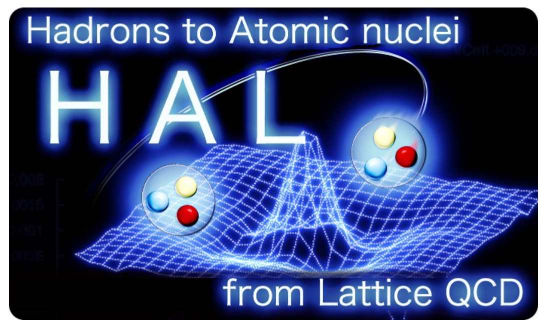
Coupled channel approach to two-baryon interactions from QCD

Kenji Sasaki (*CCS, University of Tsukuba*)

for HAL QCD collaboration



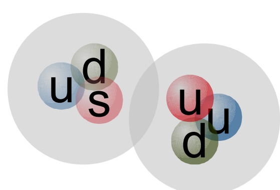
HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (*YITP*)	**T. Doi** (*RIKEN*)	**F. Etminan** (*Birjand U.*)	**T. Hatsuda** (*RIKEN*)	**Y. Ikeda** (*RIKEN*)
T. Inoue (*Nihon Univ.*)	**N. Ishii** (*RCNP*)	**K. Murano** (*RCNP*)	**H. Nemura** (*Univ. of Tsukuba*)	**T. Miyamoto** (*YITP*)
T. Iritani (*YITP*)	**S. Gongyo** (*YITP*)	**D. Kawai** (*YITP*)		

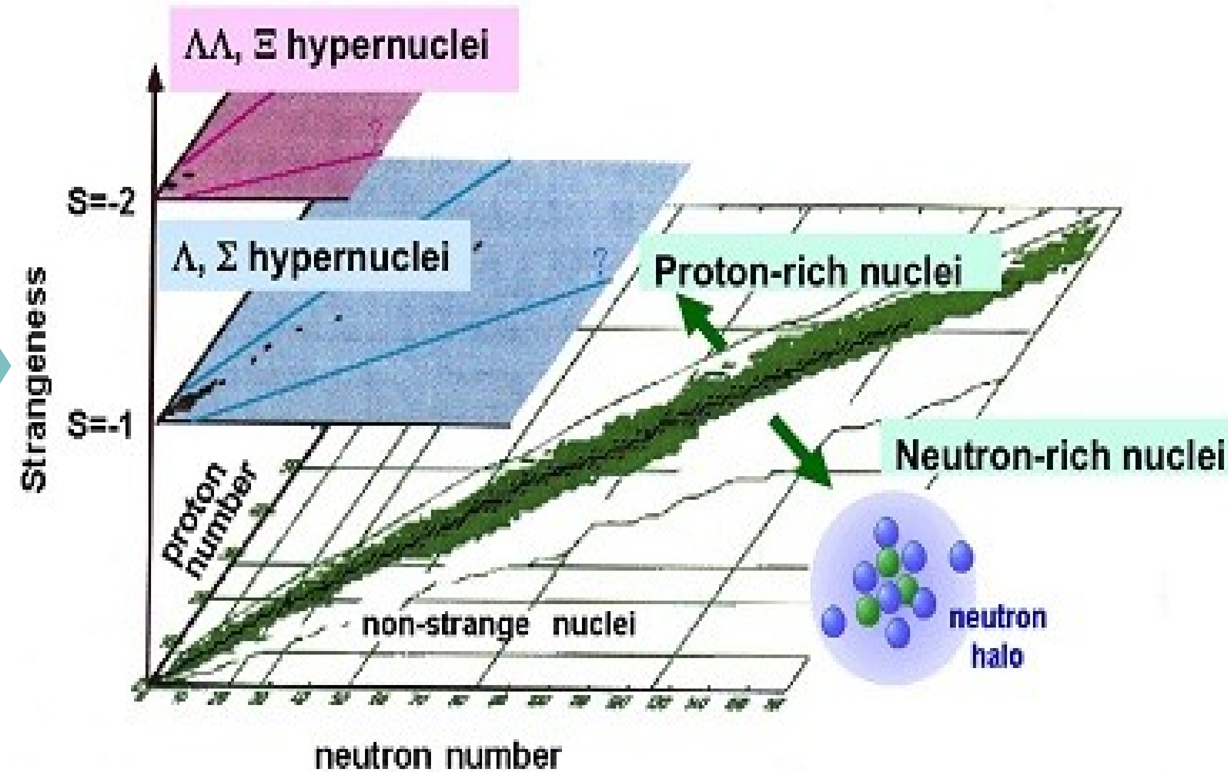
Introduction

Introduction

BB interactions are inputs to investigate the nuclear structure



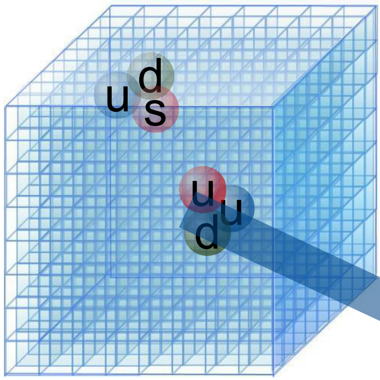
Nuclear force
Hyperon force



Once we obtain a “proper” nuclear potential,
we apply them to the structure of (hyper-) nucleus.

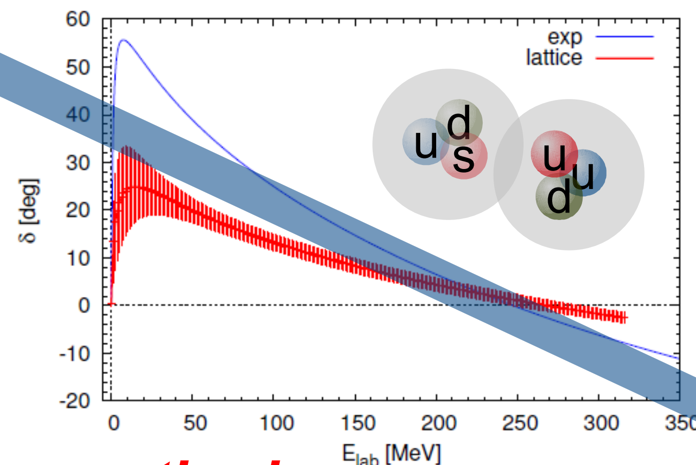
Can we derive hadronic interactions from QCD?

Strategy of HAL QCD



Technical improvements

- **Unified Contraction Algorithm,**
- **Time dependent method,**
- **Higher partial waves,**
- **Finite volume method vs potential**

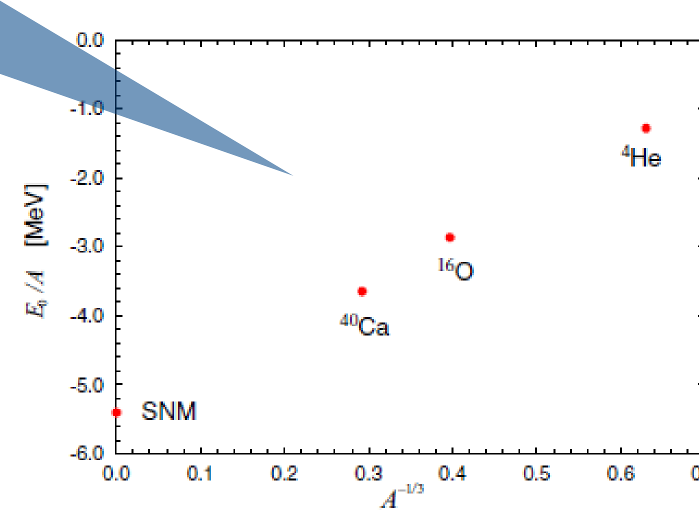


Applications

- **Few-body system**
- **Medium heavy system**
- **Neutron star EOS**

Extensions of the method

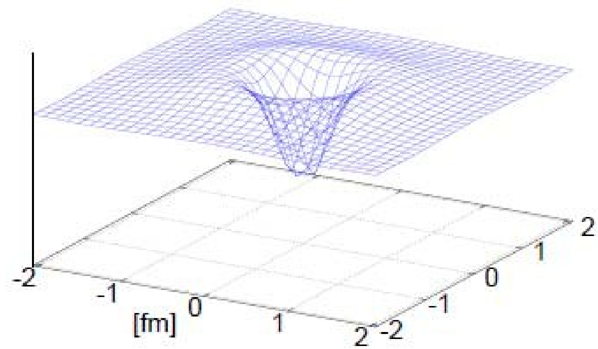
- **Generalized BB interaction**
- **N–Ω, Ω–Ω interaction**
- **Generalized Dec-Dec interaction**
- **Charmed baryon system**
- **Meson-meson,meson-baryon system**
- **Three-body interaction**



HAL QCD method

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.



$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0|H_1^{\alpha}(t,\vec{x}+\vec{r})H_2^{\alpha}(t,\vec{x})|E\rangle$$

E : Total energy of the system

Local composite interpolating operators

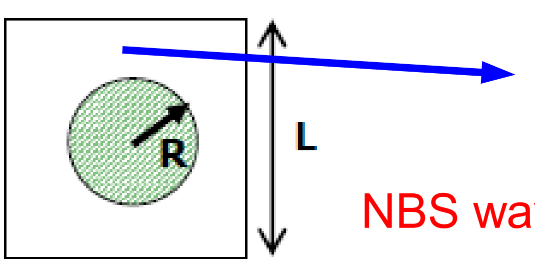
$$B_{\alpha}=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_{c\alpha} \qquad D_{\mu\alpha}=\epsilon^{abc}(q_a^T C\gamma_{\mu} q_b)q_{c\alpha}$$

$$M=(\bar{q}_a\gamma_5 q_a)$$ Etc.....

