Hadron Interactions with Strangeness

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M. Obu, Y. Akaishi,
Ngo Thi Hong Xiem

Contents

■ Hadron-Hadron Interactions
■ Baryon-Baryon Interactions
  \( \Lambda N-\Sigma N, \Sigma N, \Lambda \Lambda-\Xi N-\Sigma \Lambda-\Sigma \Sigma \)
■ Meson-Baryon Interactions
  \( K^\text{bar} N, K^\text{bar} \Lambda-\pi \Xi, K^\text{bar} \Sigma-\pi \Xi, K^\text{bar} \Xi, \pi N-\eta N-K \Lambda \)
■ Meson-Meson Interactions
  \( \pi \pi-K^\text{bar} K-\eta \eta, K\pi-K\eta \)
Hadron-Hadron (H-H) Interactions at Low Energies

**Baryon-Baryon Interactions**

- **S= 0**  \( \text{NN} \)
- **S=-1**  \( \Lambda N - \Sigma N \)
- **S=-2**  \( \Xi N - \Lambda \Lambda - \Lambda \Sigma - \Sigma \Sigma \)
- **S=-3**  \( \Xi \Lambda - \Xi \Sigma \)
- **S=-4**  \( \Xi \Xi \)

**Meson-Baryon Interactions**

- **S= 1**  \( \text{KN} \)
- **S= 0**  \( \pi N - \eta N - K \Lambda - K \Sigma \)
- **S=-1**  \( K^\text{bar} N - \pi \Lambda - \pi \Sigma - \eta \Lambda - \eta \Sigma - K \Xi \)
- **S=-2**  \( \pi \Xi - \eta \Xi - K^\text{bar} \Lambda - K^\text{bar} \Sigma \)
- **S=-3**  \( K^\text{bar} \Xi \)

**Meson-Meson Interactions**

- **S= 2**  \( \text{KK} \)
- **S= 1**  \( K \pi - K \eta \)
- **S= 0**  \( \pi \pi - K^\text{bar} K - \eta \pi - \eta \eta \)
- **S=-1**  \( K^\text{bar} \pi - K^\text{bar} \eta \)
- **S=-2**  \( K^\text{bar} K^\text{bar} \)

**Coupled-Channel Problems**

**Construction of Coupled-Channel Potentials**

- Two-body systems
- Three-body systems
- Many-body systems
Theoretical models of H-H interactions

First-principle approach by LQCD
Direct results of Fundamental Theory

Chiral Perturbation models
Reordering based on Fundamental Symmetry

Hadron-exchange models
Long-range part: hadronic mechanism
Short-range part: short-range physics
  Phenomenological Core (form factors)
  Quark-model Core
  LQCD-Core

They play complementary roles
Experimental Knowledge on H-H interactions

Two-body scattering
- $NN$, $\pi N$, $KN$, $\pi\pi$, $K\pi$ : Phase Shift Analyses
- $\Lambda N, \Sigma N-\Sigma N, \Sigma N-\Lambda N, KN, K^\text{bar} N-\pi \Lambda-\pi \Sigma$ cross sections

Hypernuclear Spectroscopy
- $\rightarrow$ Effective $\Lambda N$ interaction
- Final (intermediate) state interaction in hadron reactions
- $\rightarrow$ Off-shell HH amplitudes
- Hadron spectroscopy
  - ex. $\Lambda(1405)$ as a quasibound state of $K^\text{bar} N$
  - $f0(980)$ as a quasibound state of $K^\text{bar} K$

Model-dependent (indirect)
### Hadron-Exchange Model of H-H interactions

**Hadron-Exchange Mechanism with the Flavor SU(3) symmetric Coupling Constants**

<table>
<thead>
<tr>
<th>Location</th>
<th>Details</th>
<th>Core Type</th>
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<tbody>
<tr>
<td>Julich</td>
<td>OHEP, Form Factor, No Phenomenological Core</td>
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<td>(1980-1995)</td>
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<tr>
<td>Nijmegen</td>
<td>OHEP, TMEP, Pair Terms, Pomeron (Quark-Gluon Effects), Form factor</td>
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<td>(1977-</td>
<td></td>
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<tr>
<td>Funabashi-Gifu</td>
<td>OBEP with short-range cutoff, Phenomenological Core (2000)</td>
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<td>Gifu</td>
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<tr>
<td>GSOBEP</td>
<td>OHEP + Source Function, No Phenomenological Core</td>
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<td>(2005, 2010)</td>
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**Our Goal: A Unified Model of H-H interactions**

Shinmura Shoji
# Hadron-Hadron (H-H) Interactions at Low Energies

## Baryon-Baryon Interactions

<table>
<thead>
<tr>
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<tr>
<td>0</td>
<td>$NN$</td>
</tr>
<tr>
<td>-1</td>
<td>$ΛN−ΣN$</td>
</tr>
<tr>
<td>-2</td>
<td>$ΞN−ΛΛ−ΛΣ−ΣΣ$</td>
</tr>
<tr>
<td>-3</td>
<td>$ΞΛ−ΞΣ$</td>
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<tr>
<td>-4</td>
<td>$ΞΞ$</td>
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## Meson-Baryon Interactions

