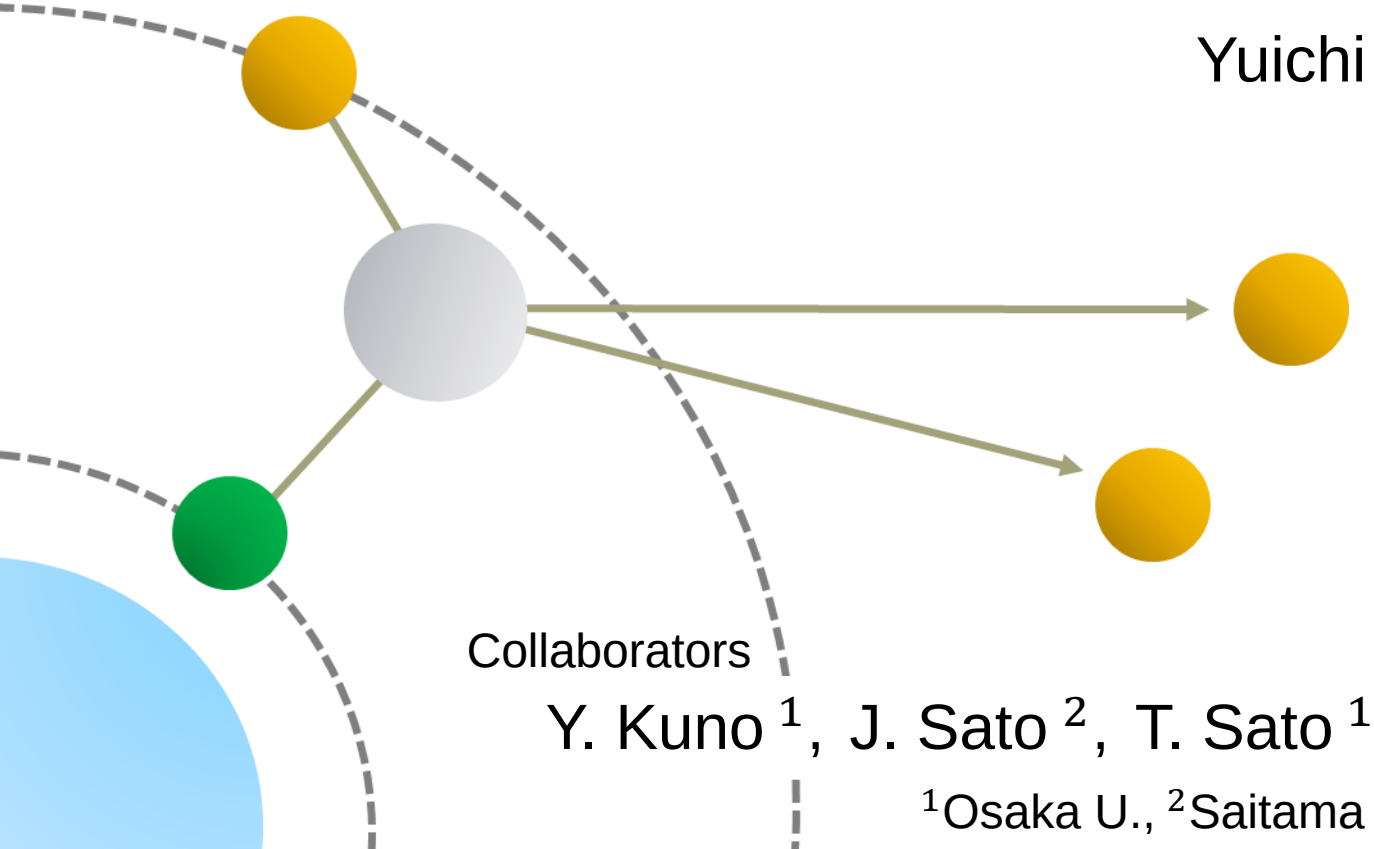
ミューオン原子中でのCLFV過程 $\mu^- e^- \to e^- e^-$

($\mu^- e^- \to e^- e^-$ in muonic atoms)

YU, Y. Kuno, J. Sato, T. Sato & M. Yamanaka, Phys. Rev. D **93**, 076006 (2016).
YU, Y. Kuno, J. Sato, T. Sato & M. Yamanaka, Phys. Rev. D **97**, 015017 (2018).

Yuichi Uesaka (Osaka U.)



Collaborators

Y. Kuno [1], J. Sato [2], T. Sato [1], M. Yamanaka [3]

[1]Osaka U., [2]Saitama U., [3]Kyoto Sangyo U.

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Charged Lepton Flavor Violation (CLFV)

- 新物理探索の有力候補 -

- process where lepton flavors are not conserved ＝ LFV process

⇨ **LFV** in charged lepton sector **＝ CLFV**

- In the standard model (SM), lepton flavors are conserved.

allowed

$\mu^- \to e^- \nu_\mu \bar{\nu}_e$

$\pi^- \to \mu^- \bar{\nu}_\mu$

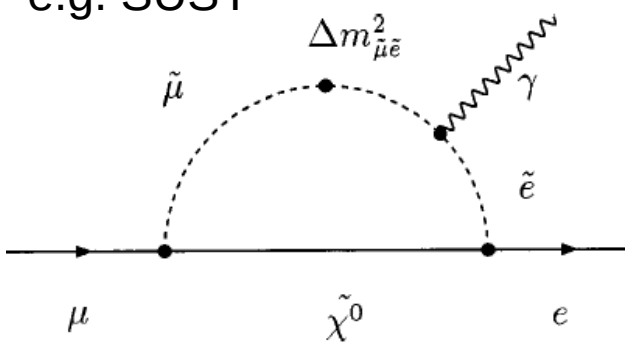
forbidden (CLFV)

$\mu^- \to e^- \gamma$

$\mu^- \to e^- e^+ e^-$

- Many "models beyond the SM" predict CLFV.

e.g. SUSY



- The contribution of ν osci. is tiny.

expected branching ratio

$\mathrm{Br}(\mu \to e\gamma) < 10^{-54}$

- Various CLFV modes have been searched, but not found yet.

CLFV searches in muon rare decay

Advantages of muon

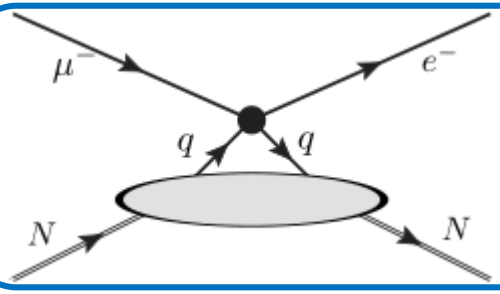
1. high intensity
2. long lifetime

- current bounds

L. Calibbi & G. Signorelli, arXiv:1709.00294 [hep-ph].

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006

μ^--e^- conversion



✓ CLFV search using muonic atom

✓ exploring μeqq interaction

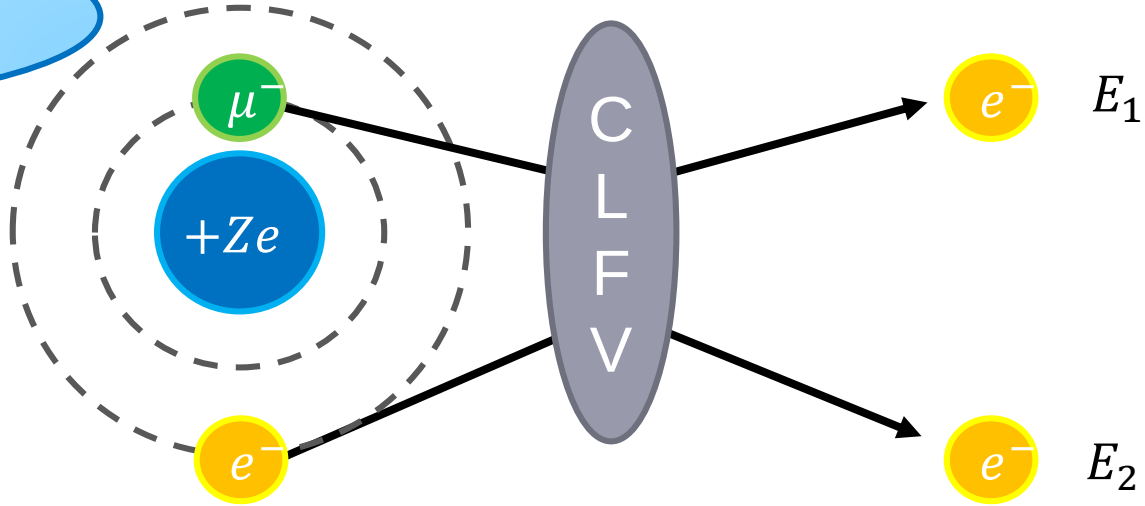
$\mu^- e^- \to e^- e^-$ in a muonic atom

M. Koike, Y. Kuno, J. Sato & M. Yamanaka,
Phys. Rev. Lett. **105**, 121601 (2010).

New CLFV search using muonic atoms

proposal in **COMET**

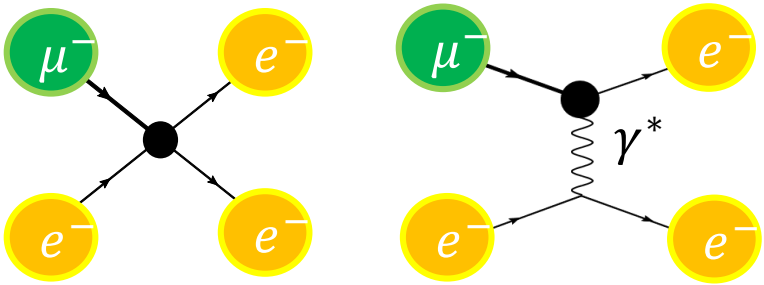
R. Abramishvili et al.,
COMET Phase-I Technical Design Report (2016).



