$K^{bar}N$ system studied with coupled-channel Complex Scaling Method

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1. **Introduction**
   - *Complex Scaling Method*
2. **Scattering problem solved with Complex Scaling Method**
   - *Formalism / Test calculation with AY $K^{bar}N$ potential*
3. **Set up for the calculation of I=0 $K^{bar}N$-πΣ system**
   - *Kinematics / Non-rela. approximation*
4. **Results**
   - *Scattering amplitude / Resonance pole*
5. **Summary and future plans**

『「ストレンジネス核の励起スペクトルとバリオン混合」第2回ミーティング』
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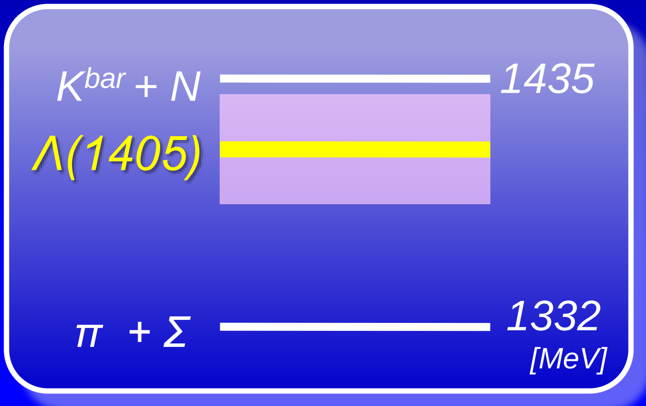
1. Introduction

1. Introduction

Λ(1405) = Important building block of K^{bar} nuclei



$J^{\pi}=1/2^{-}$, $I=0$
$K^{bar}N$-$\pi\Sigma$ coupled system

$I=0$ $K^{bar}N$ potential … very attractive

⇒ Interesting properties of K^{bar} nuclei !

coupled-channel Complex Scaling Method

- Investigate a resonant state, based on the variational approach
- Explicit treatment of the $\pi\Sigma$ channel

Importance of $\pi\Sigma N$ in $K^{-}pp$
Y. Ikeda and T. Sato, PRC 79, 035201(2009)

Chiral SU(3) potential (KSW)

N. Kaiser, P. B. Siegel and W. Weise, NPA 594, 325 (1995)

- Based on Chiral SU(3) theory
  → **Energy dependence**

+ r-space, Gaussian form

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_{\pi}^{2}} \left(\omega_i + \omega_j\right) \sqrt{\frac{M_i M_j}{s\, \omega_i \omega_j}}\; g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2} a_{ij}^{3}} \exp\left[-\left(r/a_{ij}\right)^{2}\right]$$

Complex Scaling Method

Complex rotation of coordinate

$U(\theta): \quad \mathbf{r} \to \mathbf{r}e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}e^{-i\theta}$

$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$

$E = \langle \Phi | H | \Phi \rangle = \langle \Phi_\theta | H_\theta | \Phi_\theta \rangle$

- ✓ Resonant wave function is transformed from a divergent function to a dumping function.

$$\Phi_R \sim e^{ik_R r} = e^{i(\kappa - i\gamma)r} \quad \Rightarrow \quad U(\theta)\Phi_R \sim \exp\{(\gamma + i\kappa) r e^{i\theta}\} = \exp\left[\left(\underline{\gamma\cos\theta - \kappa\sin\theta}\right)r\right]\cdot\exp\left[i\left(\gamma\sin\theta + \kappa\cos\theta\right)r\right]$$

Negative when $\tan\theta > \gamma/\kappa$

- ✓ Resonant and bound states are independent of a scaling angle θ. (ABC theorem†)

- ✓ Resonant states can be obtained by diagonalizing H_θ with Gaussian basis, in the same way as calculating bound state.

† J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269.
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

Self-consistent solutions for complex energy

Chiral potential … Energy-dependent potential

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\, g_a(r)$$

⇨ ***Self-consistency for complex energy should be considered!***

$$H_\theta(Z_{in})\ \Phi^\theta_{\bar{K}N-\pi\Sigma(I=0)} = Z_{out}\ \Phi^\theta_{\bar{K}N-\pi\Sigma(I=0)}$$

$Z = E - i\,\Gamma/2$ *: complex energy*

Self-consistent solution … $Z_{in} = Z_{out}$

Ex) KSW ; f_π = 100 MeV, a= 0.47 fm (θ=35°)

Step		1	2	3	4	5
(in)	E	−37.5	−37.5	−27.1	−27.4	−27.6
	−Γ/2	0	−60.4	−59.8	−58.3	−58.4
(out)	E	−37.5	−27.1	−27.4	−27.6	−27.6
	−Γ/2	−60.4	−59.8	−58.3	−58.4	−58.4

Self-consistency achieved after a few times iteration.

A. D., T. Inoue and T. Myo,
Proc. of BARYONS' 10,
AIP conf. proc. 1388, 549 (2010)

2. Scattering problem solved with Complex Scaling Method

Calculation of $K^{bar}N$ scattering amplitude

- *Formalism*
- *Test with AY potential*

Calc. of scattering amplitude with CSM

A. T. Kruppa, R. Suzuki and K. Katō, PRC 75, 044602 (2007)

$$H_l \psi_{l,k}^{(+)}(r) = E\psi_{l,k}^{(+)}(r)$$

$$\psi_{l,k}^{(+)}(r) = \hat{j}_l(kr) + \psi_{l,k}^{sc}(r).$$

$$\psi_{l,k}^{sc}(r) \underset{r\to\infty}{\to} k f_l(k) \hat{h}_l^{(+)}(kr),$$

- Unknown
- Non square-integrable

Point 1

Separate the incoming wave $j_l(kr)$ *!*

$$h_l^{(+)}(x) \to \exp\left[i\left(kr - \frac{l\pi}{2}\right)\right]$$

$$(E - H_l)\psi_{l,k}^{sc}(r) = V(r)\hat{j}_l(kr).$$

Complex scaling

$$r \to r\, e^{i\Theta}$$

Point 2

$$\psi_{l,k}^{sc,\theta}(r) \underset{r\to\infty}{\to} e^{i\theta/2} k f_l(k) i^{-l} e^{ikr\cos\theta - kr\sin\theta}$$

square-integrable for $0 < \theta < \pi$

$$(E - H_l(\theta))\psi_{l,k}^{sc,\theta}(r) = e^{i\theta/2} V(re^{i\theta})\hat{j}_l(kre^{i\theta}),$$

