Color Entanglement

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- Quantum Entanglement
- QCD and Color Entanglement
- QCD Factorization – Approximation
- Sign change of the Sivers’s effect,
  Factorization breaking, …
- Summary and Outlook

Workshop on Progress on Hadron structure functions in 2018
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Quantum Entanglement

Definition:

A system whose quantum state cannot be factored as a product of states of its local constituents, no matter how far they are separated

That is, all particles in such a system are not individual particles but are an inseparable whole

Particle decay – a classical example of the entanglement:

The total number of particles from the decay of a single particle form various systems of entanglement due to various conservation laws, such as momentum, angular momentum, ...

A spin-0 particle decays into two spin-1/2 particles:

Entangled quantum state:

\[
\frac{1}{\sqrt{2}} \left[ \left| \frac{1}{2}, \frac{1}{2} \right> + \left| \frac{1}{2}, -\frac{1}{2} \right> - \left| \frac{1}{2}, \frac{1}{2} \right> - \left| \frac{1}{2}, -\frac{1}{2} \right> \right]
\]

The spin of these two particles are entangled or correlated
QCD and Color Entanglement

QCD as a quantum field theory (QFT):

\[ \mathcal{L}_{\text{QCD}}(\psi, A) = \sum_f \bar{\psi}_i^f \left[ (i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f \\
- \frac{1}{4} \left[ \partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c} \right]^2 \\
+ \text{gauge fixing + ghost terms} \]

With fields/particles:

- Spin-1/2, color triplet quark fields: \( \psi_i^f(x) \quad f = u, d, s, c, b, t \)
- Spin-1, color octet gluon fields: \( A_{\mu,a}(x) \quad a = 1, 2, ..., 8 = N_c^2 - 1 \)

Number of quanta in QFT is \textbf{NOT} fixed, key difference from QM!

Microscopic entanglement in QCD:

Color of the quark and anti-quark is quantum/color entangled

\( RR, BB, GG \)

Hadronization is an entangled process:

Although hadrons are color singlet, their “distributions” are quantum/color correlated
QCD and Color Entanglement

- **Unitarity:**
  \[ \sigma_{e^+e^-\rightarrow\text{hadrons}}^{\text{tot}} \propto \left( \sigma_{e^+e^-\rightarrow\text{partons}}^{\text{tot}} \right)^2 \]

- Color entanglement between partons affect the hadron distributions.
- Without asking the details of the “distributions”, summing over all final-state, color entanglement is not a direct observable.
- Unitarity ensures that the “total” cross section is perturbatively calculable for large enough Q.

\[ \sigma_{e^+e^-\rightarrow\text{partons}}^{\text{tot}}(s = Q^2) = \sum_n \sigma^{(n)}(Q^2, \mu^2) \left( \frac{\alpha_s(\mu^2)}{\pi} \right)^n \]

- Distribution of an identified hadron(s):
  e.g., \[ E_h \frac{d\sigma_{e^+e^-\rightarrow h}}{d^3p_h}(\sqrt{s} = Q), \ E_{h_1} E_{h_2} \frac{d\sigma_{e^+e^-\rightarrow h_1 h_2}}{d^3p_{h_1} d^3p_{h_2}}(\sqrt{s} = Q) \] are not calculable since the hadronization is color entangled and nonperturbative!
QCD Factorization – Approximation

- Single hadron production – collinear factorization:

\[ QCD \text{ factorization} \rightarrow \text{approximation} : \]

When \( P_h \gg m_h \) (enhanced by some logarithms from the shower), we “neglect” the color entanglement between the long-distance hadronization processes, while keeping the color entanglement at the short-distance, calculated perturbatively

\[
E_h \frac{d\sigma_{e^+e^- \rightarrow h}}{d^3p_h}(\sqrt{s} = Q) \approx \sum_c E_c \frac{d\sigma_{e^+e^- \rightarrow c}}{d^3p_c} \otimes D_{c \rightarrow h}(z = p_h/p_c)
\]

plus power corrections in \( \mathcal{O}(m_h/P_h) \)
QCD Factorization – Approximation

- Explore the breakdown of QCD factorization:
  or the role of color entanglement in hadronization

\[ E_h \frac{d\sigma_{e^+e^-\rightarrow h}}{d^3p_h} (\sqrt{s} = Q) \]

Can be represented by a 2-D picture for each hadron with \( Q \) and \( p_h \) for the two sides, and density for the rate

- NN training and mapping out all \( \sqrt{s} = Q \) and \( z = 2p_h/Q \) dependence of the production rate (without any theory input!):

