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Phenomenology of a pseudoscalar glueball and charmed mesons in a chiral symmetric Model

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Introduction

- Quantum Chromodynamics: QCD

- Symmetries of the QCD Lagrangian.

  if all quark massless then we have chiral symmetry

$$U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$$

- Spontaneous breaking of chiral symmetry by quark condensates
- Explicit breaking of global chiral symmetry by quark masses and chiral anomaly

- Effective chiral models of QCD.
- Development of a chirally symmetric model for mesons.

  ‘Extended Linear Sigma Model (eLSM)’

Motivation

- Decay of the pseudoscalar glueball into scalar and pseudoscalar mesons.
- Linear sigma model with vector and axial vector degrees of freedom.
- Inclusion of the charmed mesons into the linear sigma model (extended Linear Sigma Model - eLSM).
- Extension from low-energy to high-energy mesons.
- Study of the model for $T = \mu = 0$ (spectroscopy in vacuum).

Fields of the model

DAAD

- **Mesons: quark-antiquark states ($q\bar{q}$)**

  (scalar, pseudoscalar, vector and axialvector quarkonia.)

Quantum number: J^{PC}

- J → TotalSpin
- P → Parity
- C → ChargeConjugation

- **Glueballs (additional mesons): The scalar and the pseudoscalar glueball.**

- **Baryons: nucleon doublet and its partner**

  (in the so-called mirror assignment)

Construction of the eLSM

The construction of the so-called Extended Linear Sigma Model based on

- dilatation invariance

Note that: The breaking of the dilatation symmetry is only included in the “gluonic part” (scalar glueball and axial anomaly)

- chiral invariance

$$SU(N_f)_r \times SU(N_f)_l \times U(1)_V$$

Furthermore, the invariance under C and P is also taken into account.

The eLSM Lagrangian with (axial-)vector mesons

$$\mathscr{L} = \mathscr{L}_{\text{meson}} + \mathscr{L}_{\text{baryon}} + \mathscr{L}_{\text{dilaton}}$$

$$\begin{aligned}\mathscr{L}_{\text{meson}} = &\operatorname{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\operatorname{Tr}(\Phi^\dagger\Phi) - \lambda_1[\operatorname{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\operatorname{Tr}(\Phi^\dagger\Phi)^2 \\ &- \frac{1}{4}\operatorname{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \operatorname{Tr}[(\frac{m_1^2}{2} + \Delta)(L_\mu^2 + R_\mu^2)] + \operatorname{Tr}[H(\Phi + \Phi^\dagger)] \\ &+ c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}\{\operatorname{Tr}(L_{\mu\nu}[L^\mu, L^\nu]) + \operatorname{Tr}(R_{\mu\nu}[R^\mu, R^\nu])\} \\ &+ \frac{h_1}{2}\operatorname{Tr}(\Phi^\dagger\Phi)\operatorname{Tr}\left(L_\mu^2 + R_\mu^2\right) + h_2\operatorname{Tr}[|L_\mu\Phi|^2 + |\Phi R_\mu|^2] \\ &+ 2h_3\operatorname{Tr}(L_\mu\Phi R^\mu\Phi^\dagger)\end{aligned}$$

+chirally invariant vector and axialvector four-point interaction vertices

$$\begin{aligned}\mathscr{L}_{\text{baryon}} = &\bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ &- \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi^\dagger\Psi_{2L})\end{aligned}$$

$$\mathscr{L}_{\text{dilaton}} = \frac{1}{2}(\partial^\mu G)^2 - \frac{1}{4}\frac{m_G}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right)$$

D.Parganlija, P.Kovacs, G.Wolf, F.Giacosa and D.H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]];
W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]].

Decays of the pseudoscalar glueball

Interaction Lagrangian for the pseudoscalar glueball:

With scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}}^{int} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^{\dagger} \right)$$