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- Using the reduction formula,

C.-J.D.Lin et al.,NPB619 (2001) 467.

$$\Psi^{\alpha}(E,\vec{r}) = \sqrt{Z_{H_1}}\sqrt{Z_{H_2}}\left(e^{i\vec{p}\cdot\vec{r}}+\int\frac{d^3q}{2E_q}\frac{T(q,p)}{4E_p(E_q-E_p-i\epsilon)}e^{i\vec{q}\cdot\vec{r}}\right)$$



$$\Psi(E,\vec{r})\simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Phase shift is defined as

$$S\equiv e^{i\delta}$$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

Nambu-Bethe-Salpeter wave function

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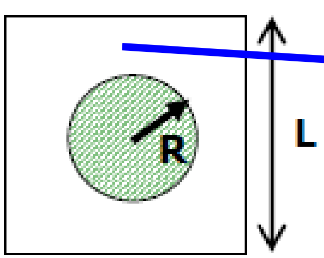

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Potential in HAL QCD method

We define potentials which satisfy Schrödinger equation

$$\left(p^2+\nabla^2\right)\Psi^\alpha(E,\vec{r}) \equiv \int d^3y\, \underline{U^\alpha_\alpha(\vec{x},\vec{y})}\, \Psi^\alpha(E,\vec{y})$$

Energy independent potential

$$\left(p^2+\nabla^2\right)\Psi^\alpha(E,\vec{r}) = K^\alpha(E,\vec{r})$$

$$\begin{aligned} K^\alpha(E,\vec{r}) &\equiv \int dE'\, K^\alpha(E',\vec{x}) \underline{\int d^3y\, \tilde{\Psi}^\alpha(E',\vec{y})\Psi^\alpha(E,\vec{y})} \\ &= \int d^3y \underline{\left[\int dE'\, K^\alpha(E',\vec{x})\, \tilde{\Psi}^\alpha(E',\vec{y})\right]}\Psi^\alpha(E,\vec{y}) \\ &= \int d^3y\, U^\alpha_\alpha(\vec{x},\vec{y})\, \Psi^\alpha(E,\vec{y}) \end{aligned}$$

We can define an energy independent potential but it is fully non-local.

This potential automatically reproduce the scattering phase shift

N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent method

Start with the normalized four-point correlator.

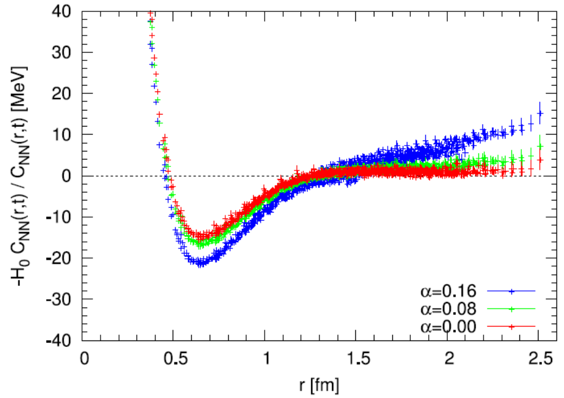
$$R_I^{B_1B_2}(t,\vec{r}) = F_{B_1B_2}(t,\vec{r})e^{(m_1+m_2)t}$$

$$= A_0\Psi(\vec{r},E_0)e^{-(E_0-m_1-m_2)t} + A_1\Psi(\vec{r},E_1)e^{-(E_1-m_1-m_2)t} + \cdots$$

Each wave functions satisfy Schrödinger eq. with proper energy

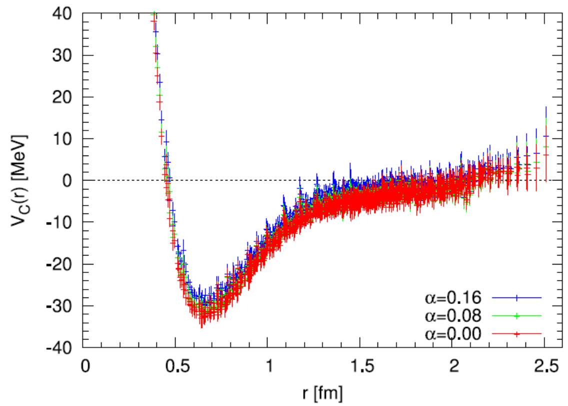
$$\left(\frac{p_0^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_0) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_0)d^3r'$$

$$\left(\frac{p_1^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_1) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_1)d^3r'$$



$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$



A single state saturation is not required!!

BB interaction from NBS wave function

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

Derivative (velocity) expansion of U is performed to deal with its nonlocality.

- For the case of oct-oct system,

$$U(\vec{r},\vec{r}\,') = \left[V_C(r)+S_{12}V_T(r)\right]+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

Leading order part

- For the case of dec-oct and dec-dec system,

$$U(\vec{r},\vec{r}\,') = \left[V_C(r)+S_{12}V_{T_1}(r)+S_{ii}V_{T_2}(r)+O(Spin\,op^3)\right]+O(\nabla^2)$$

Leading order part

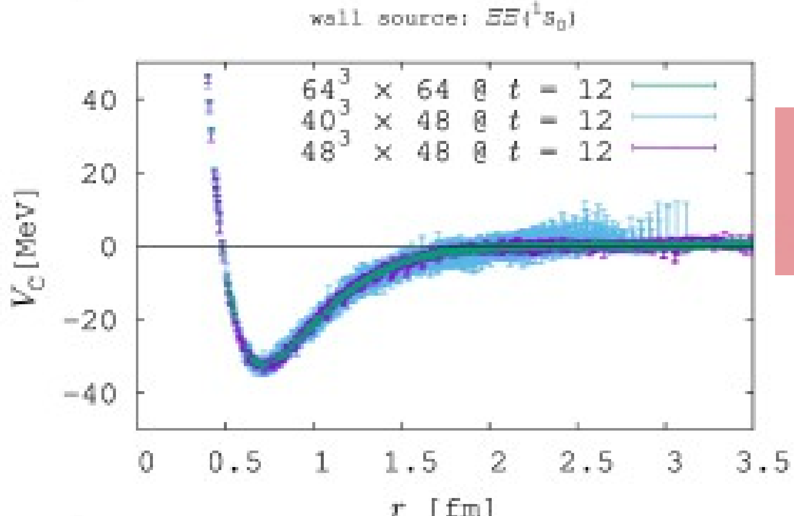
$$\equiv \left[V_C^{eff}(r)\right]+O(\nabla^2)$$

$$((\vec{r}\cdot\vec{S_1})^2-\frac{\vec{r}^2}{3}\vec{S_1}^2+(\vec{r}\cdot\vec{S_2})^2-\frac{\vec{r}^2}{3}\vec{S_2}^2)V_{T^2}(r)$$

We consider the effective central potential which contains not only the genuine central potential but also tensor parts.

Phase shift from potential and FV method

▶ Energy eigenvalues in finite volume



Solution in FV

T.Iritani(HAL QCD) Lattice2015

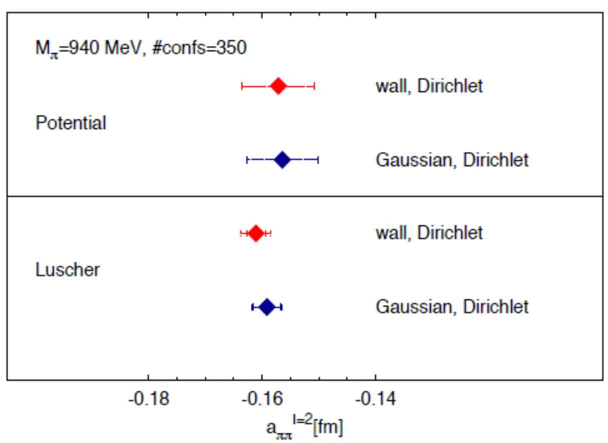
energy eigenvalues

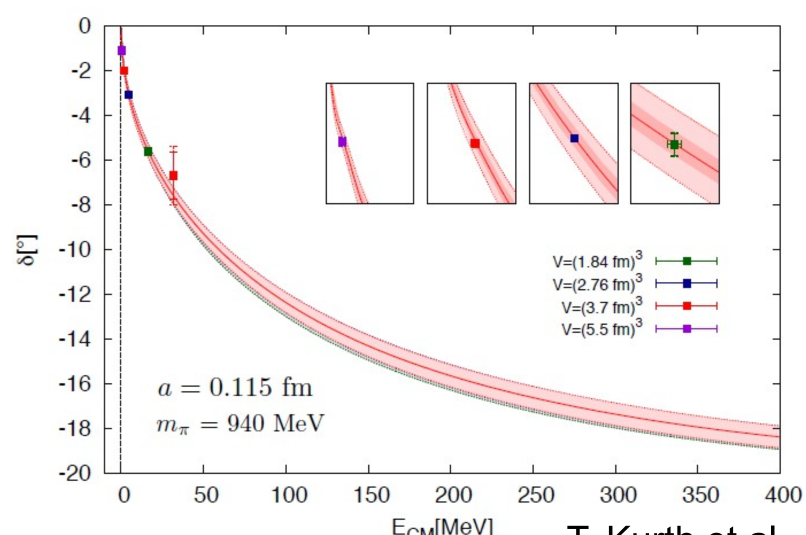
vol.	E(g.s.) [MeV]	E(1st) [MeV]
40^3	$-4.55(1.18)$	$75.63(1.31)$
48^3	$-2.58(22)$	$52.87(33)$
64^3	$-1.13(9)$	$28.71(9)$

- consistent with "wall src." $\Delta E(t)$ fit
  $-2.25(1.28)$ MeV @ $48^3 \times 48$

▶ Comparison between the potential method and Lüscher's method

π-π scattering with quench QCD





T. Kurth et al., JHEP1312 (2013)

Resulting scattering phase shifts are consistent from both methods

Coupled channel Schrödinger equation

NBS wave function with ith energy eigen state

$$\Psi^{\alpha}(E_i,\vec{r}) = \langle 0|(B_1 B_2)^{\alpha}(\vec{r})\,|E_i\rangle$$

$$\Psi^{\beta}(E_i,\vec{r}) = \langle 0|(B_1 B_2)^{\beta}(\vec{r})\,|E_i\rangle$$

