Introduction to the theory of Gravity

for Nonspecialists

Shun'ya Mizoguchi（溝口俊弥）
Theory Center, KEK

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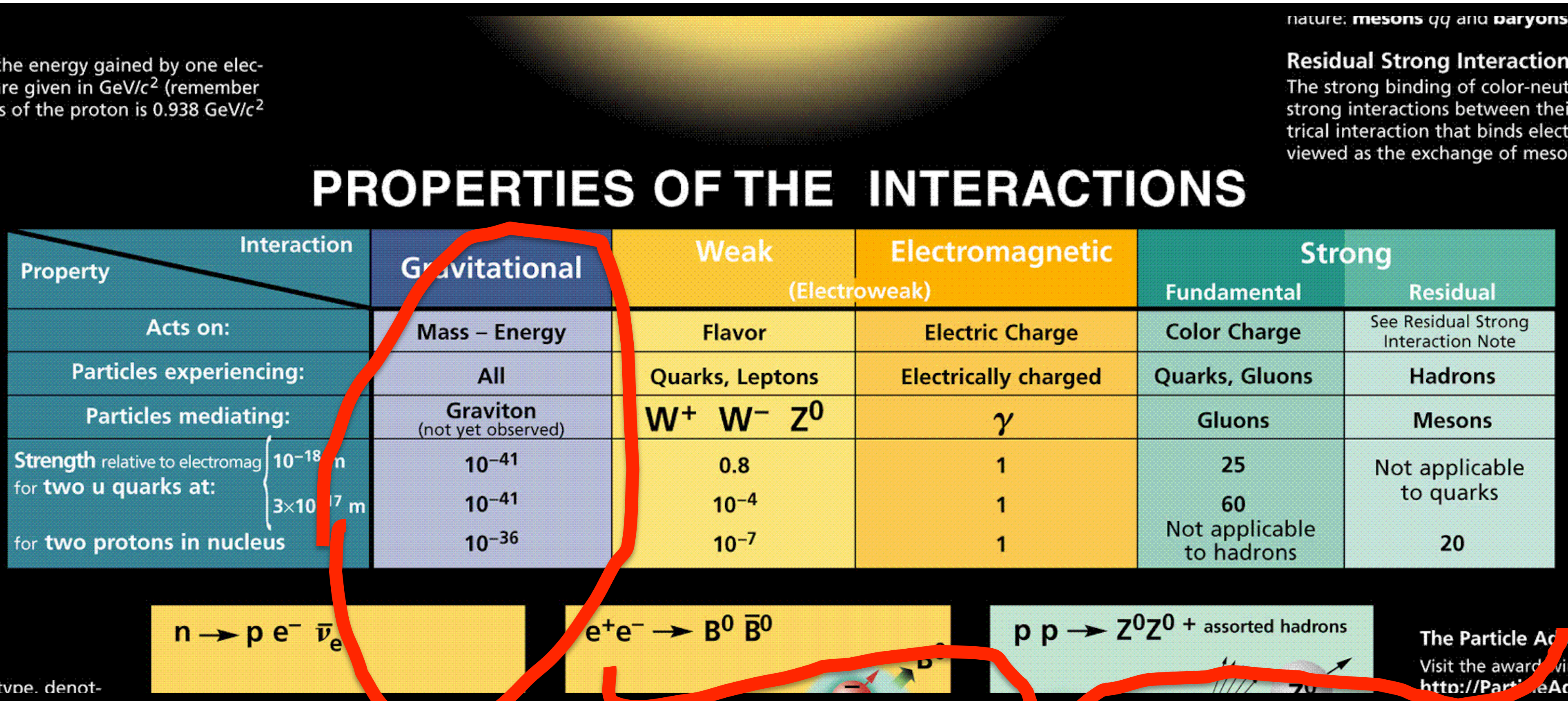
Gravity