Expanding with square-integrable basis function (ex: Gaussian basis)

$$\psi_{l,k}^{sc,\theta}(r) \approx \sum_{i=1}^{N} c_i(\theta)\phi_i(r)$$

Calc. of scattering amplitude with CSM

A. T. Kruppa, R. Suzuki and K. Katō,
PRC 75, 044602 (2007)

- *Calculation of scattering amplitude*

$$f_l(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\,\hat{j}_l(kr)V(r)\psi^{(+)}_{l,k}(r)$$

$$f_l^{Born}(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\,\hat{j}_l(kr)V(r)\hat{j}_l(kr)$$

$$f_l^{sc}(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\,\hat{j}_l(kr)V(r)\psi^{sc}_{l,k}(r)$$

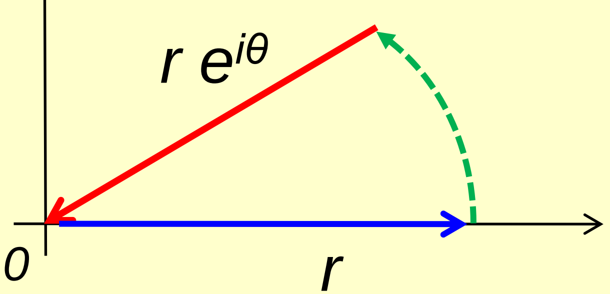
We don't have $\psi^{SC}_{l,k}(r)$ *which is a solution along r,*

but we have $\psi^{SC,\theta}_{l,k}(r)$ ***which is a solution along*** $re^{i\theta}$**.**

Point 3

Cauchy theorem

$$\oint dz\; j(kz)V(z)\psi^{SC}_{l,k}(z) = 0$$



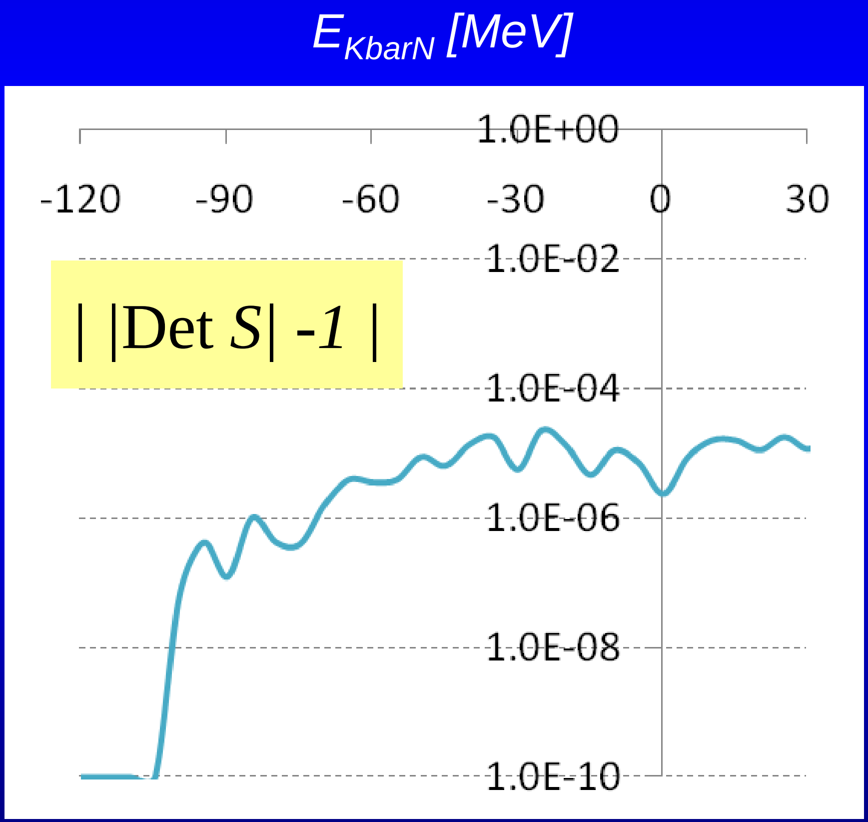
$$f_l^{sc}(k) = -\frac{2m}{\hbar^2 k^2}e^{i\theta/2}\int_0^\infty dr\,\hat{j}_l(kre^{i\theta})V(re^{i\theta})\psi^{sc,\theta}_{l,k}(r)$$