With nucleons in the framework of the so-called mirror assignment

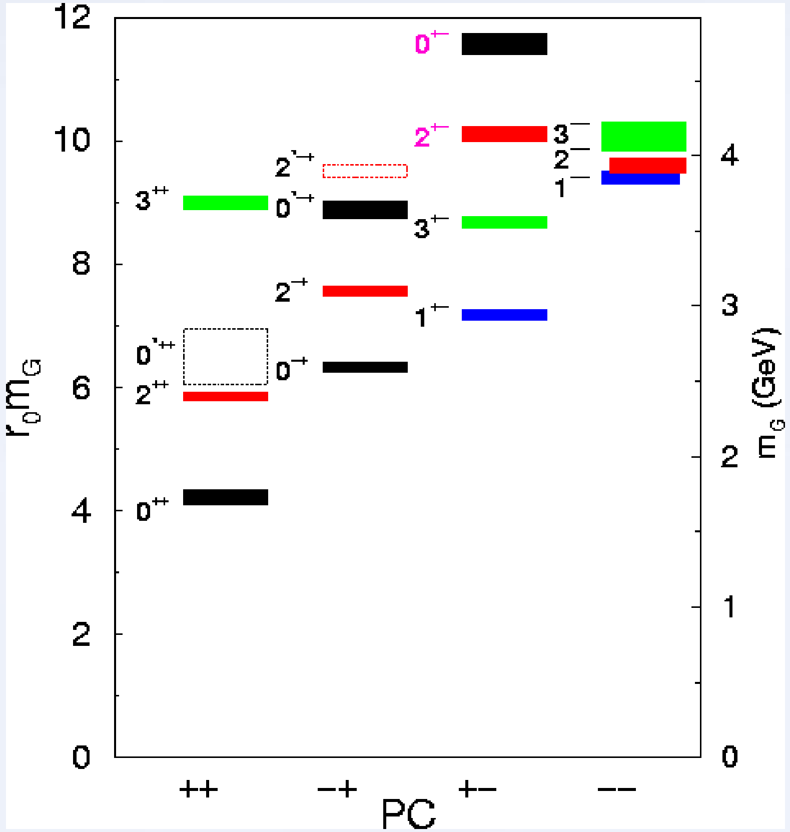
$$\mathcal{L}_{\tilde{G}-\mathrm{baryons}}^{\mathrm{int}} = i c_{\tilde{G}\Psi} \tilde{G} \left(\overline{\Psi}_2 \Psi_1 - \overline{\Psi}_1 \Psi_2 \right) .$$

- There fulfill chiral symmetry but breaks the axial anomaly.
- Only one unknown constant. All the rest is fied.

The branching ratio of decays are predicted

Mass of a pseudoscalar glueball

Lattice QCD calculation



The Pseudoscalar Glueball $\tilde{G} \equiv |gg\rangle$ at the border within light and heavy

$$M_{\tilde{G}} = 2.6 \quad \text{GeV}$$

$$J^{PC} = 0^{-+}$$

$$I = 0$$

[C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688, 220 (2004)

[arXiv:nucl-th/0309068]];

Predictions for a pseudoscalar glueball

- Predict branching ratios for decays into three pseudoscalar mesons

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma_{\tilde{G}}^{tot}$	0.049	0.043
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma_{\tilde{G}}^{tot}$	0.019	0.011
$\Gamma_{\tilde{G}\to\eta\eta\eta}/\Gamma_{\tilde{G}}^{tot}$	0.016	0.013
$\Gamma_{\tilde{G}\to\eta\eta\eta'}/\Gamma_{\tilde{G}}^{tot}$	0.0017	0.00082
$\Gamma_{\tilde{G}\to\eta\eta'\eta'}/\Gamma_{\tilde{G}}^{tot}$	0.00013	0
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma_{\tilde{G}}^{tot}$	0.47	0.47
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma_{\tilde{G}}^{tot}$	0.16	0.17
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma_{\tilde{G}}^{tot}$	0.095	0.090

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph].

PANDA/FAIR will be able to scan the energy above 2.5 GeV

BESIII has measured a candidate: X(2370)

The decay of the pseudoscalar glueball into three pions vanishes:

$$\Gamma_{\tilde{G}\to\pi\pi\pi} = 0$$

Predict branching ratios for decays into a scalar and a pseudoscalar meson

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.060	0.070
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083	0.10
$\Gamma_{\tilde{G}\to \eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.0000026	0.0000030
$\Gamma_{\tilde{G}\to \eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.039	0.026
$\Gamma_{\tilde{G}\to \eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012 (0.015)	0.0094 (0.017)
$\Gamma_{\tilde{G}\to \eta'\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0 (0.0082)	0 (0)

Could be measured by PANDA!

whereas

$$K_s = K_0^*(1430), a_0 = a_0(1450), \sigma_N \approx f_0(1370), \sigma_S \approx f_0(1710)$$

The full width of the pseudoscalar glueball is expected to be small

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph]; W.I. Eshraim and S. Janowski, PoS ConfinementX 118, (2012) [arXiv:1301.3345 [hep-ph]]; W.I. Eshraim and S. Janowski, J. Phys.Conf. Ser. 426, 012018 (2013) [arXiv:1211.7323 [hep-ph]].

