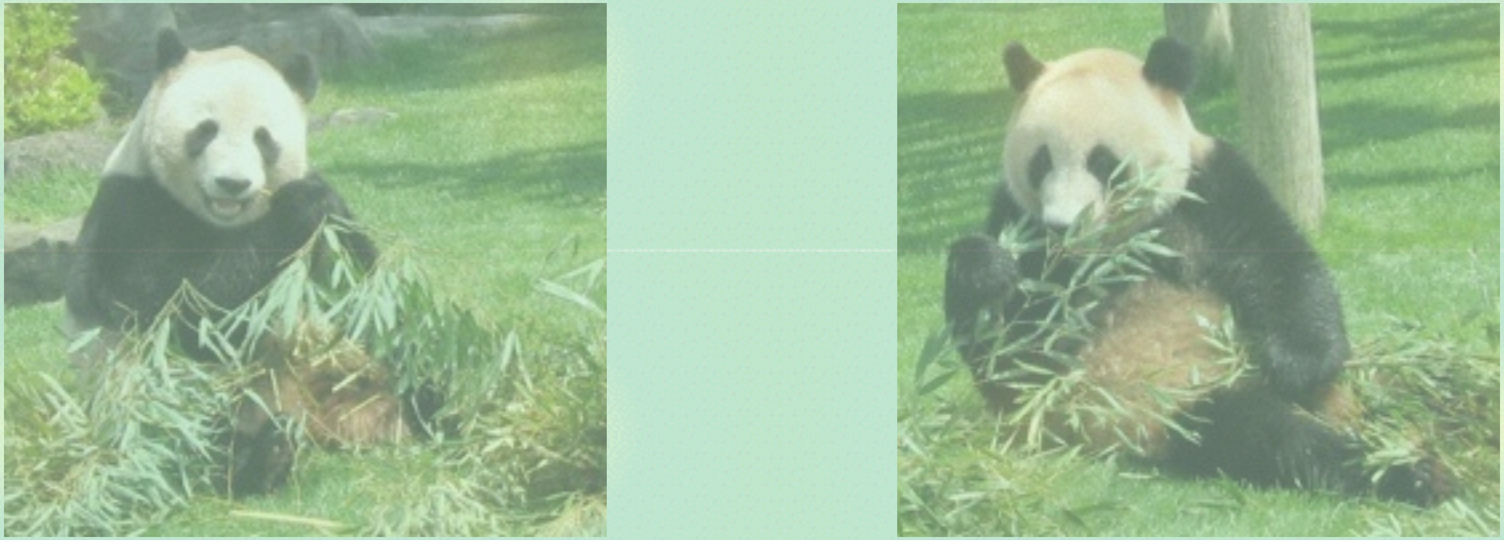
Compositeness of hadrons and near-threshold dynamics



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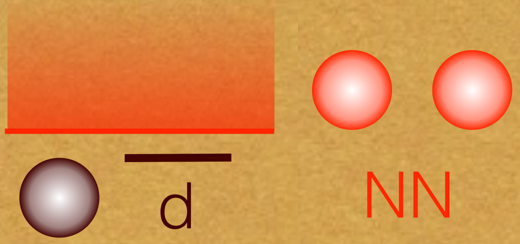
2015, Aug. 6th