expressed with Gaussian base

$f_l^{SC}(k)$ ***is independent of*** θ**.**

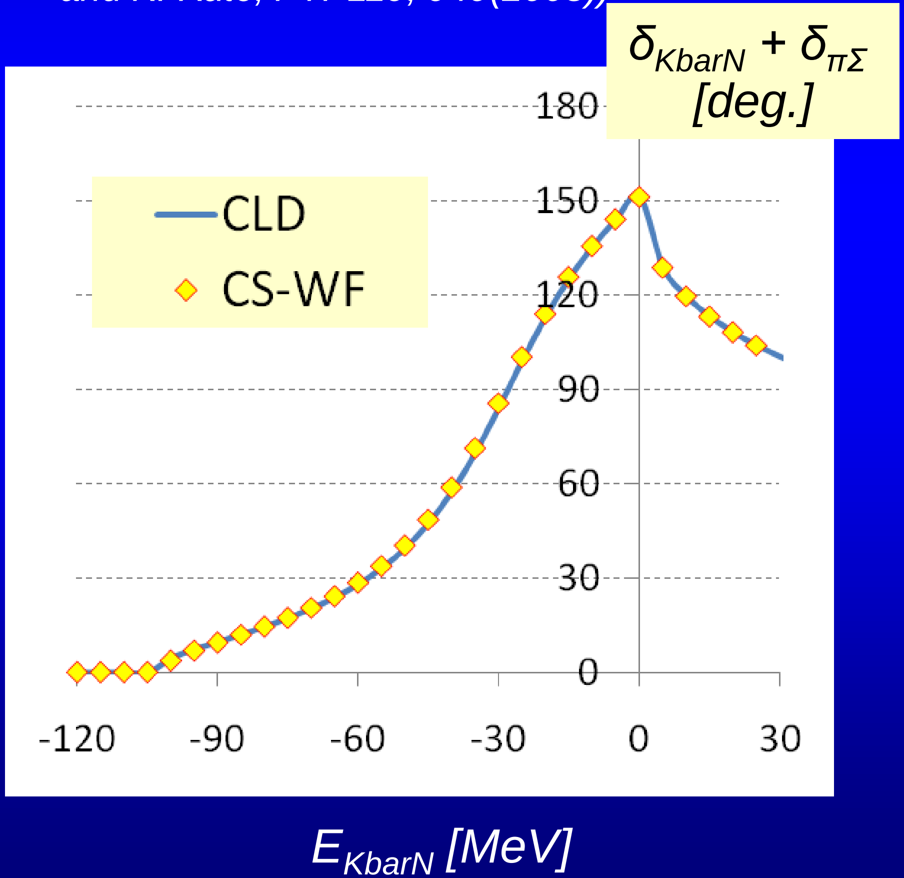
Test calculation with AY potential

Unitarity violation of S-matrix



Phase shift sum

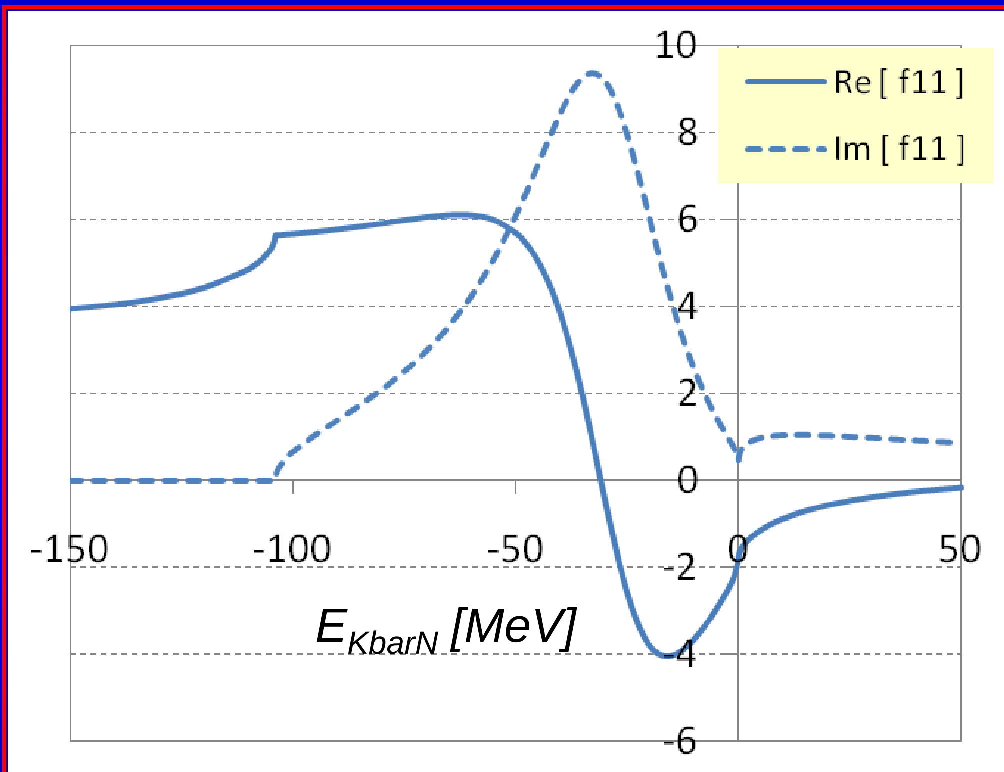
Checked by
Continuum Level Density method
(R. Suzuki, A. T. Kruppa, B. G. Giraud, and K. Katō, PTP119, 949(2008))



