Compositeness of near-threshold quasi-bound states

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Contents

- Introduction ~compositeness of bound state~
- Extension to the quasi-bound state
- Applications to exotic hadrons ~ $\Lambda(1405), a_0(980), f_0(980)$ ~
- Conclusions

Introduction ~exotic hadrons~

- Exotic hadrons

  Hadrons which do not coincide with the predictions of the quark model.

  More complicated internal structure can be expected.

  - tetra quark, penta quark
  - hadron molecule …

It is important to reveal the internal structure of exotics.

e.g. ; Λ(1405)

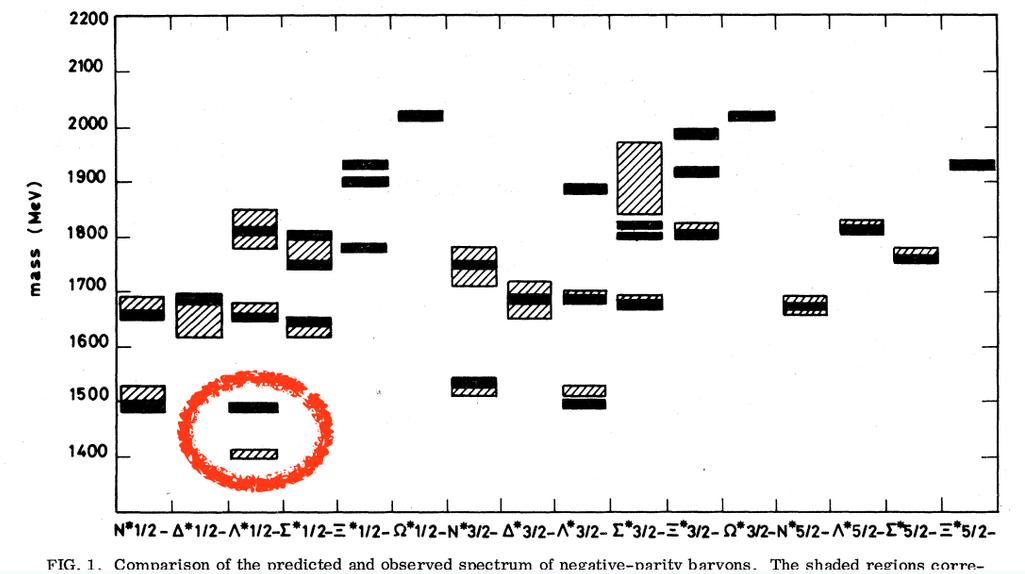
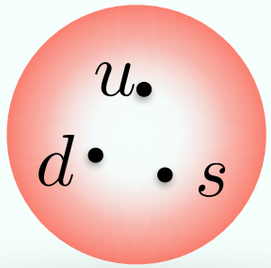


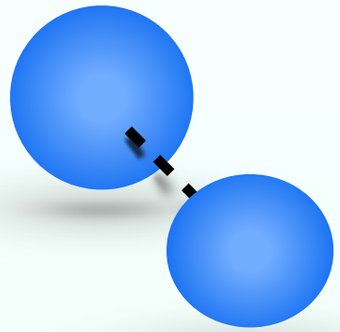
FIG. 1. Comparison of the predicted and observed spectrum of negative-parity baryons. The shaded regions corre-

Isgur, N., and G. Karl, 1978, Phys. Rev. D18, 4187

exited Λ state(uds)



$\bar{K}N$ bound state



Compositeness of bound state

- **Previous works**

  Introduced to study deuteron by Weinberg.

