Analysis of charged lepton flavor violation process
\[ \mu^- e^- \rightarrow e^- e^- \] in muonic atoms

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Contents

1. Introduction
   - Charged Lepton Flavor Violation (CLFV)
   - $\mu^- e^- \rightarrow e^- e^-$ in a muonic atom

2. Formulation
   - Partial wave expansion

3. Results
   - Decay rates and Branching ratios
   - Model-discriminating power

4. Summary
1. INTRODUCTION
### Lepton Flavor Violation

- Three lepton flavor numbers \( (L_e, L_\mu, L_\tau) \)

<table>
<thead>
<tr>
<th></th>
<th>( e^-, \nu_e )</th>
<th>( \mu^-, \nu_\mu )</th>
<th>( \tau^-, \nu_\tau )</th>
<th>others</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_e )</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( L_\mu )</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( L_\tau )</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
</tr>
</tbody>
</table>

(Anti-leptons have a minus sign.)

- In SM, each lepton flavor \# is strictly conserved. (e.g. \( \mu^- \to e^- \nu_\mu \bar{\nu}_e \))
  - Neutrino oscillation violates the conservation.
    (e.g. \( \nu_\mu \to \nu_\tau \))

- The violation in charged lepton sector has not been observed yet.
  \( \rightarrow \) Charged Lepton Flavor Violation (CLFV)

  e.g. \( \mu \to e\gamma, \tau \to e\gamma, \tau \to e\pi^0 \ldots \)

predicted in many models beyond SM

good probes of new physics!
CLFV search using muons

Advantages of using muon for rare process

1. high intensity muon beam  ($\sim 10^8$ muons per a second)
2. long lifetime and simple kinematics

Examples of CLFV processes using muons

- a) $\mu^+ \rightarrow e^+ \gamma$  
  BR $< 5.7 \times 10^{-13}$ by MEG 

- b) $\mu^+ \rightarrow e^+ e^- e^+$  
  BR $< 1.0 \times 10^{-12}$ by SINDRUM 

- c) $\mu^- N \rightarrow e^- N$  
  BR $< 7 \times 10^{-13}$ ($\mu^-\text{Au} \rightarrow e^-\text{Au}$) by SINDRUM II 

$\mu^-\text{Al} \rightarrow e^-\text{Al}$ is planned to be measured by COMET experiment @ J-PARC.

(COherent Muon to Electron Transition)
\[ \mu^- e^- \rightarrow e^- e^- \] in a muonic atom


New CLFV search using muonic atoms

Features

- clear signal: two \( e^- \)s \( (E_1 + E_2 \approx m_\mu + m_e - B_\mu - B_e) \)
- 2 type interactions
  - \( \mu e e e \) vertex
  - \( \mu e \gamma \) vertex
- atomic \# \( Z \): large \( \Rightarrow \) decay rate \( \Gamma \): large \( (\Gamma \propto (Z - 1)^3) \)

proposed to be measured in COMET

Previous estimation of decay rate


Suppose nuclear Coulomb potential is weak

\[ \Gamma \sim \sigma v_{\text{rel}} |\psi_{1S}^e(0)|^2 \propto (Z - 1)^3 \]

- \(\sigma v_{\text{rel}}\): cross section of \(\mu^- e^- \to e^- e^-\) (free particles’)
- \(\psi_{1S}^e(x)\): Schrödinger wave function of a bound electron

Branching ratio

\[ \text{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-) \]

- \(\tilde{\tau}_\mu\): lifetime of a muonic atom

- increasing as atomic # \(Z\) is larger

Using muonic atom with large \(Z\) is favored.
Improved estimation of decay rate

Approximations used in the previous work

- The spreads of bound $\mu^-, e^-$ are sufficiently large.
- emitted $e^-$: plane wave
- bound electron: non-rela
- nucleus: point charge

Those approximations are expected to be worse for large $Z$.

For more quantitative estimation

- treatment of leptons as relativistic Coulomb wave
  - distortion of emitted $e^-$s by nuclear Coulomb potential
  - relativistic treatment of bound leptons
  - nuclear charge distribution with a finite size

How will the decay rates be changed by this improvement?
2. FORMULATION
Effective Lagrangian

\[ \mathcal{L}_I = \mathcal{L}_{\text{contact}} + \mathcal{L}_{\text{photo}} \]

\[ \mathcal{L}_{\text{contact}} = g_1 (\bar{e}_L \mu_R)(\bar{e}_L e_R) + g_2 (\bar{e}_R \mu_L)(\bar{e}_R e_L) \]
\[ + g_3 (\bar{e}_R \gamma_{\mu} \mu_R)(\bar{e}_R \gamma^\mu e_R) + g_4 (\bar{e}_L \gamma_{\mu} \mu_L)(\bar{e}_L \gamma^\mu e_L) \]
\[ + g_5 (\bar{e}_R \gamma_{\mu} \mu_R)(\bar{e}_L \gamma^\mu e_L) + g_6 (\bar{e}_L \gamma_{\mu} \mu_L)(\bar{e}_R \gamma^\mu e_R) + [h.c.] \]

\[ \mathcal{L}_{\text{photo}} = g_R \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} + g_L \bar{e}_R \sigma^{\mu\nu} \mu_L F_{\mu\nu} + [h.c.] \]

Contact interaction (short range process)