Test calculation with AY potential

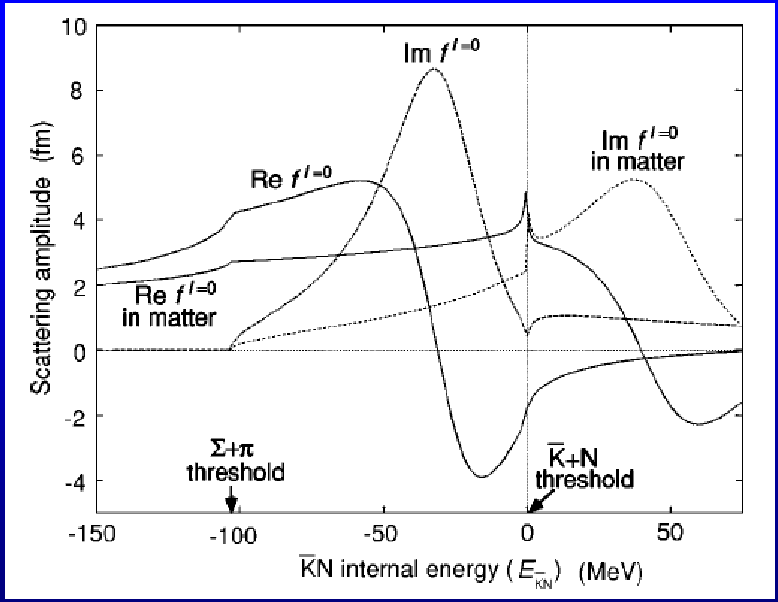
K^bar N (I=0) scattering amplitude

$$f^{I=0}_{\bar{K}N\to\bar{K}N}\ [\,fm\,]$$



Y. Akaishi and T. Yamazaki, PRC65, 044005 (2002)

Scattering length = -1.76 + i 0.46 fm



Scattering length (Scatt. amp. @ E_{KbarN}=0) = -1.77 + i 0.47 fm

3. Set up for the calculation of $I=0$ $K^{bar}N-\pi\Sigma$ system

- *Kinematics*
- *Non-rela. approximation of KSW potential*

Kinematics

1. Non-relativistic

$$H_{NR} = \sum_{c=\bar{K}N,\pi\Sigma} \left[m_c + M_c + \frac{\mathbf{p}^2}{2\mu_c} \right] |c\rangle\langle c| + V_{KSW}$$

$$E = M_c + \frac{\hbar^2 k_c^2}{2M_c} + m_c + \frac{\hbar^2 k_c^2}{2m_c}$$

$$\mu_c = \frac{M_c m_c}{M_c + m_c}$$ Reduced mass

$$\hbar k_c = \left[2\mu_c \left(E - M_c - m_c \right) \right]^{1/2}$$

2. Semi-relativistic

$$H_{SR} = \sum_{c=\bar{K}N,\pi\Sigma} \left[\sqrt{m_c^2 + \mathbf{p}^2} + \sqrt{M_c^2 + \mathbf{p}^2} \right] |c\rangle\langle c| + V_{KSW}$$

$$E = \Omega_c + \omega_c = \sqrt{M_c^2 + \hbar^2 k_c^2} + \sqrt{m_c^2 + \hbar^2 k_c^2}$$

$$\varepsilon_c = \frac{\Omega_c \omega_c}{\Omega_c + \omega_c}$$ Reduced energy

$$\hbar k_c = \left[\omega_c^2 - m_c^2 \right]^{1/2}$$

Non-rela. approximation of KSW potential

A. Original

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\; g_{ij}(r)$$

- *Normalized Gaussian*

$$g_{ij}(r) = \frac{1}{\pi^{3/2} a_{ij}^{\;3}} \exp\left[-(r/a_{ij})^2\right]$$

- *Range parameter for coupling potential*

$$a_{ij} = (a_{ii} + a_{jj})/2$$

B. Non-rela. approx. version 1

$$\omega_i \sim \mu_i$$

$$\Rightarrow\; V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8f_\pi^2}(\omega_i + \omega_j)\frac{1}{E_{Tot}}\sqrt{\frac{M_i M_j}{\mu_i\,\mu_j}}\; g_{ij}(r)$$

Comparison of the flux factor for differential cross section between non-rela. and rela.

C. Non-rela. approx. version 2

$m_i/\omega_i,\ M_i/\Omega_i \to 1$ *@ non-rela. limit (small p^2)*

$$\Rightarrow\; V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{1}{m_i\,m_j}}\; g_{ij}(r)$$

Kinematics

Potential

Semi-rela.

Non-rela.

A. Original

$$V_{ij}^{(I=0)}\left(r\right) = -\frac{C_{ij}^{(I=0)}}{8 f_{\pi}^{2}}\left(\omega_i + \omega_j\right)\sqrt{\frac{M_i M_j}{s\,\omega_i\,\omega_j}}\; g_{ij}\left(r\right)$$

B. Non-rela. approx. version 1

$$V_{ij}^{(I=0)}\left(r\right) = -\frac{C_{ij}^{(I=0)}}{8 f_{\pi}^{2}}\left(\omega_i + \omega_j\right)\frac{1}{E_{Tot}}\sqrt{\frac{M_i M_j}{\mu_i\;\mu_j}}\; g_{ij}\left(r\right)$$

C. Non-rela. approx. version 2

$$V_{ij}^{(I=0)}\left(r\right) = -\frac{C_{ij}^{(I=0)}}{8 f_{\pi}^{2}}\left(\omega_i + \omega_j\right)\sqrt{\frac{1}{m_i\; m_j}}\; g_{ij}\left(r\right)$$

4. Result

Using Chiral SU(3) potential (KSW)

… r-space, Gaussian form, Energy-dependent

- *I=0 $K^{bar}N$ scattering length and the range parameters of the KSW potential*
- *Scattering amplitude*
- *Resonance pole … wave function and size*

I=0 $K^{bar}N$ scattering length and the range parameters of KSW potential

$f_\pi = 90$ MeV

	Kinematics KSW potential	Original	Non-rela. Non-rela. v1	Non-rela. v2	Semi-rela. Original
Range parameter [fm]	a(KbarN)	0.593	0.576	0.574	0.487
	a(π Σ)	0.541	0.725	0.751	0.457
KbarN Scatt. leng. [fm]	Re	−1.701	−1.700	−1.700	−1.703
	Im	0.679	0.677	0.687	0.677

$$V_{ij}^{(I=0)}(r) = V_{ij}^{(I=0)}\left(\sqrt{s}\right) \frac{1}{\pi^{3/2} a_{ij}^{3}} \exp\left[-\left(r/a_{ij}\right)^2\right]$$

$$a_{K^{bar}N}^{I=0} = f_{K^{bar}N}^{I=0}\left(\sqrt{s}_{K^{bar}N\ thr.}\right)$$