is one of the four fundamental interactions of elementary particles

From Wikipedia

Gravity is much different from the other three interactions



PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Gravity

Other three

From :"Fundamental particles and interactions"
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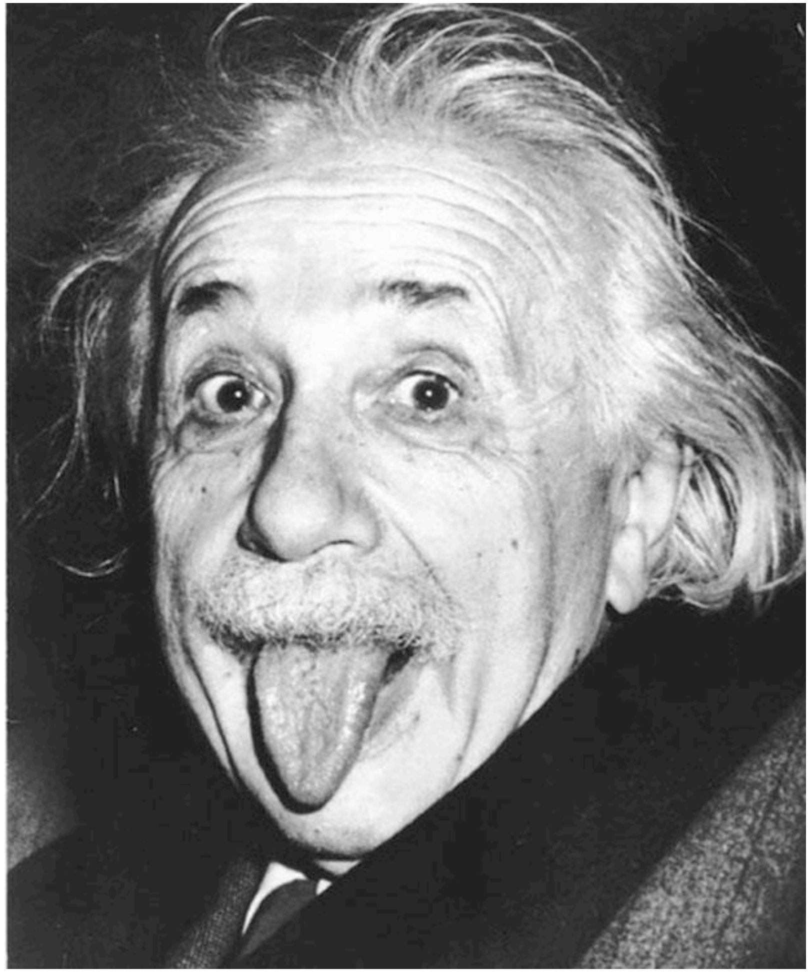
Plan

Introduction

1. Classical theory of gravity
2. Quantum theory of gravity
3. Summary and discussion

1. CLASSICAL THEORY OF GRAVITY

Classical theory of gravity



is General Relativity

Arthur Sasse Photograph

General relativity: The theory of gravity

- Not (in a narrow sense) a gauge theory; it IS a gauge theory in some sense, but is not a Yang-Mills theory
- Non-renormalizable (as we discuss later)
- Non-linear: A superposition not allowed
- Very weak as an interaction of elementary particles at any terrestrial accelerator experiment

Gauge theory vs General relativity

- A **gauge theory** is associated with a local (position dependent) symmetry of the **internal space**

