Non-dipolar Wilson links for quasi-parton distributions

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Outlines

- Quasi-parton distributions
- Breakdown of factorization
- Non-dipolar Wilson links
- Summary

Quasi-parton distributions

Difficulty on lattice

- Ordinary light-cone PDF

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

Wilson link in n_ direction

- Involve time dependence, not suitable for lattice calculation in Euclidean space
- Reason why only (few) moments (local objects) can be calculated by lattice

Quasi PDF definition

- To facilitate direct lattice calculation of PDF, i.e., all-moment calculation, modify definition in large Pz limit (Ji, 2013) $x = k^z / P^z$

$$\tilde{q}_n(x, \mu, P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P | \psi(w) W_n^\dagger(w) \gamma^z W_n(0) \psi(0) | P \rangle$$

$$W_n(w) = P \exp\left[-ig \int_0^\infty d\lambda \, T^a \, n \cdot A^a(\lambda n + w)\right] \quad w = (0, 0, 0, z)$$

- Then match Quasi PDF to light-cone PDF

$$\tilde{q}_n(x, \mu, P^z) = \int \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu)$$

lattice data

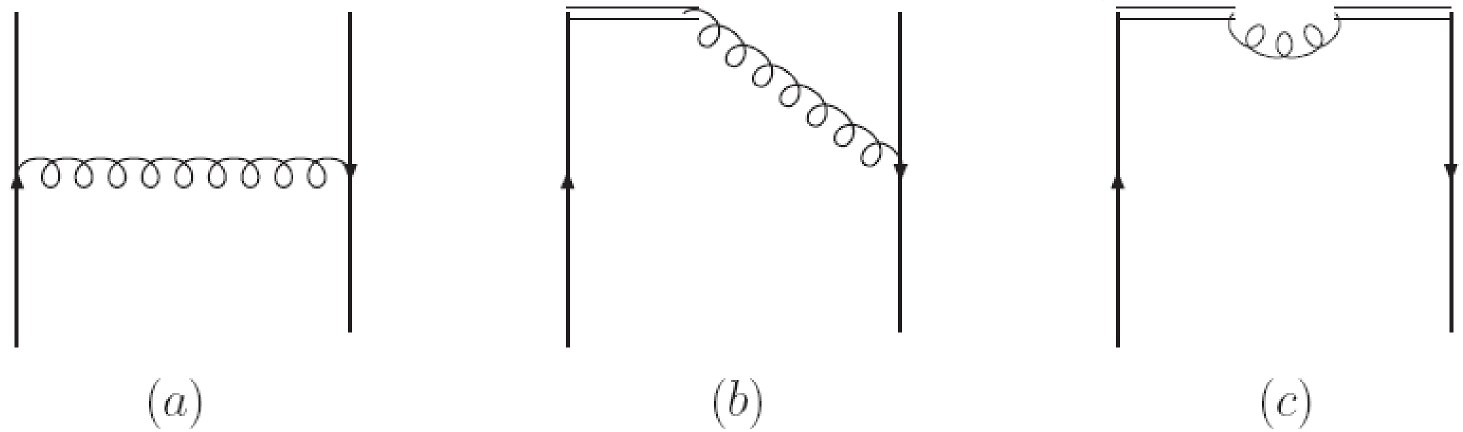
calculable in perturbation

extracted

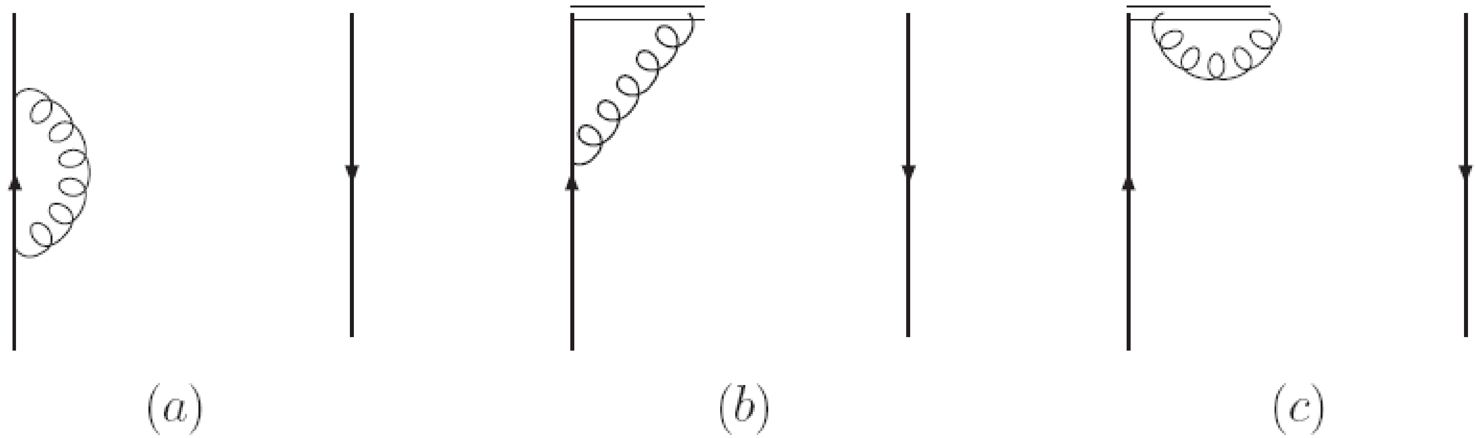