Features

- clear signal : $E_1 + E_2 \simeq m_\mu + m_e - B_\mu - B_e$
- 2 CLFV mechanisms
  - ✓ contact (μeee vertex)
  - ✓ photonic ($\mu e\gamma$ vertex)

  (similar to $\mu^+ \to e^+ e^+ e^-$)



- atomic # Z : large ⇒ decay rate Γ : large $(\Gamma \propto (Z-1)^3)$

Branching ratio of CLFV decay

How many muonic atoms decay with CLFV, compared to created # ?

$$\mathrm{BR}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$

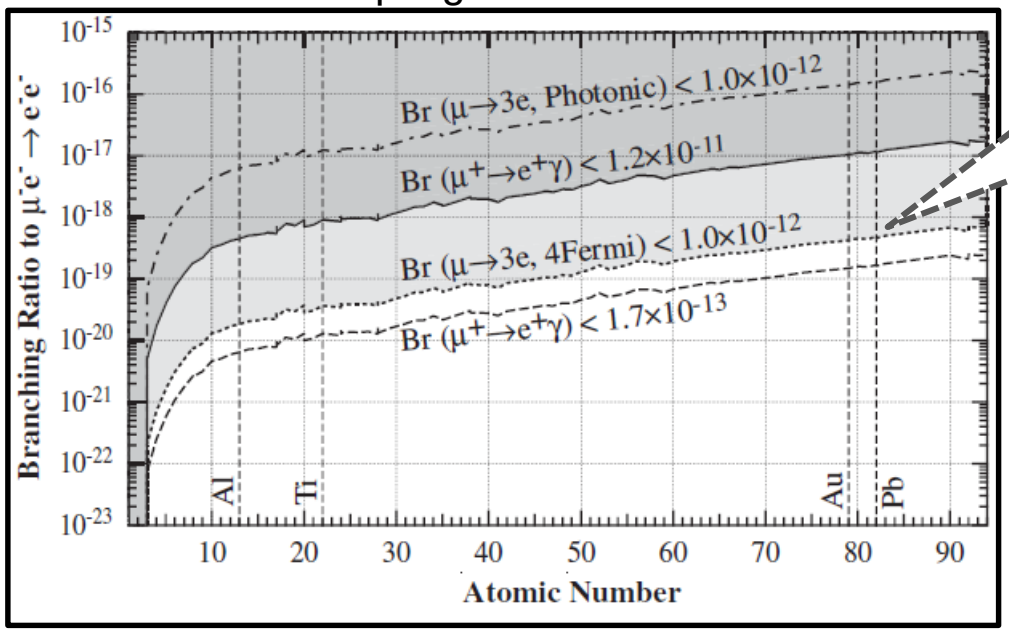
$\Gamma \propto (Z-1)^3$

due to existence prob. of bound e^- at the origin

$\tilde{\tau}_\mu$: lifetime of a muonic atom

cf. $2.2\mu\mathrm{s}$ for a muonic H ($Z=1$)
$80\mathrm{ns}$ for a muonic Pb ($Z=82$)

BR with CLFV coupling fixed on allowed maximum



e.g. $\mathrm{BR} < 5.0 \times 10^{-19}$ for Pb ($Z=82$)

if contact process is dominant

- BR increases with atomic # Z.

Using muonic atoms with large Z is favored to search for $\mu^- e^- \to e^- e^-$.

Phys. Rev. Lett. **105**,121601 (2010).

To improve calculation for decay rate

✓ previous formula of CLFV decay rate by Koike *et al.*

$$\Gamma_{\mu^- e^- \to e^- e^-} = 2\sigma v_{\mathrm{rel}} |\psi_{1S}^{e}(0)|^2 \propto (Z-1)^3$$

Note

- "Z dependence" comes from only $|\psi_{1S}^{e}(0)|^2$ (always $\Gamma \propto (Z-1)^3$)
- emitted e^-s are expected to be back-to-back with equal energies

used approximations ($Z\alpha \ll 1$)

- spatial extension of bound lepton $\gg$ wave length of emitted e^-
- bound lepton : non-relativistic
- emitted e^- : plane wave

In atoms with large Z,

← *small orbital radius*

← *relativistic (especially, e^-)*

← *Coulomb distortion*

More quantitative estimation is needed ! (important for large Z)

Improvement and expectation

✓ this work

lepton wave function : relativistic Coulomb

the improvement contains…

- finite orbit-size of bound leptons

＋

- relativistic effects for bound leptons

＋

- Coulomb distortion of emitted e^-

➡ How are CLFV decay rates modified ?

other than quantitative modifications

➢ model-dependence

➢ energy & angular distribution of emitted e^- pair

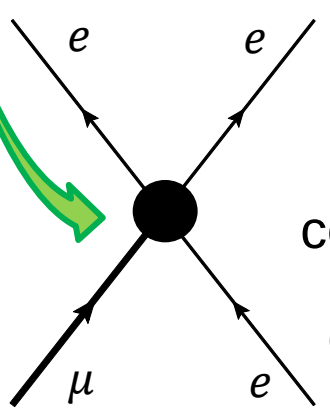
Contents

Effective Lagrangian for $\mu^- e^- \to e^- e^-$

$$\mathcal{L}_I = \underline{\mathcal{L}_{contact}} + \underline{\mathcal{L}_{\mu\to e\gamma}}$$

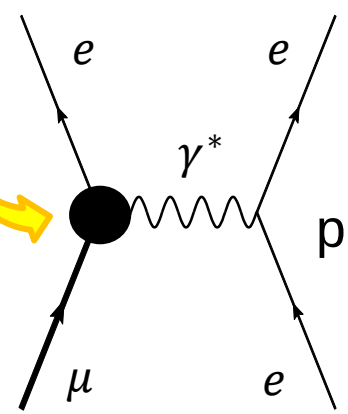
$$\mathcal{L}_{\text{contact}} = -\frac{4G_F}{\sqrt{2}}\big[g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L)$$
$$+g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L)$$
$$+g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R)\big] + [H.c.]$$

$$\mathcal{L}_{\mu\to e\gamma} = -\frac{4G_F}{\sqrt{2}} m_\mu\left[A_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + A_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu}\right] + [H.c.]$$



contact interaction

constrained by $\mu \to eee$



photonic interaction

constrained by $\mu \to e\gamma$

Our formulation for decay rate

$$\Gamma = 2\pi \sum_f \sum_{\bar{\imath}} \delta(E_f - E_i) \left| \left\langle \psi_e^{\boldsymbol{p}_1, s_1} \psi_e^{\boldsymbol{p}_2, s_2} \middle| H \middle| \psi_\mu^{1s, s_\mu} \psi_e^{1s, s_e} \right\rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_e^{\boldsymbol{p}, s} = \sum_{\kappa, \mu, m} 4\pi\, i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y^*_{l_\kappa, m}(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa, \mu}$$

κ : index of angular momentum

get radial functions by solving "Dirac eq. with ϕ" numerically

$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \big(E + m + e\phi(r)\big) f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \big(E - m + e\phi(r)\big) g_\kappa(r) = 0$$

