Lambda baryons from lattice QCD - from strange to charm -

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Our target is $\Lambda$

**Negative parity $\Lambda$ (1405)**
- difficulty in mass reproduction in quark models
- possibility of meson-baryon (NK, $\pi\Sigma$) molecule
- NK strong attraction
- Has been problematic in Lattice QCD analyses
- Recently $\Lambda(1405)$ was identified in lattice QCD calculation
  PRL 108, 112001 (2012)

**Spin-orbit partner of $\Lambda$**
- Properties of spin 3/2 $\Lambda$, which is the $\Lambda(1405)$'s partner

**$\Lambda(1405)$'s structures**
- Flavor structures
- Internal wavefunctions

**$\Lambda_c$'s properties**
- properties of $\Lambda_c$, where the strange is replaced with the charm
- What does the HQ symmetry cause?
Recent lattice QCD study
Recent Lattice study of $\Lambda$

Isolating the $\Lambda(1405)$ in Lattice QCD, Adelaide group; PRL 108, 112001 (2012)

2+1 conf. by PACS-CS
Pion mass $\rightarrow$ 156~702 MeV

\[ m^2_{\pi} (\text{GeV}^2) \]

\[ M (\text{GeV}) \]

Physical point

Brief summary

- Correlation matrix analyses with many types of interpolating fields
- lowest state lies between $KN, n\Sigma$ thresholds $\rightarrow$ Same order as the real world
- Claim to have identified $\Lambda(1405)$ signal

Still too heavy

- Adjusted the valence $s$-quark mass so as to correctly reproduce the $K$-meson mass (partially quenching)

Gets experimental value

- lowest $\rightarrow$ singlet dominant
- 2\textsuperscript{nd} lowest $\rightarrow$ octet dominant
Recent Lattice study of Λ

Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule

2+1 conf. by PACS-CS
Pion mass $\to 156\sim 702$ MeV

Electromagnetic properties

- Strange-quark magnetic moments in Λ(1405) vanishes.

$\rightarrow$ The strange quark is confined in the spin-0 Kaon in Λ(1405)

There are several issues to be clarified, but the signal can be the Λ(1405).
Why $\Lambda c$?
Our target is $\Lambda$

$\text{udS} - \Lambda$

$m_u = m_d < m_s$

SU(3)F symmetry

$\text{udC} - \Lambda$

$m_u = m_d \ll m_c$

Heavy quark spin symmetry appears

Spin-spin interactions become weak

Heavy quark spin decouples and is irrelevant

Largely broken SU(3)F symmetry

Two excitation modes may decouple

Hadron spectra would be much simpler

Manifestation of Diquark DOF?

Decouples
Diquarks

Two diquark motion (ρ and λ modes) may decouple in heavy baryons

ρ-mode excitation
Diquark’s relative motion

λ-mode excitation
Diquark’s CM motion

Then, excited $\Lambda_c$ spectra can be simply explained in terms of diquarks (?)

$\Lambda_c(2595)$

$\Lambda_c(2286)$
Diquarks

$u_dS - \Lambda$

$m_u = m_d < m_s$

SU(3)F symmetry

$u_dC - \Lambda$

$m_u = m_d \ll m_c$

HQ symmetry

Then, what is the relationship between $uds-\Lambda$ and $udc-\Lambda$?

$\rightarrow$ Internal structure change

$\rightarrow$ Manifestation of diquark’s degrees of freedom

Octet

$\Lambda(1670)$

Singlet

$\Lambda(1405)$

Octet

$\Lambda(1116)$

$\Lambda_{c}(2286)$

$\Lambda_{c}(2595)$

$\rho$-excitation?

$\lambda$-excitation?