T. Liu, N. Sato et al. @ JLab
Comparing with pQCD factorization:

Leading power factorization formalism fails at Belle energies, and near the edge of phase-space!
Semi-Inclusive DIS

- **Color entanglement – factorization:**

- **Low** $P_{hT}$ – TMD factorization:
  \[
  \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes D_{f \to h}(z, P_{\perp}) \otimes S(k_{s\perp}) + O \left[ \frac{P_{h\perp}}{Q} \right]
  \]

- **High** $P_{hT}$ – Collinear factorization:
  \[
  \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f \to h} + O \left( \frac{1}{P_{h\perp}}, \frac{1}{Q} \right)
  \]

- **$P_{hT}$ Integrated - Collinear factorization:**
  \[
  \sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f \to h} + O \left( \frac{1}{Q} \right)
  \]

**QCD factorization suppress color entanglement between hadrons**
Near the edge of phase space:

Much more sensitive to color entanglement of hadronization

\[ Q^2 : [1.7, 3] \text{ GeV}^2 \]
\[ x_B : [0.055, 0.1] \]
\[ z_h : [0.4, 0.6] \]
Semi-Inclusive DIS

- **Power corrections:**

- **Two-parton fragmentation:**

\[
D_{[q\bar{q}(1a)]}(z, \xi, \zeta, \mu_0) = \frac{1}{4N_c P_h^+} \langle 0| \bar{q}'_{c,k}(y_1^-)(\gamma \cdot n\gamma_5)_{kl} U_{c'}d'(y_1^- , 0) q_{d',l}(0)|h(P_h) \rangle
\]

\[
\approx \frac{f_h^2}{16N_c^2} z\delta(1 - z) \phi_h(\zeta, \mu_0) \phi_h(\xi, \mu_0).
\]
Semi-Inclusive DIS

- Prediction for JLab energy – photo production:

\begin{align*}
E_{\text{beam}} &= 11 \text{ GeV}, \quad x_B = 0.2, \quad Q^2 = 3 \text{ GeV}^2, \quad z_h = 0.7.
\end{align*}
Drell-Yan

- Color entanglement – factorization:

- $P_{hT}$ Integrated - Collinear factorization:
  
  \[
  \frac{d\sigma}{dQ^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') + Q \left( \frac{1}{Q} \right)
  \]

- Low $P_{hT}$ – TMD factorization:
  
  \[
  \frac{d\sigma}{dQ^2 dq_T^2} = \hat{H} \otimes \Phi(x, k_{\perp}) \otimes \Phi(x', k_{\perp}') \otimes S(k_{s\perp}) + Q \left( \frac{q_T}{Q} \right)
  \]

- High $P_{hT} \sim Q$ – Collinear factorization:
  
  \[
  \frac{d\sigma}{dQ^2 dq_T^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') + Q \left( \frac{1}{Q}, \frac{1}{q_T} \right)
  \]

- High $P_{hT} \gg Q$ – Collinear factorization:
  
  \[
  \frac{d\sigma}{dQ^2 dq_T^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') \otimes D(z) + Q \left( \frac{1}{q_T} \right)
  \]
Sign Change of Sivers Function

- Single transverse spin asymmetry – Sivers effect:

\[ f_{q/h\uparrow}(x, k_{\perp}, \vec{S}) = \int \frac{dy^- d^2y_{\perp}}{(2\pi)^3} e^{i x y^+ - i k_{\perp} \cdot y_{\perp}} \langle p, \vec{S} | \bar{\psi}(0^-, 0_{\perp}) | p, \vec{S} \rangle \]

- Gauge links:

- Process dependence:

\[ f_{q/h\uparrow}^{\text{SIDIS}}(x, k_{\perp}, \vec{S}) \neq f_{q/h\uparrow}^{\text{DY}}(x, k_{\perp}, \vec{S}) \]

Collinear factorized PDFs are process independent
Sign Change of Sivers Function

- **Parity – Time reversal invariance:**
  \[ f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp, \vec{S}) = f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp, -\vec{S}) \]

- **Definition of Sivers function:**
  \[ f_{q/h^\uparrow}(x, k_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/h^\uparrow}(x, k_\perp) \vec{S} \cdot \hat{p} \times \hat{k}_\perp \]

- **Modified universality:**
  \[ \Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp) \]

The spin-averaged part of this TMD is process independent, but, spin-averaged Boer-Mulder’s TMD requires the sign change!