ϕ : nuclear Coulomb potential

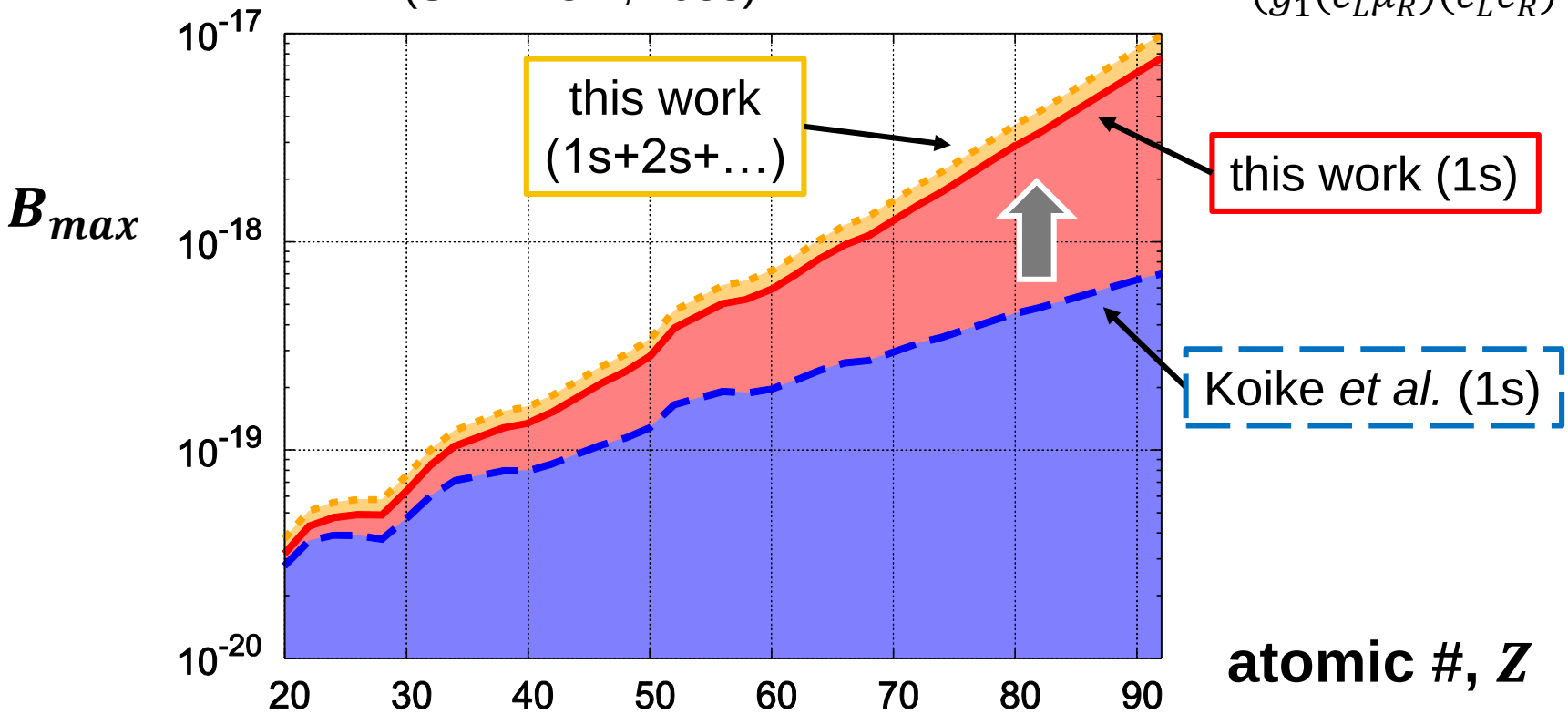
$$\psi(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

Upper limits of BR (contact process)

$BR(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12}$ → $BR(\mu^-e^- \to e^-e^-) < B_{max}$

(SINDRUM, 1988)

$(g_1(\bar{e}_L\mu_R)(\bar{e}_L e_R))$



inverse of B_{max} $(Z = 82)$

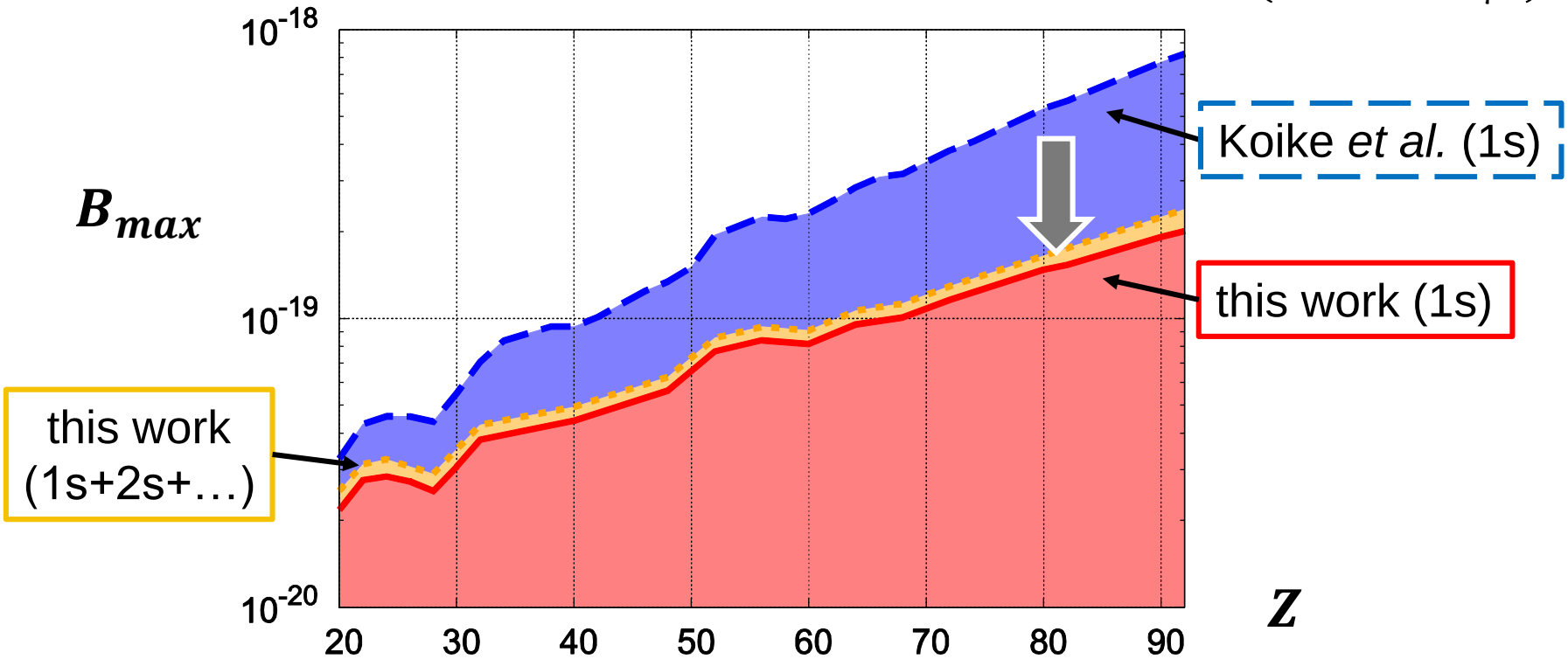
2.1×10^{18} → 3.0×10^{17}

Upper limits of BR (photonic process)

$BR(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(MEG, 2016)

$(g_L \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu})$



inverse of B_{max} $(Z = 82)$

1.8×10^{18} → 6.6×10^{18}

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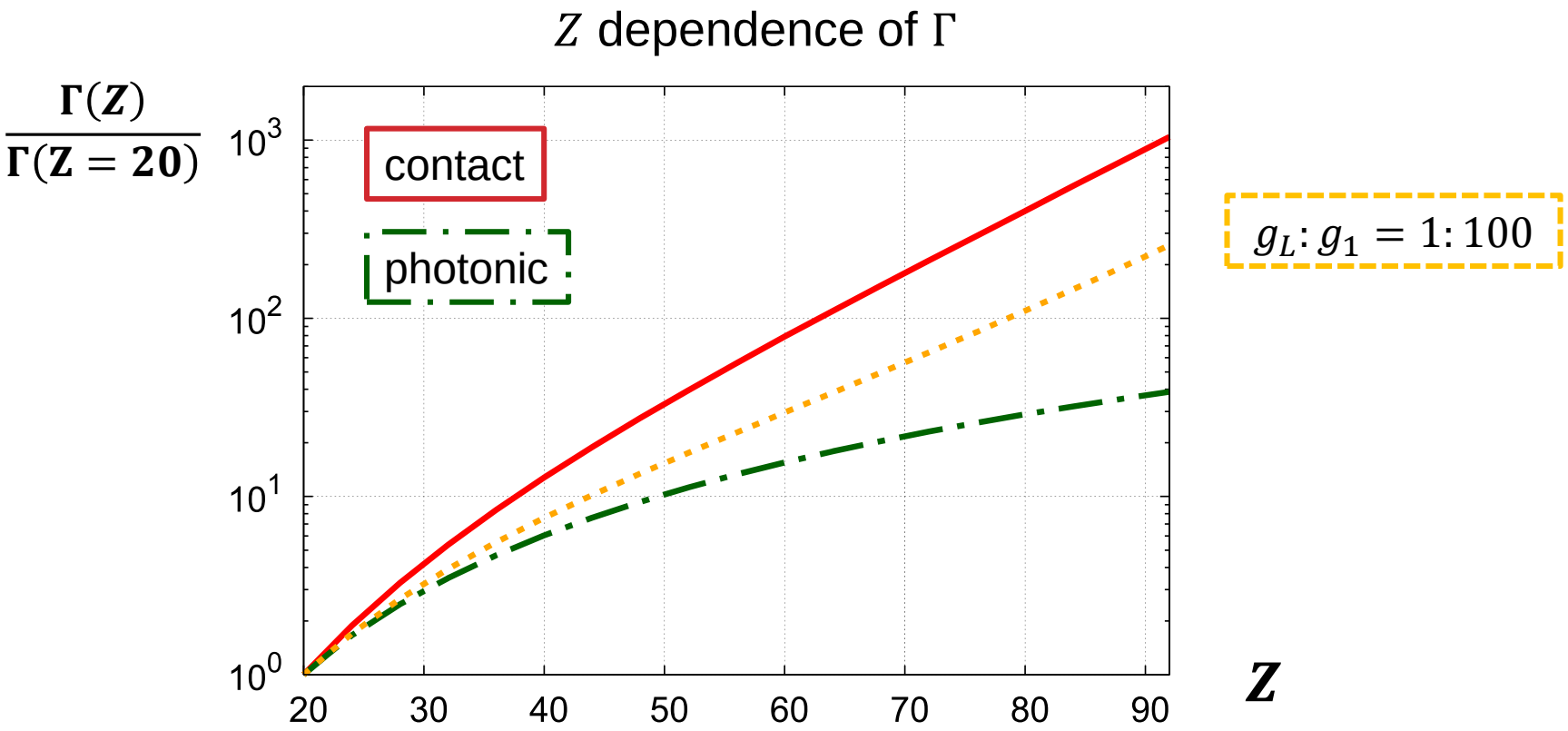
Distinguishing method 1

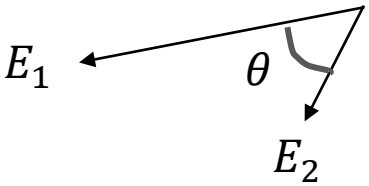
~ atomic # dependence of decay rates ~



- The Z dependences are different among interactions.
- That of contact process is strongly increasing,
  while that of photonic process is moderately increasing.