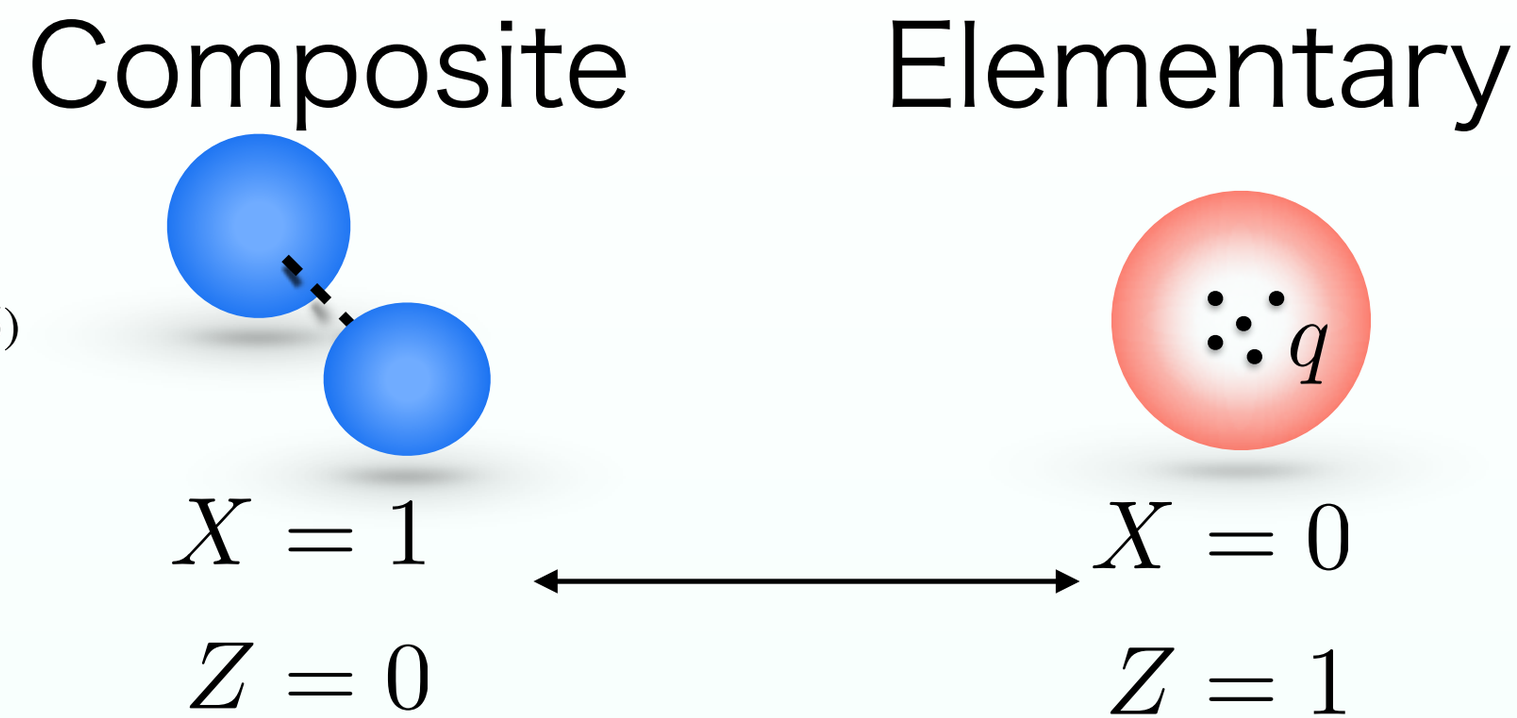
  S. Weinberg, Phys. Rev. 137, B672 (1965)

- **Condition**
  - weakly bound
  - stable state
  - s-wave

- **Output**
  - X ;weight of composite state $(0 < X < 1)$
  - Z ;wave function renormalization $(0 < Z < 1)$
  - a_0 ;scattering length
  - B ;binding energy

$$R = \frac{1}{\sqrt{2\mu B}}$$

(μ ;reduced mass of scat. state)



$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(R_{\rm typ}/R\right)\right\}$$

typical length scale

We can extract the information of the internal structure using experimental observables.

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Extension to the quasi-bound state.

Setting

- The state is weakly bound from channel 1.
- The state decays to channel 2.
- We consider the compositeness of channel 1; X .

Problem

- How does the existence of decaying channel affect the compositeness relation ?
- When can we use the relation ?

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(R_{\rm typ}/R\right)\right\} \longrightarrow ?$$

Extension to the quasi-bound state.