Photonic interaction (long range process)
Calculating method

Decay rate $\Gamma$

$$\Gamma = 2\pi \sum_f \sum_i \delta(E_f - E_i) \left| \langle \psi^s_{e1}(p_1)\psi^s_{e2}(p_2) | H | \psi^s_{\mu}(1s)\psi^s_{e}(1s) \rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi^{p,s}_{e} = \sum_{\kappa,\mu,m} 4\pi i^l_\kappa l_\kappa, m, 1/2, s|j_\kappa, \mu)Y^{*}_{l_\kappa, m}(\hat{p})e^{-i\delta_\kappa}\psi^{\kappa,\mu}_{p}$$

get radial functions by solving Dirac eq. numerically

$$\frac{dg_\kappa(r)}{dr} + \frac{1 + \kappa}{r} g_\kappa(r) - (E + m + e\phi(r))f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1 - \kappa}{r} f_\kappa(r) + (E - m + e\phi(r))g_\kappa(r) = 0$$

$\phi$ : nuclear Coulomb potential
3. RESULTS
Radial functions (scattering $e^-$)

- $\kappa = -1$ partial wave

\[ r g^{\kappa=-1}_{E_{1/2}}(r) \]
Radial functions (bound $e^-$)

$g_{e}^{1s}(r)$

$^{208}\text{Pb}$ case $\ Z = 81$

(considering $\mu^-$ screening)

<table>
<thead>
<tr>
<th>Type</th>
<th>$B_e (\text{MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rela</td>
<td>$9.88 \times 10^{-2}$</td>
</tr>
<tr>
<td>Non-rela</td>
<td>$8.93 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Relativity enhances the value near the origin.
Contact process

- bound $\mu^-$
- bound $e^-$
- scattering $e^-$

- overlap of bound $\mu^-$, bound $e^-$, and two scattering $e^-$
s

\[ r^2 g^{1s}_\mu(r) g^{1s}_e(r) g^{K=-1}_{E_{1/2}}(r) g^{K=-1}_{E_{1/2}}(r) \]

bound $e^-$: non-rela $\rightarrow$ rela
scattering $e^-$: plane $\rightarrow$ distorted

The transition rate is enhanced!
Upper limits of BR (contact process)

\[ BR(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12} \]

(SINDRUM, 1988)

\[ BR(\mu^- e^- \rightarrow e^- e^-) < B_{\text{max}} \]

\[ (g_1(\bar{e}_L\mu_R)(\bar{e}_L e_R)) \]

\[ B_{\text{max}} \]

\[ \begin{array}{c}
20 \ 30 \ 40 \ 50 \ 60 \ 70 \ 80 \ 90 \\
10^{-20} \ 10^{-19} \ 10^{-18} \ 10^{-17} \\
\end{array} \]

atomic #, \( Z \)

needed # of muonic atoms \( (Z = 82) \)

\[ 2.1 \times 10^{18} \rightarrow 3.0 \times 10^{17} \]

cf. COMET \( (\mu^- \text{Al} \rightarrow e^- \text{Al}) \)

\( O(10^{18}) \) muonic atoms

this work (1s)

previous work (1s)

this work (1s+2s+…)

needed # of muonic atoms \( (Z = 82) \)

\[ 2.1 \times 10^{18} \rightarrow 3.0 \times 10^{17} \]
Photonic process

✓ bound $\mu^-$
✓ bound $e^-$
✓ scattering $e^-$
✓ $\gamma$ propagator

◆ overlap of bound $\mu^-$, scattering $e^-$, and $\gamma$

$r^2 g_{\mu}^{1s}(r) g_{E_{1/2}}^{K=-1}(r) j_0(q_0 r)$

scattering $e^-$: plane $\rightarrow$ distorted

◆ overlap of bound $e^-$, scattering $e^-$, and $\gamma$

$r^2 g_{e}^{1s}(r) g_{E_{1/2}}^{K=-1}(r) j_0(q_0 r)$

scattering $e^-$: plane $\rightarrow$ distorted
(bound $e^-$: non-rela $\rightarrow$ rela)

The distortion makes these overlaps smaller.
Upper limits of BR (photonic process)

\[ BR(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13} \]

(MEG, 2013)

\[ BR(\mu^-e^- \rightarrow e^-e^-) < B_{\text{max}} \]

\( (g_L \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu}) \)

previous work (1s)

this work (1s)

Preliminary

needed # of muonic atoms \((Z = 82)\)

\[ 1.8 \times 10^{18} \rightarrow 7.1 \times 10^{18} \]
Can experiments discriminate those?
Discriminating method 1

~ atomic # dependence of decay rates ~

The $Z$ dependences are different among each interactions.

Compared to $(Z - 1)^3$, that of short range process is larger while that of long range process is smaller.
Discriminating method 2
~ energy and angular distributions ~

\[ E_1 : \text{energy of an emitted electron} \]

\[ \theta : \text{angle between two emitted electrons} \]

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE_1 dc\cos\theta} \quad \text{[MeV}^{-1}] \]

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE_1 dc\cos\theta} \quad \text{[MeV}^{-1}] \]

- Contact process
- Photonic process

\( Z = 82 \)
4. SUMMARY
**Summary**

- $\mu^-e^- \rightarrow e^-e^-$ process in a muonic atom
  - interesting candidate for CLFV search
  - Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron
      are important in calculating decay rates.

- contact process: decay rate Enhanced (7 times in $Z = 82$)
- photonic process: decay rate suppressed (1/4 times in $Z = 82$)

- How to identify interaction types, found by this analyses
  - atomic # dependence of the decay rate
  - energy and angular distributions of emitted electrons