Distinguishing method 2

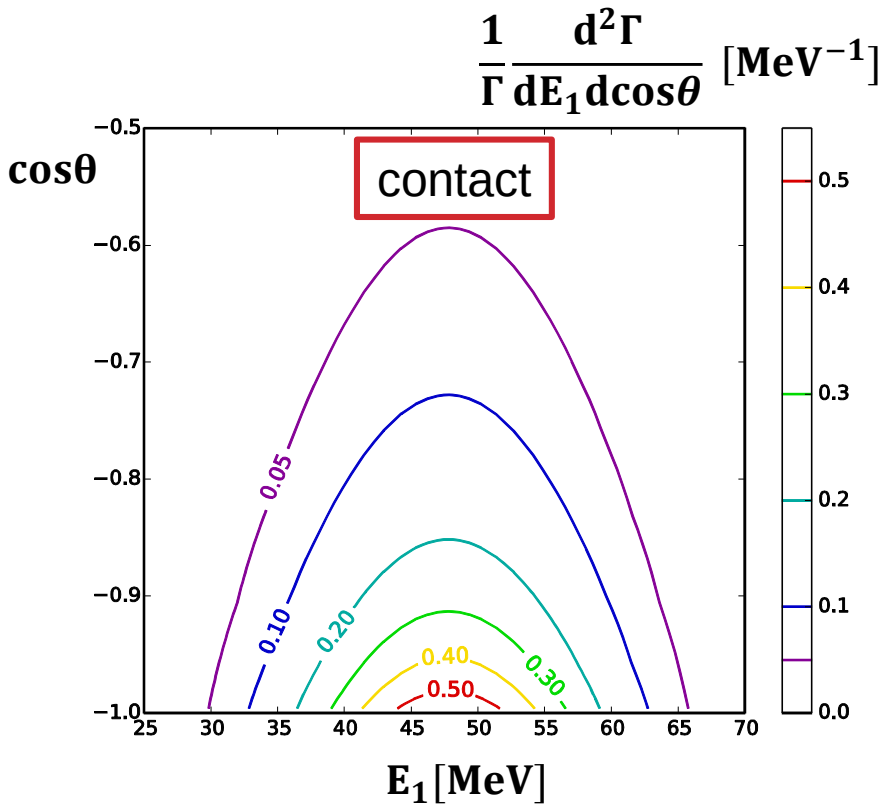
~ energy and angular distributions ~

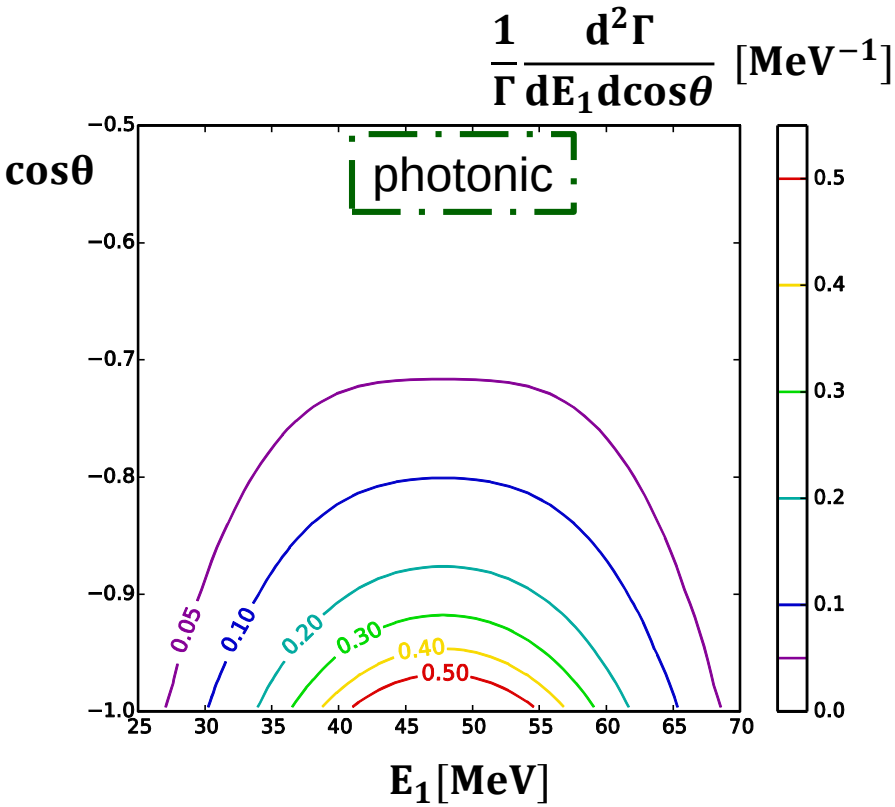


E_1 : energy of an emitted electron

θ : angle between two emitted electrons

$Z = 82$



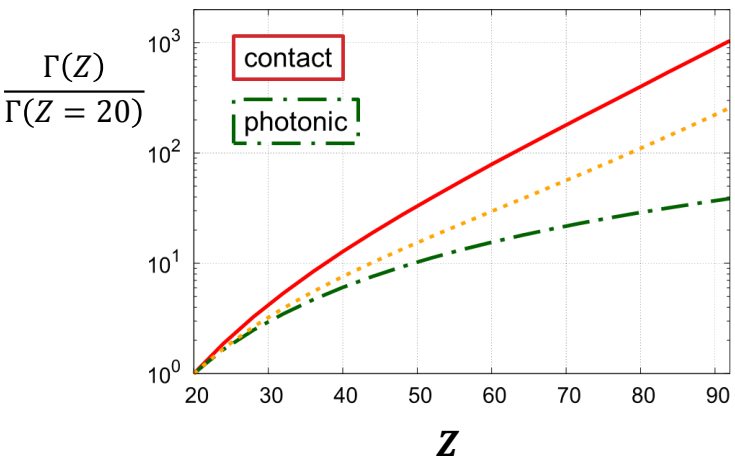


- The distributions are (a little) different among interactions.

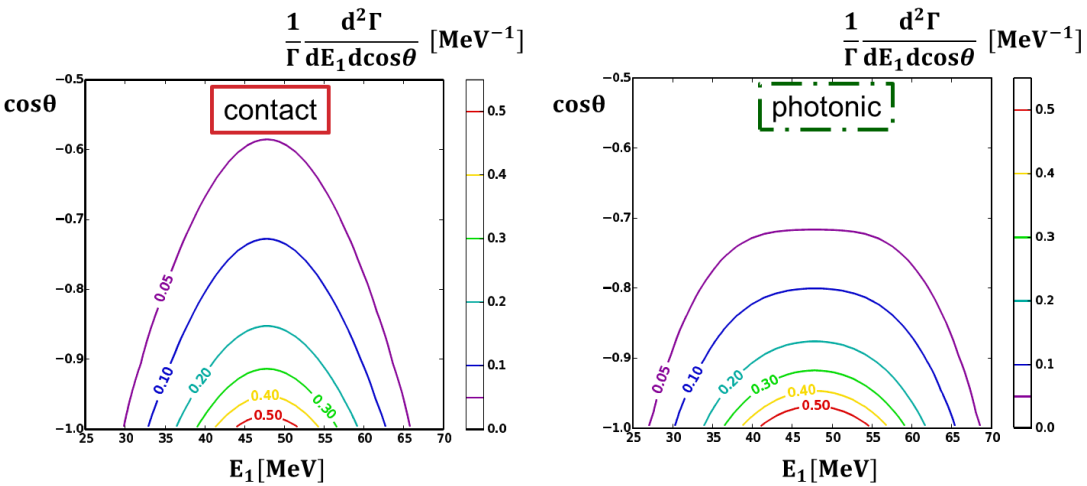
Model distinguishing power

- We can distinguish “contact” or “photonic”.

method 1. Z-dep. of decay rates



method 2. energy-angular distribution



- Can we distinguish “left” or “right” ?

e.g. $g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R)$ & $g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L)$

Distinguishing method 3

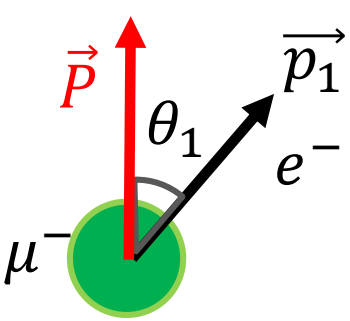
~ asymmetry of electron emission by polarized muon ~

- angular distribution of one emitted electron

$\cos\theta_1 = \hat{P} \cdot \hat{p}_1$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta_1} = \frac{1}{2}\frac{\mathrm{d}\Gamma}{\mathrm{d}E_1}\{1 + \alpha(E_1)P\cos\theta_1\}$$

