Accurate calculation of kaonic atom structure for study of the kaon-nucleus interaction

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• **Meson-Nucleus systems** are very important and useful objects to extract the meson properties at finite density.

  pion, kaon, omega, eta, eta’, phi ...

  From the meson properties at finite density, we may investigate the chiral symmetry breaking and its partial restoration in the nucleus.

**Mesic Nuclei**

$K^-, \omega, \eta, \eta', \phi$

Strong interaction

[MeV]

**Mesic Atoms**

$\pi^-, K^-$

Coulomb

[10 keV - MeV]
## Kaon-Nucleus systems

### Kaonic Nuclei
- **Strong Int.**
  - 10 – 100 MeV ?
  - Wide
  - (but, if B.E. > 100 MeV, Γ is narrow)
- **A few**

### Kaonic Atoms
- **Bound system**
  - B.E.
  - Width
- **EXP.**

### Kaonic Atoms
- **Coulomb Int.**
  - keV-MeV
  - Narrow
- **Many**

### Diagram
- **Deep type potential**
- **Nuclear radius of**
  - $^{15}\text{N}$
  - **Nuclear 1s**
  - **Nuclear 2s**
  - **Atomic 1s**
Kaonic Nuclei

Strong Int.
10 – 100 MeV ?
Wide
(but, if B.E. > 100 MeV, Γ is narrow)
A few

Kaonic Atoms

Coulomb Int.
keV-MeV
Narrow
Many

Bound system
B.E.
Width
EXP.

Deep type potential

Nuclear radius of $^{15}$N
The candidates of the kaon-nucleus interaction

**SHALLOW** -- Chiral Unitary Model (Ramos, Oset, NPA671(00)481)

**DEEP** -- Phenomenological Model
(Batty, Friedman, Gal, PR287(97)385, Mares, Friedman, Gal, NPA770(06)84)

The kaon-nucleus potential at threshold energy

How can we distinguish these potentials?
Calculated binding energy and width

Atomic State

Similar

Nuclear State

Different

The difference of two potentials could be shown in the kaonic nuclear state.
**Kaon-Nucleus system at J-PARC**

**Experiment at J-PARC**

**E-15**: A search for deeply-bound kaonic nuclear states by in-flight $^3$He(K-,n) reaction

$^3$He $(K^-, n) K^- pp, \ K^- pp \rightarrow \Lambda + p/\Sigma^0 + p$

**E-17**: Precision spectroscopy of Kaonic $^3$He 3d $\rightarrow$ 2p X-rays

**E-27**: Search for a nuclear Kbar bound state K-pp in the d(pi+,K+) reaction

$d(\pi^+, K^+) K^- pp, \ K^- pp \rightarrow \Lambda + p/\Sigma^0 + p$

**E-31**: Spectroscopic study of hyperon resonances below KbarN threshold via the (K-,n) reaction on Deuteron

$d(K^-, n) \Lambda(1405), \ \Lambda(1405) \rightarrow \pi \Sigma$
Kaon-Nucleus system at J-PARC

Our calculated results related to J-PARC experiments

J-PARC E-15

J-PARC E-27

J-PARC E-31

input parameter:
Bound state of $\Lambda^* p$: 20 MeV
Decay width of $\Lambda^* p \rightarrow Y p$: 10 MeV

From this work, we found the bound state could make enough structure to observe.
• We try to obtain the information on the kaon-nucleus interaction from the **kaonic atoms**.

**X-ray spectroscopy of kaonic atoms**

\[ \Delta E(2p) = E_X^{\exp} - E_X^{\text{EM}} \]

• By measuring the shift \( \Delta E \) and width, we could obtain the information on the strong interaction.
High-resolution hadronic-atom x-ray spectroscopy with cryogenic detectors

Shinji OKADA (RIKEN)

The HEATES collaboration

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NIST’s TES array system for x-rays

NIST’s standard TES

- 1 pixel: 300 x 320 μm²
- 240 array: total ~ 23 mm²
- 2~3 eV (FWHM) @ 6 keV

well established system!

e.g., soft-X-ray spectroscopy @ BNL

NSLS U7A:
soft-X-ray (200 – 800 eV)
spectroscopy beamline.

