Constraint on higher order symmetry energy parameters and its relevance to neutron star properties

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  [arXiv:1611.07133]
• AO, Kolomeitsev, Lattimer, Tews, X.Wu, in prog.
QCD Phase Diagram

RHIC, LHC, Early Universe
Lattice QCD

Heavy-Ion Collisions
(BES, FAIR, NICA, J-PARC)

QGP

CSC
\[ \delta = \frac{(N-Z)}{A} \quad \text{(or } Y_Q \text{ (hadron)} = \frac{Q_h}{B} \sim \frac{1}{2} - \frac{1}{2} \delta ) \]
Symmetry Energy Parameters & Neutron Star Radius

Nuclear Matter Symmetry Energy parameters \((S_0, L)\) are closely related to Neutron Star Properties, e.g. \(R_{1.4} = R_{NS}(M = 1.4M_\odot)\)

How can we constrain \((S_0, L)\)?
→ Nuclear Exp't. & Theory, Astro. Obs., Unitary gas

Conjecture: UG gives the lower bound of neutron matter energy.
*Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)*

\[
S(n) = E_{PNM} - E_{SNM} \geq E_{UG} - E_{SNM}
\]

\[
E_{UG} = \xi E_{FG} \quad (\xi \approx 0.38)
\]

Sym. Nucl. Matter EOS is relatively well known.

→ For a given \(L\), lower bound of \(S_0\) exists
Constraint on \((S_0, L)\) from Lower Bound of PNM Energy

- Unitary gas + 2 \(M_\odot\) constraints rule out 5 EOSs out of 10 numerically tabulated and frequently used in astrophys. calc.
Further Constraints on Higher-Order Sym. E. parameters

- $K_n$ and $Q_n$ are correlated with $L$ in “Good” theoretical models.
  
  \[
  K_n = 3.534L - (74.02 \pm 21.17)\text{MeV} \\
  Q_n = -7.313L + (354.03 \pm 133.16)\text{MeV}
  \]
Questions:
What are the effects of these higher-order symmetry energy parameters on the MR curve of NS?

This work:
TLOK + 2 $M_\odot$ constraints + $k_F$ expansion $\rightarrow R_{1.4}$

Contents
- Introduction
- Symmetry Energy Parameters, Nuclear Matter EOS, and Neutron Star Radius
- Implications to quark-hadron physics in cold dense matter
  - Neutron chemical potential, QCD phase transition
- Summary
Symmetry Energy Parameters, Nuclear Matter EOS, and Neutron Star Radius
Saturation & Symmetry Energy Parameters

\[ E_{\text{NM}}(u, \alpha) = E_{\text{SNM}}(u) + \alpha^2 S(u) \]

\[ E_{\text{SNM}}(u) \simeq E_0 + \frac{K_0}{18}(u-1)^2 + \frac{Q_0}{162}(u-1)^3 \]

\[ S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_s}{18}(u-1)^2 + \frac{Q_s}{162}(u-1)^3 \]

\((u = n/n_0, \alpha = (n_n - n_p)/n)\)

Energy does not approach zero at \(n \to 0\).

Fermi momentum expansion (\(\sim\) Skyrme type EDF)

Generated many-body force is given by \(k_F \propto u^{1/3}\)

\[ E_{\text{SNM}}(u) \simeq T_0 u^{2/3} + a_0 u + b_0 u^{4/3} + c_0 u^{5/3} + d_0 u^2 \]

\[ S(u) \simeq T_s u^{2/3} + a_s u + b_s u^{4/3} + c_s u^{5/3} + d_s u^2 \]

Kin. E. Two-body Density-dep. pot.
Expansion Coefficients

Coefficients \((a,b,c,d)\) are represented by Saturation and Symmetry Energy Parameters

\[ a_0 = -4T_0 + 20E_0 + K_0 - Q_0/6 \]
\[ b_0 = 6T_0 - 45E_0 - 5K_0/2 + Q_0/2 \]
\[ c_0 = -4T_0 + 36E_0 + 2K_0 - Q_0/2 \]
\[ d_0 = T_0 - 10E_0 - K_0/2 + Q_0/6 \]

\[ a_s = -4T_s + 20S_0 - 19L/3 + K_s - Q_s/6 \]
\[ b_s = 6T_s - 45S_0 + 15L - 5K_s/2 + Q_s/2 \]
\[ c_s = -4T_s + 36S_0 - 12L + 2K_s - Q_s/2 \]
\[ d_s = T_s - 10S_0 + 10L/3 - K_s/2 + Q_s/6 \]

\[
\left( T_0 = \frac{3}{5} \frac{\hbar^2 k_F (n_0)^2}{2m}, \quad T_s = T_0 (2^{1/3} - 1) \right)
\]