Contents

- **Introduction: compositeness of hadrons**
- **Near-threshold bound state**

  S. Weinberg, Phys. Rev. 137, B672 (1965);
  T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)



- **Near-threshold resonance**

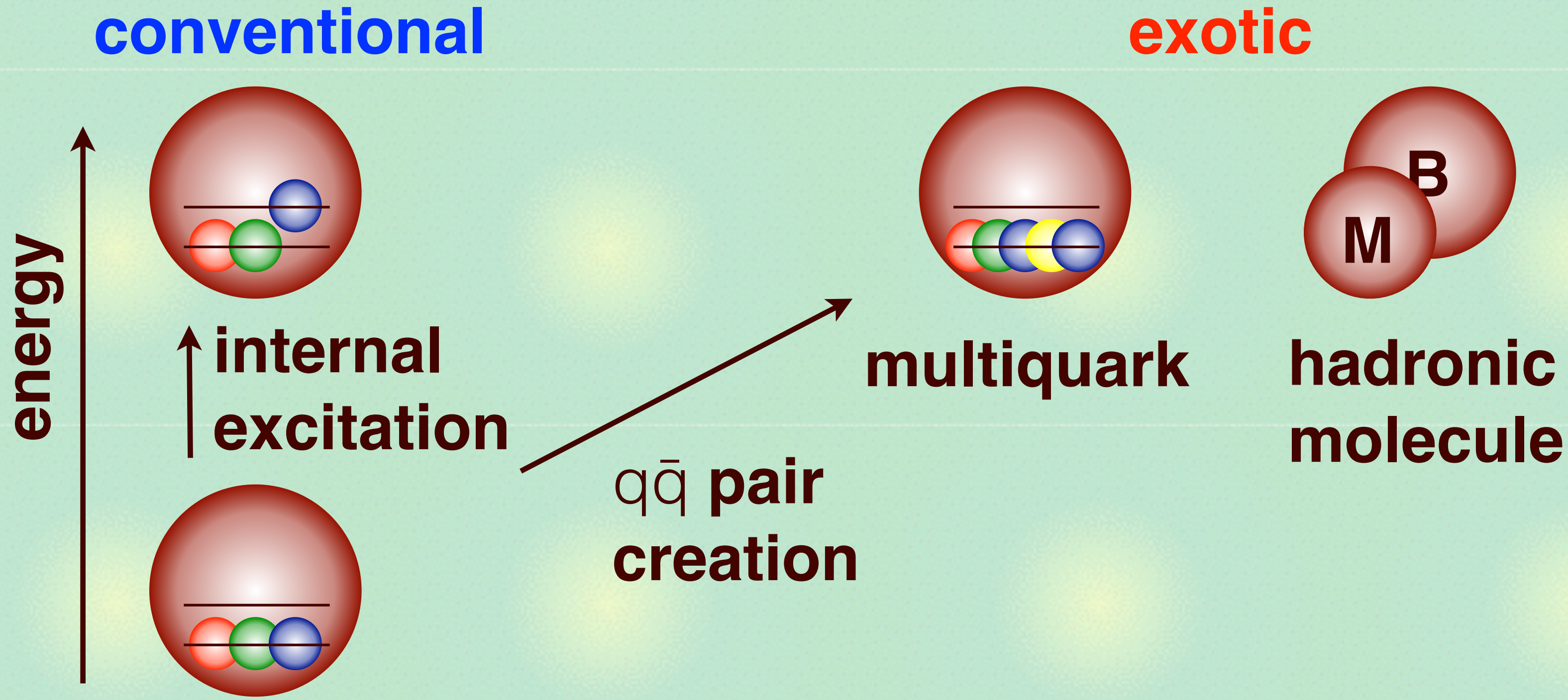
  T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

- **New things:**
  - reformulation with effective field theory
  - interpretation of complex compositeness
  - quasi-bound state (coupled-channel) —> next talk

  Y. Kamiya, T. Hyodo, in preparation
- **Summary**

Exotic structure of hadrons

Various excitations of baryons



conventional

exotic

energy

internal excitation

$q\bar{q}$ pair creation

multiquark

hadronic molecule

Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\ q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

Is this relevant strategy?

Ambiguity of definition of hadron structure

Decomposition of hadron “wave function”

$$|\,\Lambda(1405)\,\rangle = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\ q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

- N_X = **probability?**

Conditions for “probability”

- **Basis must be orthogonal with each other.**

$$\langle\,udsq\bar{q}\,|\,\bar{K}N\,\rangle \neq 0 \qquad \langle\,uds\,|\,udsq\bar{q}\,\rangle \neq 0$$

- **The coefficients must be real and positive.**

$|\,\Lambda(1405)\,\rangle = ?$ **complex weight** N_X **for unstable particles**

Questions:

- **What is the appropriate basis?**
- **How can we interpret the complex weights?**

Strategy

Elementary/composite nature of bound states near the lowest energy two-body threshold

elementary Z ●

- uududd
- ΔΔ
- $uududdu\bar{u}$
- NN(d-wave)
- …

composite X ● ●

- NN(s-wave)

- orthogonality <— eigenstates of bare Hamiltonian

- normalization <— eigenstate of full Hamiltonian

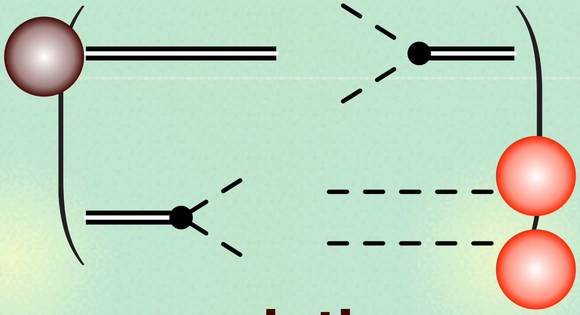
- model independence <— low-energy universality