Extension of the system

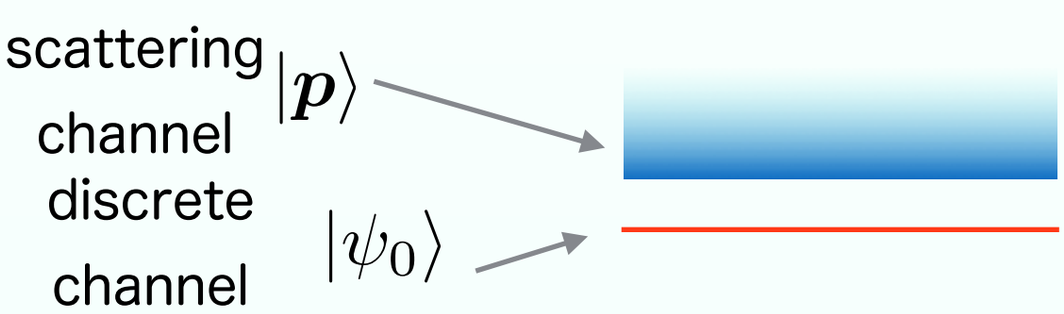
- We introduce the decaying mode channel,

  → estimate the contribution of decaying mode.

Bound state

c.f. T. Sekihara, T. Hyodo and D. Jido, arXiv:1411.2308

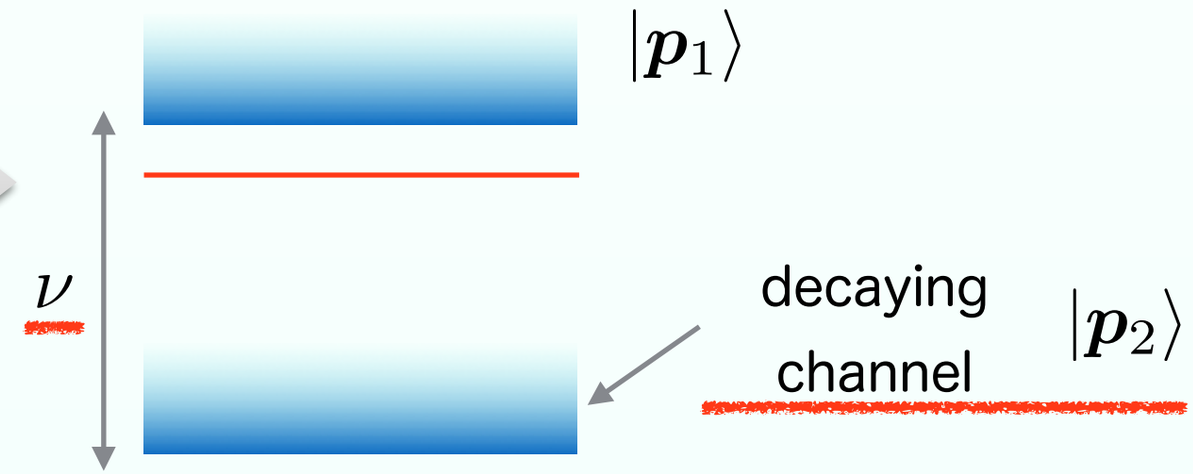
$$\begin{pmatrix} \hat{H}_{\psi_0} & 0 \\ 0 & \hat{H}_{\rm sc} \end{pmatrix} + \begin{pmatrix} 0 & \hat{V} \\ \hat{V} & 0 \end{pmatrix}$$



$$H\,|\Psi\rangle = E_h\,|\Psi\rangle$$
$$E_h = -B \text{ ;real}$$

Quasi-bound state

$$\begin{pmatrix} \hat{H}_{\psi_0} & 0 & 0 \\ 0 & \hat{H}_{{\rm sc},1} & 0 \\ 0 & 0 & \hat{H}_{{\rm sc},2} \end{pmatrix} + \begin{pmatrix} 0 & \hat{V}_1 & \hat{V}_2 \\ \hat{V}_1 & 0 & 0 \\ \hat{V}_2 & 0 & 0 \end{pmatrix}$$



$$H\,|\Psi\rangle = E_h\,|\Psi\rangle$$
$$E_h = -B - i\Gamma/2 \text{ ;complex}$$

Extension to the quasi-bound state.