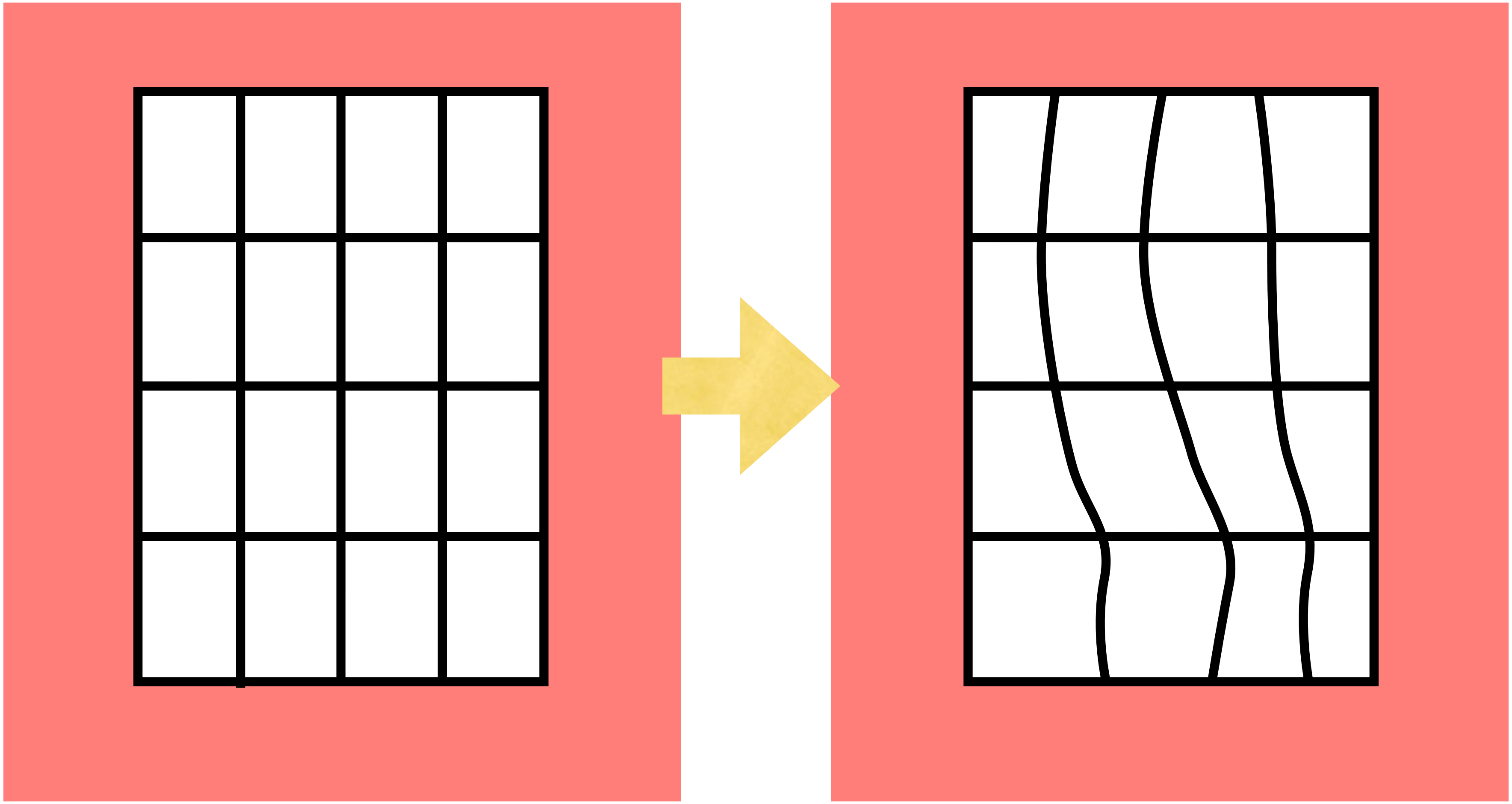
Ex. U(1) gauge theory

$$\mathcal{L} = \bar{\psi} i \not{D} \psi \quad \psi \to e^{i\alpha(x)}\psi \quad \bar{\psi} \to \bar{\psi} e^{-i\alpha(x)}$$

is invariant if $D_\mu = \partial_\mu - iA_\mu \quad A_\mu \to A_\mu + \partial_\mu \alpha$

- On the other hand, the essential property of **general relativity** is the invariance with respect to a general coordinate transformation, which is a symmetry of the **space time itself**

General coordinate transformation



cf. Special relativity

assumes the invariance with respect to **global** (position independent) **Lorentz transformation** as the fundamental principle:

$$x^{\mu} = (ct, x, y, z) \to x'^{\mu} = (ct', x', y', z')$$

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu} = \eta_{\mu\nu} dx'^{\mu} dx'^{\nu}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

General relativity: Introduction of a “metric” $g_{\mu\nu}$

$x^\mu \to x'^\mu = x'^\mu(x)$: General functions of x

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \neq \eta_{\mu\nu} dx'^\mu dx'^\nu$$

$$\underline{ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu}$$

$$x^\mu \to x'^\mu = x'^\mu(x), \quad g'_{\mu\nu} = g_{\rho\sigma} \frac{\partial x^\rho}{\partial x'^\mu} \frac{\partial x^\sigma}{\partial x'^\nu}$$

$$\Rightarrow ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g'_{\mu\nu} dx'^\mu dx'^\nu$$

The metric $g_{\mu\nu}$ plays the main role of describing gravity in general relativity

The metric $g_{\mu\nu}$ determines gravity in the space time

Ex. A flat Lorentzian metric:"Zero gravity"

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Ex. A black-hole metric (the Schwarzschild black-hole, in the polar coordinates)

$$g_{\mu\nu} = \begin{pmatrix} -(1-\frac{a}{r}) & & & \\ & \frac{1}{1-\frac{a}{r}} & & \\ & & r^2 & \\ & & & r^2\sin^2\theta \end{pmatrix}$$