*** “Elementary” stands for anything other than the composite channel of interest (missing channels, CDD pole, …).**

Compositeness and elementariness

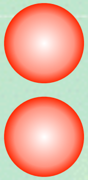
Coupled-channel Hamiltonian (bare state + continuum)

$$\begin{pmatrix} M_0 & \hat{V} \\ \hat{V} & \frac{p^2}{2\mu} \end{pmatrix} |\,\Psi\,\rangle = E|\,\Psi\,\rangle, \quad |\,\Psi\,\rangle = \begin{pmatrix} c(E)|\,\psi_0\,\rangle \\ \chi_E(\boldsymbol{p})|\,\boldsymbol{p}\,\rangle \end{pmatrix}$$



- Bound state normalization + completeness relation

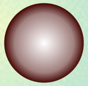
$$\langle\,\Psi\,|\,\Psi\,\rangle = 1 \qquad 1 = |\,\psi_0\,\rangle\langle\,\psi_0\,| + \int d^3q|\,\boldsymbol{q}\,\rangle\langle\,\boldsymbol{q}\,|$$



$$1 = \underbrace{\left|\langle\,\Psi\,|\begin{pmatrix} |\,\psi_0\,\rangle \\ 0 \end{pmatrix}\right|^2}_{\text{bare state contribution}} + \underbrace{\int d^3q\left|\langle\,\Psi\,|\begin{pmatrix} 0 \\ |\,\boldsymbol{q}\,\rangle \end{pmatrix}\right|^2}_{\text{continuum contribution}} \equiv Z + X \longleftarrow$$

compositeness

elementariness (field renormalization constant)

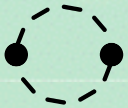


Z, X: **real and nonnegative —> probabilistic interpretation**

Z in model calculations

In general, Z is model dependent (~ potential, wave function)

$$Z(B) = \frac{1}{1 - \frac{d}{dE}\int \frac{|\langle \psi_0 | \hat{V} | \boldsymbol{q} \rangle|^2}{E - q^2/(2\mu) + i0^+} d^3q \Big|_{E=-B}} \equiv \frac{1}{1 - \Sigma'(-B)}$$

$\Sigma(E) \sim$ 

- Z can be calculated by employing models.

Baryons	Z	$\lvert Z\rvert$	Mesons	Z	$\lvert Z\rvert$
$\Lambda(1405)$ higher pole (Ref. 58)	$0.00 + 0.09i$	0.09	$f_0(500)$ or σ (Ref. 58)	$1.17 - 0.34i$	1.22
$\Lambda(1405)$ lower pole (Ref. 58)	$0.86 - 0.40i$	0.95	$f_0(980)$ (Ref. 58)	$0.25 + 0.10i$	0.27
$\Delta(1232)$ (Ref. 60)	$0.43 + 0.29i$	0.52	$a_0(980)$ (Ref. 58)	$0.68 + 0.18i$	0.70
$\Sigma(1385)$ (Ref. 60)	$0.74 + 0.19i$	0.77	$\rho(770)$ (Ref. 55)	$0.87 + 0.21i$	0.89
$\Xi(1535)$ (Ref. 60)	$0.89 + 0.99i$	1.33	$K^*(892)$ (Ref. 59)	$0.88 + 0.13i$	0.89
Ω (Ref. 60)	0.74	0.74			
$\Lambda_c(2595)$ (Ref. 56)	$1.00 - 0.61i$	1.17			

[55] F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012), [56] T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013), [58] T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2012), [59] C.W. Xiao, F. Aceti, M. Bayar, Eur. Phys. J. A49, 22 (2013), [60], F. Aceti, *et al.*, Eur. Phys. J. A50, 57 (2014).

Model-independent determination?

Weak binding limit

Z **of weakly-bound (**$R \gg R_{\text{typ}}$**) s-wave state <— observables.**

S. Weinberg, Phys. Rev. 137, B672 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

a **: scattering length,** r_e **: effective range**
$R = (2\mu B)^{-1/2}$ **: radius <— binding energy**
R_{typ} **: typical length scale of the interaction**

- **Deuteron is** NN **composite (**$a \sim R \gg r_e$**), only from observables, without referring to the nuclear force/wave function.**

- **Derivation for general separable interaction:**
  T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

How is it possible to related Z **with observables?**

Scaling limit

Scaling (zero-range) limit: scattering length a ≠ 0, R_{typ} -> 0

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006)