Violation of the “sign” change should be the break of factorization, that is, a much stronger color entanglement!
Sign Change of Sivers Function

- **RHIC:**
  PRL 116(2016)132301:

- **COMPASS:**
  PRL 119(2017)112002
Hadronic Scattering

- Color entanglement – factorization:

  \[
  \frac{d\sigma}{dp_T^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') \otimes D(z) + \mathcal{O}\left(\frac{1}{p_T}\right)
  \]

  when \( p_T \gg m_h \)

- Breaking of TMD factorization for di-jet production:

  \[ H_1(p_A) + H_2(p_B) \Rightarrow \text{Jet}(p_1) + \text{Jet}(p_2) + X \]

  ✧ Dominated kinematic region:

  \[ p_1 = \frac{P}{2} + q \quad p_2 = \frac{P}{2} - q \quad \text{with} \quad P \gg q \]

  ✧ Proposal: if the TMD factorization is valid in this region, di-jet momentum imbalance is an excellent observable to test the universality of the Sivers function

  ✧ Unfortunately, TMD factorization was not valid for this process due to color entanglement

Boer and Vogelsang
Collins and Qiu
Vogelsang and Yuan
Momentum Imbalance of di-Jet Production

- Single spin-asymmetry could be generated by both initial- and final-state interaction – needs a phase

- Very simple representation of $qq' \rightarrow qq'$ channel:

\[
\frac{d\Delta \sigma}{dy_1 dy_2 dP_T^2 d^2 q_T} \propto q'(x) q_T^{\text{SIDIS}}(x, q_T) \left( C_I + C_{F1} + C_{F2} \right) \frac{s^2 + u^2}{t^2}
\]

when $k$ is parallel to the polarized hadron

Perturbatively generated Sivers’ function at $g^2$

Initial- and final-state interactions differ only by a color factor
Momentum Imbalance of di-Jet Production

- Test the TMD factorization by studying long-distance physics of partonic scattering cross section:

  If the factorization is valid, all factorized long-distance information should be process independent.

- Consider the poles from collinear gluon attachment to the lowest order partonic diagram in the TMD approach.

  ! Initial-state and final-state have different color flow! If one keeps the color difference in the hard part, one could get the same leading order hard part – necessary, not sufficient!

Qiu, Vogelsang, and Yuan
Momentum Imbalance of di-Jet Production

- A simple model:
  - simplifying the derivation while keeping the same physics

- Hadron is made of a fermion $\psi$ and a scalar $\phi$
- There are two hadrons, $H_1$ and $H_2$
- Gauge field (Abelian) couples $g_i$ to $\psi_i$ and $-g_i$ to $\phi_i$

$$\lambda_i \left( \bar{H}_i \psi_i \phi_i^\dagger + \bar{\psi}_i H_i \phi_i \right)$$

- Basic idea:
  - If the TMD factorization is valid,
  - Gauge link of hadron $H_1$ should not depend on the property of hadron $H_2$, or any details of the subprocesses
  - Leading contribution from multiple gluon interaction should be expressed in terms of gauge link times the same lowest order hard part
  - Otherwise, the TMD factorization is violated
Momentum Imbalance of di-Jet Production

Leading contribution to SSA:

Phase from the leading pole: 
\[ (i\pi)(g_1 + 2g_2)\delta(1^+) \]

can we keep the \( g_2 \) dependence in the hard part?

We find that the \( (i\pi)^2 \) from two gluon exchange also depends on \( g_2 \), which cannot be factored into the same lowest order hard part with \( g_2 \).

That is, we found an example in which the gauge link of hadron 1 depends on the property of hadron 2, which signals the failure of the TMD factorization.

Vogelsang and Yuan, so as Rogers and Mulders obtained the same conclusion.
Momentum Imbalance in di-Jet Production

- Color flow of TMD factorization:

- Color flow breaks TMD factorization:

  The color flow can’t be separated into two loops, each of them depends on only one-hadron

  Color is entangled!

This is consistent with the general rule that Qiu & Sterman found:

Only the first subleading power term could be factorized when observables involve multiple hadrons
Quantum entanglement is a very interesting phenomenon

- Separates quantum theory from classical ones

Observables involving multiple identified hadrons could not be calculated perturbatively, without making approximations – leading to the factorization, …

- QCD factorization is an approximation to suppress the color entanglement

Breaking of QCD factorization reduces our predictive power, but, might give us new opportunities to explore the even richer phenomena of color entanglement

QCD dynamics is rich – We only learned very little of it!

Thanks!
Backup slides