The branching ratio of the decay processes $\Gamma_{\tilde{G}\to\overline{N}N}$ and $\Gamma_{\tilde{G}\to\overline{N}^{*}N+\mathrm{h.c.}}$

$$\frac{\Gamma_{\tilde{G}\to\overline{N}N}}{\Gamma_{\tilde{G}\to\overline{N}^{*}N+\mathrm{h.c.}}} = 1.94\,.$$

Remark: The pseudoscalar glueball can be produced directly through a fusion process in proton-proton collision

Charmed mesons
in the extended Linear Sigma Model

Fields in the model

- **Mesons: quark-antiquark states ($q\bar{q}$)**

$$4N_f^2 + 2 \text{ fields}$$

- **For $N_f = 4$ there are *66* mesons: *64* quark-antiquark fields + one pseudoscalar glueball $\tilde{G}$ +one scalar glueball *G***

Quantum numbers (J^{PC})

Pseudoscalar mesons: 0^{-+}	Scalar mesons: 0^{++}	Vector mesons: 1^{--}	Axial-vector mesons: 1^{++}
D^0	$D_0^{*0} \Rightarrow D_0^*(2400)^0$	$D^{*0} \Rightarrow D^*(2007)^0$	$D_1^0 \Rightarrow D_1(2420)^0$
$D^\pm$	$D_0^{*\pm} \Rightarrow D_0^*(2400)^\pm$	$D^{*\pm} \Rightarrow D^*(2010)^\pm$	$D_1^\pm \Rightarrow D_1(2420)^\pm$
$D_s^\pm$	$D_{S0}^{*\pm} \Rightarrow D_{S0}^*(2317)^\pm$	$D_S^\pm$	$D_{S1}^\pm \Rightarrow D_{S1}(2536)^\pm$
$\eta_C \Rightarrow \eta_C(1S)$	$\chi_{c0} \Rightarrow \chi_{c0}(1P)$	$J/\psi \Rightarrow J/\psi(1S)$	$\chi_{c0} \Rightarrow \chi_{c0}(1P)$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]]; W. I. Eshraim, F. Giacosa, and D. H. Rischke , [arXiv:1405.5861 [hep-ph]].

Including charm degree of freedom

1) Pseudoscalar fields:

$$P=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_N+\pi^0) & \pi^+ & K^+ & D^0 \\ \pi^- & \frac{1}{\sqrt{2}}(\eta_N-\pi^0) & K^0 & D^- \\ K^- & \bar{K}^0 & \eta_S & D_S^- \\ \bar{D}^0 & D^+ & D_S^+ & \eta_c \end{pmatrix}$$

$\eta=\eta_N\cos\phi+\eta_S\sin\phi$ and $\eta'=-\eta_N\sin\phi+\eta_S\cos\phi$

with mixing angle $\phi=-44.6^o$ [W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph]].

2) Scalar fields:

$\bar{n}n\propto\bar{u}u+\bar{d}d$

$$S=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma_N+a_0^0) & a_0^+ & K_0^{*+} & D_0^{*0} \\ a_0^- & \frac{1}{\sqrt{2}}(\sigma_N-a_0^0) & K_0^{*0} & D_0^{*-} \\ K_0^{*-} & \bar{K}_0^{*0} & \sigma_S & D_{S0}^{*-} \\ \bar{D}_0^{*0} & D_0^{*+} & D_{S0}^{*+} & \chi_{c0} \end{pmatrix}$$

$\bar{s}s$

$\bar{c}c$

The multiplet of the scalar and pseudoscalar quark-antiquark states: $\Phi=S+iP$

3) Vector fields:

$$V^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(\omega_N+\rho^0) & \rho^+ & K^*(892)^+ & D^{*0} \\ \rho^- & \frac{1}{\sqrt{2}}(\omega_N-\rho^0) & K^*(892)^0 & D^{*-} \\ K^*(892)^- & \bar{K}^*(892)^0 & \omega_S & D_S^{*-} \\ \bar{D}^{*0} & D^{*+} & D_S^{*+} & J/\psi \end{pmatrix}^\mu$$