W.B. Doraise, TES Workshop @ ASC (Portland), Oct 8, 2012

~ 200 eV (FWHM) @ 6 keV
... a typical Silicon detector used in the previous K-atom exp.
Is 240 pixel (~23 mm²) enough?

estimated K-⁴He Kα yield w/ realistic setup
~ 20 events / day

<table>
<thead>
<tr>
<th></th>
<th>K-⁴He Kα events</th>
<th>Energy resolution in FWHM</th>
<th>Stat. accuracy of ene. determining (6 keV)</th>
</tr>
</thead>
</table>
| KEK-E570 with SDD | 1500 events     | 190 eV                    | 2 eV
  (190 / 2.35 / sqrt(1500)) |
| TES              | 150 events (~ a-week beam) | 2 - 3 eV               | ~ 0.1 eV
  (2 - 3 / 2.35 / sqrt(150)) |
We improve our calculation with high precision EM term.

Nucleus: 4He, 7Li

Density distribution:

4He -- 3 parameter Fermi distribution

\[
\rho(r) = \rho_0 \left(1 + \omega \left(\frac{r}{c}\right)^2\right) \frac{1}{1 + \exp((r - c)/z)}
\]

Set (A): \(c=1.008, z=0.327, \omega=0.445 \text{ [fm]}\)
Set (B): \(c=0.964, z=0.322, \omega=0.517 \text{ [fm]}\)

7Li -- Modified Harmonic Oscillator

\[
\rho(r) = \rho_0 \left(1 + \alpha \left(\frac{r}{a}\right)^2\right) \exp\left(-\left(\frac{r}{a}\right)^2\right)
\]

\(a=1.623, \alpha=0.429 \text{ [fm]}\)
• The meson energy is obtained by solving the Klein-Gordon equation.

\[
\left[-\nabla^2 + \mu^2 + 2\mu V_{\text{opt}}(r)\right]\phi(\vec{r}) = \left[E_{\text{meson}} - V_{\text{coul}}(r)\right]^2 \phi(\vec{r})
\]

\(\mu\) : reduced mass of kaon-nucleus system  
\(V_{\text{opt}}(r)\) : the optical potential of kaon-nucleus system

- Chiral Unitary Model  
  (Ramos, Oset, NPA671(00)481)

- Phenomenological Model  
  (Batty, Friedman, Gal, PR287(97)385, Mares, Friedman, Gal, NPA770(06)84)

\(V_{\text{coul}}(r)\) : finite-size coulomb potential

\[
V_{\text{coul}}(r) = -e^2 \int \frac{\rho_p(r')}{|\vec{r} - \vec{r}'|}
\]
Calculated results of 4He

**Phenomenological Model**
(Mares, Friedman, Gal, NPA770(06)84)

\[ V_{opt}(r = 0) \sim -(180 + 73i) \text{ MeV} \]

set(A)
set(B)

**Chiral Unitary Model**
(Ramos, Oset, NPA671(00)481)

\[ V_{opt}(r = 0) \sim -(40 + 55i) \text{ MeV} \]

set(A)
set(B)

\[ \text{SHIFT} = (E_K(3d) - E_K(2p)) - (E^{EM}(3d) - E^{EM}(2p)) \]
Calculated results of $^4$He

Phenomenological Model
(Mares, Friedman, Gal, NPA770(06)84)
$V_{\text{opt}}(r = 0) \sim -(180 + 73i) \text{ MeV}$

set(A)
set(B)

PHENO: estimated with fit errors
$V_{\text{opt}}(r = 0) \sim -(170 - 187) - (70 - 75)i \text{ MeV}$

Chiral
Unitary
Model
(Ramos, Oset, NPA671(00)481)

$V_{\text{opt}}(r = 0) \sim -(36 - 44) - (50 - 60)i \text{ MeV}$

CHIRAL: assuming 10% errors

Chiral Unitary Model
(Ramos, Oset, NPA671(00)481 )
$V_{\text{opt}}(r = 0) \sim -(40 + 55i) \text{ MeV}$

set(A)
set(B)
Calculated results of 4He

Phenomenological Model
(Mares, Friedman, Gal, NPA770(06)84)
\[ V_{\text{opt}}(r = 0) \sim -(180 + 73i) \text{ MeV} \]
set(A)
set(B)

High-resolution experiment is planned at J-PARC. (RIKEN, Okada-san)
Is it possible to distinguish these two potentials from the data of kaonic atoms?

Chiral Unitary Model
(Ramos, Oset, NPA671(00)481)
\[ V_{\text{opt}}(r = 0) \sim -(40 + 55i) \text{ MeV} \]
set(A)
set(B)
Calculated results of $^7$Li

Difference of energy shift is larger for $^7$Li than for $^4$He. Can we distinguish these two potentials?