<table>
<thead>
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<th>Interaction</th>
</tr>
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<tbody>
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<td>$KN$</td>
</tr>
<tr>
<td>0</td>
<td>$πN−ηN−KΛ−KΣ$</td>
</tr>
<tr>
<td>-1</td>
<td>$K^{\text{bar}}N−πΛ−πΣ−ηΛ−ηΣ−KΞ$</td>
</tr>
<tr>
<td>-2</td>
<td>$πΞ−ηΞ−K^{\text{bar}}Λ−K^{\text{bar}}Σ$</td>
</tr>
<tr>
<td>-3</td>
<td>$K^{\text{bar}}Ξ$</td>
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## Meson-Meson Interactions

<table>
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<th>Interaction</th>
</tr>
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<tbody>
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<td>2</td>
<td>$KK$</td>
</tr>
<tr>
<td>1</td>
<td>$Kπ−Kη$</td>
</tr>
<tr>
<td>0</td>
<td>$ππ−K^{\text{bar}}K−ηπ−ηη$</td>
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<tr>
<td>-1</td>
<td>$K^{\text{bar}}π−K^{\text{bar}}η$</td>
</tr>
<tr>
<td>-2</td>
<td>$K^{\text{bar}}K^{\text{bar}}$</td>
</tr>
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</table>

## Coupled-Channel Problems

### Construction of Coupled-Channel Potentials

- **Two-body systems**
- **Three-body systems**
- **Many-body systems**

OBEP with cutoff (at r=0.4fm) + SU(3)-symmetric Phenomenological SRC
(Different scalar-meson masses are used in FG-A and FG-B)

\[ V = V_{\text{core}}(r) + [1-\exp(-(r/r_c)^2)]^4 V_{\text{OBEP}} \quad r_c=0.4\text{fm} \]

\[ V_{\text{core}}(r) = V_c \exp(-(r/r_g)^2) \quad r_g=0.5(0.49147) \text{ fm for FG-A(FG-B)} \]

Phenomenological cores : \( V_c \) (MeV)

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<th>FG-B</th>
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<tbody>
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<td>{27}</td>
<td>{27}</td>
</tr>
<tr>
<td>Even</td>
<td>{10*}</td>
<td>{10*}</td>
</tr>
<tr>
<td></td>
<td>{10}</td>
<td>{10}</td>
</tr>
<tr>
<td></td>
<td>{8a}</td>
<td>{8a}</td>
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<td>2822</td>
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<td>2277</td>
<td>2473</td>
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<td></td>
<td>40</td>
<td>75</td>
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<td></td>
<td>1162</td>
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<td></td>
<td>3579</td>
<td>256</td>
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<td></td>
<td>173</td>
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<td>Odd</td>
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<td></td>
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<td>267</td>
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<td>3603</td>
</tr>
<tr>
<td></td>
<td>-</td>
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</table>
BB potentials derived from LQCD calc.

T. Inoue et al.  
(HAL QCD collaboration)  
PTP124(2010)591

Flavor SU(3) Limit  
Even(1S0,3S1) states

pion mass = 1014 MeV red  
pion mass = 835 MeV green

Relative Strengths:

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<thead>
<tr>
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<th>{27}</th>
<th>{10*}</th>
<th>{10}</th>
<th>{8a}</th>
<th>{8s}</th>
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<td>1.1</td>
<td>0.2</td>
<td>4.1</td>
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<td>FG-A</td>
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<td>0.01</td>
<td>0.4</td>
<td>1.3</td>
<td>0.06</td>
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<tr>
<td>FG-B</td>
<td>1</td>
<td>0.9</td>
<td>0.03</td>
<td>0.08</td>
<td>0.09</td>
<td>0.01</td>
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</table>

---NN---  --------YN---------  --YY--

---NN---  --------YN---------  --YY--

Fig. 2. The six independent BB potentials for S-wave in the flavor SU(3) limit, extracted from the lattice QCD simulation at $m_\pi = 1014$ MeV (red bars) and $m_\pi = 835$ MeV (green crosses).