- **All (2-body) quantities are expressed by** a: **universality**

$$\psi(r)=\frac{e^{-r/a}}{\sqrt{2\pi a r}} \qquad B=1/(2\mu a^2) \quad \Rightarrow \quad R=a \quad \Rightarrow \quad Z=0$$

- **Bound state is always composite in the scaling limit.**

T. Hyodo, Phys. Rev. C90, 055208 (2014)

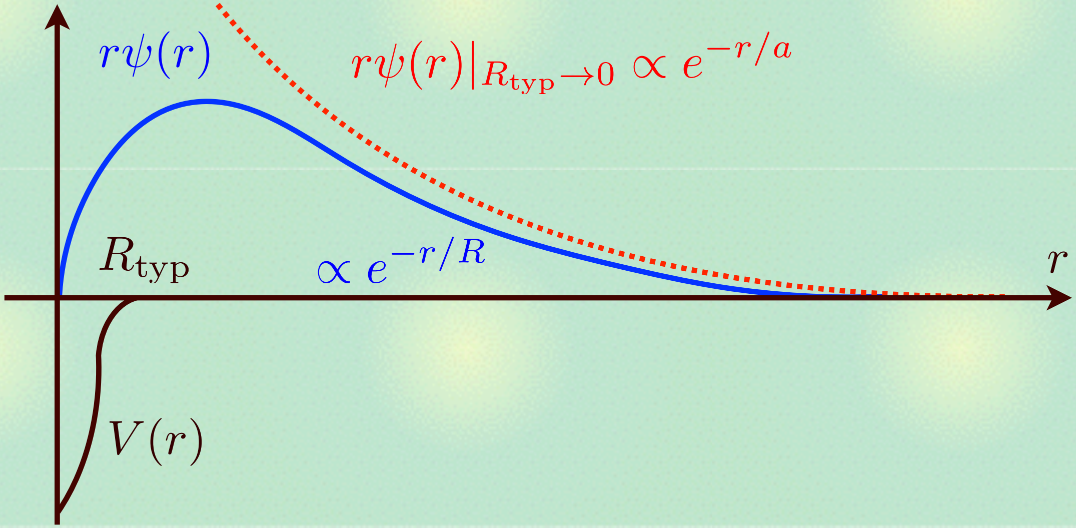
C. Hanhart, J.R. Pelaez, G. Rios, Phys. Lett. B 738, 375 (2014)

Finite R_{typ}**:** Z **expresses the violation of the scaling**

$$a=\frac{2(1-Z)}{2-Z}R+\mathcal{O}(R_{\text{typ}})$$

model independent

model dependent



Effective field theory

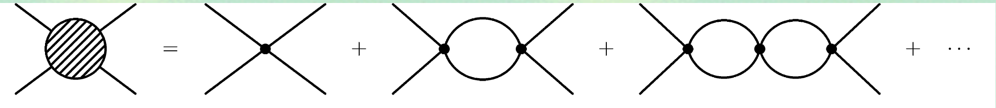
Universal description of low-energy scattering $p \ll \Lambda \sim 1/R_{typ}$

- **nonrelativistic quantum field theory with contact int.**

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

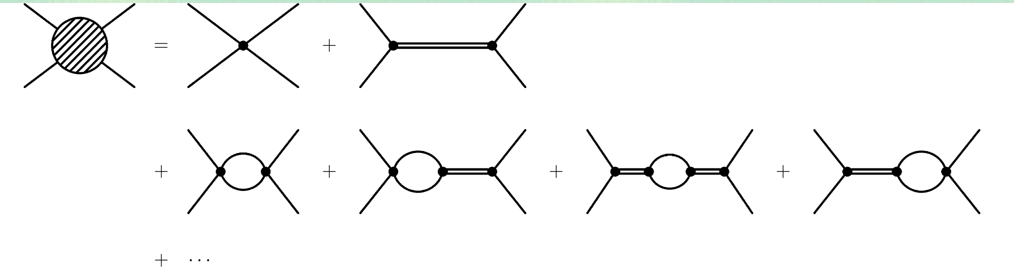
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

- **zero-range model —> scaling limit :** Z=0



$$f(p) = \frac{1}{-\frac{1}{a} + ip + \mathcal{O}(\frac{p^2}{\Lambda^2})}$$

- **resonance model :** Z≠0



$$f(p) = \frac{1}{-\frac{1}{a} + \frac{r_e}{2}p^2 + ip + \mathcal{O}(\frac{p^4}{\Lambda^4})}$$