Definition of compositeness

Bound state

Bound state $|\Psi\rangle$ is normalized with $\langle\Psi|\Psi\rangle = 1$

- $X + Z = 1$
- $0 < X, Z < 1$

$$X = \langle\Psi|\psi_0\rangle\langle\psi_0|\Psi\rangle = |\langle\psi_0|\Psi\rangle|^2$$

The probabilistic interpretation is valid for X and Z.

Quasi-bound state

To normalize unstable state, we introduce Gamow state $|\tilde{\Psi}\rangle$. Normalization condition becomes,

$$\langle\tilde{\Psi}|\Psi\rangle = \langle\Psi^*|\Psi\rangle = 1.$$

T. Berggren, Nucl. Phys. A 109 (1968).

The expectation value of the any operator becomes complex number.

- $X + Z = 1$
- $\cancel{0 < X, Z < 1}$ $X, Z \in C$

$$X = \langle\tilde{\Psi}|\psi_0\rangle\langle\psi_0|\Psi\rangle$$

The probabilistic interpretation is not valid !

Extension to the quasi-bound state.

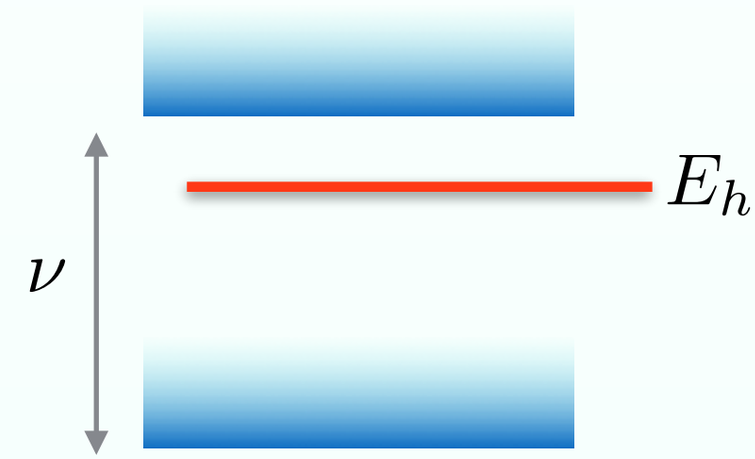
- Assuming $|E_h|$ is small, we expand a_0 with respect to $1/R$.

$$R = \frac{1}{\sqrt{-2\mu_1 E_h}}$$

$$l = \frac{1}{\sqrt{2\mu_1 \nu}}$$

$$a_0 = R\left\{\frac{2X}{1+X} + \underbrace{\mathcal{O}\left(|R_{\rm typ}/R|\right)}_{\text{original}} + \underbrace{\mathcal{O}\left(|l/R|^3\right)}_{\text{new}}\right\}$$

If $|R_{\rm typ}/R|$ and $|l/R|^3$ are sufficiently smaller than 1, we can extract X from a_0 and E_h.



- Notice
  - a_0 、E_h 、X are all complex number, then above relation is established among them.
  - If the the contribution of decaying mode is neglected, the compositeness relation is same to the one for bound state.

Interpretation of X

- X is a complex number.
- close to bound state case

$$\begin{cases} X = 0.8 - 0.1i \\ Z = 0.2 + 0.1i \end{cases}$$

small cancelation → The probabilistic interpretation is seemed to be possible.

- When real part is not in [0,1]

$$\begin{cases} X = 1.9 - 0.2i \\ Z = -0.9 + 0.2i \end{cases}$$

- When imaginary part is large.

$$\begin{cases} X = 0.9 - 0.8i \\ Z = 0.1 + 0.8i \end{cases}$$

large cancelation

→ It is difficult to interpret X as a probability.

→ Is there any good prescription to interpret the complex value?

Interpretation of X

Our proposal

c.f. T. Berggren, Phys. Lett. B 33 (1979) 8

For probabilistic interpretation we define following real quantity.