**OHEP + QCD-based Cores**  
(New versions)

---

**Relative Strengths of Short Range Cores in Even States**  
(Strengths in Odd States are free parameters)

<table>
<thead>
<tr>
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<th>{10^*}</th>
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<th>{8a}</th>
<th>{8s}</th>
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<td>0.2</td>
<td>4.1</td>
<td>-0.6</td>
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<tr>
<td>FG-A</td>
<td>1</td>
<td>0.8</td>
<td>0.01</td>
<td>0.4</td>
<td>1.3</td>
<td>0.06</td>
</tr>
<tr>
<td>FG-B</td>
<td>1</td>
<td>0.9</td>
<td>0.03</td>
<td>0.08</td>
<td>0.09</td>
<td>0.01</td>
</tr>
<tr>
<td>FG-C</td>
<td>1</td>
<td>0.8</td>
<td>1.1</td>
<td>0.2</td>
<td>3.0</td>
<td>-0.6</td>
</tr>
<tr>
<td>FG-D</td>
<td>1</td>
<td>0.8</td>
<td>1.1</td>
<td>0.2</td>
<td>3.0</td>
<td>-0.6</td>
</tr>
<tr>
<td>FG-D'</td>
<td>1</td>
<td>0.8</td>
<td>1.1</td>
<td>0.2</td>
<td>4.1</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

---NN------  ---YN------  ----YY----

NN sector is almost unchanged. This means that rich Phenomenological data inevitably lead us to the lattice QCD calculations.
NN phase shifts with FG-A to -D
(Comparison with PSA)

BE(d) MeV
(Exp=2.22590)
FG-A=2.22578
FG-B=2.22576
FG-C=2.22593
FG-D=2.22590
YN scattering cross sections (Comparison with experimental values)

\[
\begin{array}{c|c}
\text{calc} & \text{exp} \\
0.465 & 0.33 \pm 0.05(1958) \\
0.474 \pm 0.016(1968), & 0.465 \pm 0.011(1970)
\end{array}
\]

Inelastic Capture Ratio

\[
\frac{\sigma(\Sigma-p \rightarrow \Sigma 0n)}{\sigma(\Sigma-p \rightarrow \Sigma 0n) + \sigma(\Sigma-p \rightarrow \Lambda n)}
\]

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Comparison with OHEP and LQCD

ΛΛ−ΛΛ(1S0)

Our models: weak attraction

HAL-QCD calc. by T, Inoue et al. weak attraction

Fig. 3. BB potentials in baryon basis for S=-2, I=0, \(^1S_0\) sector. Three diagonal(off-diagonal) potentials are shown in left(right) panel. Phase of off-diagonal ones in the right panel are arranged in zoom-out plot. Their true signs are shown in the insertion.
Comparison with OHEP and LQCD

$\Sigma^+ p(1S0)$

- Our models: attraction
- HAL-QCD calc.: attraction

$\Sigma^+ p(3S1)$

- Our models: Strong repulsion
- HAL-QCD calc.: repulsion
Comparison with OHEP and LQCD

$E^{-n(1S0)}$

Our models: strong repulsion

HAL-QCD calc.: strong repulsion

$E^{-n(3S1)}$

Our models: repulsion

HAL-QCD calc.: repulsion?
Some Problems in BB interactions

(1) CSB in $\Lambda N$ interaction
   No direct evidence
   s-shell hypernuclei $\rightarrow$ CSB/CS=1.8% !
     (weak spin-dependence)
   p-shell hypernuclei $\rightarrow$ Even-Odd cancellation?
     (Many-body effects and $\Sigma$-components may be important)

(2) Effects of $\Lambda N-\Sigma N$ coupling
   Coherent $\Sigma$-mixing in neutron-rivh hypernuclei and
   in dense hadronic matter

(3) Properties (ex. spin-isospin dependence) of $\Sigma N$ interaction

(4) $\Lambda\Lambda-\Xi N$ interaction
   Attractive $\Lambda\Lambda$ interaction $\leftarrow$ double hypernuclei
   Attractive $\Xi N$ interaction $\leftarrow$ $\Xi$-hypernuclei

(5) S=-3,-4 interactions
   Existence of Deuteron-like bound states
Scattering length $a_{\Lambda\Lambda}(^{1}S_0)$ and $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})$ from double hypernuclei and theoretical $\Lambda\Lambda$ potentials

All models predict attractive $\Lambda\Lambda$ potential in $^{1}S_0$ state

<table>
<thead>
<tr>
<th>Potential</th>
<th>$a_{\Lambda\Lambda}(^{1}S_0)$</th>
<th>$\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSC97a</td>
<td>-0.33(-0.11)</td>
<td></td>
</tr>
<tr>
<td>NSC97e</td>
<td>-0.50(-0.25)</td>
<td></td>
</tr>
<tr>
<td>ESC04a</td>
<td>-1.15</td>
<td>1.36</td>
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<tr>
<td>ESC04d</td>
<td>-1.32</td>
<td>0.98</td>
</tr>
<tr>
<td>ESC08a</td>
<td>-0.88</td>
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</tr>
<tr>
<td>FSS</td>
<td></td>
<td>3.66</td>
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<td>fss2</td>
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<td>Ch-EFT</td>
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<tr>
<td>fg2014z</td>
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<td></td>
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<tr>
<td>fg2014w</td>
<td>-0.65</td>
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Nagara and Mikage Events (KEK E373)