Range parameters:

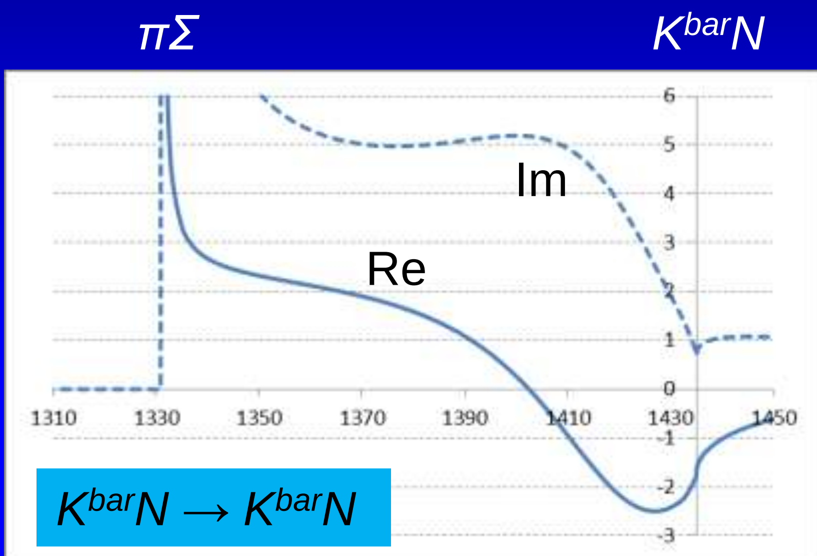
$$a_{ij} = \left\{ a_{K^{bar}N,K^{bar}N},\ a_{\pi\Sigma,\pi\Sigma},\ a_{K^{bar}N,\pi\Sigma} \equiv \left(a_{K^{bar}N,K^{bar}N} + a_{\pi\Sigma,\pi\Sigma}\right)/2 \right\}$$

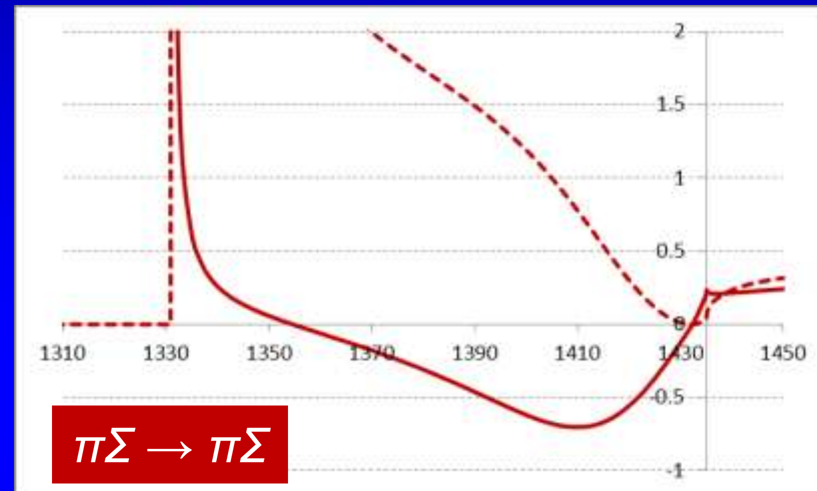
Exp. : $a^{I=0}_{KbarN}$ = -1.70 + i 0.68 fm (A. D. Martin)

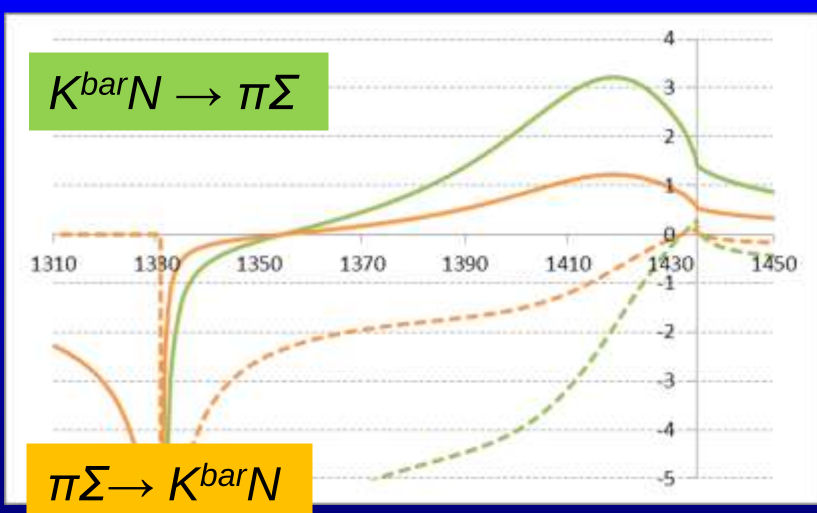
Scattering amplitude
… KSW org. – Non. rela.