Replace S with C
Strategies
Our strategy

We investigate **masses and flavor structures** of $udQ$-$\Lambda$ baryons

We investigate the $Q$-dependences of
(1) $\Lambda$ mass spectra
(2) Internal flavor structures

- **Real world**
  - $Q=$Strange quark ($\sim 100\text{MeV}$)
    - $uds$-$\Lambda$

- **Virtual world**
  - Interpolation using lattice QCD by changing $Q$-quark mass

- **Real world**
  - $Q=$Charm quark ($\sim 1.3\text{GeV}$)
    - $udc$-$\Lambda$
Our strategy

-- Lattice QCD setups --

2+1 gauge configuration by PACS-CS Iwasaki gauge action, Wilson quark action 32^3 x 64, a~0.1 fm, cut off ~2.2GeV

→ Well reproduces light hadron mass spectra

We do not employ Relativistic Heavy Quark actions

→ cut off of 2.2 GeV may be insufficient for mQ > 1GeV
→ not good, but use a common quark action for u, d, Q quarks in order to see the internal structure changes
Our strategy

We employ operators classified in terms of Cubic group irreducible rep. → 4x4 correlation matrix (We can extract ground~3rd excited states.)

u, d, Q quark operator sizes are the same.
→ Operator mixing vanishes in the flavor-SU(3) limit (μu=md=mQ)
→ Easy to see the flavor structures of Λ particles

--- Hadronic operators ---

PRD72 (2005) 074501 LHPC group

Operator examples
Correlation matrix analyses

-- Mass and flavor structures --

2pt cross correlator

\[ \langle \Lambda^i(T)\Lambda^j(0) \rangle = \langle 0 | \Lambda^i e^{-H T} \Lambda^j | 0 \rangle = \langle 0 | \Lambda^i | k \rangle \exp(-E^*_k T) \langle k | \Lambda^j | 0 \rangle = c^T \Lambda(E_k) c \]

where \( i, j \) are operator indices.

\( \Lambda(E_k) = \text{diag}(-E_1 T, -E_2 T, ...) \)

Eigenvalue of correlation matrix \( \rightarrow \) mass of the state

Eigenvector of correlation matrix \( \rightarrow \) operator overlaps with the state

We can investigate internal structures.
Mass spectra
Mass spectra

UDS

$\frac{1}{2}$ negative $\Lambda$ channel

UDC

$\frac{1}{2}$ negative $\Lambda_c$ channel

Spectra are similar in $\Lambda$ and $\Lambda_c$ channel

Still heavier than 1405 MeV → But very close

2nd state’s behavior is a bit different from the $\Lambda_s$ channel → Difference in internal structures
Mass spectra

Hadron mass dependences on Q-quark mass

$
\begin{align*}
\Lambda(1405) \\
\Lambda_c(2595) \\
\Lambda(1670) \\
\Lambda_c(\sim 2900) \\
\Lambda(1800) \\
\Lambda_c(\sim 2950) \\
\Lambda_c(2595)
\end{align*}
$
Flavor structures
Flavor structures

We consider “flavor symmetry” for $u, d, Q$ quarks.
(even if $Q=$charm, we classify states still in terms of SU(3) )

Singlet component

$$C_s = \langle \Lambda | O_s | vac \rangle$$

$$O_s = \frac{1}{\sqrt{3}} (u(dQ) + d(Qu) + Q(ud))$$

Octet component

$$C_o = \langle \Lambda | O_o | vac \rangle$$

$$O_o = \frac{1}{\sqrt{6}} (u(dQ) + d(Qu) - 2Q(ud))$$
Flavor structures

Lowest 3 states in $\frac{1}{2}-$ channel

\[ \text{uds } \Lambda(1405) \quad \text{uds } \Lambda(2595) \quad \text{uds } \Lambda(1670) \quad \text{uds } \Lambda(1800) \quad \text{uds } \Lambda(\sim 2900) \quad \text{uds } \Lambda(\sim 2950) \]

\[ \text{udc } \Lambda(2595) \quad \text{udc } \Lambda(\sim 2900) \]

preliminary
Flavor structures

Lowest 3 states in $\frac{1}{2}$- channel

- **Singlet** $\rightarrow$ equally mixed
  - $uds \Lambda(1405)$
  - $uds \Lambda(1800)$