**Phenomenological Model**
(Mares, Friedman, Gal, NPA770(06)84)

**Chiral Unitary Model**
(Ramos, Oset, NPA671(00)481)

Graph showing the width vs. shift with different symbols representing different models and potentials.
SUMMARY

• We try to obtain the information on the kaon-nucleus interaction from the kaonic atoms.

• We calculated the energy shift for 4He/7Li corresponding to the X-ray spectroscopy experiment.

• From the calculated results, we found that high-resolution experiment is needed to distinguish two potentials. We expect high resolution for new experiment at J-PARC!

• We should not use the optical potential to a light nuclear case for more accurate calculation?

• We search the kaon-nucleus systems (kaonic atom, kaonic nucleus) which gives us an answer for the potential strength.
Possibility of nuclear deformation by anti-kaon in Thomas-Fermi model

Junko Yamagata-Sekihara (RCNP, Osaka Univ.)
Satoru Hirenzaki (Nara Women’s Univ.)
In this work

- We study the possibility of nuclear deformation by the existence of meson, especially anti-kaon as a typical example. For a kaonic nuclear state, anti-kaon is mainly bound with the strong interaction to the nucleus. The strength of the interaction is still controversial.

- The Thomas-Fermi model is used in this work. We could obtain the nuclear density all over the nuclear chart in a systematic manner.

- The nuclear shape is determined to minimize the total energy of the meson-nucleus system determined by the following equation.

\[ E_{\text{total}}[\rho] = E_{\text{nucleus}}[\rho] + E_{\text{meson}}[\rho] \]

Meson(Kaon) Energy : Klein-Gordon equation
In this work

STEP 1
We obtain the nuclear density distribution \((\rho_N)\) by the Thomas-Fermi model by minimizing the energy of nucleus \(E_{\text{nucleus}}[\rho_N]\).

\[
E_{\text{nucleus}} \quad : \text{The energy of nucleus is mainly determined with an equation of state (EOS) of uniform nuclear matter.}
\]

STEP 2
We obtain the nuclear density \((\rho)\) of meson-nucleus system by minimizing the total energy \(E_{\text{total}}[\rho]\). The total energy is determined as

\[
E_{\text{total}}[\rho] = E_{\text{nucleus}}[\rho] + E_{\text{meson}}[\rho]
\]

\(E_{\text{nucleus}} \quad : \text{The energy of nucleus is mainly determined with a equation of state (EOS) of uniform nuclear matter.}\)

\(E_{\text{meson}} \quad : \text{The meson energy is obtained by solving the Klein-Gordon equation.}\)
**STEP 1 : The nuclear density**

**Thomas-Fermi model**

Oyamatsu, NPA561(1993)181

- This model could describe well nuclear property (mass, radius, ...) and is applied systematically to unstable nucleus, supernova matter, neutron star matter, not only stable nucleus.

\[
E_{nucleus}[\rho] = \int d^3r \epsilon(\rho_n(\vec{r}), \rho_p(\vec{r})) + F_0 \int d^3r |\nabla \rho(\vec{r})|^2 + \frac{e^2}{2} \int d^3r \int d^3r' \frac{\rho_p(\vec{r}) \rho_p(\vec{r}')}{|\vec{r} - \vec{r}'|}
\]

- **Main term** (determined by an equation of state (EOS) of uniform nuclear matter)
- **Density inhomogeneity term** (the most important term in determining the shape of the nuclear surface)
- **Coulomb term**

We use the Woods-Saxon form as the density distribution. Two parameters (c, a) are obtained by minimizing the energy of nucleus.

\[
\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{(r - c)}{a}\right)}
\]
STEP 1: The nuclear density

- **EOS (equation of state)**
  Oyamatsu, NPA561(1993)181

\[
\rho = \rho_p + \rho_n \\
\rho = \rho_n, \; \rho_p = 0 \\
\rho_p = \rho_n
\]

- **Density distribution**

  Woods-Saxon form

\[
\rho(r) = \frac{\rho_0}{1 + \exp((r - c)/a)}
\]

  \(c, a\) : parameter
STEP 1: The nuclear density

- EOS (equation of state)
  Oyamatsu, NPA561(1993)181

\[ \rho = \rho_p + \rho_n \]
\[ \rho = \rho_n, \quad \rho_p = 0 \]

\[ \rho_p = \rho_n \]

- Density distribution
  Woods-Saxon form
  \[ \rho(r) = \frac{\rho_0}{1 + \exp((r - c)/a)} \]
  c, a : parameter

- Results of nuclear density
  Nucleus: $^{12}\text{C}$, $^{40}\text{Ca}$, $^{90}\text{Zr}$, $^{120}\text{Sn}$, $^{208}\text{Pb}$

- Good systematics from light to heavy nuclei
- Central densities tend to be small.
We search the parameters of nuclear density of meson-nucleus system which satisfy **the minimum total energy condition**.