Two-channel coupling case

$$\int dr\, \tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\,\delta_{\beta}{}^{\gamma}$$

We define potentials which satisfy a coupled channel Schrodinger equation

$$\begin{pmatrix}(p_\alpha^2+\nabla^2)\Psi^{\alpha}(E_i,\vec{r})\\ (p_\beta^2+\nabla^2)\Psi^{\beta}(E_i,\vec{r})\end{pmatrix} = \int dr' \begin{pmatrix} U^{\alpha}{}_{\alpha}(\vec{r},\vec{r}\,') & U^{\alpha}{}_{\beta}(\vec{r},\vec{r}\,') \\ U^{\beta}{}_{\alpha}(\vec{r},\vec{r}\,') & U^{\beta}{}_{\beta}(\vec{r},\vec{r}\,')\end{pmatrix}\begin{pmatrix}\Psi^{\alpha}(E_i,\vec{r}\,')\\ \Psi^{\beta}(E_i,\vec{r}\,')\end{pmatrix}$$

Leading order of velocity expansion and time-derivative method

$$\begin{pmatrix}\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\alpha}\right)R^{\alpha}_{E_0}(t,\vec{r})\\ \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta}\right)R^{\beta}_{E_0}(t,\vec{r})\end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r})\end{pmatrix}\begin{pmatrix}R^{\alpha}_{E_0}(t,\vec{r})\\ R^{\beta}_{E_0}(t,\vec{r})\end{pmatrix}$$

$$\Delta_{\beta}^{\alpha} = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

Considering two different energy eigen states

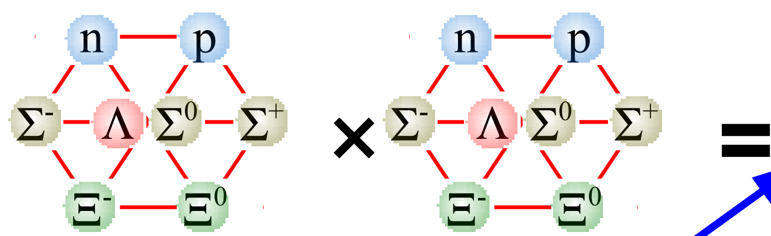
$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r})\end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E1}(t,\vec{r}) \\ \left(\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t}\right)R^{\beta}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t}\right)R^{\beta}_{E1}(t,\vec{r})\end{pmatrix}\begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r})\end{pmatrix}^{-1}$$

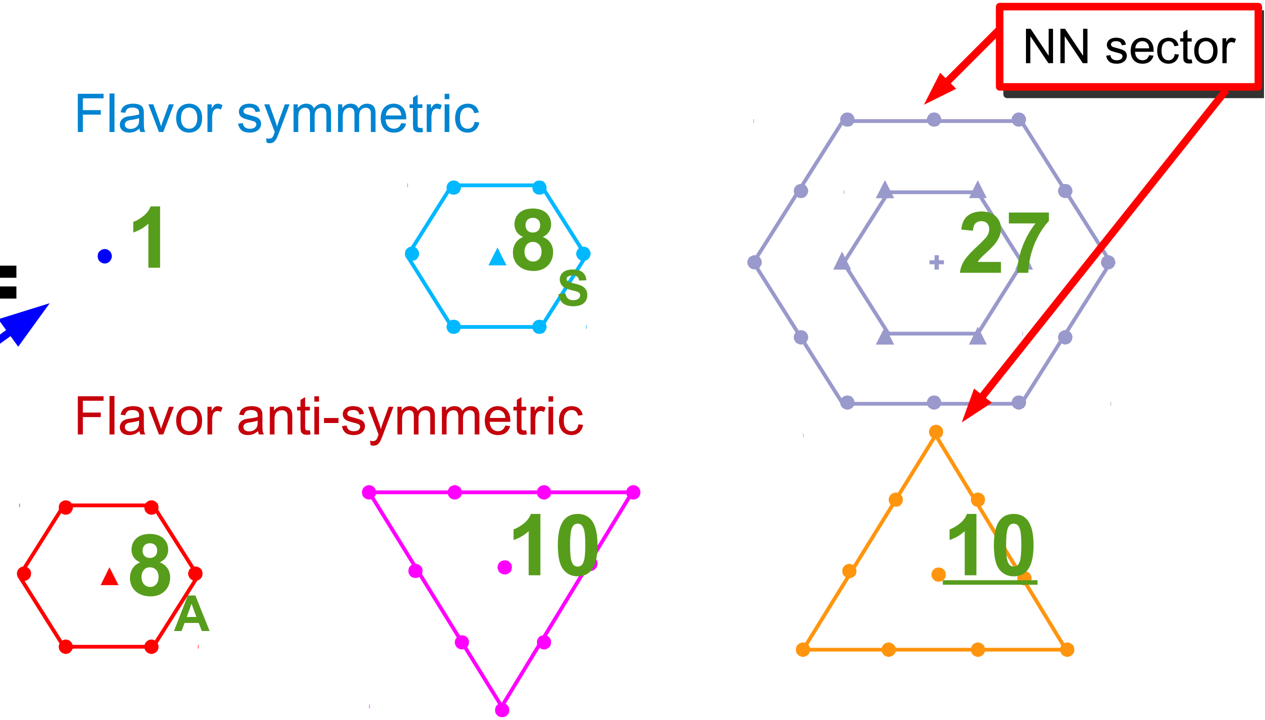
S=-2 BB interaction

--- focus on the H-dibaryon ---

SU(3) feature of BB interaction

Three flavor (u,d,s) world





Strong attraction is expected.

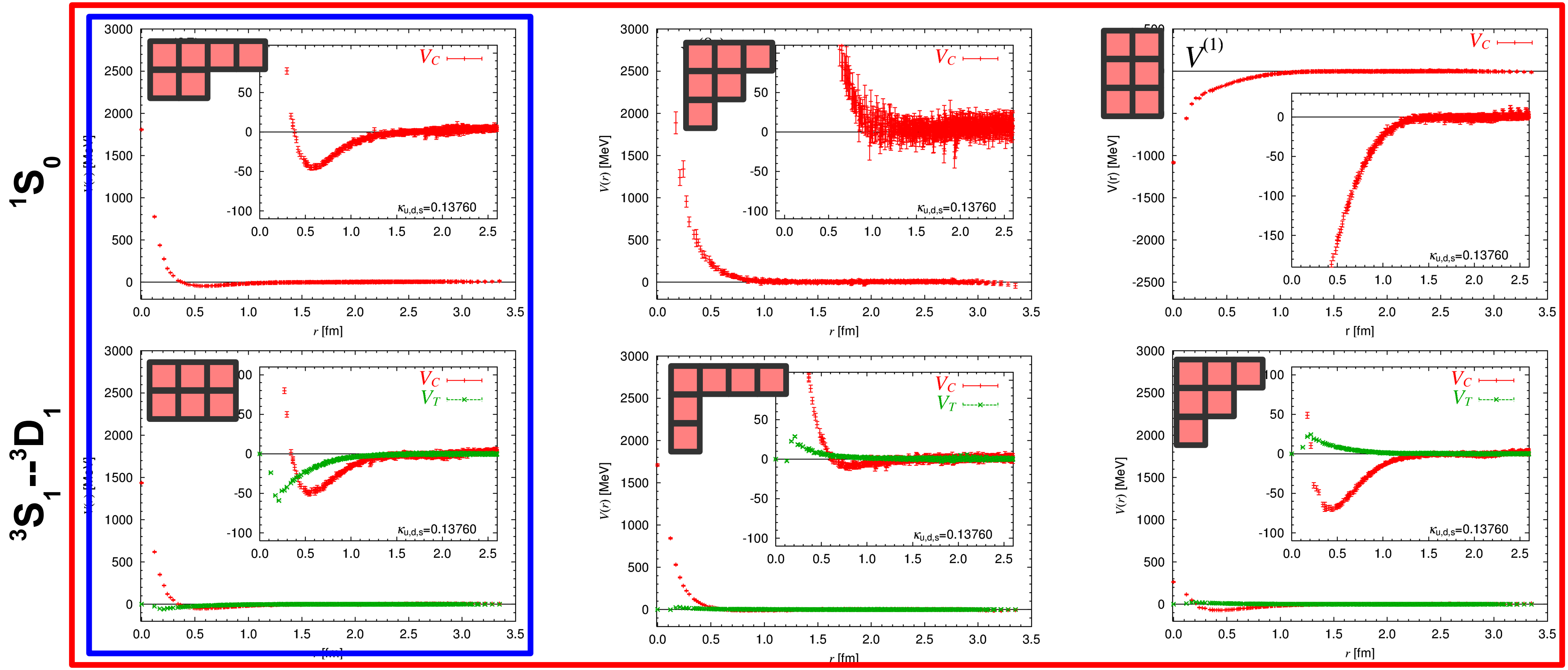
In view of quark degrees of freedom

Oka, Shimizu and Yazaki NPA464 (1987)

- Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.
- Strengths of repulsive core in YN and YY interaction are largely depend on their flavor structures.
- For the s-wave BB system, no repulsive core is predicted in flavor singlet state which is known as H-dibaryon channel.