$\vec{P}$: polarization vector



asymmetry factor α

$$\alpha(E_1)P\cos\theta_1 = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

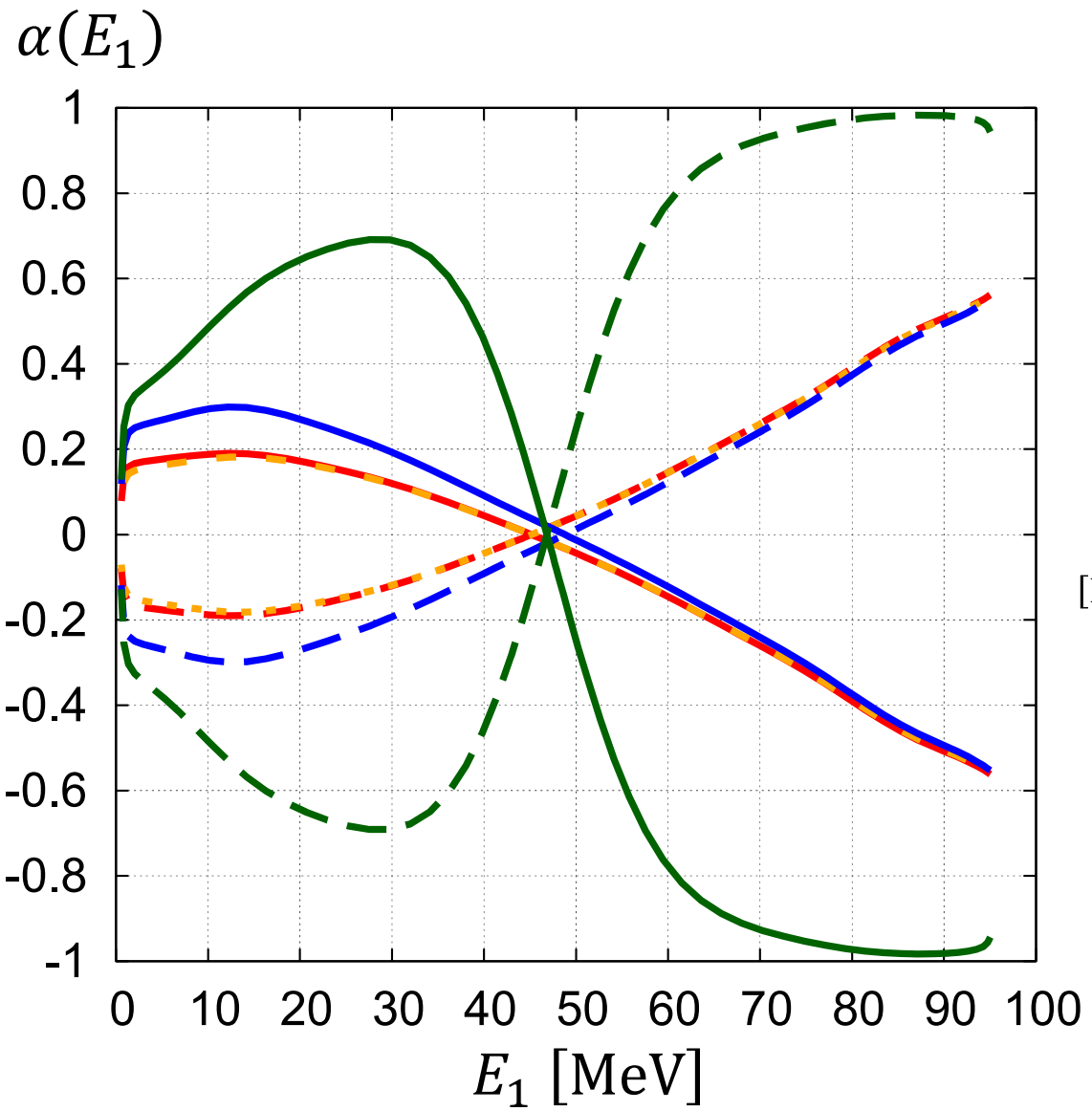
$$\sigma^{s_\mu} \equiv \int \boxed{d^3p_2} \sum_{s_1,s_2,s_e} \left|\left\langle \psi_e^{\boldsymbol{p}_1,s_1}\psi_e^{\boldsymbol{p}_2,s_2}\middle|\mathcal{L}_I\middle|\psi_\mu^{1s,s_\mu}\psi_e^{1s,s_e}\right\rangle\right|^2 \delta(E_f - E_i)$$

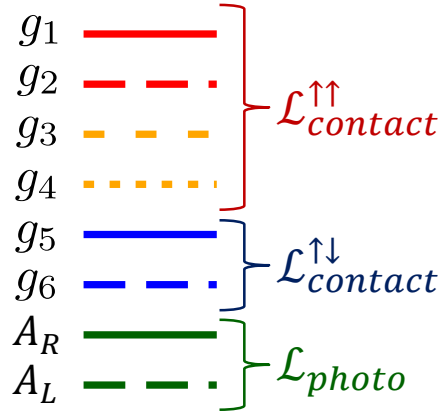
($s_\mu = \uparrow, \downarrow$)

(integrate the angle of the other electron)

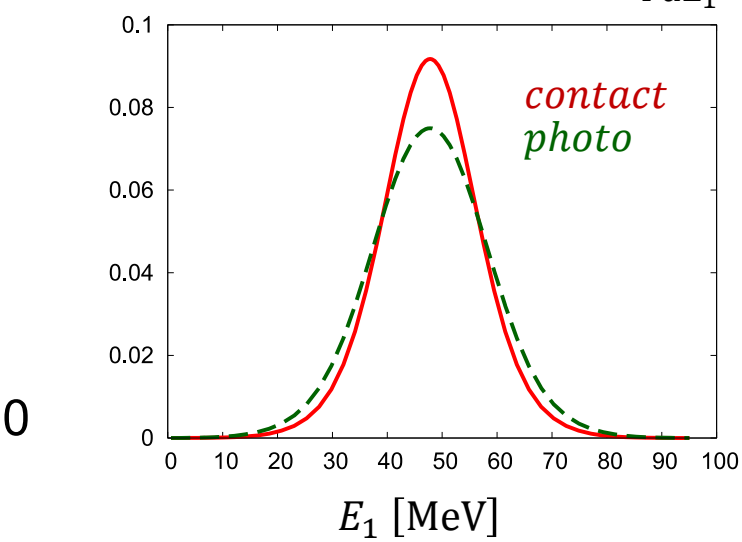
E_1 dependence of electron asymmetry

$Z = 82$





cf. E_1 distribution $\frac{d\Gamma}{\Gamma dE_1}$



Contents

Summary

- $\mu^- e^- \to e^- e^-$ process in a muonic atom
  - ✓ interesting candidate for CLFV search
  - ✓ Our finding
    - <u>Distortion</u> of emitted electrons
    - <u>Relativistic treatment</u> of a bound electron

are important in calculating decay rates.

⇩ Distortion makes difference between 2 processes.

- ■ contact process : decay rate Enhanced (7 times Γ_0 in $Z = 82$)
- ■ photonic process : decay rate suppressed (1/4 times Γ_0 in $Z = 82$)

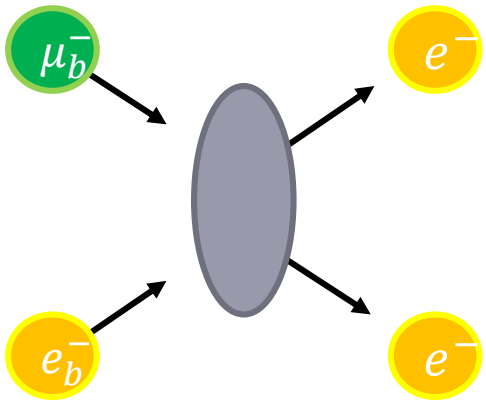
◆ How to discriminate interactions, found by this analyses

- ✓ atomic # dependence of the decay rate
- ✓ energy and angular distributions of emitted electrons
- ✓ asymmetry of electron emission by polarized muon

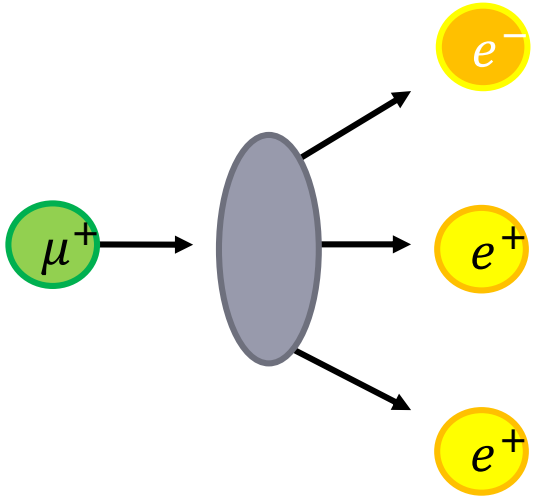
BACKUP

Comparison to $\mu^+ \to e^+e^+e^-$

$\mu^- e^- \to e^- e^-$ in a muonic atom



$\mu^+ \to e^+e^+e^-$



difference 1 : signal

2 e^-s

(approximately) two-body decay

1 e^- & 2 e^+s

three-body decay

difference 2 : interference among interactions

Interference appears differently.

