^{\rm CLEO}_{Z_c^0(3900)} = 34 \pm 29\ \text{MeV}.$$

§. Summary

- **Two different tetra-quark models:**
  - ∗ **Diquarkonium model** — (**Scalar and axial-vector mesons with the same flavor structure**)

    Predicted mass of hidden-charm scalar $\sim m_{X(3872)}$
  - ∗ **Our tetra-quark model** — (**Scalar** $\sim [qq][\bar{q}\bar{q}]$, **Axial-vector** $\sim [qq](\bar{q}\bar{q}) \oplus [(qq)[\bar{q}\bar{q}]$)

    Large mass difference between scalar and axial-vector mesons

    Predicted mass of $m_{\hat{\delta}^c}$ **is close to** $m_{\hat{\delta}^c(3.2)}$ ($\Leftarrow$ **the** $\eta\pi^0$ **peak**)

    and is much lower than $m_{X(3872)}$

  $\Rightarrow$ **Confirmation of existence of** $\hat{\delta}^c(3.2)$ **is awaited.**
- **Within the framework of the present prescription to decompose tetra-quark states into products of** $\{q\bar{q}\}$ **pairs,**
  - ∗ **Simple tetra-quark model:**
    - ○ **All the necessary couplings to ordinary mesons,**
    - ○ **No unnatural coupling**
  - ∗ **Diquarkonium model:**
    - △ **Lack of a part of necessary couplings to ordinary mesons,**
    - △ **Inclusion of unnatural couplings**

- **Crude estimate of rates for two-body decays of partners of $X(3872)$**
  - **Estimated $\Gamma_{X_I^0(-)}$ has been compared with $\Gamma^{\mathrm{CLEO}}_{Z_c^0(3900)}$.**
  - **$X(-)$, $X^s(-)$ are expected to be considerably broad.**

    $\Rightarrow$ Their observation at the next stage of Belle experiments (?)
  - **Probably no OZI-rule-allowed decay of $X^s(+)$**

    $\Rightarrow$ $\left\{ \begin{array}{l} * \ X^s(+) \text{ is probably narrow.} \\ * \ \text{Radiative decays will be important.} \end{array} \right.$