$f_\pi = 90\ \mathrm{MeV}$

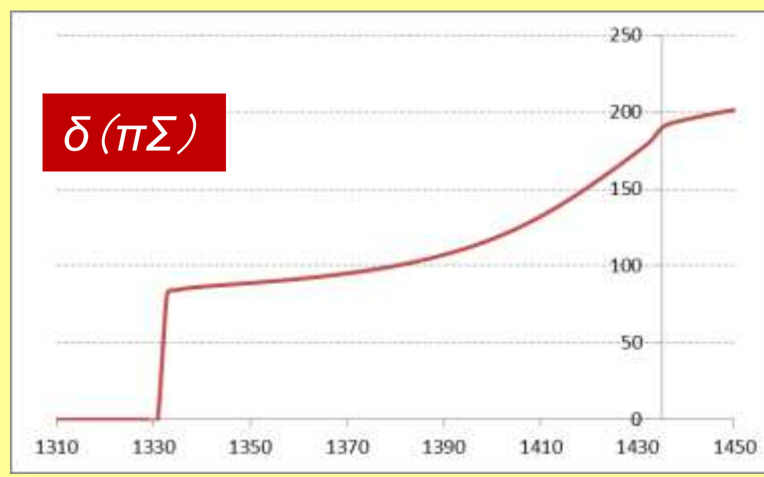
$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}\left(\omega_i + \omega_j\right)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\; g_{ij}(r)$$







- Phase shift (πΣ)



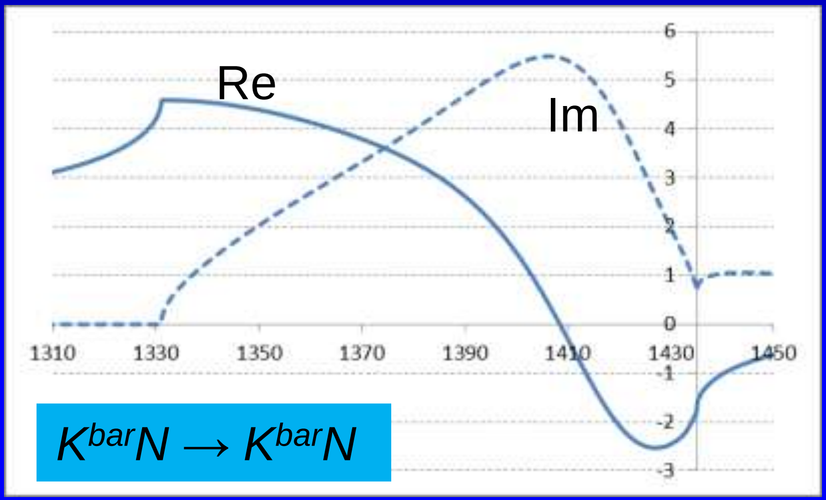
Scattering amplitude

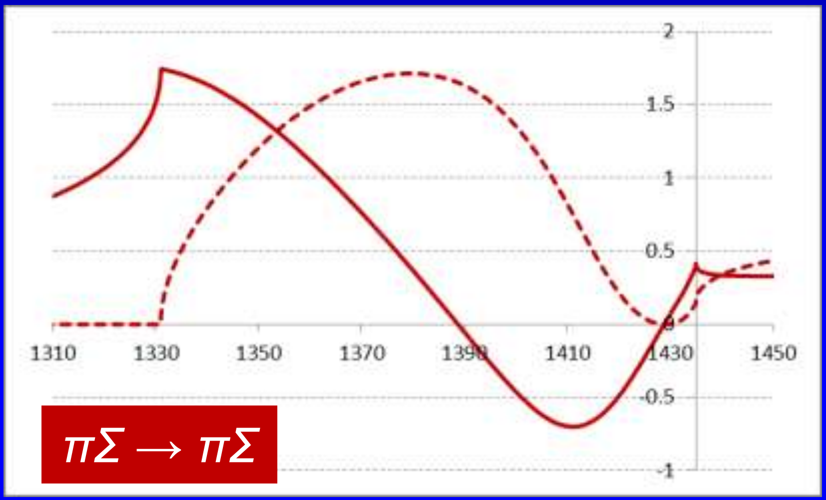
… KSW NRv1 – Non. rela.

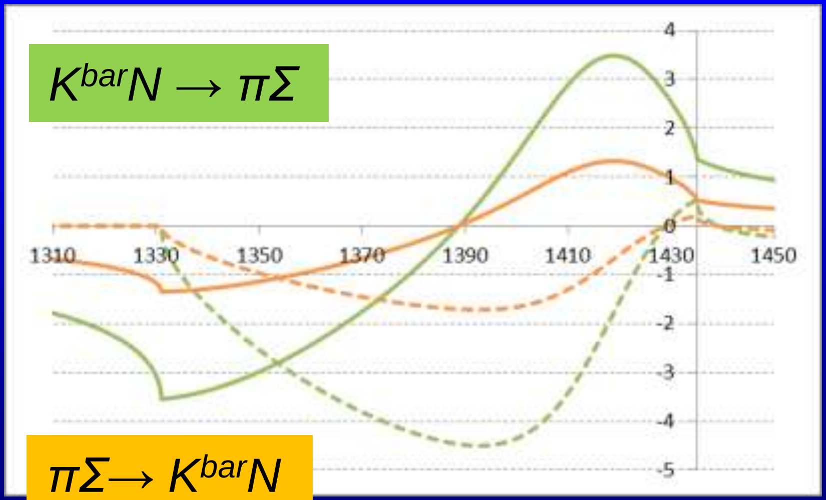
f_π=90 MeV

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\frac{1}{E_{Tot}}\sqrt{\frac{M_i M_j}{\mu_i \mu_j}}\, g_{ij}(r)$$

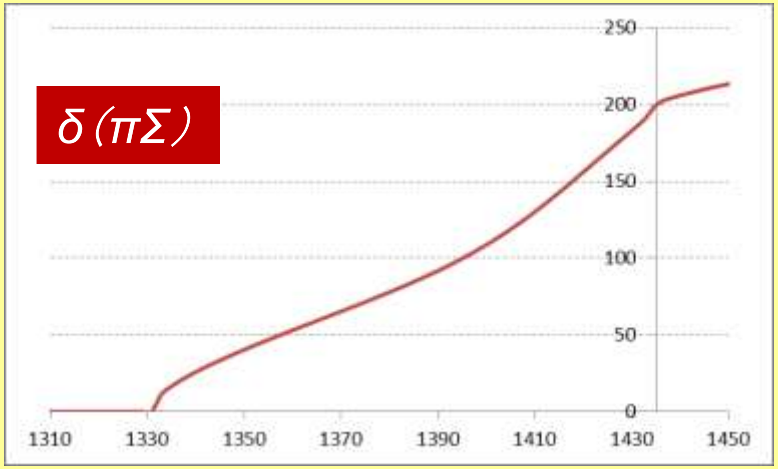
πΣ $K^{bar}N$







- Phase shift (πΣ)