4) Axial vector fields:

$$A^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(f_{1,N}+a_1^0) & a_1^+ & K_1^+ & D_1^0 \\ a_1^- & \frac{1}{\sqrt{2}}(f_{1,N}-a_1^0) & K_1^0 & D_1^- \\ K_1^- & \bar{K}_1^0 & f_{1,S} & D_{S1}^- \\ \bar{D}_1^0 & D_1^+ & D_{S1}^+ & \chi_{c,1} \end{pmatrix}^\mu$$

The left-handed matrix: $L^\mu=V^\mu+A^\mu$ and the right-handed matrix: $R^\mu=V^\mu-A^\mu$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]]; W. I. Eshraim; Giacosa; and D. H. Rischke; [arXiv:1405.5861 [hep-ph]]

Spontaneous Symmetry Breaking (SSB)

Shifting the fields

$$G \to G + G_0 \,, \quad \sigma_N \to \sigma_N + \phi_N \,, \quad \sigma_S \to \sigma_S + \phi_S$$

where

$$\phi_N = Z_\pi f_\pi \qquad \phi_S = \frac{2Z_k f_k - \phi_N}{\sqrt{2}}$$

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph].

For ***N**$_f$**= 4*** new shift

$$\chi_{C0} \to \chi_{C0} + \phi_C$$

where

$$\phi_C = \frac{2Z_D f_D - \phi_N}{\sqrt{2}} = \sqrt{2} Z_{D_s} f_{D_s} - \phi_S = \frac{Z_{\eta_C} f_{\eta_C}}{\sqrt{2}}$$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]];W. I. Eshraim, F. Giacosa and D. H. Rischke, , arXiv:1405.5861 [hep-ph]].

There are 29 eqs. for the squared masses of mesons with 15 unknown parameters.

Parameters

The values of the $N_f = 3$ parameters :

Parameter	Value	Parameter	Value
m_1^2	0.413×10^6 MeV^2	m_0^2	-0.918×10^6 MeV^2
$\phi_C^2 c/2$	$450 \cdot 10^{-6}$ MeV^{-2}	δ_S	0.151×10^6MeV^2
g_1	5.84	h_1	0
h_2	9.88	h_3	3.87
ϕ_N	164.6 MeV	ϕ_S	126.2 MeV
λ_1	0	λ_2	68.3

[D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]].

⇨ $\chi^2/d.o.f = 1.23$

The new three parameters for $N_f = 4$ are $\phi_C, \delta_C, \varepsilon_C$.

By fit with $\chi^2/d.o.f = 1$:

$$\phi_C = (176 \pm 28)\ \text{MeV},\ \ \delta_C = (3.91 \pm 0.36) \times 10^6\ \text{MeV}^2,\ \varepsilon_C = (2.23 \pm 0.71) \times 10^6\ \text{MeV}^2\ .$$

[W. I. Eshraim, F. Giacosa and D. H. Rischke, , arXiv:1405.5861 [hep-ph]].

Results

Masses of light mesons:

Observable	our Value [MeV]	Experimental Value [MeV]
$m_{f_{1N}}$	1186	1281.8 ± 0.6
m_{a_1}	1185	1230 ± 40
$m_{f_{1S}}$	1372	1426.4 ± 0.9
m_{K^*}	885	891.66 ± 0.26
m_{K_1}	1281	1272 ± 7
m_{σ_1}	1362	(1200-1500)-i(150-250)
m_{a0}	1363	1474 ± 19
m_{σ_2}	1531	1720 ± 60
m_{ω_N}	783	782.65 ± 0.12
m_{ω_S}	975	1019.46 ± 0.020
m_ρ	783	775.5 ± 38.8
m_η	509	547.853 ± 0.024
m_π	141	139.57018 ± 0.00035
$m_{\eta'}$	962	957.78 ± 0.06
$m_{K_0^*}$	1449	1425 ± 50
m_K	485	493.677 ± 0.016

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]; D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph].