Tedious but straightforward calc.
TLOK+2M\(\odot\) constraints

TLOK constraints

- \((S_0, L)\) is in Pentagon.
- \((K_n, Q_n)\) are from TLOK constraint.
- \(K_0=(190-270)\) MeV
- \((n_0, E_0)\) is fixed
  \(n_0=0.164\) fm\(^{-3}\), \(E_0=-15.9\) MeV (small uncertainties)
- \(Q_0\) is taken to kill \(d_0\) parameter
  (Coef. of \(u^2\). Sym. N. M. is not very stiff at high-density)

2 M\(\odot\) constraint

- EOS should support 2 M\(\odot\) neutron stars.

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.
2$M_\odot$ constraint narrows the range of EOS.

Consistent with FP and TT(Togashi-Takano) EOSs.

APR and GCR(Gandolfi-Carlson-Reddy) EOSs seems to have larger $S_0$ values.

OKLTW, in prog.
**Neutron Star MR curve**

TLOK + 2 $M_{\odot}$ constraints $\rightarrow R_{1.4}=(10.6-12.2)$ km

- E and P are linear fn. of Sat. & Sym. E. parameters $\rightarrow$ Min./Max. appears at the corners of pentagon (ABCDE).

- For a given $(S_0, L)$,
  - unc. of $R_{1.4}$ $\sim$ 0.5 km
  - = unc. from higher-order parameters

- Unc. from $(S_0, L)$ $\sim$ 1.1 km
  $\rightarrow$ We still need to fix $(S_0, L)$ more precisely.
Impact of GW from binary neutron star merger

GW170817 from NS-NS → Multi messenger astrophysics
(Kyutoku's talk)

Neutron Star Radius
- Inspiral region → Tidal deformability ($\Lambda$) → NS radius (e.g. $R_{1.4}$)

Neutron Star Maximum Mass
- No GW signal from Hyper Massive NS → $M_{\text{max}}$
  - $M_{\text{max}}(T=0,\omega=0) < M_{\text{max}}(T=0,\omega) < M < M_{\text{max}}(T,\omega)$

Nucleosynthesis site of r-process nuclei
- kilonova/macronova from decay energy of the synthesized elements
- r-process nucleosynthesis seems to occur in BNSM!

Central Engine of (Short) Gamma-Ray Bursts
- GW as standard siren (Hubble constant)

Courtesy of Y. Sekiguchi @ YKIS2018b
Various Constraints

Annala+, PRL120('18)172703

Abbott+, 1805.11579

I. Tews, J. Margueron, S. Reddy, PRC98 ('18)045804

Lattimer, Prakash PRep.621('16),127
Neutron Star MR curve

Our constraint is consistent with many of previous ones.

\[ R_{1.4} = (10.6-12.2) \text{ km} \]

Present work (TLOK + 2 \( M_{\odot} \)) \( \text{OKLTW, in prog.} \)

LIGO-Virgo (Tidal deformability \( \Lambda \) from BNSM)

(10.5-13.3) km \( \text{Abbott+('18b)} \)
(9.1-14.0) km \( \text{De+'18 (A)} \)

Theoretical Estimates

(10.7-13.1) km

\( \text{Lattimer, Prakash('16)} \)

(10.0-13.6) km

\( \text{Annala+'18 (\( \chi \text{EFT}+p\text{QCD}) \)} \)

(10-13.6) km

\( \text{Tews+'18(\( \chi \text{EFT}+c_s \))} \)

(12.0-13.6) km

\( \text{Fattoyev+'18 (PREX)} \)

12.7 ± 0.4 km

\( \text{Margueron+'18 (n expansion)} \)
Implications to quark-hadron physics in cold dense matter (1)
Neutron Chemical Potential and Hyperon Puzzle
Neutron Chemical Potential in NS

- Λ appears in neutron stars if $E_\Lambda (p=0) = M_\Lambda + U_\Lambda < \mu_n$

- W. Weise's conjecture: $U_\Lambda$ in $\chi$EFT (2+3 body) is stiff enough.