$\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He}) = 0.70\pm0.17\text{MeV}$

H. Takahashi, PRL87(2001)212502
K. Nakazawa, NPA835(2010)207

$\Lambda\Lambda(1S0) \sim -0.8 \text{ fm}$

$\Lambda\Lambda(1S0) = -1.2\pm0.6\text{fm}$


J-PARC E07 experiments

NSC97, ESC-models:
Fg2014z,w : S.Shinmura, Ngo Thi Hong Xiem,(2014)
**Is ΞN interaction attractive or repulsive?**

### Ξ in symmetric nuclear matter

<table>
<thead>
<tr>
<th>potentials</th>
<th>$U_\Xi$</th>
<th>$U_\Xi$(n)</th>
<th>$U_\Xi$(p)</th>
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<tbody>
<tr>
<td>NSC97e</td>
<td>44.7(35.8)</td>
<td>9.69</td>
<td>34.98</td>
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<tr>
<td>ESC04a</td>
<td>15.1</td>
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<td>ESC04b</td>
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<td>ESC04d</td>
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<td>ESC08a</td>
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<td>FG-B</td>
<td>43.6(34.9)</td>
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<td>21.8</td>
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<td>GSOBEP</td>
<td>28.1(22.5)</td>
<td>17.6</td>
<td>10.5</td>
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<td>fg2014z</td>
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<td>fg2014w</td>
<td>15.6(12.5)</td>
<td>11.9</td>
<td>3.7</td>
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</tbody>
</table>

Theoretical ΞN Interactions give very scattered results.

Isospin dependence: strongly model-dependent

Recent observation (Kiso event)

$B_\Xi(^{15}_\Xi C)=4.38\pm0.25\text{MeV}$

K. Nakazawa, JPS-meeting,HAW2014, 2WF7(2014)

$U_\Xi \sim U_\Lambda/2$

Calculations (Shell model)

$B_\Xi(^{12}_\Xi \text{Be})=4.38\text{MeV}(\text{ESC04d})$

T. Motoba, S. Sugimoto NPA(2010)223

(J-PARC E05 (K$^-$,K$^+$), E07)
**$\Xi^- - n$ interaction**

$\Xi^- -$neutron part ← $V_{\Xi N}(\text{Isospin}=1)$

<table>
<thead>
<tr>
<th>$\text{pw}$</th>
<th>nsc97e</th>
<th>FG-A</th>
<th>FG-B</th>
<th>GSOBEPE</th>
<th>35w01p</th>
<th>35z01p</th>
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<tbody>
<tr>
<td>1S0</td>
<td>+4.40</td>
<td>+5.91</td>
<td>+8.22</td>
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<tr>
<td>3S1-3D1</td>
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<td>-0.58</td>
<td>-0.46</td>
<td>-0.79</td>
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<tr>
<td><strong>Total</strong></td>
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<td>-0.17</td>
<td>+21.80</td>
<td>+17.56</td>
<td>+16.42</td>
<td>+11.94</td>
</tr>
</tbody>
</table>

s-wave: repulsive!  
p-wave: attractive!

引力的( attractive ) 斥力的( repulsive )

Shinmura Shoji
\[ \Xi^- - p \text{ interaction} \]

\[ \Xi^- - \text{proton part} \leftarrow \frac{1}{2} V_{\Xi N}(\text{Isospin}=1) + \frac{1}{2} V_{\Xi N}(\text{Isospin}=0) \]

<table>
<thead>
<tr>
<th>pw</th>
<th>nsc97e</th>
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<th>FG-B</th>
<th>GSOBE P</th>
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<th>35z01p</th>
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<tr>
<td>3S1-3D1</td>
<td>+17.88</td>
<td>+2.66</td>
<td>+14.54</td>
<td>-0.26</td>
<td>+8.48</td>
<td>+8.44</td>
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<tr>
<td>3P0</td>
<td>+4.72</td>
<td>-0.34</td>
<td>+0.08</td>
<td>-0.16</td>
<td>-0.03</td>
<td>-0.46</td>
</tr>
<tr>
<td>1P1</td>
<td>+0.24</td>
<td>-2.61</td>
<td>-1.36</td>
<td>+1.26</td>
<td>-2.37</td>
<td>-1.95</td>
</tr>
<tr>
<td>3P1</td>
<td>+2.43</td>
<td>-3.85</td>
<td>+1.22</td>
<td>+2.54</td>
<td>-0.38</td>
<td>-1.64</td>
</tr>
<tr>
<td>3P2</td>
<td>+1.55</td>
<td>-4.69</td>
<td>-1.03</td>
<td>-4.42</td>
<td>-1.92</td>
<td>-4.01</td>
</tr>
<tr>
<td>L&gt;1</td>
<td>+0.17</td>
<td>-1.69</td>
<td>-0.17</td>
<td>+0.12</td>
<td>-0.73</td>
<td>-0.71</td>
</tr>
<tr>
<td>Total</td>
<td>+34.98</td>
<td>-6.48</td>
<td>+21.81</td>
<td>+10.53</td>
<td>+5.69</td>
<td>+3.69</td>
</tr>
</tbody>
</table>