Masses of (open and hidden) charmed mesons:

Resonance	Quark content	J^P	Our Value [MeV]	Experimental Value [MeV]
D^0	$u\bar{c}, \bar{u}c$	0^-	1981 ± 73	1864.86 ± 0.13
$D_S^\pm$	$s\bar{c}, \bar{s}c$	0^-	2004 ± 74	1968.50 ± 0.32
$\eta_c(1S)$	$c\bar{c}$	0^-	2673 ± 118	2983.7 ± 0.7
$D_0^*(2400)^0$	$u\bar{c}, \bar{u}c$	0^+	2414 ± 77	2318 ± 29
$D_{S0}^*(2317)^\pm$	$s\bar{c}, \bar{s}c$	0^+	2467 ± 76	2317.8 ± 0.6
$\chi_{c0}(1P)$	$c\bar{c}$	0^+	3144 ± 128	3414.75 ± 0.31
$D^*(2007)^0$	$u\bar{c}, \bar{u}c$	1^-	2168 ± 70	2006.99 ± 0.15
D_s^*	$s\bar{c}, \bar{s}c$	1^-	2203 ± 69	2112.3 ± 0.5
$J/\psi(1S)$	$c\bar{c}$	1^-	2947 ± 109	3096.916 ± 0.011
$D_1(2420)^0$	$u\bar{c}, \bar{u}c$	1^+	2429 ± 63	2421.4 ± 0.6
$D_{S1}(2536)^\pm$	$s\bar{c}, \bar{s}c$	1^+	2480 ± 63	2535.12 ± 0.13
$\chi_{c1}(1P)$	$c\bar{c}$	1^+	3239 ± 101	3510.66 ± 0.07

W. I. Eshraim, F. Giacosa and D. H. Rischke, [arXiv:1405.5861 [hep-ph]];
W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph].

Mass difference and decay constants

The mass difference of the squared charmed (axial-)vector mesons:

mass difference	theoretical value MeV^2	experimental value MeV^2
$m^2_{D_1} - m^2_{D^*}$	$(1.2 \pm 0.6) \times 10^6$	1.82×10^6
$m^2_{\chi_{C1}} - m^2_{J/\psi}$	$(1.8 \pm 1.3) \times 10^6$	2.73×10^6
$m^2_{D_{S1}} - m^2_{D^*_S}$	$(1.2 \pm 0.6) \times 10^6$	1.97×10^6

Weak decay constant of *D*, *D*s, and f_{η_C} .

$$f_D = (254 \pm 17)\ \text{MeV}\ ,\ f_{D_S} = (261 \pm 17)\ \text{MeV}\ ,\ f_{\eta_C} = (314 \pm 39)\ \text{MeV}.$$

[Exp. value = 206.7 ± 8.9] *MeV*, [Exp. value = 260.5 ± 5.4] *MeV*, [Exp. value = 335 ± 75] *MeV*

W. I. Eshraim, F. Giacosa and D. H. Rischke, arXiv:1405.5861 [hep-ph].

Decay widths of open charmed mesons:

Decay Channel	Theoretical result [MeV]	Experimental result [MeV]
$D_0^*(2400)^0 \to D\pi = D^+\pi^- + D^0\pi^0$	139^{+243}_{-114}	$D^+\pi^-$ seen; full width $\Gamma = 267 \pm 40$
$D_0^*(2400)^+ \to D\pi = D^+\pi^0 + D^0\pi^+$	51^{+182}_{-51}	$D^+\pi^0$ seen; full width: $\Gamma = 283 \pm 24 \pm 34$
$D^*(2007)^0 \to D^0\pi^0$	0.025 ± 0.003	seen; < 1.3
$D^*(2007)^0 \to D^+\pi^-$	0	not seen
$D^*(2010)^+ \to D^+\pi^0$	$0.018^{+0.002}_{-0.003}$	0.029 ± 0.008
$D^*(2010)^+ \to D^0\pi^+$	$0.038^{+0.005}_{-0.004}$	0.065 ± 0.017
$D_1(2420)^0 \to D^*\pi = D^{*+}\pi^- + D^{*0}\pi^0$	65^{+51}_{-37}	$D^{*+}\pi^-$ seen; full width: $\Gamma = 27.4 \pm 2.5$
$D_1(2420)^0 \to D^0\pi\pi = D^0\pi^+\pi^- + D^0\pi^0\pi^0$	0.59 ± 0.02	seen
$D_1(2420)^0 \to D^+\pi^-\pi^0$	$0.21^{+0.01}_{-0.015}$	seen
$D_1(2420)^0 \to D^+\pi^-$	0	not seen; $\Gamma(D^+\pi^-)/\Gamma(D^{*+}\pi^-) < 0.24$
$D_1(2420)^+ \to D^*\pi = D^{*+}\pi^0 + D^{*0}\pi^+$	65^{+51}_{-36}	$D^{*0}\pi^+$ seen; full width: $\Gamma = 25 \pm 6$
$D_1(2420)^+ \to D^+\pi\pi = D^+\pi^+\pi^- + D^+\pi^0\pi^0$	0.56 ± 0.02	seen
$D_1(2420)^+ \to D^0\pi^0\pi^+$	0.22 ± 0.01	seen
$D_1(2420)^+ \to D^0\pi^+$	0	not seen; $\Gamma(D^0\pi^+)/\Gamma(D^{*0}\pi^+) < 0.18$
$D_{S1}(2536)^+ \to D^*K = D^{*0}K^+ + D^{*+}K^0$	25^{+22}_{-15}	seen; full width $\Gamma = 0.92 \pm 0.03 \pm 0.04$
$D_{S1}(2536)^+ \to D^+K^0$	0	not seen
$D_{S1}(2536)^+ \to +D^0K^+$	0	not seen