The nuclear density distribution with the minimum total energy is the solution we look for.

\[ \rho(r) = \frac{\rho_0}{1 + \exp((r - c)/a)} \]

\[ E_{\text{total}}[\rho] = E_{\text{nucleus}}[\rho] + E_{\text{meson}}[\rho] \]
We search parameters in the nuclear density.

We use the Woods-Saxon form as the density distribution and look for two parameters \((c, a)\) satisfying the total energy minimum condition.

The energy of nucleus is determined in the same manner as the Thomas-Fermi model.

\[
E_{\text{nucleus}}[\rho] = \int d^3r \epsilon(\rho_n(\vec{r}), \rho_p(\vec{r})) + F_0 \int d^3r |\nabla \rho(\vec{r})|^2 + \frac{e^2}{2} \int d^3r \int d^3r' \frac{\rho_p(\vec{r})\rho_p(\vec{r}')}{|\vec{r} - \vec{r}'|}.
\]

- **Main term** (determined by an equation of state (EOS) of uniform nuclear matter)
- **Density inhomogeneity term** (the most important term in determining the shape of the nuclear surface)
- **Coulomb term**

Oyamatsu, NPA561(1993)181
STEP 2: The nuclear density of meson-nucleus system

• The meson energy is obtained by solving the Klein-Gordon equation.

\[ [-\nabla^2 + \mu^2 + 2\mu V_{\text{opt}}(r)] \phi(r) = [E_{\text{meson}} - V_{\text{coul}}(r)]^2 \phi(r) \]

\( \mu \): reduced mass
\( V_{\text{opt}}(r) \): the optical potential of kaon-nucleus system

\[
\text{Re} V_{\text{opt}}(r) = V_0 \frac{\rho(r)}{\rho_N(0)}
\]

\( V_0 \): parameter (-50 to -150 MeV)
\( \rho_N \): nuclear density without kaon

(\text{obtained by minimizing only } E_{\text{nucl}})

\[
\text{Im} V_{\text{opt}}(r, E) : \text{Chiral Unitary Model}
\]
Ramos, Oset, NPA671(06)84

\( V_{\text{coul}}(r) \): finite-size coulomb potential

\[
V_{\text{coul}}(r) = -e^2 \int \frac{\rho_p(r')}{|r - r'|} \text{ d}r'
\]

We solve the KG equation to obtain a kaon energy of the deepest state.
We search the parameters of nuclear density of meson-nucleus system which satisfy the **minimum total energy condition**.

The nuclear density distribution with the minimum total energy is the solution we look for.

\[
E_{\text{total}}[\rho] = E_{\text{nucleus}}[\rho] + E_{\text{meson}}[\rho]
\]

\[
\rho(r) = \frac{\rho_0}{1 + \exp((r - c)/a)}
\]
The central nuclear density becomes higher for the deeper kaon state in each nucleus.
\( \Delta \rho(0) \equiv \rho(0) - \rho_N(0) \)

In the case of 12C, the change of the central nuclear density is the largest.

We find the effect of meson existence is larger for lighter nucleus.

We also find the stronger potential causes deeper kaon states and higher nuclear density in each nucleus.

However, the density distribution is fixed to the Woods-Saxon functional form in this work.

Very ‘local’ meson effect could not be expressed by this form.

We should use more flexible density form to Thomas-Fermi model in the next step.
• We report the possibility of nuclear deformation by the existence of meson, especially anti-kaon as a typical example.

• The Thomas-Fermi model is used in this work. We could obtain the nuclear density all over the nuclear chart systematically.

• The density form we assumed is the Woods-Saxon functional form.

• From the calculated results, we find the strong potential causes deeper kaon states and higher nuclear density in each nucleus and the effect of meson existence is larger for lighter nucleus.

• We should use a flexible density form to Thomas-Fermi model to consider the ‘local’ deformation. We will use the mixing form of two types of functions which has four parameters.

\[
\rho_{WSG}(r) = \frac{(1 - F')\rho_0}{1 + \exp((r - c)/a_1)} + F \exp\left(-\frac{r^2}{2a_2}\right)
\]
• This is a first step to investigate proper meson-nucleus systems to study the aspect of symmetry at various nuclear densities !!
• In case with mesons of quite strong interaction, the system could be also used to investigate the equation of state itself.

Thank you for your attention !