B-B potentials in SU(3) limit

m_π= 469MeV

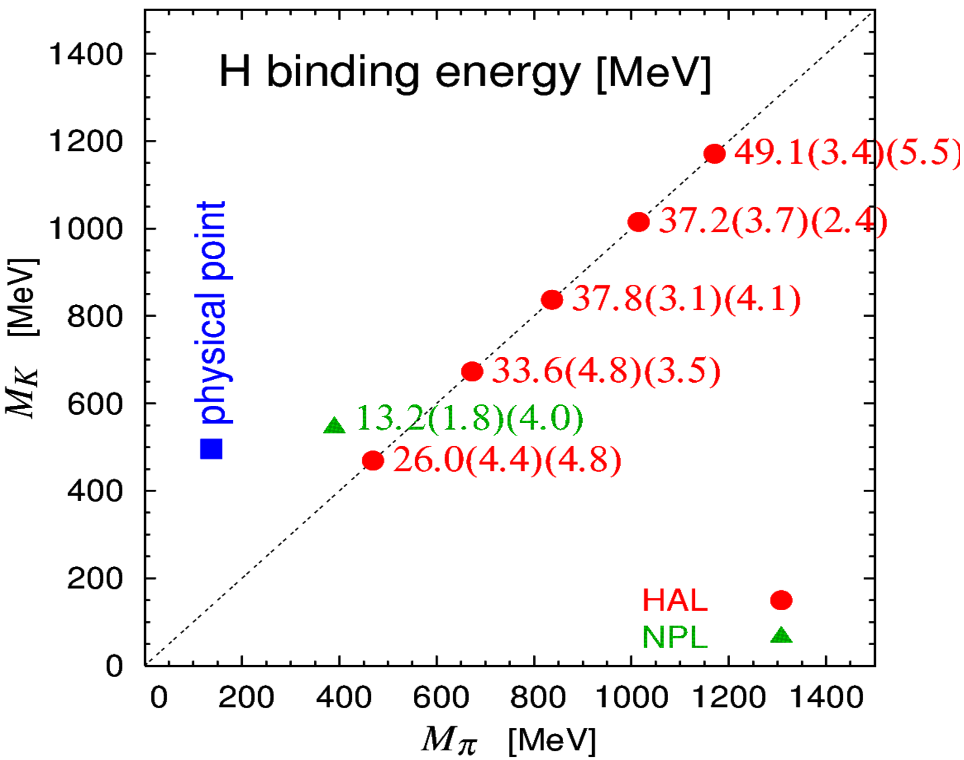


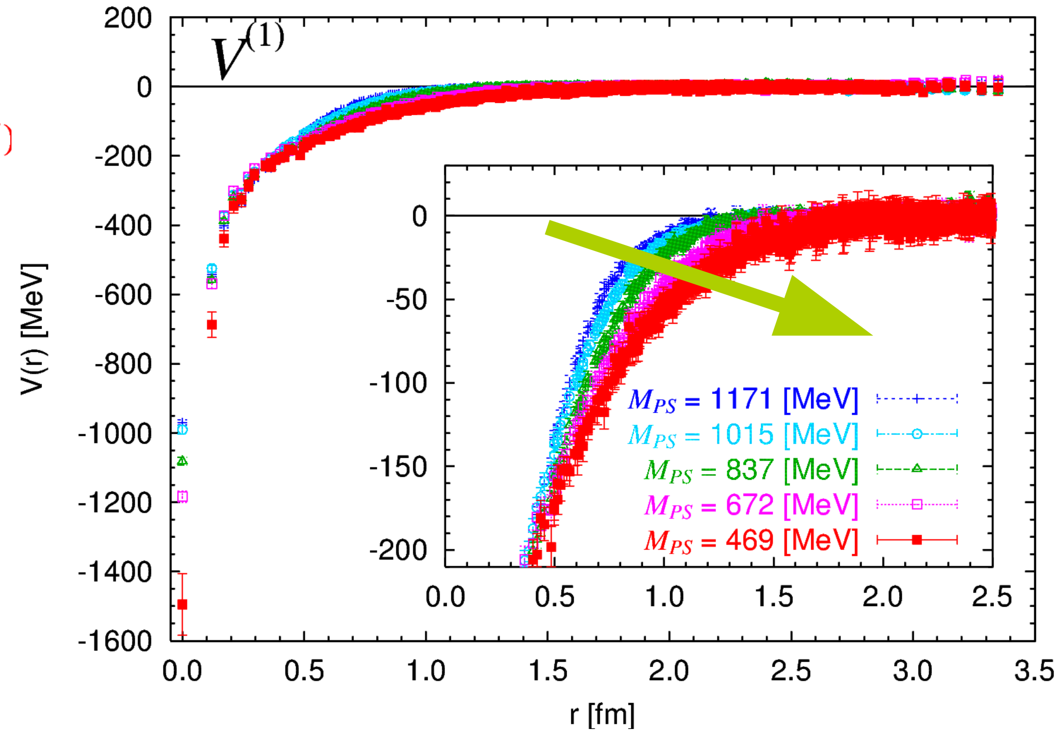
Two-flavors

Three-flavors

- Quark Pauli principle can be seen at around short distances
  - No repulsive core in flavor singlet state
  - Strongest repulsion in flavor 8s state
- Possibility of bound H-dibaryon in flavor singlet channel.

H-dibaryon (unphysical situation)





HAL : PRL106(2011)162002
NPL : PRL106(2011)162001

- Both results shows the bound H-dibaryon state in heavy pion region.
- Potential in flavor singlet channel is getting more attractive as decreasing quark masses

Does the H-dibaryon state survive on the physical point?

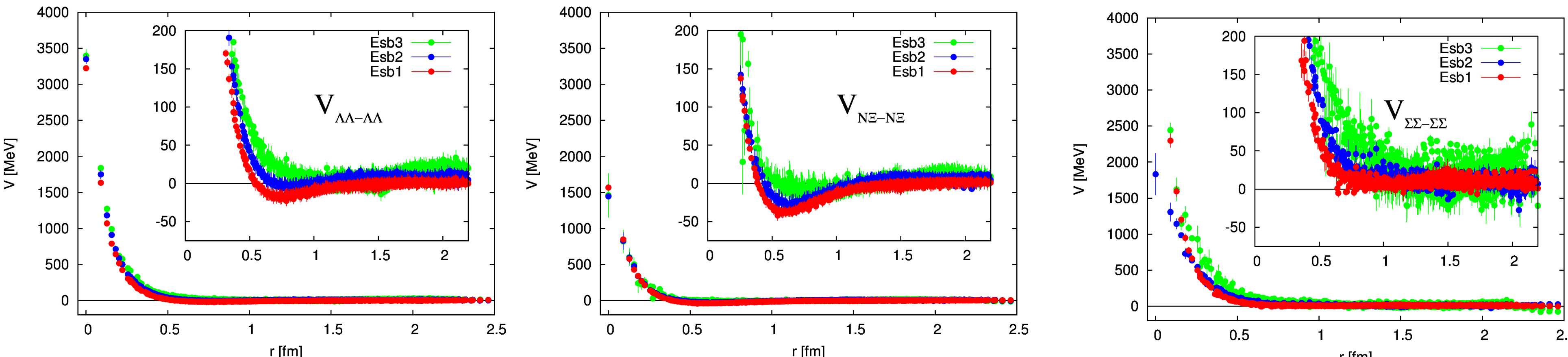
→ **Go to the SU(3) broken situation.**

$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

$m\pi$= 701 MeV
$m\pi$= 570 MeV
$m\pi$= 411 MeV

- **N_f = 2+1 full QCD with L = 2.9fm**

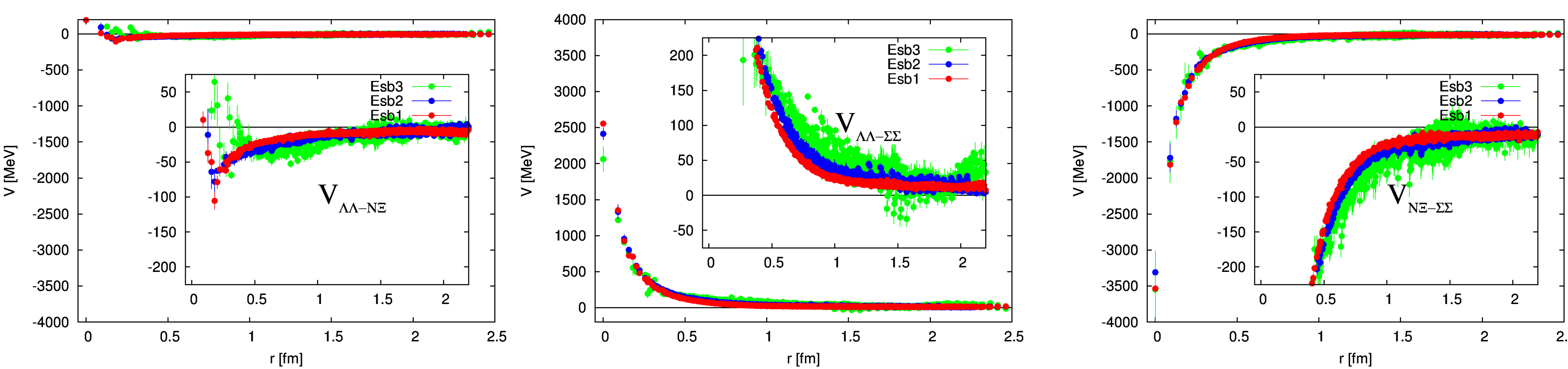
Diagonal elements



shallow attractive pocket — Deeper attractive pocket — Strongly repulsive

All channels have repulsive core

Off-diagonal elements

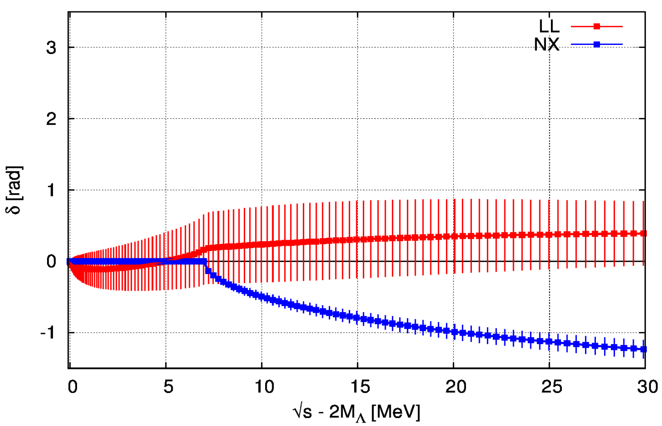


ΛΛ and NΞ phase shifts

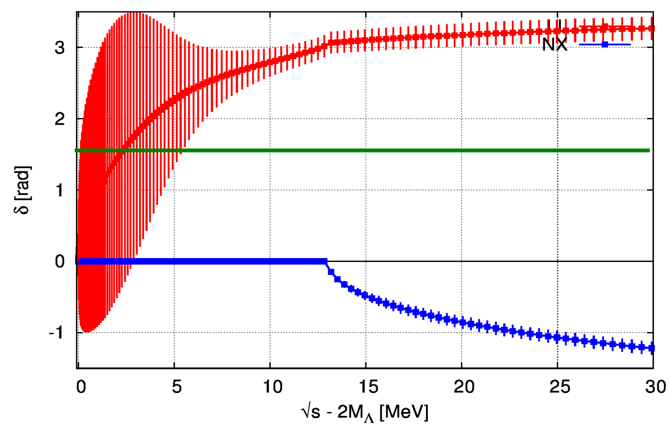
- **N_f = 2+1 full QCD with L = 2.9fm**

Preliminary!