W. I. Eshraim, F. Giacosa and D. H. Rischke, arXiv:1405.5861 [hep-ph].

Decay widths of hidden charmed mesons:

- The decay widths of charmonium state depend on the parameters λ_1 and h_1.

Using fit including the decay widths of charmonium state χ_{C0}, we obtain

$$\lambda_1 = -0.16 \quad \text{and} \quad h_1 = 0.046.$$

W. I. Eshraim and D. H. Rischke, in preparation, preliminary!

Mixing matrix of the three scalar fields (σ_N, σ_S, G)

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.94 & -0.17 & 0.29 \\ 0.21 & 0.97 & -0.12 \\ -0.26 & 0.18 & 0.95 \end{pmatrix} \begin{pmatrix} \sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \\ \sigma_S \equiv \bar{s}s \\ G \equiv gg \end{pmatrix}$$

where G is a scalar glueball.

S. Janowski, F. Giacosa and D. H. Rischke, Phys.Rev. D90 (2014) 114005 [arXiv:1408.4921 [hep-ph]].

Decay widths of hidden charmed mesons:

1) Decay widths of (axial-)vector charmonium states:

$$\Gamma_{J/\psi} = 0 \quad \text{and} \quad \Gamma_{\chi_{c1}} = 0$$

2) Decay widths of scalar charmonium state (η_C):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\eta_c \to \overline{K}_0^* K}$	0.01	-
$\Gamma_{\eta_c \to a_0 \pi}$	0.01	-
$\Gamma_{\eta_c \to f_0(1370)\eta}$	0.00018	-
$\Gamma_{\eta_c \to f_0(1500)\eta}$	0.006	-
$\Gamma_{\eta_c \to f_0(1710)\eta}$	0.000032	-
$\Gamma_{\eta_c \to f_0(1370)\eta'}$	0.027	-
$\Gamma_{\eta_c \to f_0(1500)\eta'}$	0.024	-
$\Gamma_{\eta_c \to f_0(1710)\eta'}$	0.0006	-
$\Gamma_{\eta_c \to \eta\eta\eta}$	0.052	-
$\Gamma_{\eta_c \to \eta'\eta'\eta'}$	0.0023	-
$\Gamma_{\eta_c \to \eta'\eta\eta}$	0.44	-
$\Gamma_{\eta_c \to \eta'\eta'\eta}$	0.0034	
$\Gamma_{\eta_c \to \eta K\overline{K}}$	0.15	0.32±0.17
$\Gamma_{\eta_c \to \eta' KK}$	0.41	
$\Gamma_{\eta_c \to \eta\pi\pi}$	0.12	0.54±0.18
$\Gamma_{\eta_c \to \eta'\pi\pi}$	0.08	1.3±0.0.6
$\Gamma_{\eta_c \to KK\pi}$	0.095	-