- But $\mu_n$ is larger with TLOK+2$M_\odot$ constraints

W. Weise, NFQCD2018 (2018.06); Gerstung, Kaiser, Weise, in prog.

APR $\mu_n$

OKLTW, in prog.
Neutron Chemical Potential

\[ \mu_n + M_N = \frac{\partial (nE)}{\partial n_n} = E + u \frac{\partial E}{\partial u} + 2\alpha(1 - \alpha)S(u) \]

Single particle potential

\[ U_\Lambda(u) = \frac{\partial (nV)}{\partial n_\Lambda} \]

\[ \simeq U_{0\Lambda} + \frac{L_\Lambda}{3}(u - 1) \]

\[ U_{0\Lambda} \simeq -30 \text{ MeV} \]

\[ L_\Lambda = ??? \]

(L_\Lambda < 0 in most of RMF before 2010)

Sym. E. and \( L_\Lambda \) determine the onset density of \( \Lambda \).

(Already mentioned in Millener, Dover, Gal paper)
Implications to quark-hadron physics in cold dense matter (2)

QCD phase transition density and order in cold dense matter
QCD phase transition in cold dense matter

Transition to quark matter in cold-dense matter
1st order or crossover?

Crossover: Masuda, Hatsuda, Takatsuka, Kojo, Baym, ...

1st order p.t.

Many effective models predict, e.g. Asakawa-Yazaki CP

Recent phenomenological support: Negative Directed Flow in HIC
Y. Nara, H. Niemi, AO, H. Stoecker, PRC94('16)034906.
Y. Nara, H. Niemi, AO, J. Steinheimer, X.-F. Luo, H. Stoecker, EPJA 54 ('18)18

The phase transition density may be above NS central density
**Directed Flow** \( v_1 = \langle \cos \phi \rangle = \langle p_x / p_T \rangle, \quad \text{Slope} = d v_1 / dy \)

**Negative Directed Flow**

- **Directed Flow** slope at \( \sqrt{s_{NN}} = 11.5 \text{ GeV} \) (STAR ('14))
- Strong softening of EOS is necessary at \( n > (5-10) n_0 \)

\[ Y. Nara, H. Niemi, AO, H. Stoecker, \\ PRC94('16)034906. \]

\[ Y. Nara, H. Niemi, AO, J. Steinheimer, \\ X.-F. Luo, H. Stoecker \\ EPJA 54 ('18)18 \]
Isospin & Hypercharge Sym. $E$ in quark matter

- Two types of vector int. in NJL
  
  \[ \mathcal{L}_v = -G_0 (\bar{q} \gamma_\mu q)^2 - G_v \sum_i \left[ (\bar{q} \gamma_\mu \lambda_i q)^2 + (\bar{q} i \gamma_5 \gamma_\mu \lambda_i q)^2 \right] \]

- Isospin & Hypercharge Sym. $E$
  \[ E = \alpha^2 S(n) + \alpha_Y^2 S_Y(n) \, , \, \alpha = -2 \langle T_z \rangle / B \, , \, \alpha_Y = \langle B + S \rangle / B \]
$(\rho, T, Y_e)$ during SN, BH formation, BNSM

See also Oertel+16

AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11), 284
Reservations and Prospects
**Reservations**

- Only massless electrons are considered and Crust EOS is ignored.
  - With $\mu$, chemical potential may be reduced a little.
- Non-relativistic kinetic energy is used.
  - With rel. K.E., $E$ per nucleon is modified by 0.03 MeV @ $10 \ n_0$ as long as Sat. and Sym. $E$ parameters are fixed.
- Function form is limited to $k_F$ expansion with $u^{k/3} \ (k=2-6)$.
  - $R_{1.4}$ range becomes narrower with $k=2-5$.
  - Density expansion gives EOSs very sensitive to parameters.
- Smooth $E(u)$ (= No phase transition) is assumed.
  - We expect QCD phase transition at $(5-10) \ n_0$ from recent BES data of directed flow *Nara, Niemi, AO, Stoecker ('16)*
  - Transition to quark matter may not soften EOS drastically.
- Causality is violated at high densities, $n > (4-6) \ n_0$. 
To Do (or Prospect)