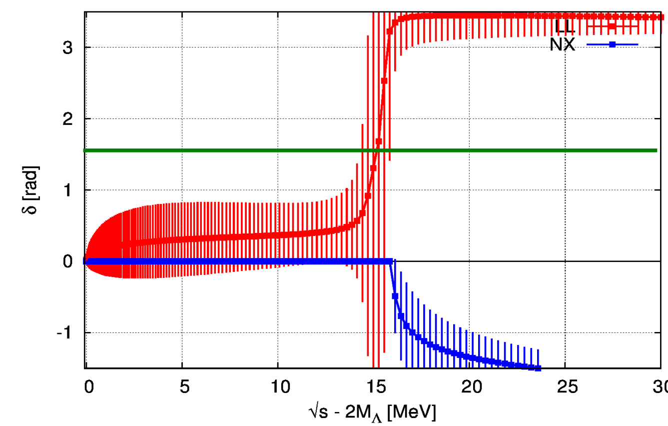
mπ = 700 MeV



mπ = 570 MeV

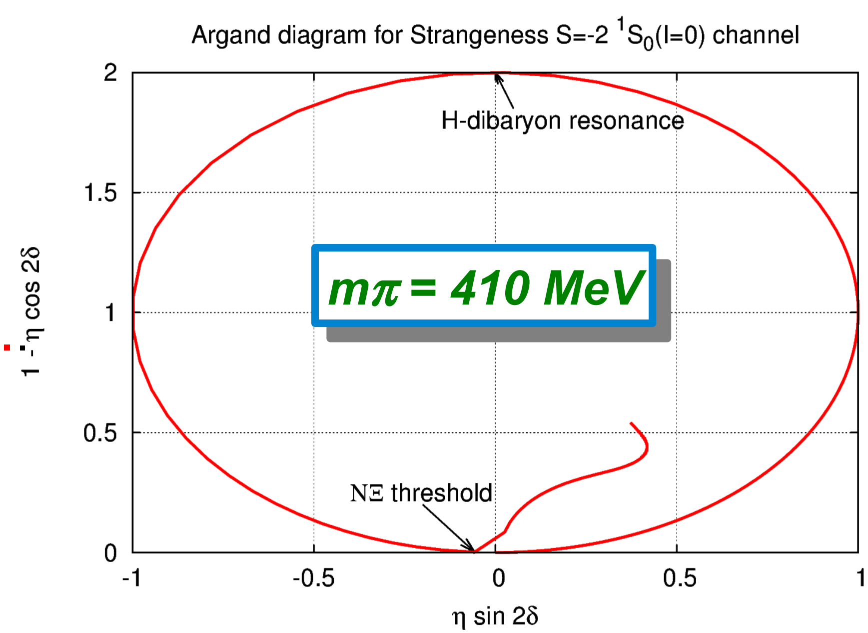


mπ = 410 MeV



- ***mπ = 700 MeV :*** bound state
- ***mπ = 570 MeV*** : resonance near ΛΛ threshold
- ***mπ = 410 MeV*** : resonance near NΞ threshold..

H-dibaryon is unlikely bound state

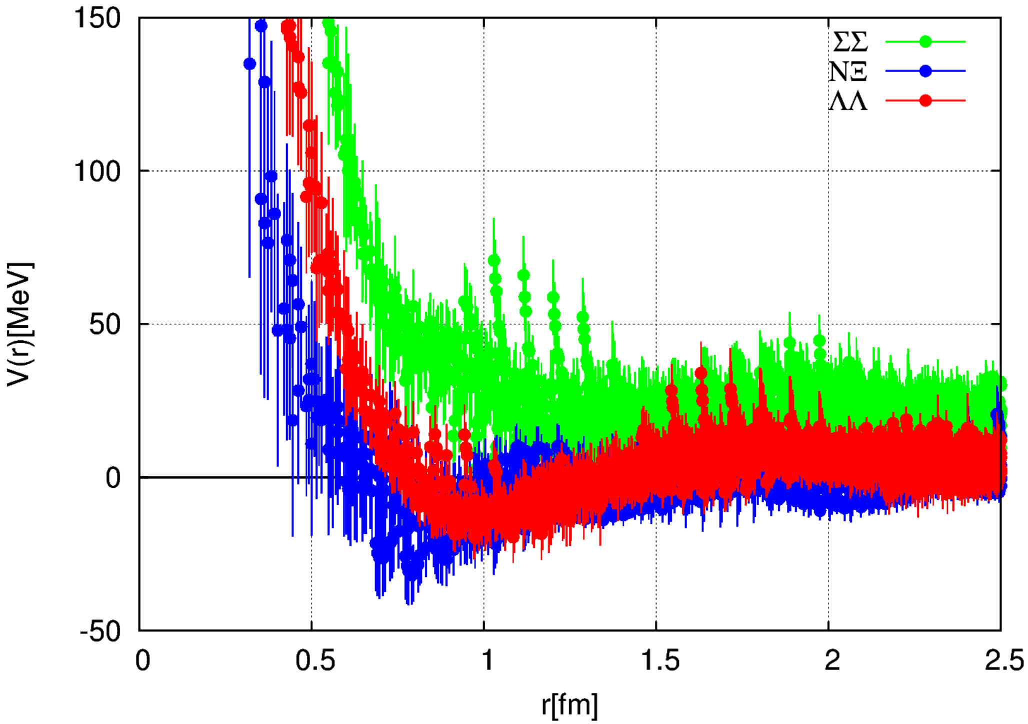


ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ *channel near the physical point*

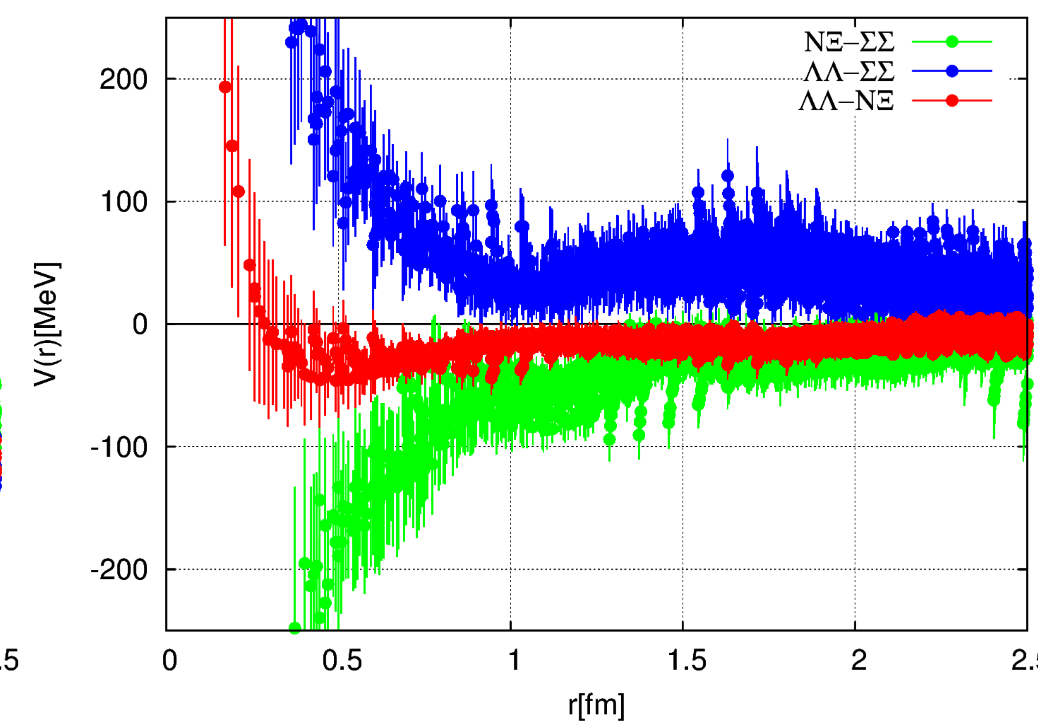
- **N_f = 2+1 full QCD with L = 8fm, *mπ = 145 MeV***

Preliminary!

Diagonal elements



Off-diagonal elements



- All diagonal element have a repulsive core ΣΣ–ΣΣ potential is strongly repulsive.
- Off-diagonal potentials are relatively strong except for ΛΛ–NΞ transition
- We need more statistics to discuss physical observables through this potential.

Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

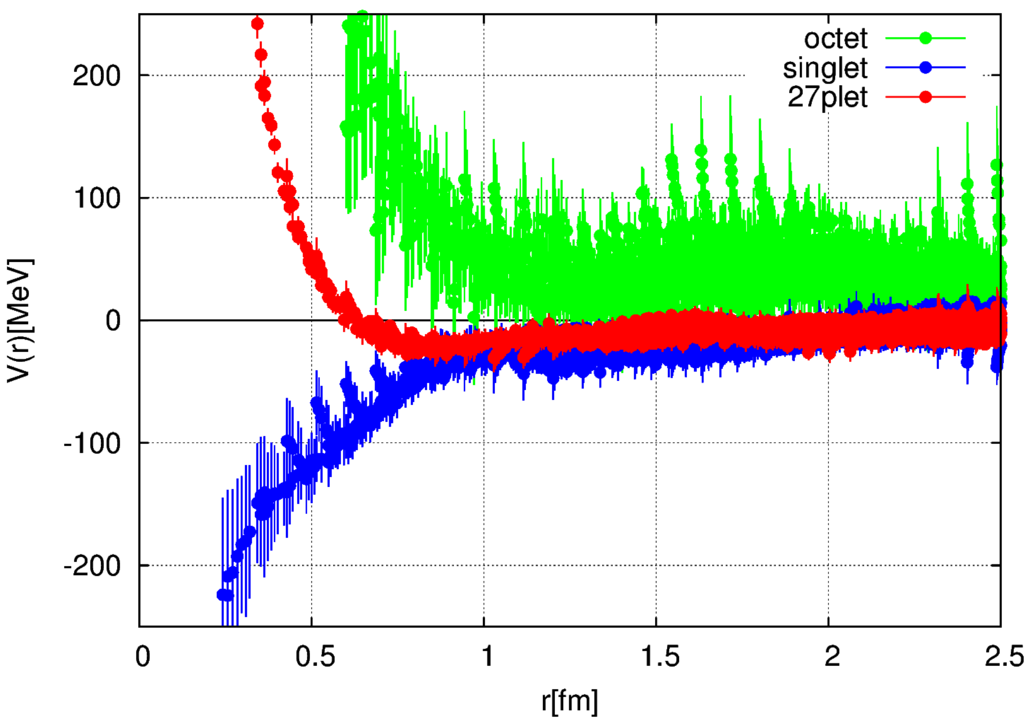
Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effectual measure of the SU(3) breaking effect.

Potentials in $^1S_0$ channel with SU(3) basis

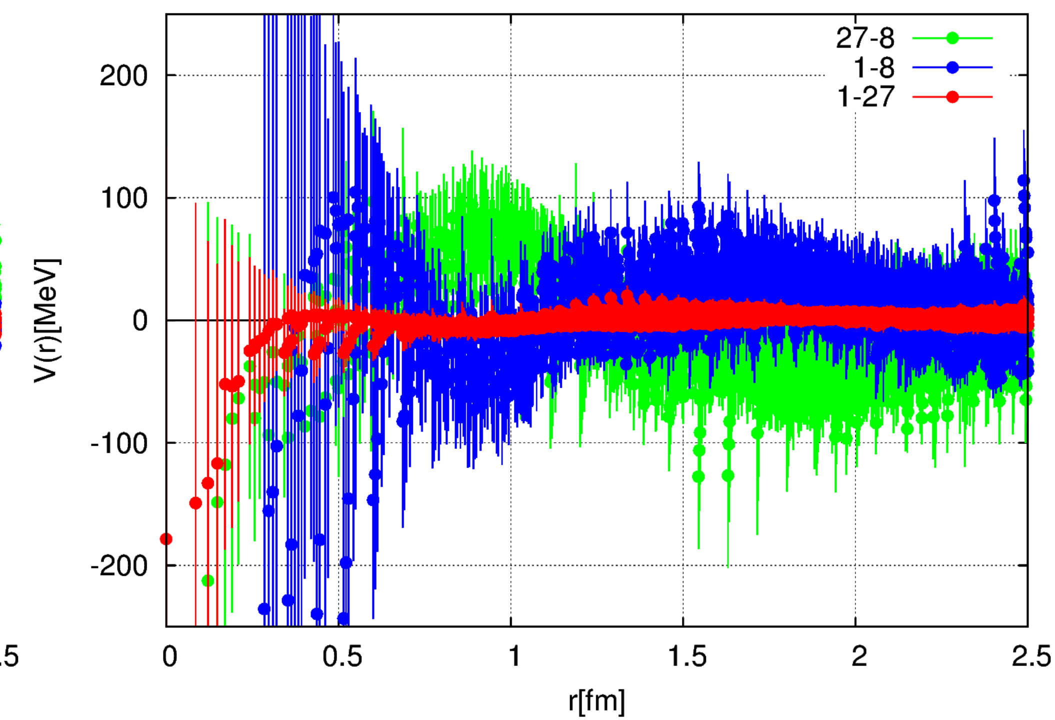
▶ **N_f = 2+1 full QCD with L = 8fm, *m_π = 145 MeV***

Preliminary!

Diagonal elements



Off-diagonal elements



- Potential of flavor singlet channel does not have a repulsive core
- Potential of flavor octet channel is strongly repulsive which reflects Pauli effect.
- Off-diagonal potentials are visible only in r<1fm region.

Interactions of decuplet baryons

SU(3) aspects of BB interaction

We have succeeded to evaluate potentials
between ground state baryons directly from QCD.

$$8\otimes 8 = 1\oplus 8_s \oplus \boxed{27} \oplus 8_a \oplus 10 \oplus \boxed{\overline{10}}$$

H-dibaryon (1)

Nuclear force (27, $\overline{10}$)

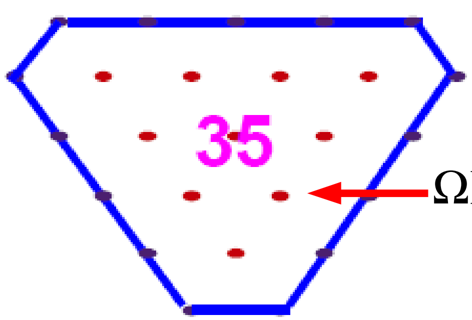
Inclusion of decuplet baryons

- **For decuplet-octet system**

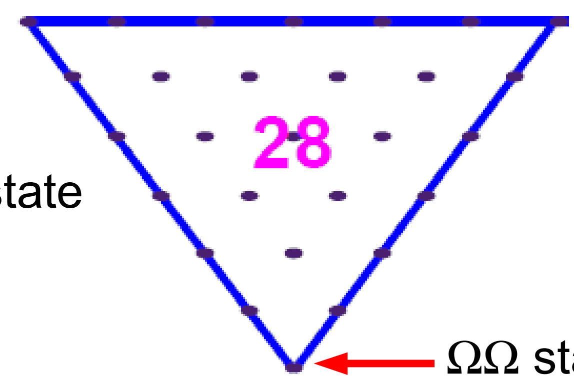
$$10\otimes 8 = \boxed{35} \oplus 8 \oplus 10 \oplus 27$$

- **For decuplet-decuplet system**

$$10\otimes 10 = \boxed{28} \oplus 27 \oplus \boxed{35} \oplus \overline{10}$$



ΩN state



ΔΔ state (I=0,J=3)

ΩΩ state (pure 28plet)

Alternative source of generalized baryon-baryon interactions

$N\Omega$ *interaction*

Octet-Decuplet interaction

- **Flavor symmetry aspect**

  Octet-Decuplet interaction can be classified as

$$8 \otimes 10 = 35 \oplus \boxed{8} \oplus 10 \oplus 27$$

Strongly attractive CMI is expected

Flavor octet dibaryon is predicted

M.Oka PRD38-298

NΩ $J^p(I) = 2^+(1/2)$ is considered

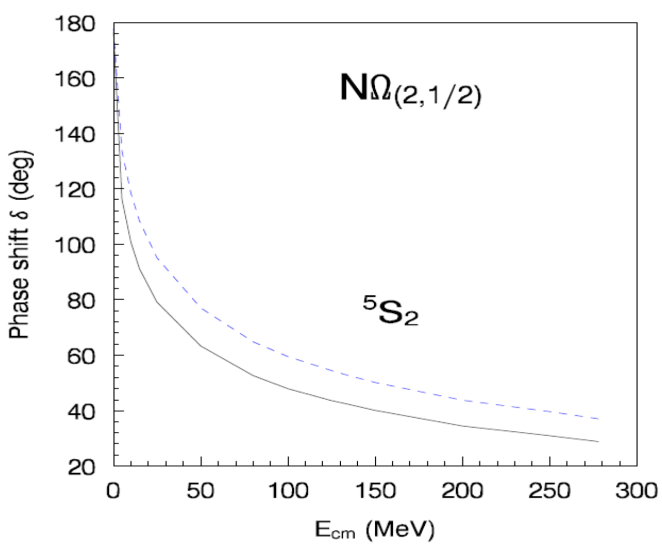
- **Easy to tackle it by lattice QCD simulation**
  - **Lowest state in J=2 coupled channel**
    - **NΩ–ΛΞ*–ΣΞ*–ΞΣ***
  - **Multi-strangeness reduces a statistical noise**
  - **Wick contraction is very simple**

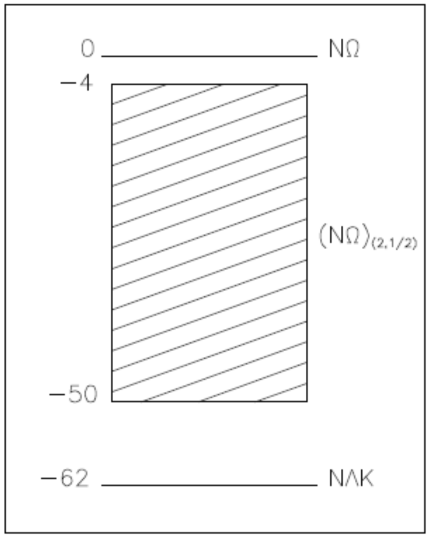
$N\Omega$ system from quark model

- One of di-baryon candidate — T.Goldman et al PRL59(1987)627
- (Quasi-)Bound state is reported with J=2, I=1/2

- Constituent quark model — M.Oka PRD38(1988)298
  - CMI does not contribute for this system because of no quark exchange between baryons.
  - Coupled channel effect is important.