⇨ There is possibility to test the relative phase of couplings.

Radial wave function (bound μ^-)

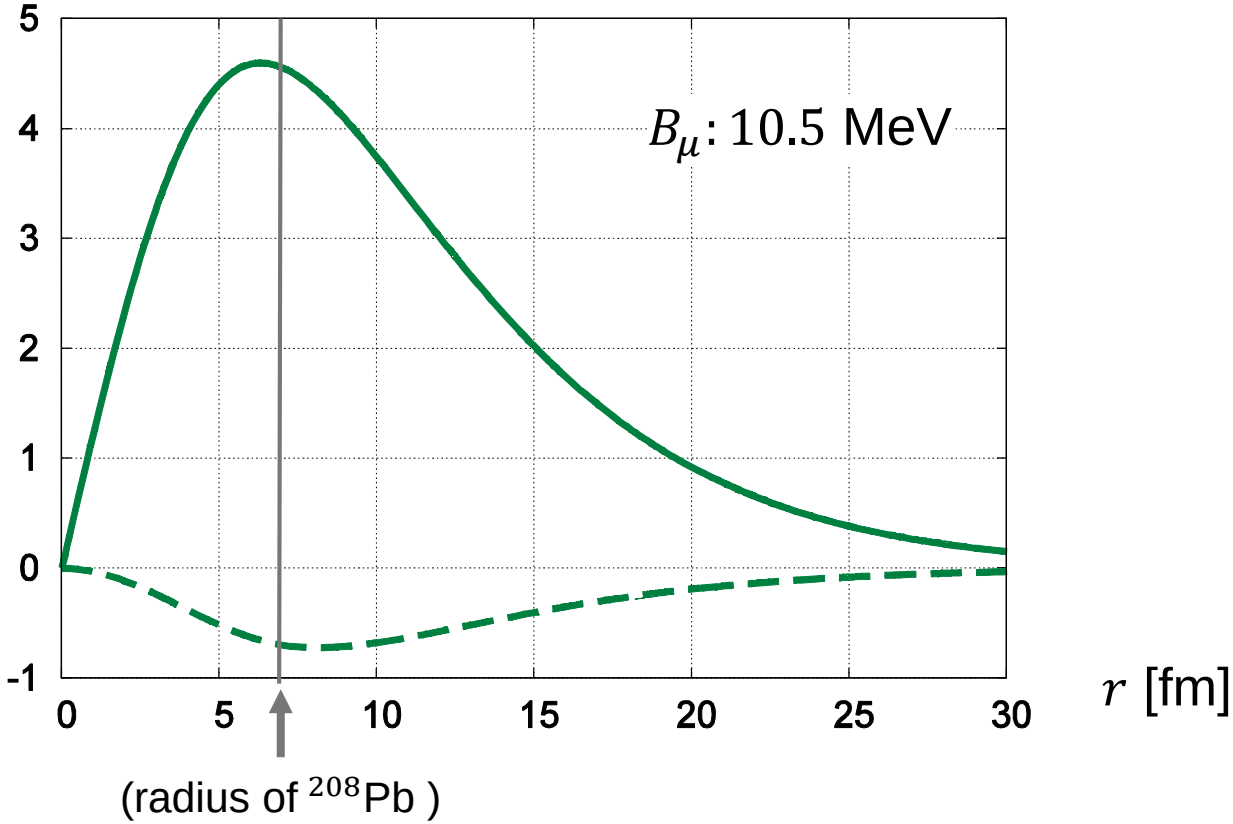
^{208}Pb case $Z = 82$

$[\mathrm{MeV}^{-1/2}]$

$rg_\mu^{1s}(r), rf_\mu^{1s}(r)$

$rg_\mu^{1s}(r)$: solid

$rf_\mu^{1s}(r)$: dotted

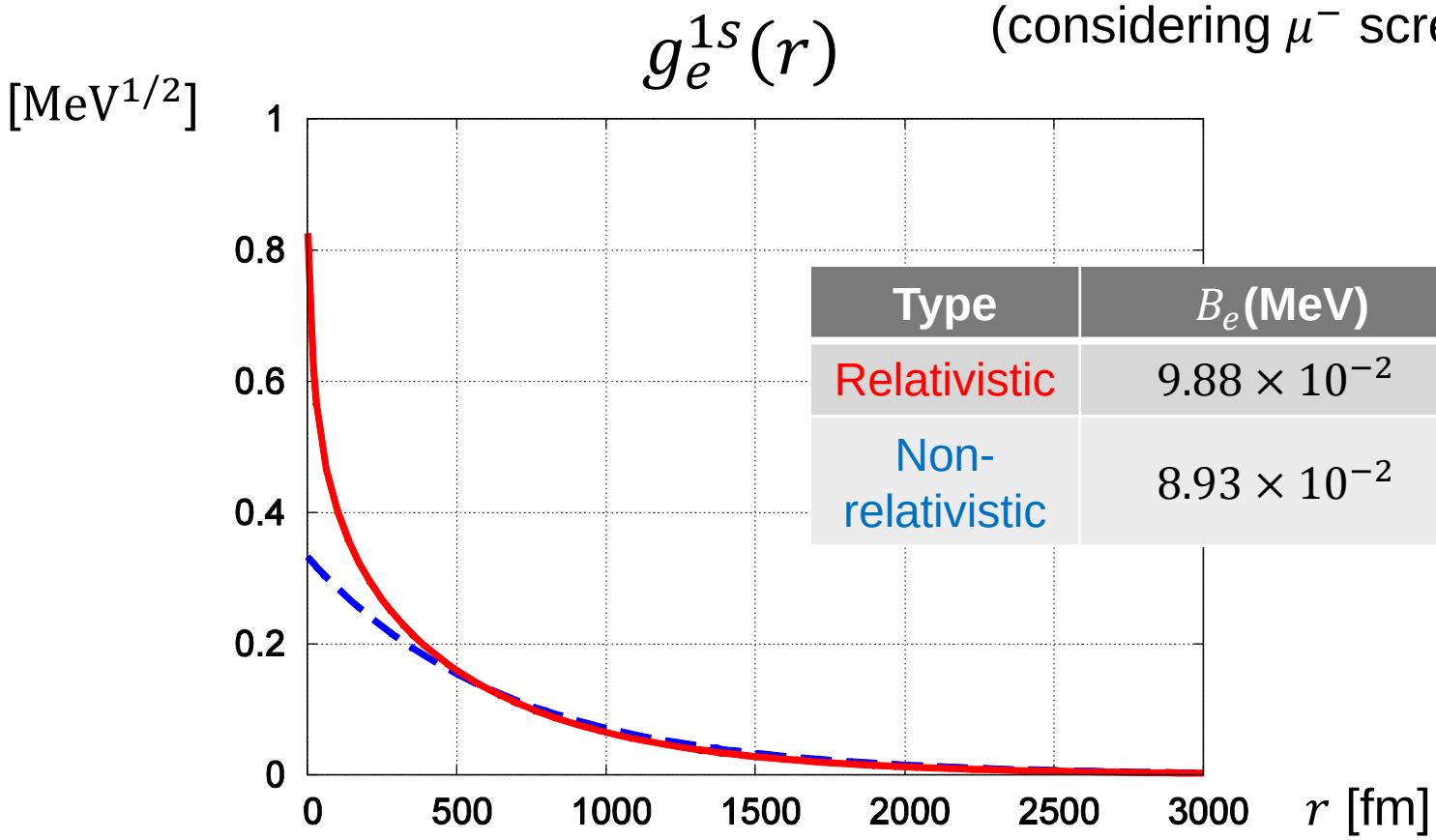


✓ It is important to consider finite nuclear charge radius.

Radial wave function (bound e^-)

^{208}Pb case $Z = 81$

(considering μ^- screening)



Type	B_e(MeV)
Relativistic	9.88×10^{-2}
Non-relativistic	8.93×10^{-2}

Relativity enhances the value near the origin.