W. I. Eshraim and D. H. Rischke, in preparation preliminary!

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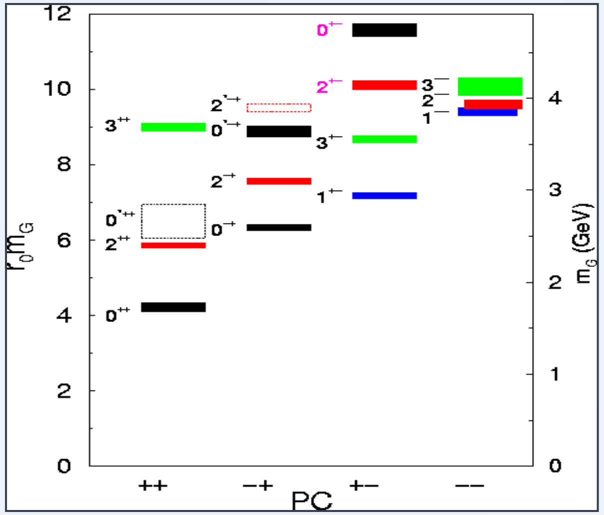
Decay width of η_C into a pseudoscalar glueball

BESIII: $m_{\tilde{G}} = 2370\ MeV$

$$\Gamma_{\eta_{C\to\tilde{G}\pi\pi}} = \begin{cases} = 0.16\ MeV \\ = 0.124\ MeV \end{cases}$$

Could be measured by

Lattice QCD calculations:



$m_{\tilde{G}} = 2600\ MeV$

W. I. Eshraim and D. H. Rischke, in preparation

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Decay width of χ_{c0}

3) Decay widths of a pseudoscalar charmonium state (χ_{c0}):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0}\to a_0 a_0}$	0.004	-
$\Gamma_{\chi_{c0}\to k_1 \overline{K}_1}$	0.005	-
$\Gamma_{\chi_{c0}\to \eta\eta}$	0.022	0.031 ± 0.0039
$\Gamma_{\chi_{c0}\to \eta'\eta'}$	0.02	0.02 ± 0.0035
$\Gamma_{\chi_{c0}\to \eta\eta'}$	0.004	<0.0024
$\Gamma_{\chi_{c0}\to K^* K_0^*}$	0.00007	-
$\Gamma_{\chi_{c0}\to \rho\rho}$	0.01	-
$\Gamma_{\chi_{c0}\to f_0(1370) f_0(1370)}$	0.005	<0.003
$\Gamma_{\chi_{c0}\to f_0(1500) f_0(1500)}$	0.004	<0.0005
$\Gamma_{\chi_{c0}\to f_0(1370) f_0(1500)}$	0.000004	<0.001
$\Gamma_{\chi_{c0}\to f_0(1370) f_0(1710)}$	0.0003	0.0069 ± 0.004
$\Gamma_{\chi_{c0}\to f_0(1500) f_0(1710)}$	0.00004	<0.0007
$\Gamma_{\chi_{c0}\to K_0^* K\eta}$	0.008	-
$\Gamma_{\chi_{c0}\to K_0^* K\eta'}$	0.004	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta}$	0.0004	-
$\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta}$	0.003	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta'\eta'}$	0.0027	-
$\Gamma_{\chi_{c0}\to f_0(1370)\eta\eta'}$	0.000089	-
$\Gamma_{\chi_{c0}\to f_0(1500)\eta\eta'}$	0.011	-
$\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta}$	0.00008	-
$\Gamma_{\chi_{c0}\to f_0(1710)\eta\eta'}$	0.00003	-

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0}\to \overline{K}_0^{*0} K_0^{*0}}$	0.01	0.01 ± 0.0047
$\Gamma_{\chi_{c0}\to K^- K^+}$	0.059	0.061 ± 0.007
$\Gamma_{\chi_{c0}\to \pi\pi}$	0.089	0.088 ± 0.0092
$\Gamma_{\chi_{c0}\to \overline{K}^{*0} K^{*0}}$	0.0175	0.017 ± 0.0072
$\Gamma_{\chi_{c0}\to ww}$	0.01	0.0099 ± 0.0017
$\Gamma_{\chi_{c0}\to \phi\phi}$	0.004	0.0081 ± 0.0013
$\Gamma_{\chi_{c0}\to k_1^+ K^-}$	0.005	0.063 ± 0.0233

W. I. Eshraim and D. H. Rischke, in preparation, preliminary!

Conclusions and Outlook

1. In the case of $N_f = 3$: Decay of a pseudoscalar glueball with a mass above 2 GeV.

2. Linear sigma model with $N_f = 4$ and vector and axial-vector mesons.

3. Masses of (open and hidden) charmed mesons.

4. Charm-anticharm condensate and decay constants.

5. Decay widths of (open and hidden) charmed mesons.