Covariant derivative

- Covariant derivative in general relativity (for vectors):

$$\nabla_\mu v_\nu = \partial_\mu v_\nu + \Gamma^\lambda{}_{\nu\mu} v_\lambda$$

$$\Gamma^\lambda{}_{\nu\mu} = \frac{1}{2} g^{\lambda\rho}(\partial_\nu g_{\mu\rho} + \partial_\mu g_{\nu\rho} - \partial_\rho g_{\nu\mu})$$

cf. Abelian gauge theory

$$D_\mu \psi = \partial_\mu \psi - i A_\mu \psi$$

ゲージ場がポテンシャル、電磁場はその曲率
重力ではgがポテンシャルでゲージ場Γが重力場！

Curvature(field strength)

- Curvature tensor: Riemann tensor

$$[\nabla_\mu,\ \nabla_\nu]v_\rho \equiv R^\lambda{}_{\rho\mu\nu}v_\lambda$$

$$R^\lambda{}_{\rho\mu\nu} = \partial_\mu\Gamma^\lambda{}_{\rho\nu} - \partial_\nu\Gamma^\lambda{}_{\rho\mu} + \Gamma^\lambda{}_{\sigma\mu}\Gamma^\sigma{}_{\rho\nu} - \Gamma^\lambda{}_{\sigma\nu}\Gamma^\sigma{}_{\rho\mu}$$

- Ricci tensor

$$R_{\mu\nu} \equiv R^\lambda{}_{\mu\lambda\nu}$$

Einstein's equation is written in terms of Ricci tensor

Note: Curvatures of gauge and gravity theories

- In gauge theories, the curvature (field strength) $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (or for non-abelian case $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$) is 1st order in derivative

$$\Rightarrow \quad \mathcal{L} \sim F^2$$

- In general relativity, $R^\lambda{}_{\rho\mu\nu} = \partial_\mu \Gamma^\lambda{}_{\rho\nu} - \partial_\nu \Gamma^\lambda{}_{\rho\mu} + \cdots$ is 2nd order in derivative

$$\Rightarrow \quad \mathcal{L} \sim R$$

Einstein's action:

$$S = \int d^4x \, \sqrt{-g} R$$

Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

$T_{\mu\nu}$: Energy-momentum tensor

$$R \equiv g^{\mu\nu} R_{\mu\nu}$$

$$g^{\mu\lambda} g_{\lambda\nu} = \delta^{\mu}_{\nu}$$

Is Einstein's equation compatible with Newton's law?: The Newtonian limit

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \Rightarrow R_{\mu\nu} = \kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

Non-relativistic matter: $T_{\mu\nu} = \rho c^2 u_\mu u_\nu$

$$\Rightarrow R_{00} = \frac{4\pi G \rho}{c^2}$$

Weak gravity limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$R_{00} = -\frac{1}{2} \Delta h_{00}$$

$\Rightarrow$ If $h_{00} = -\frac{2}{c^2}\phi$, $\underline{\Delta \phi = 4\pi G \rho \text{ !!!}}$

Newton's law

$\Delta\phi = 4\pi G\rho$ From Green's function:

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67408 \times 10^{-11}[\mathrm{kg}^{-1}\mathrm{m}^3\mathrm{s}^{-2}]$$

$$\sqrt{\frac{\hbar c^5}{G}} = 1.2 \times 10^{19}[\mathrm{GeV}]$$

cf. Coulomb's law

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = 8.9876 \times 10^9 [\mathrm{Nm}^2\mathrm{C}^{-2}]$$

$$\frac{ke^2}{\hbar c} = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$$

Gravity versus electro-magnetism

Gravity

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67408 \times 10^{-11}[\mathrm{kg}^{-1}\mathrm{m}^3\mathrm{s}^{-2}]$$

$$\sqrt{\frac{\hbar c^5}{G}} = 1.2 \times 10^{19}[\mathrm{GeV}]$$

Electro-magnetism

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = 8.9876 \times 10^{9}[\mathrm{Nm}^2\mathrm{C}^{-2}]$$

$$\frac{ke^2}{\hbar c} = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$$

- One cannot write a dimensionless combination from $G, \hbar$ and c ⇨ The strength of the coupling depends on the energy, even at the classical level
- Gravity is not relevant until the energy reaches around 10^{19} GeV ⇨ Never important in any terrestrial accelerator experiment

Inertial mass versus gravitational mass

- In the Newtonian mechanics

$$F = m_I a \qquad m_I : \text{inertial mass}$$

- In a uniform gravitational field

$$F = m_G g \qquad m_G : \text{gravitational mass}$$

If they are always equal $m_I = m_G$, everything falls with the same acceleration

"Universality of Free Fall(UFF)"

Inertial mass versus gravitational mass

But is there any a priori reason for the equality?