- Chiral quark model — Q.B.Li, P.N.Shen, EPJA8(2000)
  - Strong attraction yielded by scalar exchange

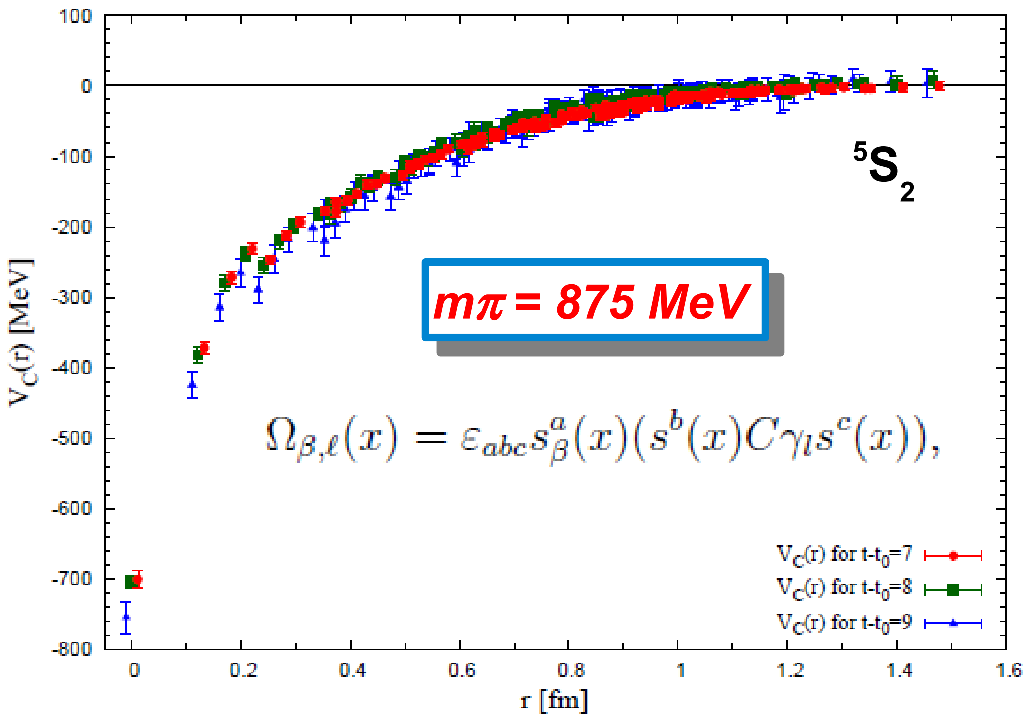


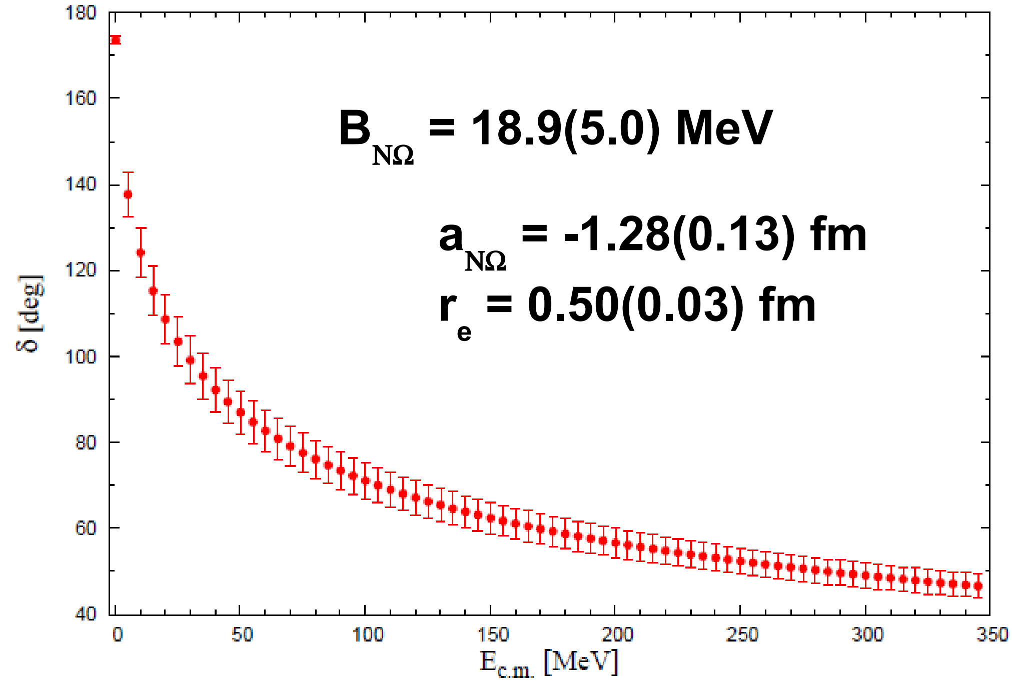


$N\Omega$ system $J^p(I) = 2^+(1/2)$

- N_f = 2+1 full QCD with L = 1.9fm

F.Etminan(HAL QCD), NPA928(2014)89





$N\Omega$ state cannot decay into $\Lambda\Xi$ (D-wave) state in this setup

- **Strongly attractive S-wave effective potential in $J^p(I) = 2^+(1/2)$**
- **Good baseline to explore S=-3 baryonic system**

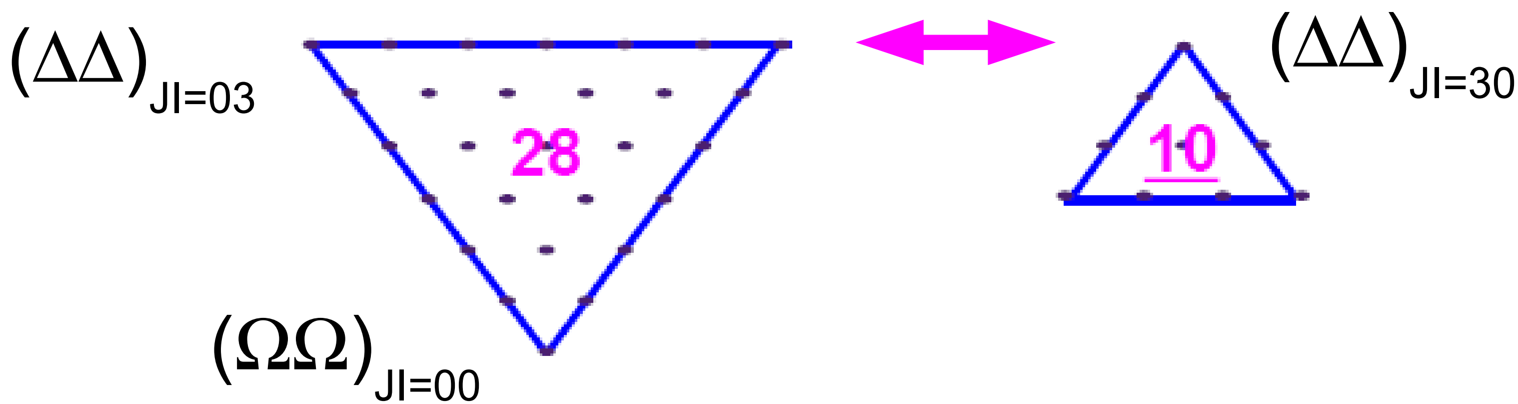
Decuplet-Decuplet interactions

Decuplet-Decuplet interaction

- **Flavor symmetry aspect**

Decuplet-Decuplet interaction can be classified as

$$10\otimes 10 = 28 \oplus \cancel{27} \oplus \cancel{35} \oplus \overline{10}$$



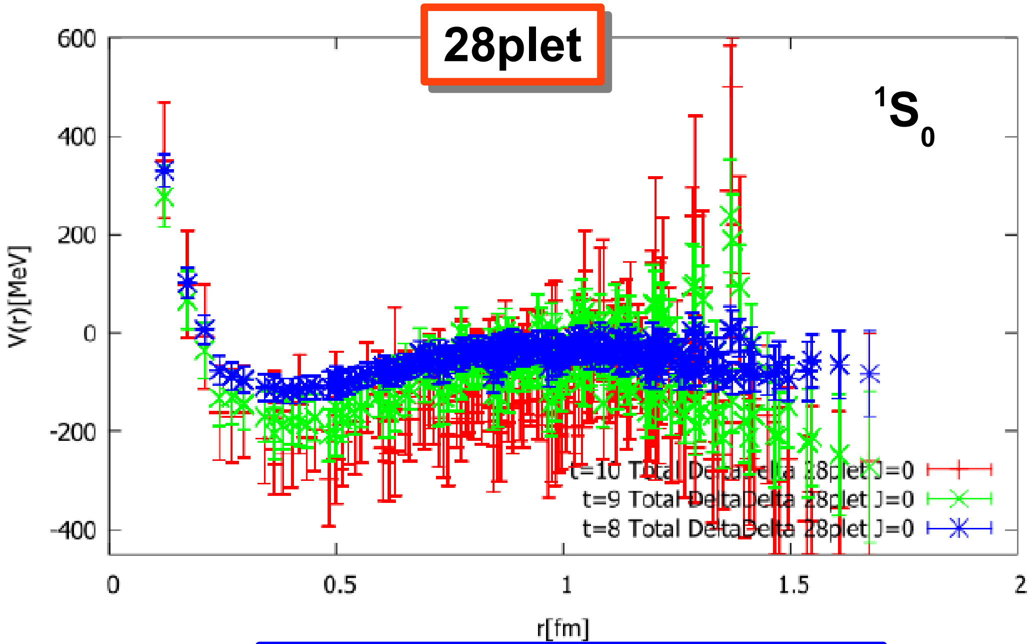
	28plet (0^+)	28plet (2^+)	10*plet (1^+)	10*plet (3^+)
Pauli	**allowed**	**forbidden**	---	**allowed**
CMI	**repulsive**	---	---	**Not attractive**