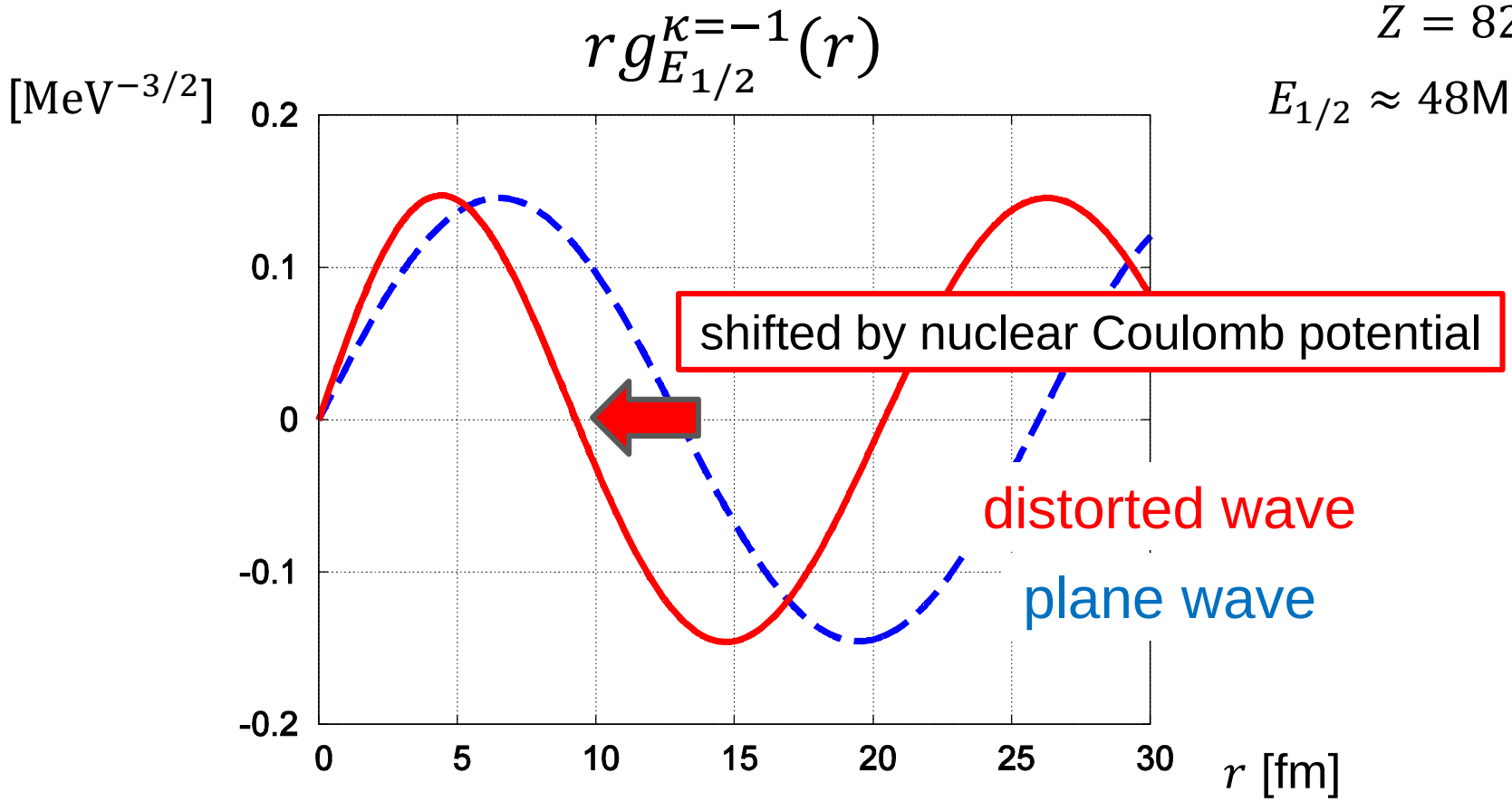
Radial wave function (scattering e^-)

e.g. $\kappa = -1$ partial wave

^{208}Pb case

$Z = 82$

$E_{1/2} \approx 48$MeV



① enhanced value near the origin

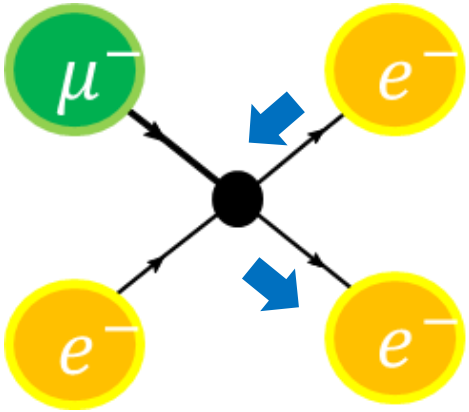
② local momentum increased effectively

Effect of distortion

scat. e^- : **plane wave**

(Assuming momentum conservation at each vertex)

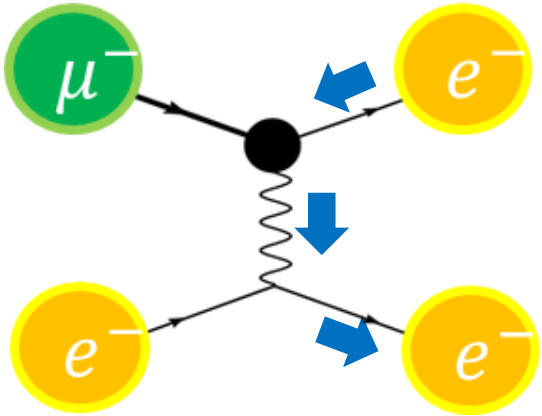
bound μ^- emitted e^-



bound e^- emitted e^-

contact process

bound μ^- emitted e^-



bound e^- emitted e^-

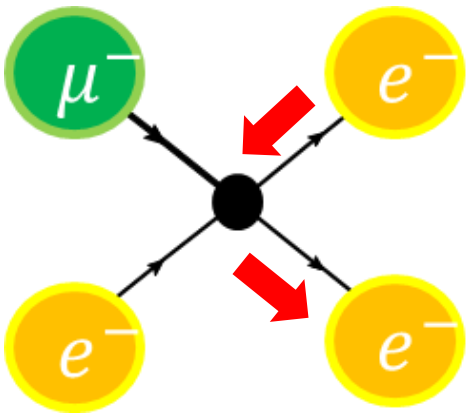
photonic process

Effect of distortion

scat. e^- : **distorted wave**

(Assuming momentum conservation at each vertex)

bound μ^- emitted e^-

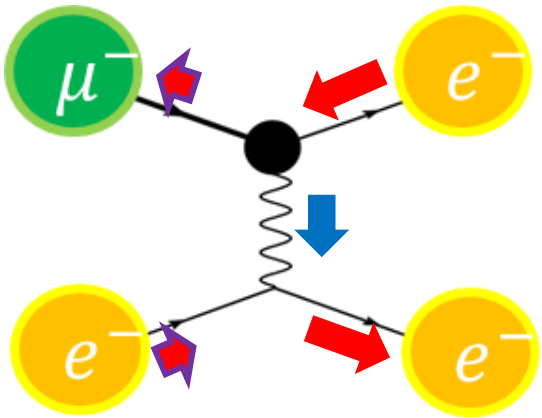


bound e^- emitted e^-

contact process

- no momentum mismatches

bound μ^- emitted e^-



bound e^- emitted e^-

photonic process

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

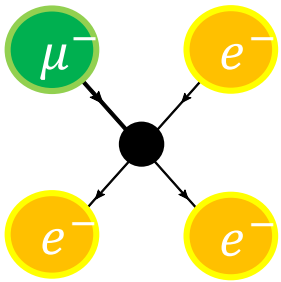
enhanced !!

suppressed…

Contact process

✓ bound μ^-

✓ scattering e^-

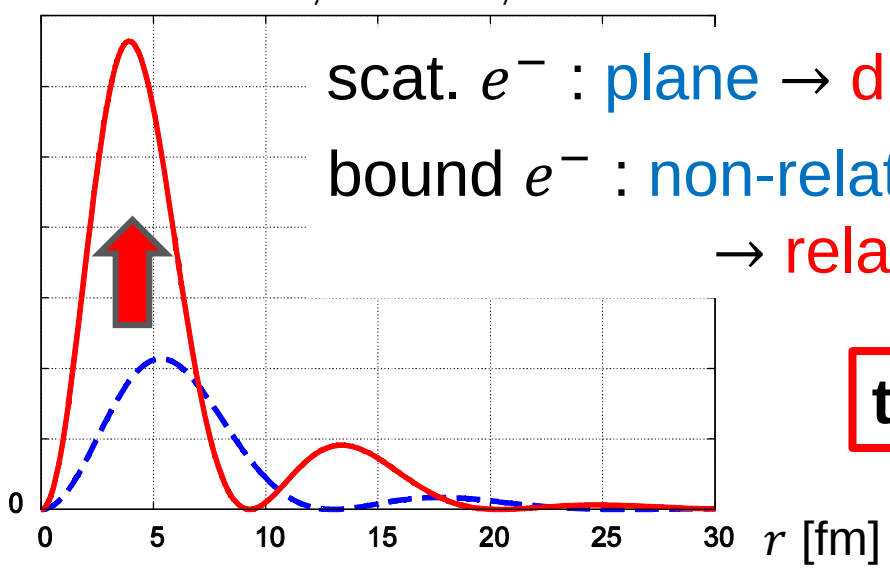


✓ bound e^-

✓ scattering e^-

◆ overlap of bound μ^-, bound e^-, and two scattering e^-s

$r^2 g_\mu^{1s}(r) g_e^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) g_{E_{1/2}}^{\kappa=-1}(r)$ (^{208}Pb)