In general relativity there is no distinction between m_I and m_G!

Action

$$S = \frac{1}{2c\kappa}\int d^4x\,\sqrt{-g}R\; + S_{matter}$$

$$\frac{\delta}{\delta g^{\mu\nu}} \Rightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

Take

$$S_{matter} = -mc\int ds$$

Point particle action

$$= -mc\int d\tau\sqrt{g_{\mu\nu}(x(\tau))\frac{dx^\mu(\tau)}{d\tau}\frac{dx^\nu(\tau)}{d\tau}}$$

$$\frac{\delta}{\delta x^\mu} \Rightarrow \frac{d^2x^\mu(\tau)}{d\tau^2} + \Gamma^\mu{}_{\nu\rho}(x(\tau))\frac{dx^\nu(\tau)}{d\tau}\frac{dx^\rho(\tau)}{d\tau} = 0$$

"Geodesic" equation, independent of m

Mass curves the spacetime

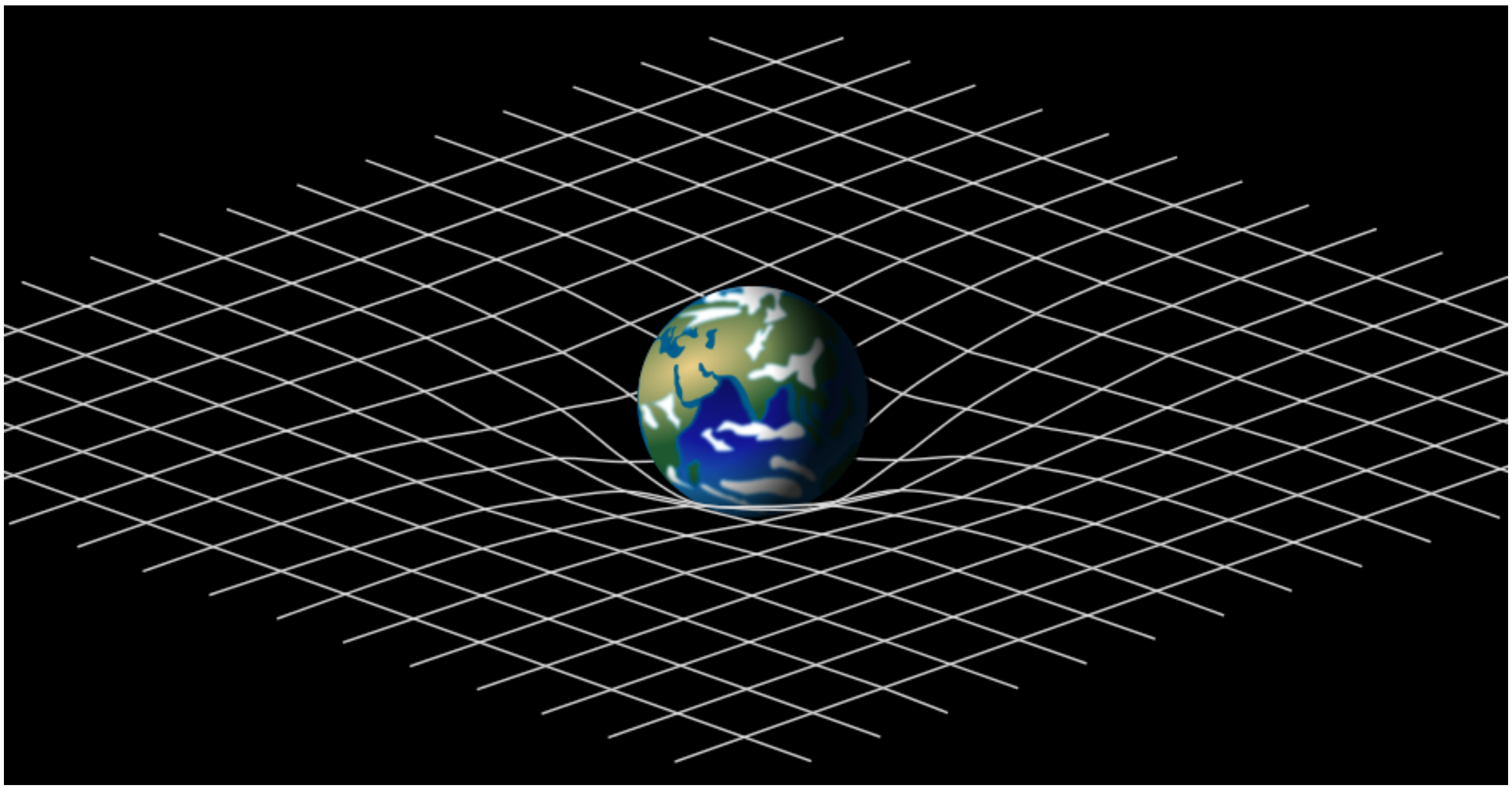
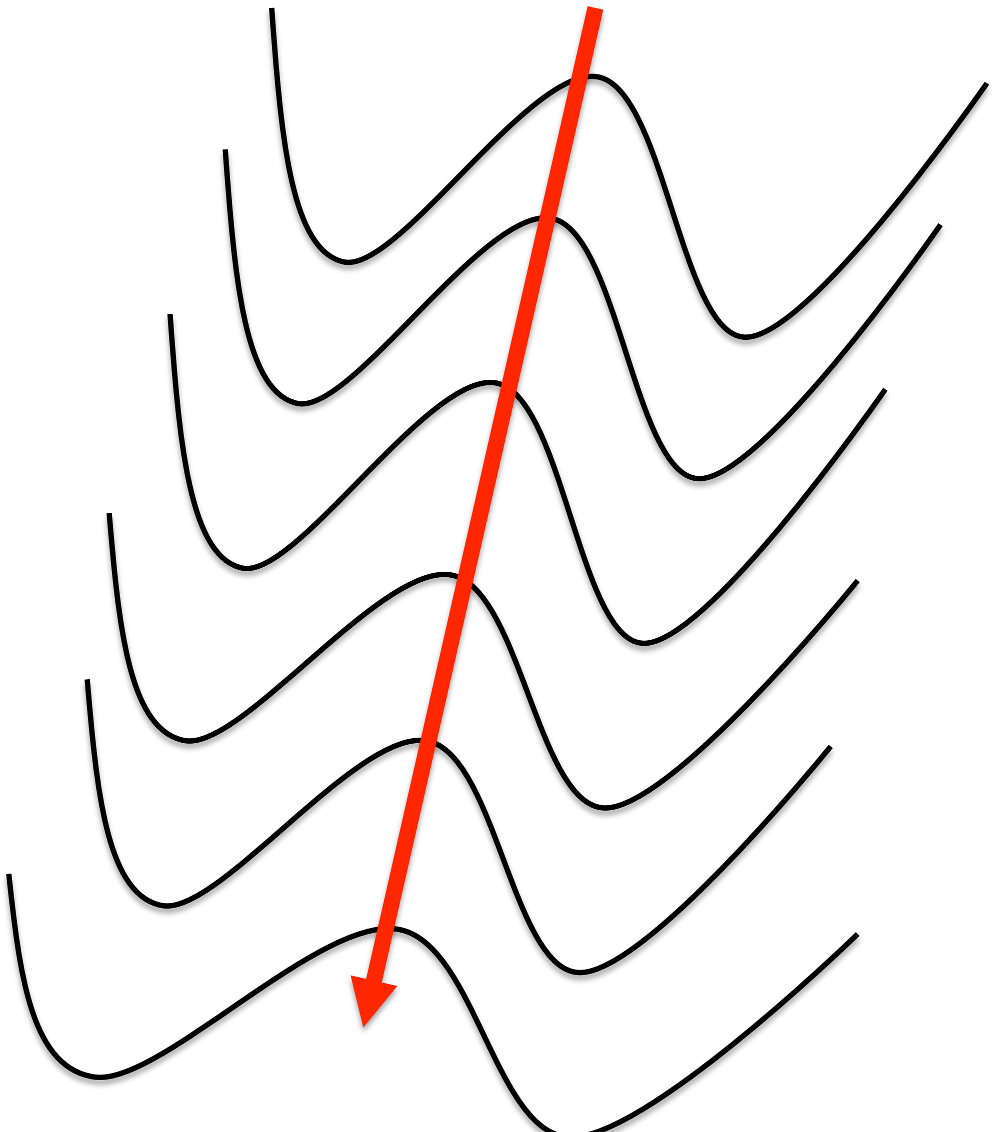