引力的( attractive ) 斥力的( repulsive )

s-wave: repulsive!
p-wave: attractive!
### Hadron-Hadron (H-H) Interactions at Low Energies

#### Baryon-Baryon Interactions

<table>
<thead>
<tr>
<th>S</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$NN$</td>
</tr>
<tr>
<td>-1</td>
<td>$\Lambda N - \Sigma N$</td>
</tr>
<tr>
<td>-2</td>
<td>$\Xi N - \Lambda \Lambda - \Lambda \Sigma - \Sigma \Sigma$</td>
</tr>
<tr>
<td>-3</td>
<td>$\Xi \Lambda - \Xi \Sigma$</td>
</tr>
<tr>
<td>-4</td>
<td>$\Xi \Xi$</td>
</tr>
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</table>

#### Meson-Baryon Interactions

<table>
<thead>
<tr>
<th>S</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$KN$</td>
</tr>
<tr>
<td>0</td>
<td>$\pi N - \eta N - K \Lambda - K \Sigma$</td>
</tr>
<tr>
<td>-1</td>
<td>$K^\text{bar} N - \pi \Lambda - \pi \Sigma - \eta \Lambda - \eta \Sigma - K \Xi$</td>
</tr>
<tr>
<td>-2</td>
<td>$\pi \Xi - \eta \Xi - K^\text{bar} \Lambda - K^\text{bar} \Sigma$</td>
</tr>
<tr>
<td>-3</td>
<td>$K^\text{bar} \Xi$</td>
</tr>
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</table>

#### Meson-Meson Interactions

<table>
<thead>
<tr>
<th>S</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$KK$</td>
</tr>
<tr>
<td>1</td>
<td>$K \pi - K \eta$</td>
</tr>
<tr>
<td>0</td>
<td>$\pi \pi - K^\text{bar} K - \eta \pi - \eta \eta$</td>
</tr>
<tr>
<td>-1</td>
<td>$K^\text{bar} \pi - K^\text{bar} \eta$</td>
</tr>
<tr>
<td>-2</td>
<td>$K^\text{bar} K^\text{bar}$</td>
</tr>
</tbody>
</table>

### Coupled-Channel Problems

- Construction of Coupled-Channel Potentials
- Two-body systems
- Three-body systems
- Many-body systems
One-hadron-exchange model of meson-baryon interaction

Long-range part of potential: One Hadron Exchange

SU(3) symmetric Interaction Lagrangian
(mBB coupling constants are predetermined in BB potential model)

Gaussian Form factor with a common range
(Cutoff range is the same with the range of short-range potential.)

Short-range part of potential: Phenomenological
The SU(3)-symmetric Strengths
Common range for all mB pairs
We consider two cases of range

<table>
<thead>
<tr>
<th>pot I</th>
<th>pot II</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_G</td>
<td>0.4</td>
</tr>
</tbody>
</table>

V = (SU(3) symmetric strengths)×exp(-q^2/Λ^2) + V(one-hadron-exchange potential)×exp(-q^2/Λ^2)

where, L=2/r_G
**Results with our πN potential**

(Comparison with experimental values)

### πN Phase Shifts

![Phase Shift Graphs]

### πN Scattering Lengths

<table>
<thead>
<tr>
<th></th>
<th>calc</th>
<th>exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>rG</td>
<td>0.40</td>
<td>0.45</td>
</tr>
<tr>
<td>S11</td>
<td>+0.2458</td>
<td>+0.2482</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+0.2473±0.0043</td>
</tr>
<tr>
<td>S31</td>
<td>−0.1496</td>
<td>−0.1466</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−0.1444±0.0057</td>
</tr>
<tr>
<td>P11</td>
<td>−0.2359</td>
<td>−0.2340</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−0.2368±0.0058</td>
</tr>
<tr>
<td>P31</td>
<td>−0.1375</td>
<td>−0.1290</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−0.1316±0.0058</td>
</tr>
<tr>
<td>P13</td>
<td>−0.0862</td>
<td>−0.0894</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−0.0877±0.0058</td>
</tr>
<tr>
<td>P33</td>
<td>+0.6238</td>
<td>+0.6235</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+0.6257±0.0058</td>
</tr>
</tbody>
</table>

πN : t-channel exch. σ, f₀, ρ
u-channel exch. N, Δ, N’(1440), S₁₁(1567)
s-channel exch. N, Δ, N’(1440), S₁₁(1567)
Results with our KN potential
(Comparison with experimental values)