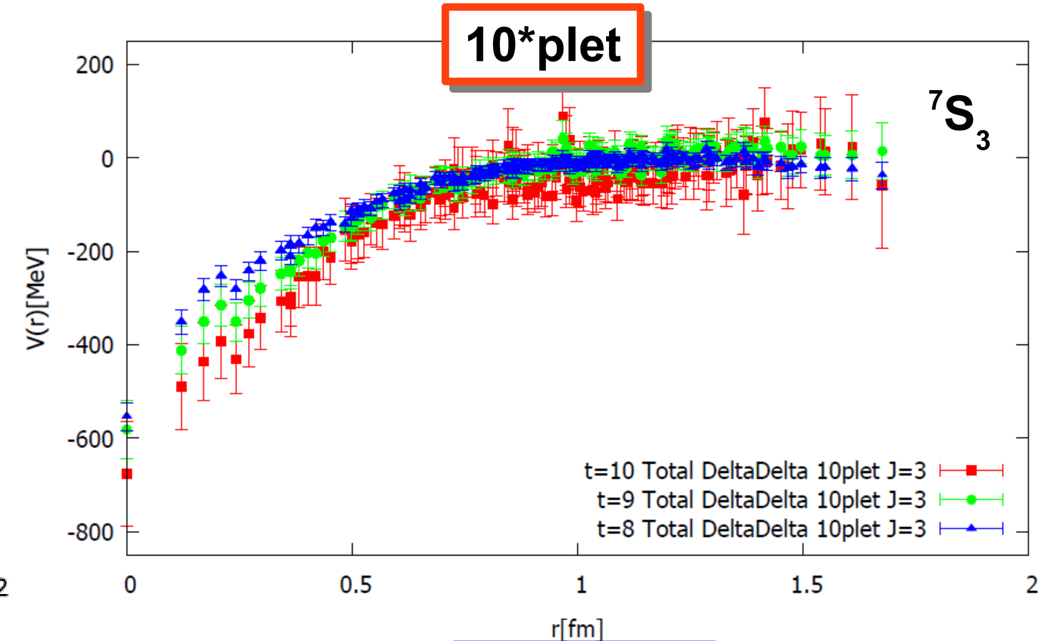
- **Δ–Δ(J=3) : Bound (resonance) state was found in experiment.**
- **Δ–Δ(J=0) [and Ω–Ω(J=0)] : Mirror of Δ–Δ(J=3) state**

Decuplet-Decuplet interaction in SU(3) limit

- N_f = 2+1 full QCD with L = 1.93fm, ***mπ = 1015 MeV***

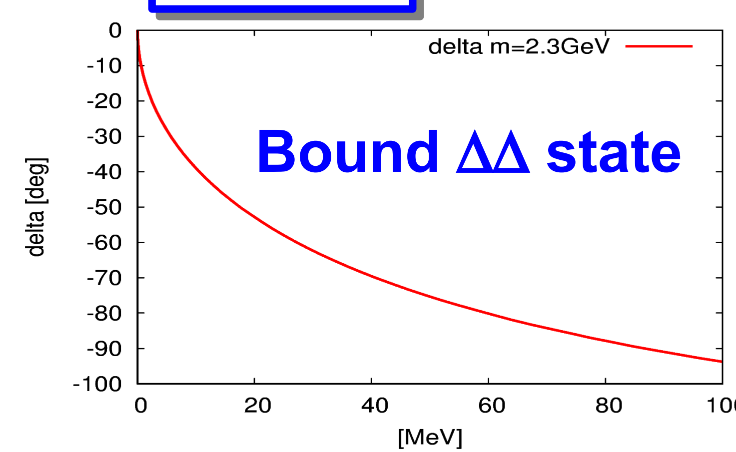
Preliminary!





Δ–Δ(J=0) and Ω–Ω(J=0)

Δ–Δ(J=3)



- **Short range repulsion and attractive pocket are found in 28plet.**
- **10*plet [$J^P(I)=3^+(0)$] is strongly attractive.**

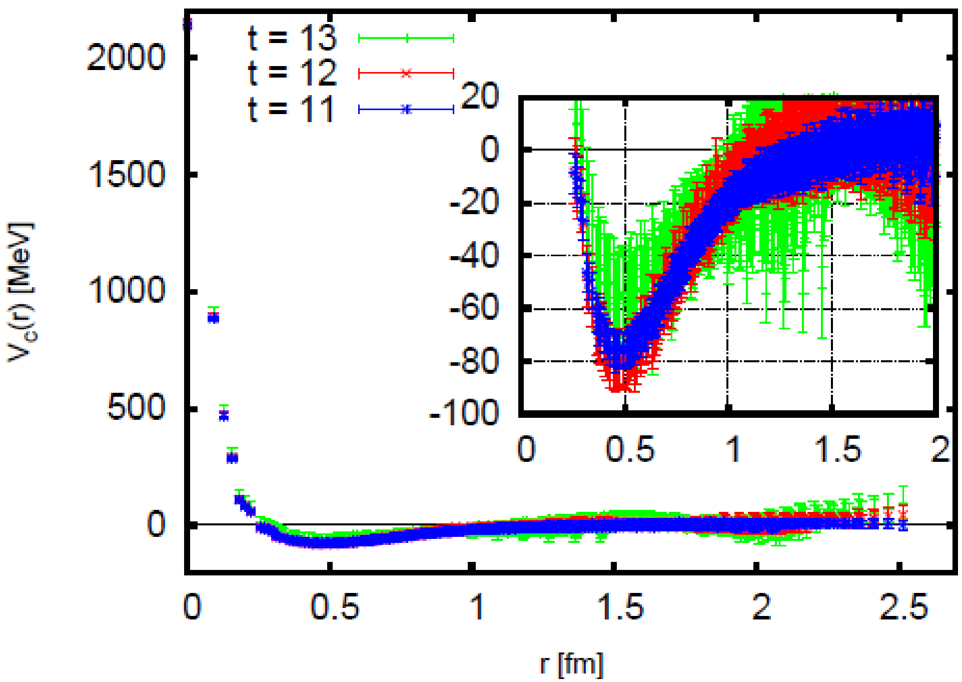
M. Yamada et al(HAL QCD), accepted in PTEP

ΩΩ $J^p(I) = 0^+(0)$ state in unphysical region

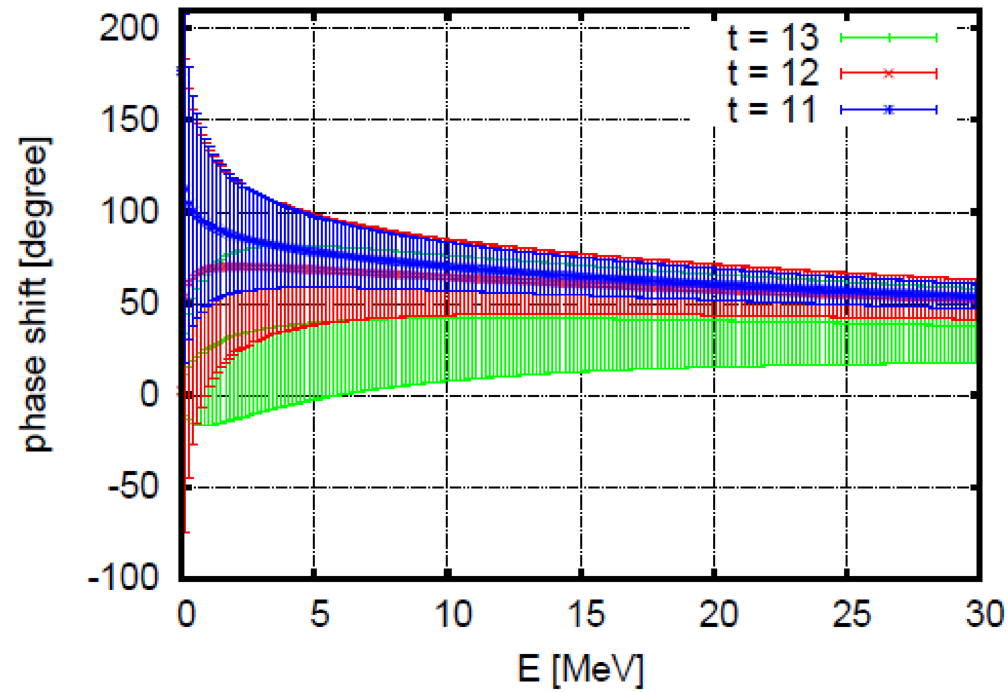
- **N_f = 2+1 full QCD with L = 3fm, *m_π = 700 MeV***

The ΩΩ state is stable against the strong interaction.

Potential



Phase shift



- **Short range repulsion and attractive pocket are found.**
- **Potential is nearly independent on "t" within statistical error.**
- **The system may appear close to the unitary limit.**

Summary and outlook

- We have investigated coupled channel hadronic interactions from lattice QCD.

- We have studied exotic candidate states
  - H-dibaryon channel
    - There is strongly attractive potential in flavor singlet state.
    - It is not enough statistics to calculate several observables and to discuss the fate of H-dibayon.

  - $N\Omega$ state with $J^p=2^+$
    - It is strongly attractive without short range repulsion.
    - It forms a bound state with about 20MeV B.E..
    - ΩN–$\Xi*\Sigma$–$\Xi*\Lambda$–$\Sigma*\Xi$ coupled channel calculation is necessary

  - $\Delta\Delta$ and $\Omega\Omega$ states
    - $\Delta\Delta$(I=0) have strongly attractive potential
    - $\Delta\Delta$(I=3) and $\Omega\Omega$ potential have repulsive core and attractive pocket
    - Both channels form (quasi-)bound state???