T. Sekihara, T. Hyodo, D. Jido, PTEP2015, 063D04 (2015)

↓

Contact interaction —> separable —> weak binding formula

Y. Kamiya, T. Hyodo, in preparation

Short summary for bound states

- **Appropriate basis for bound states**

elementary Z

- uududd
- ΔΔ - πNN - …

composite X

- NN(s-wave)

- **Conditions for model-independent formula:**
  - **stable s-wave bound state near threshold**
- **Applicability:**
  - **Deuteron only!**

Application to exotic hadrons

—> Generalization to unstable particles

Generalization to resonances

Compositeness of bound states

$$Z(B) = \frac{1}{1 - \Sigma'(-B)}$$

Naive generalization to resonances:

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

$$\underbrace{Z(E_R)}_{\text{complex}} = \frac{1}{1 - \underset{\uparrow\,\text{complex}}{\Sigma'(E_R)}}$$

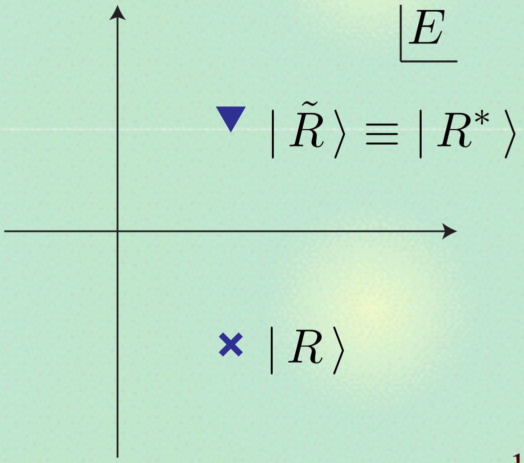
- Problem of interpretation (not probability!)

<— Normalization of resonances

$$\langle\, R \,|\, R \,\rangle \to \infty, \quad \langle\, \tilde{R} \,|\, R \,\rangle = 1$$

$$1 = \underbrace{\langle\, \tilde{R} \,|\, \psi_0 \,\rangle\langle\, \psi_0 \,|\, R \,\rangle}_{\text{complex}} + \int d\boldsymbol{p}\langle\, \tilde{R} \,|\, \boldsymbol{p} \,\rangle\langle\, \boldsymbol{p} \,|\, R \,\rangle$$

$$\langle\, \tilde{R} \,|\, \psi_0 \,\rangle = \langle\, \psi_0 \,|\, R \,\rangle \neq \langle\, \psi_0 \,|\, R \,\rangle^*$$



T. Berggren, Nucl. Phys. A 109, 265 (1968)

Interpretation of complex Z

Suppose that we obtain…

case	Z	X	U=|Z|+|X|-1
1	0.94 + i 0.01	0.06 - i 0.01	0.00088
2	0.94 + i 5.3	0.06 - i 5.3	9.7
3	4.45 + i 0.01	-3.45 - i 0.01	6.9

Ideal case 1: Z **dominance, elementary.**

<— wave function is similar to the bound state with Z = 0.94

Problematic cases: large imaginary part (case 2) and/or large cancellation of the real part (case 3)

Possible measure of uncertainty: U=|Z|+|X|-1

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

- When U **is large, then** Z **and** X **should not be interpreted.**

Alternative measure of compositeness

Magnitude of effective range

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\text{typ}}),$$

$$\begin{cases} a \sim R_{\text{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\text{typ}} & \text{(composite dominance)}. \end{cases}$$

- **elementary dominance** (Z~1) **—> large negative** r_e

Pole counting (bound or quasi-bound state)

D. Morgan, M.R. Pennington, Phys. Lett. B258, 444 (1991)
V. Baru *et al.*, Phys. Lett. B586, 53 (2004)

- **two poles in the effective range expansion**

$$k_1 = i\sqrt{2\mu E}, \quad k_2 = -i\sqrt{2\mu E}\frac{2-Z}{Z}$$

- **elementary** (Z~1) **—>** $k_1 \sim -k_2$, **composite** (Z~0) **—>** $|k_1| \ll |k_2|$