**KN phase shifts**

<table>
<thead>
<tr>
<th></th>
<th>calc</th>
<th>exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>rG</td>
<td>0.40</td>
<td>0.45</td>
</tr>
<tr>
<td>S01</td>
<td>-0.008</td>
<td>-0.013</td>
</tr>
<tr>
<td>S11</td>
<td>-0.365</td>
<td>-0.369</td>
</tr>
<tr>
<td>P01</td>
<td>+0.166</td>
<td>+0.179</td>
</tr>
<tr>
<td>P11</td>
<td>-0.106</td>
<td>-0.103</td>
</tr>
<tr>
<td>P03</td>
<td>-0.058</td>
<td>-0.071</td>
</tr>
<tr>
<td>P13</td>
<td>+0.047</td>
<td>+0.040</td>
</tr>
</tbody>
</table>

**KN scattering lengths**

- t-channel exch. : σ, f₀, a₀, ρ, ω, φ
- u-channel exch. : Λ, Σ
(No s-channel exchange diagram)
Results for $K^{\bar{\text{p}}}$ N scattering quantities

**$K^{-}\text{p}$ threshold data:**

<table>
<thead>
<tr>
<th></th>
<th>calc</th>
<th>exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rG$</td>
<td>0.40</td>
<td>0.45</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.35</td>
<td>2.36 $^{\pm}$.04</td>
</tr>
<tr>
<td>$R_C$</td>
<td>0.660</td>
<td>0.700 $^{\pm}$.011</td>
</tr>
<tr>
<td>$R_n$</td>
<td>0.189</td>
<td>0.172 $^{\pm}$.015</td>
</tr>
<tr>
<td>Re(a)</td>
<td>-0.666</td>
<td>-1.019 figure</td>
</tr>
<tr>
<td>Im(a)</td>
<td>0.462</td>
<td>0.398 figure (fm)</td>
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</tbody>
</table>

If Isospin-symmetric masses are used

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Re(a)</td>
<td>-0.354</td>
</tr>
<tr>
<td>Im(a)</td>
<td>0.453</td>
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</table>

New parameters are only two!

<table>
<thead>
<tr>
<th></th>
<th>{27}</th>
<th>{10*}</th>
<th>{10}</th>
<th>{8-1}+5/9</th>
<th>{8-2}</th>
<th>{8-2}</th>
<th>{1}</th>
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<tbody>
<tr>
<td>$\pi\text{N}$</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\text{KN}$</td>
<td>○</td>
<td>○</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$K^{\bar{\text{p}}}$ $\text{N}$</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Our potentials provide $\Lambda(1405)$ resonance as a single resonance

$\sqrt{s}=1393-16i$ (\(M=1393\text{MeV}, \Gamma=32\text{MeV}\))
for potential I (\(r_G=0.40\text{ fm}\))

$\sqrt{s}=1406-6i$ (\(M=1406\text{MeV}, \Gamma=12\text{MeV}\))
for potential II (\(r_G=0.45\text{ fm}\))
We constructed a potential model describing simultaneously Baryon-Baryon and Meson-Baryon Scattering. Based on SU(3)-symmetry and One-hadron-exchange mechanism

NN, YN, YY, πN, KN, K^{bar}N interactions at low energies,

We extend the potential to

\[
\begin{align*}
S=-2 & & \pi^{\bar{\Xi}} - K^{\bar{\Xi}} \Lambda^{\bar{\Xi}} - K^{\bar{\Xi}} \Sigma^{\bar{\Xi}} - \eta^{\bar{\Xi}} \\
S=-3 & & K^{\bar{\Xi}}
\end{align*}
\]

and discuss existence of S-wave resonances
Application to $K^{\bar{b}ar}$-hyperon scattering

Isospin Basis

$S=-2$ and $I=1/2$

$\pi\Xi + K^{\bar{b}ar}\Lambda + K^{\bar{b}ar}\Sigma + \eta\Xi$

(1458) (1611) (1688) (1867)

$S=-2$ and $I=3/2$

$\pi\Xi + K^{\bar{b}ar}\Sigma$

(1458) (1688)

$S=-3$ and $I=0$

$K^{\bar{b}ar}\Xi$

(1815)

$S=-3$ and $I=1$

$K^{\bar{b}ar}\Xi$

(1815)

Physical Mass Spectrum

(Charge Basis)
$S=-2$ and $I=1/2,3/2$

$\pi\Xi + K^{\text{bar}}\Lambda + K^{\text{bar}}\Sigma$

**S-wave phase shifts**

<table>
<thead>
<tr>
<th>$rG=0.40$, $I=1/2$</th>
<th>$rG=0.40$, $I=3/2$</th>
<th>$rG=0.45$, $I=1/2$</th>
<th>$rG=0.45$, $I=3/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{11}$</td>
<td>$\delta_{33}$</td>
<td>$\delta_{11}$</td>
<td>$\delta_{11}$</td>
</tr>
<tr>
<td>$\delta_{44}$</td>
<td>$\delta_{44}$</td>
<td>$\delta_{44}$</td>
<td>$\delta_{44}$</td>
</tr>
</tbody>
</table>