$\tilde{X}$; probability to find the scattering state in physical state

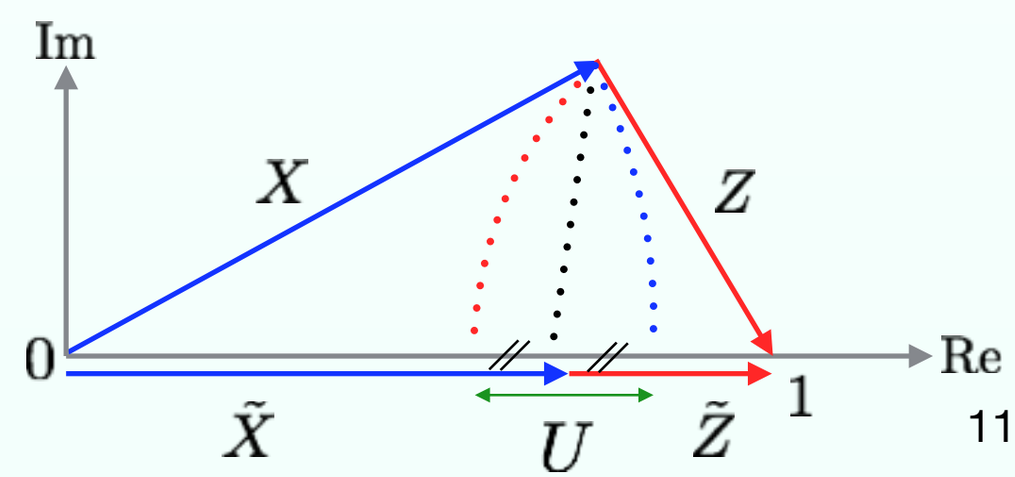
$\tilde{Z}$; probability to find the other states

U ; degree of ambiguity

$$\tilde{X} \equiv \frac{1-|Z|+|X|}{2}$$
$$\tilde{Z} \equiv \frac{1-|X|+|Z|}{2}$$
$$U \equiv |Z|+|X|-1$$

conditions :

- $\tilde{X}+\tilde{Z}=1$
- $0 \leq \tilde{X}, \tilde{Z} \leq 1$
- When there is no cancelation, $\tilde{X}=X, \tilde{Z}=Z, U=0$.
- U becomes large when the cancelation becomes large.



If U is small, we can interpret $\tilde{X}$ as the compositeness.

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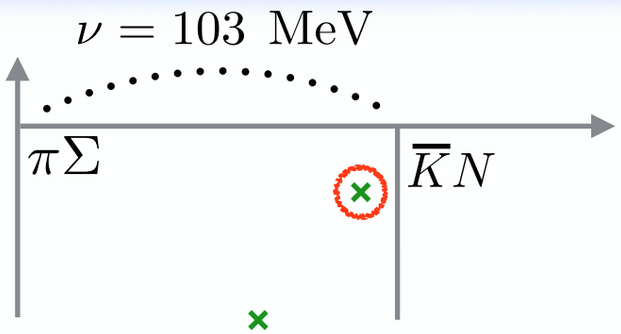
Compositeness of exotics

- $\Lambda(1405)$ in $I=0$ $\bar{K}N$ scattering

  We use E_h and a_0 in the following papers.



1) Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881 (2012) 98

2,3) Z. H. Guo and J. A. Oller, Phys. Rev. C 87, no. 3, 035202 (2013)

4) M. Mai and U. G. Meissner, Nucl. Phys. A 900, 51 (2013)

Set	E_h	a_0	X	$\tilde{X}$	U	$\left\lvert\frac{R_{\rm typ}}{R}\right\rvert$	$\left\lvert\frac{l}{R}\right\rvert^3$
1)	-11-i26	1.39-i0.85	1.3+i0.1	1.0	0.5	0.2	0.1
2)	-15-i24	1.74-i1.27	2.4-i1.2	0.9	3.4	0.2	0.1
3)	-11-i19	1.58-i1.27	1.4-i0.6	0.9	1.2	0.2	0.1
4)	-4-i8	1.81-i0.92	0.6+i0.1	0.6	0.0	0.1	0.0

- $|R_{\rm typ}/R|$ and $|l/R|^3$ are small enough. $a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\rm typ}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$
- U is small for Set 1) and 4) but large for 2) and 3).
- $\tilde{X}$ is close to 1.