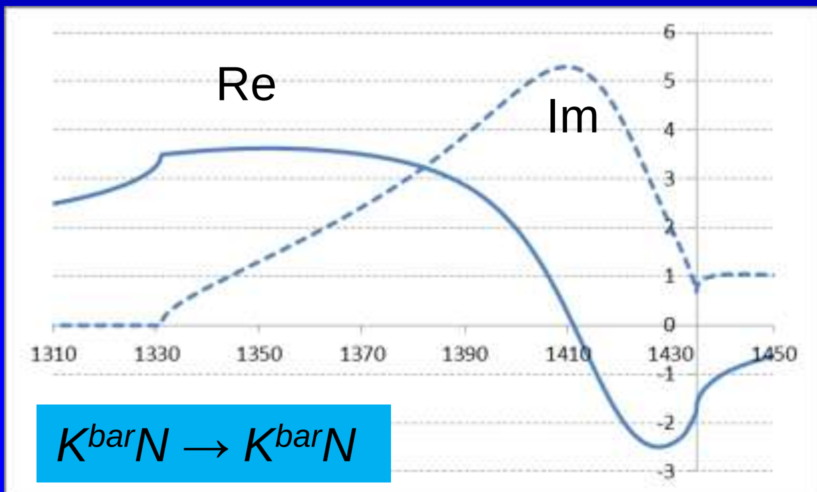
Scattering amplitude

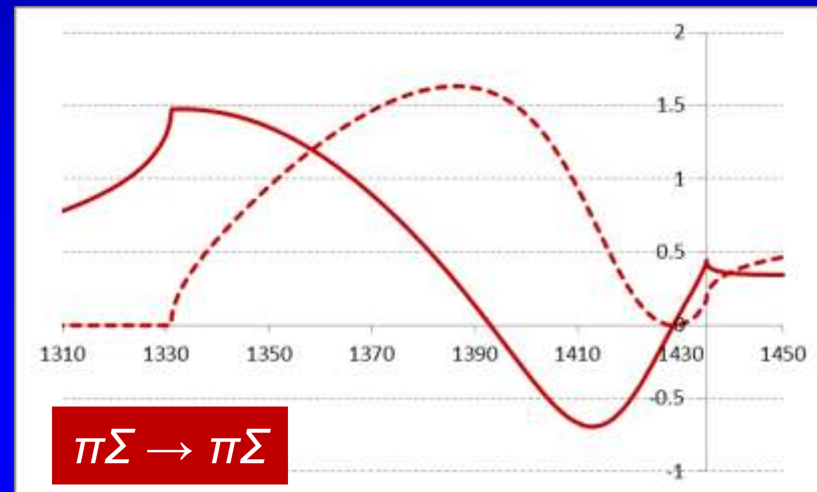
… KSW NRv2 – Non. rela.

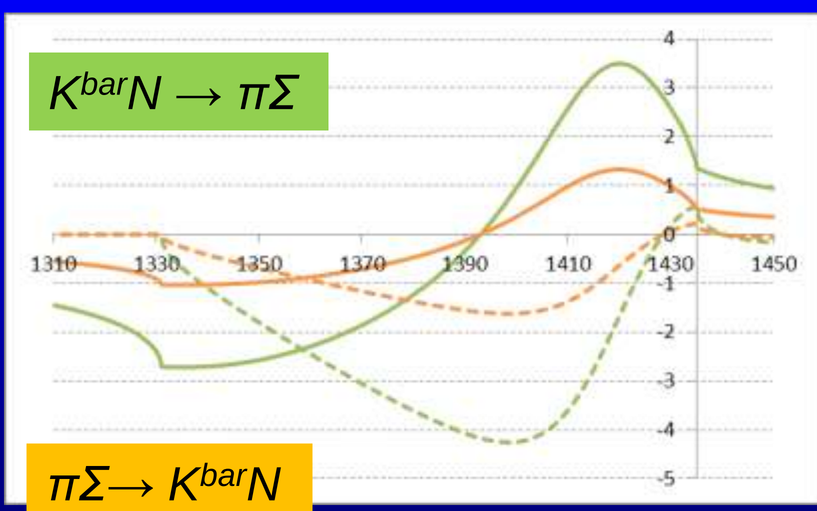
f_π=90 MeV

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{1}{m_i\, m_j}}\, g_{ij}(r)$$

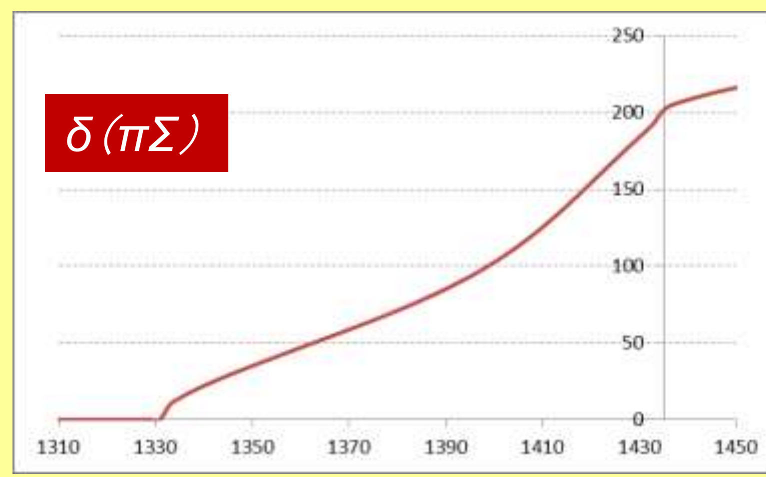
$\pi\Sigma$ $K^{bar}N$







- Phase shift (πΣ)