- $\delta_{11}$
- $\delta_{33}$
- $\delta_{44}$
Resonance Poles $\Xi^*(I=1/2, J^\pi=1/2^-)$

Cross sections $\sigma_{11}, \sigma_{33}, \sigma_{44}$

$r_G = 0.40$
$I = 1/2$

1: $\pi\Xi$
3: $K^{\text{bar}}\Lambda$
4: $K^{\text{bar}}\Sigma$

$s = 1510 - 73i$

$r_G = 0.45$
$I = 1/2$

1: $\pi\Xi$
3: $K^{\text{bar}}\Lambda$
4: $K^{\text{bar}}\Sigma$

$s = 1495 - 84i$
S=-3 and I=0 and 1 : $K^\text{bar} \Xi$ scattering

Isospin=0 state

S-Wave Phase Shifts

- Bound state pole ($\text{Im}(q)>0$) for $r_G=0.40$ at $E=1796\text{MeV}(\text{BE}=19\text{MeV})$
- Virtual state pole ($\text{Im}(q)<0$) for $r_G=0.45$ at $E=1802\text{MeV}(\text{"BE"}=13\text{MeV})$
The origin of the $K^\text{bar}$-Baryon Attractions

$= \text{Isospin-dependent } \rho\text{-contribution} + \text{large attractive } \omega\text{ contribution}$

On-shell Potential Values ($V/4\pi$) at 50MeV above each $K^\text{bar}$-Baryon threshold

<table>
<thead>
<tr>
<th>$K^\text{bar}$-B</th>
<th>Isospin</th>
<th>$\rho$</th>
<th>$\omega$</th>
<th>$\phi$</th>
<th>scalar</th>
<th>Baryon</th>
<th>Short</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\text{bar}N$</td>
<td>0</td>
<td>-42.4</td>
<td><strong>-91.7</strong></td>
<td>20</td>
<td>-25.0</td>
<td>22.4</td>
<td>-44.9</td>
<td>-161.5</td>
</tr>
<tr>
<td>$K^\text{bar}N$</td>
<td>1</td>
<td>14.1</td>
<td><strong>-91.7</strong></td>
<td>20</td>
<td>-25.3</td>
<td>156.5</td>
<td>11.5</td>
<td>85.2</td>
</tr>
<tr>
<td>$K^\text{bar}\Lambda$</td>
<td>1/2</td>
<td>0</td>
<td><strong>-84.7</strong></td>
<td>49.6</td>
<td>-28.7</td>
<td>6.5</td>
<td>-12.1</td>
<td>-69.4</td>
</tr>
<tr>
<td>$K^\text{bar}\Sigma$</td>
<td>1/2</td>
<td>-78.4</td>
<td><strong>-87.5</strong></td>
<td>55.9</td>
<td>-27.1</td>
<td>30.7</td>
<td>18.5</td>
<td>-87.9</td>
</tr>
<tr>
<td>$K^\text{bar}\Sigma$</td>
<td>3/2</td>
<td>39.2</td>
<td><strong>-87.5</strong></td>
<td>55.9</td>
<td>-30.0</td>
<td>0</td>
<td>-2.2</td>
<td>-24.6</td>
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<tr>
<td>$K^\text{bar}\Xi$</td>
<td>0</td>
<td>-69.4</td>
<td><strong>-70.3</strong></td>
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<td>-28.7</td>
<td>14.2</td>
<td>7.1</td>
<td>-56.4</td>
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<tr>
<td>$K^\text{bar}\Xi$</td>
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<td>23.1</td>
<td><strong>-70.3</strong></td>
<td>90.7</td>
<td>-32.7</td>
<td>5.2</td>
<td>14.2</td>
<td>30.4</td>
</tr>
</tbody>
</table>

Scalar mesons provide almost constant attraction ($\sim -30\text{MeV}$)
# Hadron-Hadron (H-H) Interactions at Low Energies

## Baryon-Baryon Interactions

| S= 0 | NN |
| S= -1 | ΛN–ΣN |
| S= -2 | ΞN–ΛΛ–ΛΣ–ΣΣ |
| S= -3 | ΞΛ–ΞΣ |
| S= -4 | ΞΞ |

## Meson-Baryon Interactions

| S= 1 | KN |
| S= 0 | πN–ηN–ΚΛ–ΚΣ |
| S= -1 | K\(^\text{bar}\)N–πΛ–πΣ–ηΛ–ηΣ–ΚΞ |
| S= -2 | πΞ–ηΞ–Κ\(^\text{bar}\)Λ–Κ\(^\text{bar}\)Σ |
| S= -3 | K\(^\text{bar}\)Ξ |