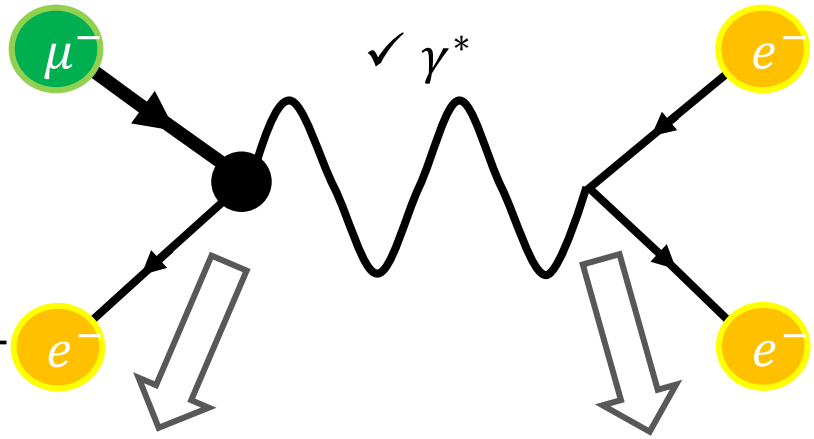
scat. e^- : plane → distorted

bound e^- : non-relativistic → relativistic

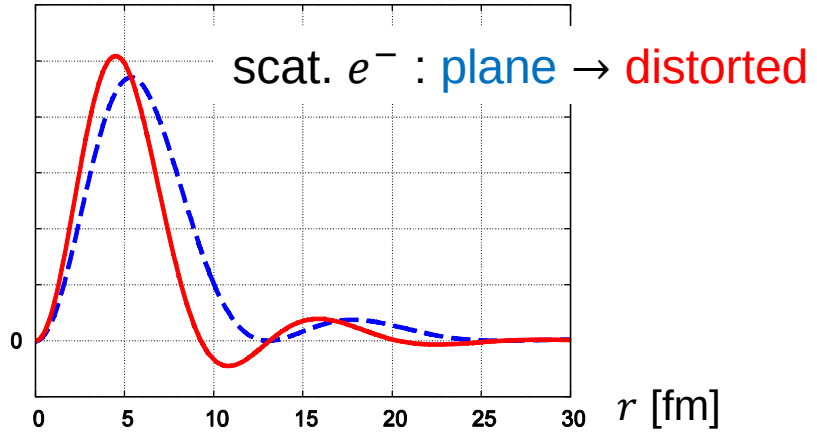
wave functions shift to the center

transition rate increases!

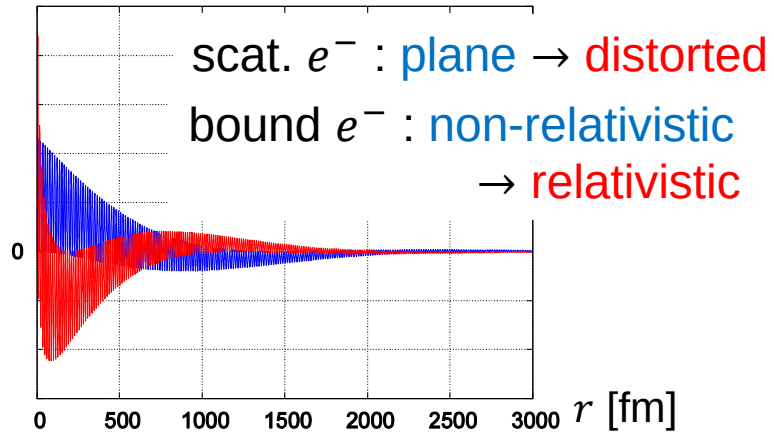
Photonic process



$r^2 g_\mu^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)$



$r^2 g_e^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)$ (^{208}Pb)



distortion of scattering e^- ⇒ overlap integral decreases

(Rough) Estimation of decay rate

"flux"

$$\Gamma = \sigma v_{\mathrm{rel}} \int \mathrm{d}V \rho_\mu \rho_e$$

Suppose nuclear Coulomb potential is weak,

$$\Gamma_{\mu^- e^- \to e^- e^-} = 2\sigma v_{\mathrm{rel}} |\psi^e_{1S}(0)|^2$$

(sum of two $1S$ e^-s)

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σ : cross section of $\mu^- e^- \to e^- e^-$ (free particles')

v_{rel} : relative velocity of μ^- & e^-

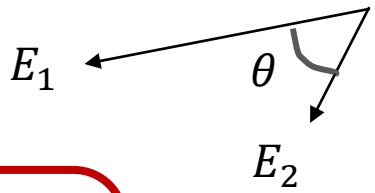
$$\psi^e_{1S}(\vec{x}) = \sqrt{\frac{(m_e(Z-1)\alpha)^3}{\pi}} \exp(-m_e(Z-1)\alpha|\vec{x}|)$$

: wave function of $1S$ bound electron (non-relativistic)

$$\Gamma \propto (Z-1)^3$$

(the same Z dependence in the both contact & photonic cases)

Decay rate



$$\Gamma(\mu^-(1S)e^-(\alpha) \to e^-e^-) = \frac{1}{2}\int_{m_e}^{m_\mu - B_\mu^{1S} - B_e^\alpha} \mathrm{d}\,E_1 \int_{-1}^{1} \mathrm{d}\cos\theta \frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta}$$

E_1 : energy of an emitted electron

θ : angle between two emitted electrons

differential decay rate :

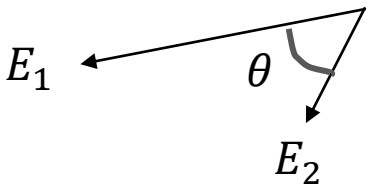
P_l : Legendre polynomial

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta} = \sum_{\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l} M(E_1,\kappa_1,\kappa_2,J)M^*(E_1,\kappa_1',\kappa_2',J) \times w(\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l)P_l(\cos\theta)$$

$$M(E_1,\kappa_1,\kappa_2,J) = \underbrace{\sum_{i=1,\cdots,6} g_i\, M^i_{\mathrm{contact}}(E_1,\kappa_1,\kappa_2,J)}_{\text{contact}} + \underbrace{\sum_{j=L,R} g_j\, M^j_{\mathrm{photo}}(E_1,\kappa_1,\kappa_2,J)}_{\text{photonic}}$$

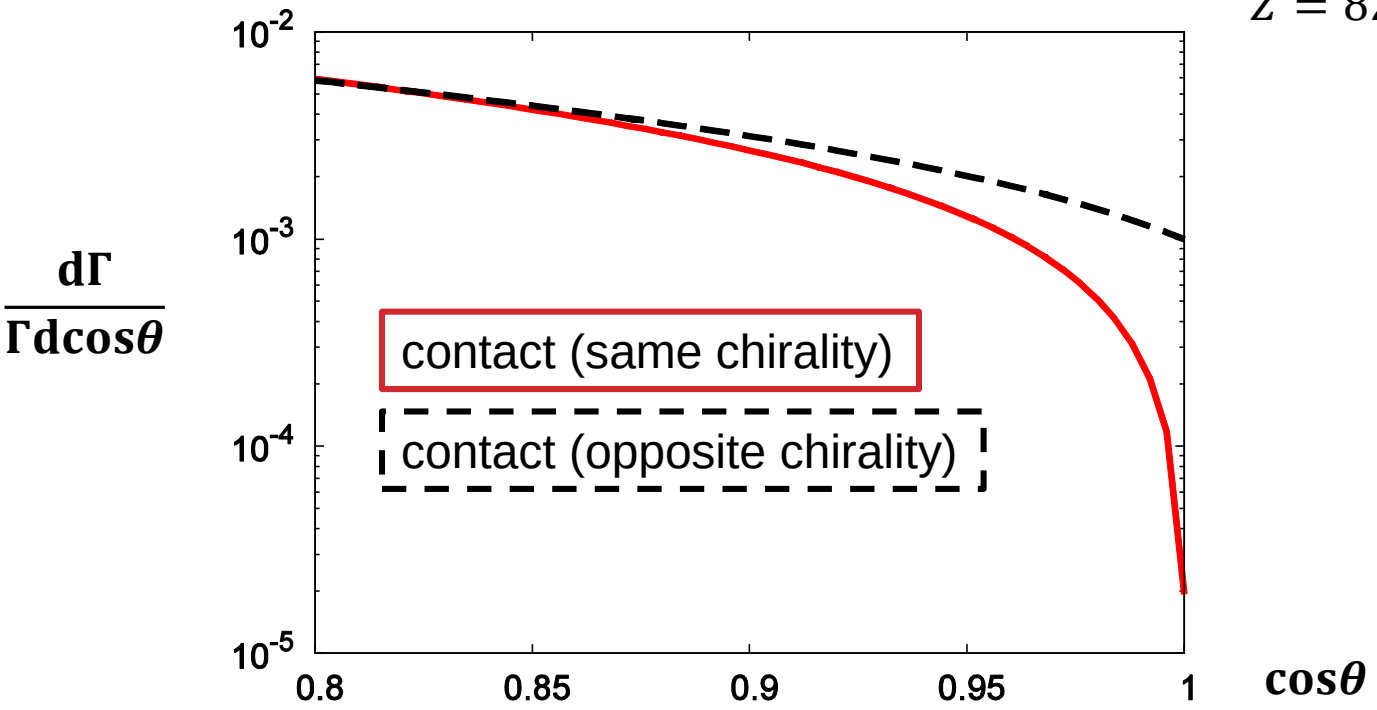
Discriminating method 2

θ : angle between two emitted electrons



angular distribution ($\cos\theta \approx 1$)

$Z = 82$



- e^- pair has same chirality ⇒ e^- pair cannot emit same momentum

(due to Pauli principle)

Contribution from all bound e^-s

normalize the contribution of $1S$ e^- to 1

contact (g_1)

1S	2S	2P	3S	3P	3D	4S	Total
1	0.17	6.2×10^{-3}	5.1×10^{-2}	3.1×10^{-3}	2.3×10^{-9}	2.1×10^{-2}	1.25

photonic (g_L)

1S	2S	2P	3S	3P	3D	4S	Total
1	0.15	7.3×10^{-3}	4.3×10^{-2}	2.6×10^{-3}	2.4×10^{-5}	1.8×10^{-2}	1.21

- it is sufficient to consider about S electrons for both cases