One loop in covariant gauge

- Fig. 1

n_ for LCPDF, n for QPDF



- Fig. 2



- Necessary for derivation of Z

One-loop result

- Quoted from Ji 2013

splitting function

$$q(x,\mu^2,P^z) = \frac{\alpha_s}{2\pi}T(x)\ \frac{\Lambda}{P^z} + \frac{\alpha_s}{2\pi}P(x)\ \ln\frac{\Lambda P^z}{\lambda^2}$$

- Same collinear divergence as LCPDF
- Rotation of Wilson links does not change collinear structure: $1/n\cdot l \approx 1/n^- l^+$
- But…

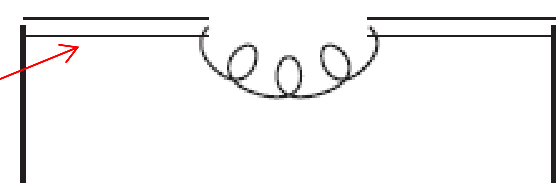
Breakdown of factorization

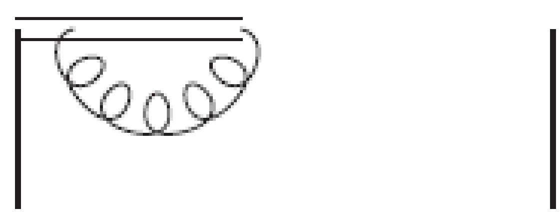
Linear divergence

- Linear (power) divergence exists in QPDF from self-energy corrections to Wilson links
- Caused by large l^0 of gluon momentum

$$
\begin{aligned}
\tilde{q}_n^{1c} &= -\frac{1}{2} g^2 C_F \int \frac{d^2 l_T}{(2\pi)^3} \frac{P^z}{l^0 (l^z)^2} \\
&= -\frac{\alpha_s}{2\pi} C_F \frac{\mu}{(1-x)^2 P^z}.
\end{aligned}
$$

$$
l^0 = \sqrt{l_T^2 + (l^z)^2 + m_g^2}
$$



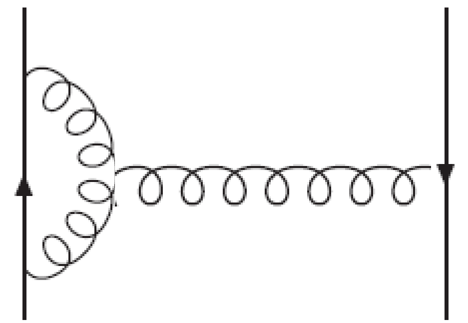


Induced collinear divergence

- First gluon, on-shell and energetic with $l^0 \sim l_T >> l^z$, moves in direction perpendicular to Wilson links (z axis)
- Attach second gluon to it
- Additional collinear divergence (along perpendicular direction) may be induced at two loops