- Baryons other than nucleons Λ, Δ, Ξ, Σ, ...
- Connecting to Hadron Resonance Gas (HRG) EOS
  - HRG EOS
    - mass and kinetic E of hadrons with M<2 GeV + simple potential E
      \[ \varepsilon_{HRG} = \mathcal{T} + cn^2 \]
    - or Lattice EOS in HIC (No saturation, No constraint from NS).
  - We need to guess the potential energy density more seriously for consistent understanding of HIC, Nuclear, and NS physics.
    \[ \varepsilon = \mathcal{T} + V \]
    Nuclear and NS physics
- Connecting to Quark(-Gluon) matter EOS
  - Embed model-H singularities E.g. Nonaka, Asakawa ('04)
  - “Interpolation” of nuclear and quark matter EOS
Summary

- Tews-Lattimer-AO-Kolomeitsev ('17) constraints (S0, L, K_n, Q_n) and 2 M_☉ constraint with the aid of Fermi momentum (k_F) expansion lead to the constraint on 1.4 M_☉ neutron star radius of (10.6-12.2) km.
  - Consistent with many of other constraint.

- Onset density of hyperons may be sensitive to the symmetry energy in addition to potential parameters, (U_{0B}, L_{B}).
  - We need to know the slope of potential in addition to the depth.

- Global EOS (HIC and Nuclear/NS matter) needs to be given in a way where HIC physicists and NS physicists admit. E.g. “Hadron Resonance Gas (HRG)+Potential from NS”

Thank you for your attention.
Further Constraint on $Q_n$

- $2 \, M_\odot$ requirement constrains $Q_n$ further.

$$Q_n > -9.3L + 480 \text{ MeV}$$

FIG. 4. Constraint on $Q_n$

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.
Neutron star – Is it made of neutrons?

- Possibilities of various constituents in neutron star core
  - Strange Hadrons
    - $d^* u^* d u^* s$
      - proton
      - $\Lambda$ hyperon
  - Meson condensate ($K$, $\pi$)
    - $\bar{d} u \bar{u} d^* s$
      - $\pi$
      - anti kaon
  - Quark matter
  - Quark pair condensate (Color superconductor)
    - $d d^* u$
      - 2SC

$NS \text{ core} = \text{Densest stable matter existing in our universe.}$

$R \sim 10 \text{ km}$

$M \sim 1.4 \, M_\odot, \quad \rho_c \sim (3-10) \, \rho_0$
$(\rho, T)$ during SN & BH formation

Ishizuka, AO, Tsubakihihara, Sumiyoshi, Yamada, JPG 35('08) 085201; AO et al., NPA 835('10) 374.

Shen EOS + hyperons
QCD phase transition is not only an academic problem, but also a subject which would be measured in HIC or Compact Stars.
Unitary Gas Constraint

Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)

Conjecture:
Unitary gas gives the lower bound of neutron matter energy.

\[
S(n) = E_{\text{PNM}} - E_{\text{SNM}} \geq E_{\text{UG}} - E_{\text{SNM}}
\]

\[
E_{\text{UG}} = \xi E_{\text{FG}} \quad (\xi \simeq 0.38)
\]

- \(a_0 = \infty\) in unitary gas
  → lower bound energy of \(a_0 < 0\) systems (w/o two-body b.s.)?
- Supported by (most of) ab initio calc.

Sym. Nucl. Matter EOS is relatively well known.
Potential Energy Density

Potential Energy Density in the Fermi momentum expansion

\[ \mathcal{V} = nV = \sum_{i,j \in B} n_i n_j v_{ij}(n) \]

Density-dependent NN interactions \( v_{ij} \) (i, j=p or n) are known.

Single particle potential

\[ U_i = \frac{\partial \mathcal{V}}{n_i} = \sum_j n_j v_{ij}(n) + \sum_{jk} n_j n_k \frac{\partial v_{jk}(n)}{\partial n_i} \]

\[ = U_{0i} + \frac{L_i}{3} (u - 1) + \mathcal{O}((u - 1)^2) \]

\[ \simeq au + bu^{4/3} \]

Again, a and b are given as a linear function of \( U_{0i} \) and \( L_i \).