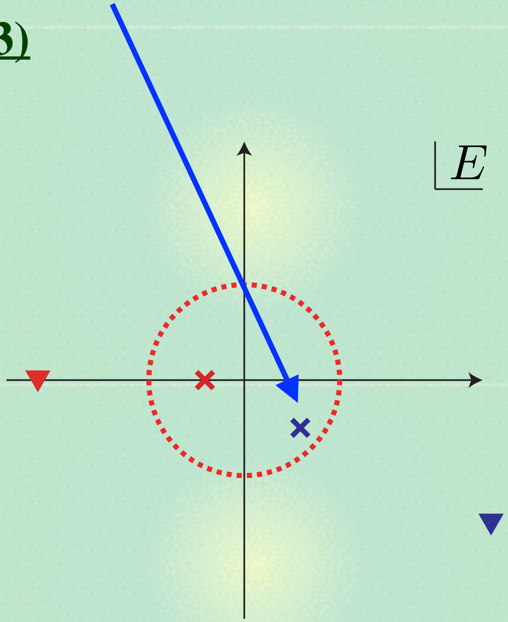
Near-threshold resonances

Weak binding limit for bound states

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables

What about near-threshold resonances (~ small binding)?

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)



Effective range expansion

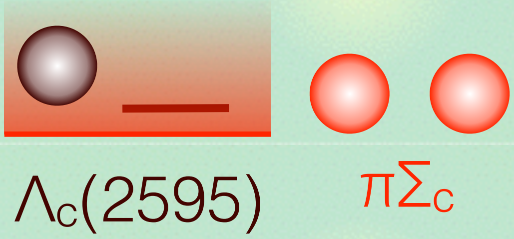
$$f(p) = \left(-\frac{1}{a} + \frac{r_e}{2}p^2 - ip\right)^{-1}$$

$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a} - 1}$$

Resonance pole position —> (a, r_e) —> elementariness

Application: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering



- central values in PDG

$$E = 0.67 \text{ MeV}, \quad \Gamma = 2.59 \text{ MeV} \qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^{+} + p^{-}}{ip^{+}p^{-}} = -10.5 \text{ fm}, \quad r_e = \frac{2i}{p^{+} + p^{-}} = \boxed{-19.5 \text{ fm}}$$

- Field renormalization constant cannot be interpreted.

$$Z = 1 - 0.608i, \quad U = |Z| + |X| - 1 = 0.78$$

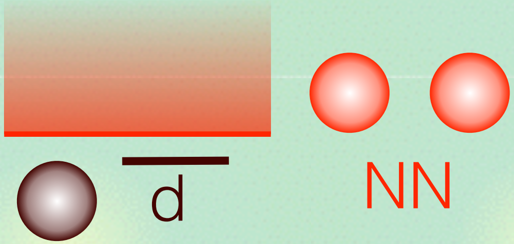
Large negative effective range

<— substantial elementary contribution other than $\pi\Sigma_c$
(three-quark, other meson-baryon channel, or ...)

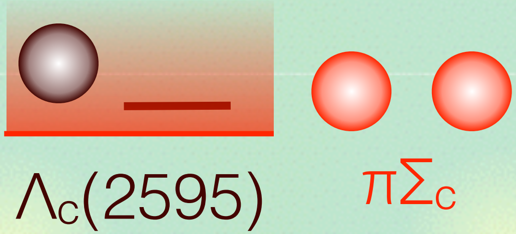
$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ composite**

Generalization to quasi-bound states

So far, we consider the lowest energy threshold.



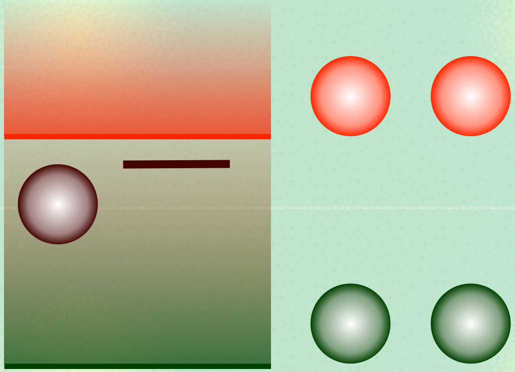
bound state



resonance

- **Scattering length is real.**

Quasi-bound state



channel 1: $\bar{K}N$

quasi-bound state: $\Lambda(1405)$

channel 2 (decay): $\pi\Sigma$

- **Scattering length of channel 1 is complex.**

—> see next talk

Summary

Model-independent aspect of compositeness

Compositeness X **and elementariness** Z

- appropriate basis for hadron structure

- determined by observables

- formulated in effective field theory

Complex Z **and** X **for unstable particles:**

- Uncertainty: $U=|Z|+|X|-1$

- Elementariness <— magnitude of r_e

Near-threshold resonance:

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)

- $\Lambda_c(2595)$ **is not a** $\pi\Sigma_c$ **molecule.**