$\Lambda(1405)$: $\bar{K}N$ composite dominance

Compositeness of exotics

- $\Lambda(1405)$ in $I=0$ $\bar{K}N$ scattering

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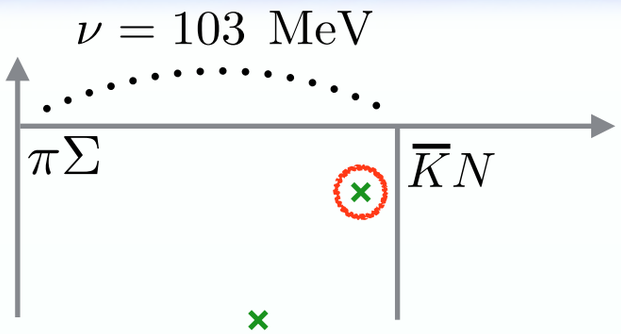

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- $|R_{\rm typ}/R|$ and $|l/R|^3$ are small enough. $a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\rm typ}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$
- U is small for Set 1) and 4) but large for 2) and 3).
- $\tilde{X}$ is close to 1.

$\Lambda(1405)$: $\bar{K}N$ composite dominance

Compositeness of exotics

- $a_0(980)$ in $K\overline{K}$ scattering

We use E_h and a_0 in the following papers.

V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B 586, 53 (2004)

Set	E_h	a_0	X	$\tilde{X}$	U	$\left\lvert\frac{R_{\rm typ}}{R}\right\rvert$	$\left\lvert\frac{l}{R}\right\rvert^3$
1	15.7-i23.3	-0.16-i0.59	0.13-i0.16	0.16	0.09	0.08	0.03
2	25.7-i38.4	-0.07-i0.69	0.24-i0.24	0.27	0.13	0.10	0.05
3	30.0-i22.7	-0.16-i1.05	0.40-i0.35	0.42	0.23	0.09	0.04
4	21.8-i-37.6	-0.01-i0.76	0.27-i0.24	0.30	0.13	0.10	0.04
5	1.91-i39.9	0.18-i0.61	0.23-i0.16	0.25	0.07	0.10	0.04

- $|R_{\rm typ}/R|$ and $|l/R|^3$ are small enough.
- U is small for each set.
- $\tilde{X}$ is close to 0.

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\rm typ}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$$

$a_0(980)$: small $K\bar{K}$ fraction

Compositeness of exotics

- $f_0(980)$ in $K\overline{K}$ scattering

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Set	E_h	a_0	X	$\tilde{X}$	U	$\left\lvert \frac{R_{\rm typ}}{R} \right\rvert$	$\left\lvert \frac{l}{R} \right\rvert^3$
1	-16.7-i25.3	1.15-i0.74	0.73-i0.10	0.72	0.02	0.08	0.06
2	-10.6-i25.6	0.99-i0.88	0.63-i0.14	0.62	0.04	0.08	0.06
3	-11.0-i28.1	1.09-i0.89	0.76-i0.12	0.75	0.04	0.08	0.06
4	15.5-i6.08	-0.14-i1.99	0.72-i0.36	0.68	0.27	0.06	0.04

- $|R_{\rm typ}/R|$ and $|l/R|^3$ are small enough.
- U is small for each set.
- $\tilde{X}$ is close to 1.

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\rm typ}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$$

$\Rightarrow$ $f_0(980)$: $K\overline{K}$ composite dominance

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Conclusions

- We extend the compositeness relation to quasi-bound states.

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\rm typ}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$$

If the energy gap is sufficiently large, we can ignore the effect of decaying channel to calculate compositeness.

- We construct interpretation of complex X.

$$\tilde{X} \equiv \frac{1-|Z|+|X|}{2}, \qquad U \equiv |X|+|Z|-1$$

If the ambiguity U is small, we interpret $\tilde{X}$ as the compositeness.

- We apply the method to exotic hadrons and discussed the internal structures.

$\Lambda(1405)$: $\bar{K}N$ composite dominance

$a_0(980)$: small $K\bar{K}$ fraction

$f_0(980)$: $K\bar{K}$ composite dominance