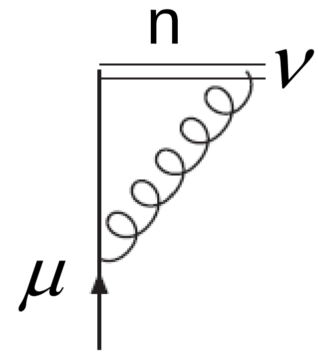
Light-cone gauge

- In light-cone gauge $n_{-} \cdot A = 0$, collinear divergence appears only in diagrams without Wilson links
- Two loop example:



- Gluon propagator

$$\propto -g^{\mu\nu} + \frac{n_{-}^{\mu} l^{\nu} + n_{-}^{\nu} l^{\mu}}{n_{-} \cdot l}$$



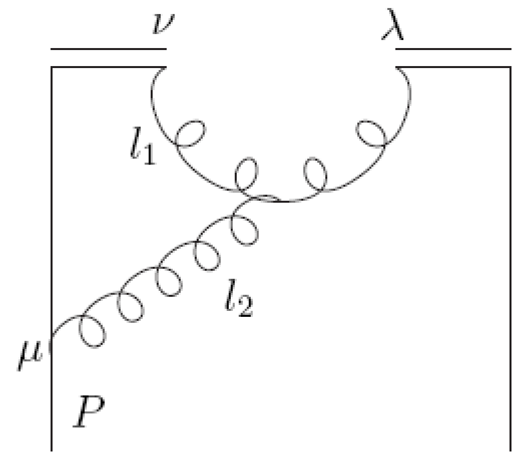
- Collinear divergence cancels between

$$-\frac{n^{\mu}}{n \cdot l} + \frac{n_{-}^{\mu}}{n_{-} \cdot l}$$

The two-loop diagram

- Only troublesome diagram is
- Integration over 1st gluon

$$I = \frac{\pi l_1^z l_2^2}{(l_2^z)^3} \ln \frac{l_1^z (l_2^0)^2 m_g^2}{4\mu^2 (l_2^z)^3} + \cdots$$



- No collinear divergence from 2nd gluon
- This region with large loop momentum perpendicular to z axis is usually power suppressed, but enhanced by linear divergence in QPDF

Breakdown of factorization

- This induced collinear divergence in QPDF cannot go into LCPDF, but into Z
- Matching kernel Z become infrared divergent, and not calculable in perturbation theory
- LCPDF cannot be extracted reliably from lattice data of QPDF!

Non-dipolar Wilson links

Modified quasi PDF

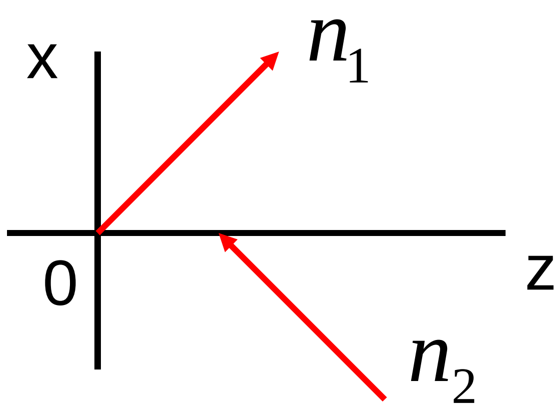
- Introduce non-dipolar Wilson links with the two pieces of Wilson links rotated into different directions

$$\tilde{q}(x,\mu,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\psi(w) W^{\dagger}_{n_2}(w) \gamma^z W_{n_1}(0)\psi(0)|P\rangle$$