Photo credit: Wikipedia

Mass curves the spacetime

http://ribbon.a-thera.jp/article/118439.html

Geodesic = Shortest route



Geodesic equation in the Newtonian limit

The i-component: $\dfrac{d^2x^i}{d\tau^2} + \Gamma^i{}_{\nu\rho}\dfrac{dx^\nu}{d\tau}\dfrac{dx^\rho}{d\tau} = 0$

1st order in $h_{\mu\nu}$, $|v/c| << 1$

$$\Rightarrow \frac{d^2x^i}{dt^2} - \frac{c^2}{2}\partial_i h_{00} = 0$$

cf. Newtonian mechanics

$$\frac{d^2x^i}{dt^2} + \partial_i \phi = 0$$

$\Rightarrow$ $h_{00} = -\dfrac{2}{c^2}\phi$ in agreement with the Newtonian limit of Einstein's equation!

2. QUANTUM THEORY OF GRAVITY

Questions about quantum gravity

- There is no a priori-reason why gravity must be quantized
  - Unlike at the dawn of quantum mechanics where there were various puzzles (such as ones about the photoelectric effect or the spectrum of a black body), no inconsistency has been reported to date in general relativity

Questions about quantum gravity

- Still, there have been many efforts (besides the ordinary perturbation theory) to quantize gravity in various approaches (canonical gravity, loop quantum gravity,…)

(Einstein)Gravity is non-renormalizable

$$S = \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}R$$

$$\kappa = [M^{-2}]$$

Loop expansion of the effective action:

$$\Gamma = \int d^4x\left(\mathcal{L}^{(0)} + \kappa\mathcal{L}^{(1)} + \kappa^2\mathcal{L}^{(2)} + \kappa^3\mathcal{L}^{(3)} + \cdots\right)$$

$\mathcal{L}^{(k)} = [M^k] \sim \Lambda^k \quad \Lambda:$ cutoff scale

⇨ Infinitely many new divergences

⇨ Infinitely many counter terms needed

General relativity as an effective theory Donoghue (94)

- In renormalizable theories, divergences only shift the values of parameters, so once if these are measured by experiments and determined, low-energy effects are isolated from the (unknown) UV theory
- Likewise, even in non-renormalizable theories, predictions can be made by an expansion in the energy, where the effect of the UV theory appears as shifts in a finite number of parameters Donoghue (94)

Example: The gravitational potential Donoghue (94)

$$V(r) = -G\frac{m_1 m_2}{r}\left(1 - \underbrace{\frac{G(m_1+m_2)}{rc^2}}_{\text{post-Newtonian}} + \underbrace{\frac{127}{30\pi^2}\frac{G\hbar}{r^2c^3}}_{\text{quantum correction}}\right)$$

Ex. 2nd term for a typical neutron star:

$$r = 7[\mathrm{km}], \quad \rho = 1.4\times 10^{15}[\mathrm{g/cm^3}] \ \Rightarrow\ \frac{Gm}{rc^2} = 0.2$$

cf. $l_P = 1.616229(38)\times 10^{-35}$ [m]

3rd term is extremely small; relevant in the early universe?

3. SUMMARY AND DISCUSSION

Summary

- Classical theory of gravity is described by general relativity, which assumes that physical laws be invariant under arbitrary general coordinate transformations
- The fundamental variable of general relativity is the metric g, which measures the distance between two infinitesimally separated points
- The basic equation is Einstein's equation, which is expressed in terms of Ricci tensor and is reduced to Newton's law in the Newtonian limit

Summary

- Gravitational coupling is weak, and not relevant until the energy reaches around 10^{19} GeV
- The Einstein gravity is a non-renormalizable theory but can be made some prediction by using the technique of effective field theory

Discussion

- Perhaps it is not a QUANTUM gravity, a priori it is unclear whether a quantum mechanical system may "adopt" a gravitational BACKGROUND as the potential in the Schrodinger equation; it will be interesting to see this in the ultra cold neutron experiment

Discussion

- Donoghue’s prediction is too tiny to be confirmed in a terrestrial accelerator experiment, but it may have left a trace during the inflation; can one see this in the CMB or the primordial gravitational wave observation?