## Meson-Meson Interactions

| S= 2 | KK |
| S= 1 | Kπ–Κη |
| S= 0 | ππ–K\(^\text{bar}\)K–ηπ–ηη |
| S= -1 | K\(^\text{bar}\)π–K\(^\text{bar}\)η |
| S= -2 | K\(^\text{bar}\)K\(^\text{bar}\) |

---

**Coupled-Channel Problems**

Construction of Coupled-Channel Potentials

- Two-body systems
- Three-body systems
- Many-body systems
One-meson-exchange model of meson-meson interactions

SU(3) symmetric 3-meson interaction Lagrangian
Consistent with meson-baryon (mB) potential model

ps-ps-vector (used in mB potential)

\[ L_{ppv} = g_{ppv} \text{Tr} \left[ \left( \left( \partial^\mu P \right) P - P \left( \partial P^\mu \right) \right) V_\mu \right] \]

ps-ps-scalar (used in mB potential)

\[ L_{pps} = \left( f_{pps} / m_\pi \right) \text{Tr} \left[ \left( \partial^\mu P \partial P_\mu \right) S \right] \]

ps-ps-tensor (not used in mB potential)

\[ L_{ppt} = \left( 2g_{ppt} / m_\pi \right) \text{Tr} \left[ \left( \partial^\mu P \partial P^\nu \right) T_{\mu \nu} \right] \]

Exchange Diagrams:
  t- and u-channel exchange
  s-channel exchange

Form factor
  Monopole type
  Gaussian type
# Cutoff Form factor

## Monopole form factors

For t-, u-channel exchange of vector mesons

\[ F(q) = \frac{\Lambda^2 + m_v^2}{\Lambda^2 + q^2} \]

For s-channel exchange of vector mesons

\[ F(\omega_p) = \frac{\Lambda^2 + m_v^2}{\Lambda^2 + \omega_p^2} \]

For s-channel exchange of scalar and tensor mesons

\[ F(\omega_p) = \frac{\Lambda^4 + m_v^4}{\Lambda^4 + \omega_p^4} \]

## Gaussian form factors

For t-, u-channel exchange

\[ F(q) = \exp\left(-\frac{q^2}{\Lambda^2}\right) \]

For s-channel exchange

\[ F(\omega_p) = \exp\left(-\frac{\omega_p^2}{\Lambda^2}\right) \]
**ππ-K^\bar{\text{K}}$$\eta$$** Isospin=0, s- and d-waves

Resonances:

- **monopole**
  - \( f_0 \): 1000-i20, 1075-i170, (970-1010)-i(20-50)
  - \( \sigma_1 \): 580-i380, 430-i380, (400-550)-i(200-350)
  - \( \sigma_2 \): 410-i560, 390-i500

- **gaussian**
  - \( f_0 \): 1000-i20, 1075-i170, (970-1010)-i(20-50)
  - **exp**

Resonance:

- **monopole**
  - \( f_2 \): 1270-i110, 1250-i90, (1275.1±1.2)-i93

- **gaussian**
  - **exp**
S=0, Isospin=0 and 1, p-wave

$K^{\bar{b}ar}K(\text{Isospin}=0)$

resonance:
- monopole
- gaussian
- $\exp$

$\phi$ 1016.5-i1.6 1022.5-i1.6 1019-i2.1

$\pi\pi-K^{\bar{b}ar}K-\pi\eta(\text{Isospin}=1)$

resonance:
- monopole
- gaussian
- $\exp$

$\rho$ 800-i70 800-i60 775-i74
\( \pi \eta - K^{\text{bar}} K \) Isospin=1, s-wave

resonance:
- monopole
- gaussian
- \( a_0 \) 845-i15 800-i15 980-i(25-50)
$K\pi-K\eta$, Isospin=1/2, s- and p-wave

resonances:
- Monopole: $\kappa$ 1450-i75, 1440-i35, (1375-1475)-i(95-175)
- Gaussian: $\kappa$ 650-i230, 650-i190, (653-711)-i270

resonance:
- Monopole: $K^*$ 907-i20, 910-i18, 892-i25
Summary

(1) Model of hadron-hadron interactions:
   Long-range part: One-hadron-exchange mechanism with the SU(3) symmetry
   Short-range part: QCD-based potential gives a good description of baryon-baryon, meson-baryon, meson-meson interactions.

(2) Coupled-channel dynamics in two-body, three-body and many-body systems provides interesting physics.

(3) Some Problems
   (a) CSB component in $\Lambda N$ interaction
   (b) Effect of $\Lambda - \Sigma$ mixing in hypernuclei and dense hadronic matter
   (c) Properties of $\Lambda \Lambda - \Xi N$ interaction
   (d) Existence of hadron resonances (two-body or three-body)
       $K^{\text{bar}} N, \ K^{\text{bar}} \Lambda, \ K^{\text{bar}} \Sigma, \ K^{\text{bar}} \Xi, \ K^{\text{bar}} K, \ K^{\text{bar}} \pi, \ \pi \pi$