$$n_1 = (0,1,0,1) \text{ and } n_2 = (0,-1,0,1) \qquad n_1 \cdot n_2 = 0$$

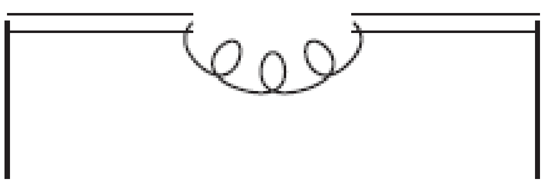
- Remove linear divergence

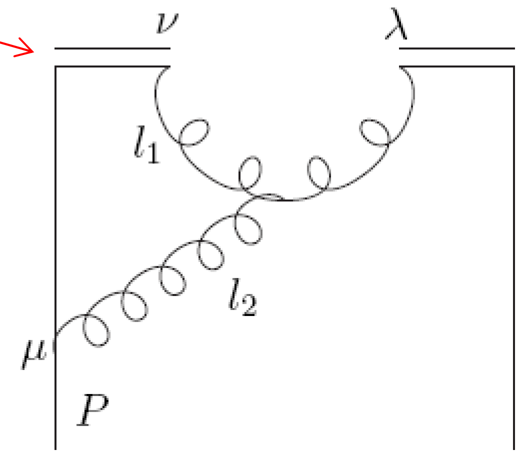
$$\tilde{q}^{1c} = 0$$



The key

- The key is the introduction of additional l_T into eikonal propagators





- Additional $1/l_T^2$ suppresses troublesome region. No induced collinear divergence, or induced collinear divergence is power suppressed, and neglected

Alternatives

- Consider “cross section” defined by space-like correlator, measure this cross section on lattice, and then extract LCPDF
- Aglietti, Ciuchini, Corbo, Franco, Martinelli and Silvestrini, 1998
- Abada, Boucaud, Herdoiza, Leroy, Micheli, Pene and Rodriguez-Quintero, 2001
- Braun and Mueller, 2007
- Ma and Qiu, 2014

Results of Fig.1(b)

- Confirm the same collinear divergence

MQPDF

$$\tilde{q}^{1b} = \begin{cases} \frac{\alpha_s}{4\pi} C_F \left[\frac{2x}{1-x} \ln \frac{2(1-x)P^z \mu}{x m_g^2} - \frac{1+x}{1-x} \ln \frac{\mu}{(1-x)\sqrt{2}P^z} - \frac{\pi}{4} \right], & \text{for } 0 < x < 1; \\ \frac{\alpha_s}{4\pi} C_F \left[\ln \frac{\mu}{2(x-1)P^z} + \frac{1+x}{1-x} \ln \sqrt{2} + \frac{\pi}{4} \right], & \text{for } x > 1. \end{cases}$$

LCPDF

$$q^{1b} = \frac{\alpha_s}{4\pi} C_F \frac{2x}{1-x} \ln \frac{\mu^2}{x m_g^2},$$

- Rotation of Wilson links does not change collinear structure

Modified matching kernel

- Matching MQPDF to LCPDF gives

$$
\begin{aligned}
Z\left(\xi, \frac{\mu}{P^z}\right) \;=\; & \left[1 - \frac{\alpha_s}{4\pi} C_F \left(6 \ln \frac{\mu}{2P^z} + 6 + \ln^2 2 + \ln 2 + \frac{\pi^2}{6}\right)\right] \delta(1-\xi) \\
& + \frac{\alpha_s}{4\pi} C_F \left\{ 2(2+\xi+\xi^2) \left[\frac{\ln(2P^z/\mu)}{(1-\xi)_+} + \left(\frac{\ln(1-\xi)}{1-\xi} \right)_+ \right] - \frac{1+\xi}{(1-\xi)_+} \ln 2 - \frac{\pi}{2} \right\}
\end{aligned}
$$

$$0 \;<\; \xi \;\leq\; 1$$

- No linear divergence and infrared finite

Summary

- Naïve QPDF violates factorization due to additional collinear divergence induced by linear divergence
- MQPDF with the non-dipolar Wilson links is free of the linear divergence, and maintains infrared structure of
- All-order proof of factorization follows Ma and Qiu, 2014
- Should put MQPDF on lattice