- **Octet** $\rightarrow$ equally mixed
  - $udc \Lambda(2595)$
  - $udc \Lambda(\sim 2900)$

Preliminary

Amplitudes here are normalized so that squared sum is 1. The ratio $1/\sqrt{2} : 1/\sqrt{2}$ implies 50:50 mixture
Flavor structures to Diquark picture

SU(3) wave functions can be expressed in terms of diquark wave functions

**ORBITAL**
- $R_\rho \rightarrow \rho$ mode (diquark p-wave excitation)
- $R_\lambda \rightarrow \lambda$ mode (diquark’s CM p-wave excitation)

**SPIN**
- $\chi_\rho \rightarrow$ Diquark has spin 1 (total 1/2)
- $\chi_\lambda \rightarrow$ Diquark has spin 0 (total 1/2)
- $\chi_s \rightarrow$ Diquark has spin 1 (total 3/2)

**Singlet**
$$|\Lambda ; 1 \rangle = \frac{1}{\sqrt{2}} (R_\lambda \chi_\rho - R_\rho \chi_\lambda)$$

**Octet**
$$|\Lambda ; 8 \rangle = \frac{1}{\sqrt{2}} (R_\lambda \chi_\rho + R_\rho \chi_\lambda)$$

$$|\Lambda ; 8 \rangle = \frac{1}{\sqrt{2}} (R_\rho \chi_s)$$
Flavor structures to Diquark picture

Spin ½ negative parity channel

Replace S with C

$\Lambda(1405)$

Singlet dominant

$\Lambda(1800)$

Octet dominant

$\Lambda(1670)$

Octet dominant

$\Lambda(1405)$

Singlet dominant

$m_u = m_d < m_s$

SU(3)F symmetry

Broken SU(3)F

$m_u = m_d \ll m_c$

$\Lambda_c(\sim 2950)$

$\rho$-mode dominant

$\Lambda_c(\sim 2900)$

$\rho$-mode dominant (spin total 3/2)

$\Lambda_c(2595)$

$\lambda$-mode dominant

Diquark DOF seem to appear around $m_Q \sim 700$ MeV
Flavor structures to Diquark picture

Spin $\frac{1}{2}$ negative parity channel

Replace S with C

$\Lambda(1405)$
- Singlet dominant

$\Lambda(1670)$
- Octet dominant

$\Lambda(1800)$
- Octet dominant

$m_u = m_d < m_s$
- SU(3)F symmetry

O:S=1:-1

$\Lambda_c(\sim 2950)$
- $\rho$-mode dominant

Possibility of level crossing (or misidentification?)

$\Lambda(2900)$
- $\Lambda$-mode dominant (spin total $3/2$)

Broken SU(3)F

udS $\rightarrow$ $\Lambda$

udC $\rightarrow$ $\Lambda$

$m_u = m_d \ll m_c$

$m_u = m_d < m_s$

$\Lambda(1800)$
- Octet dominant

$\Lambda(1405)$
- Singlet dominant

$\Lambda_c(\sim 2950)$
- $\rho$-mode dominant

$\Lambda_c(2595)$
- $\Lambda$-mode dominant

$SU(3)F$ symmetry

Replace S with C

$O:S=1:0$

$O:S=1:1$

$O:S=1:-1$
Flavor structures to Diquark picture

Level crossing occurs?

![Graph showing masses and couplings as functions of 1/kappa_HQ.](image)
We investigated the flavor structure of the low-lying Λ’s in lattice QCD.

**Mass**

The mass spectra of Λc and Λs are similar though their flavor structures are completely different.

**Flavor structure**

Ground state: (UDS) singlet dominant state $\rightarrow$ (UDC) $\lambda$-mode excitation

Excited state: (UDS) octet dominant state $\rightarrow$ (UDC) $\rho$- and $\lambda$-mode excitations

Well classified in terms of diquark excitations above (current) $m_Q > 700$ MeV

**Future work?**

Other channels?
More sophisticated analysis on the internal structures of Λ baryons