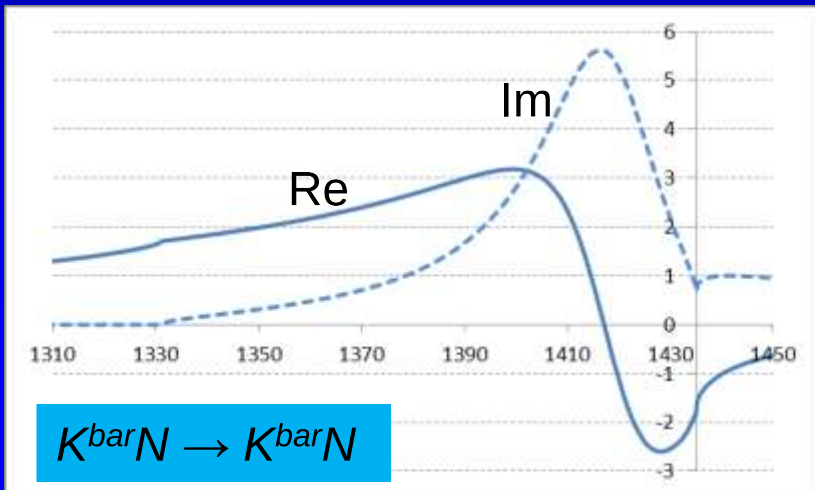


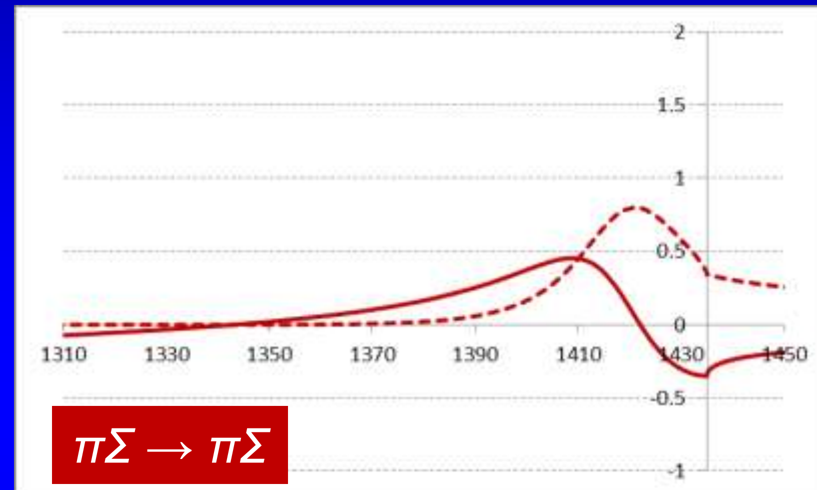
Scattering amplitude
… KSW org. – Semi rela.

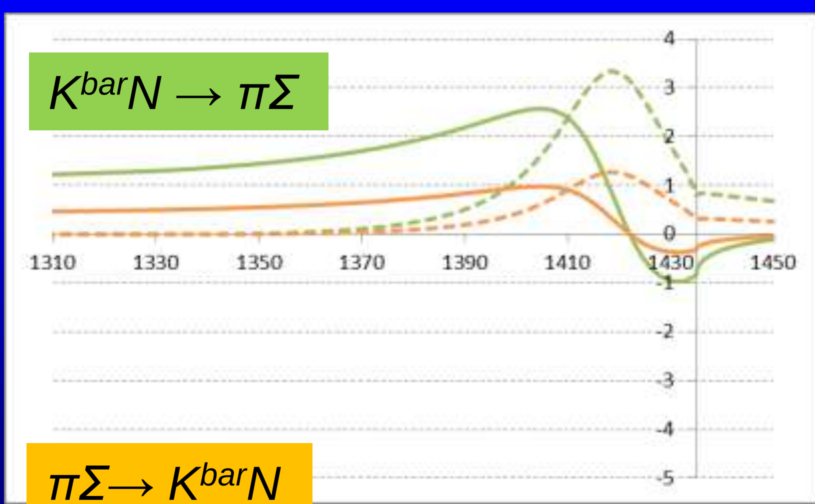
f_π=90 MeV

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\; g_{ij}(r)$$

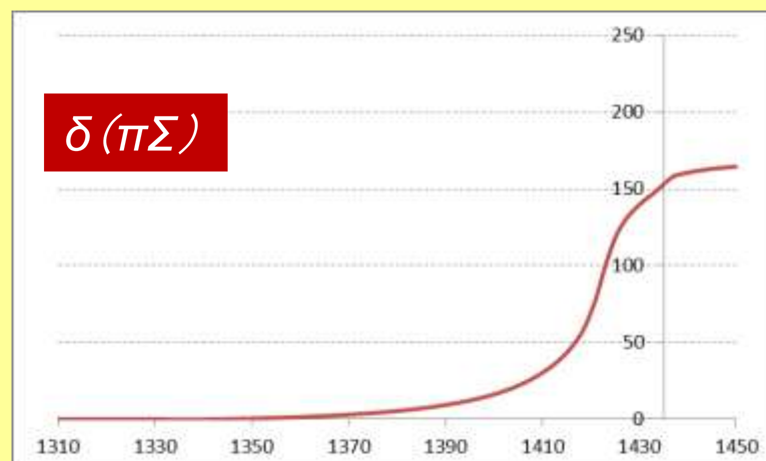
$\pi\Sigma$ $\quad\quad$ $K^{bar}N$







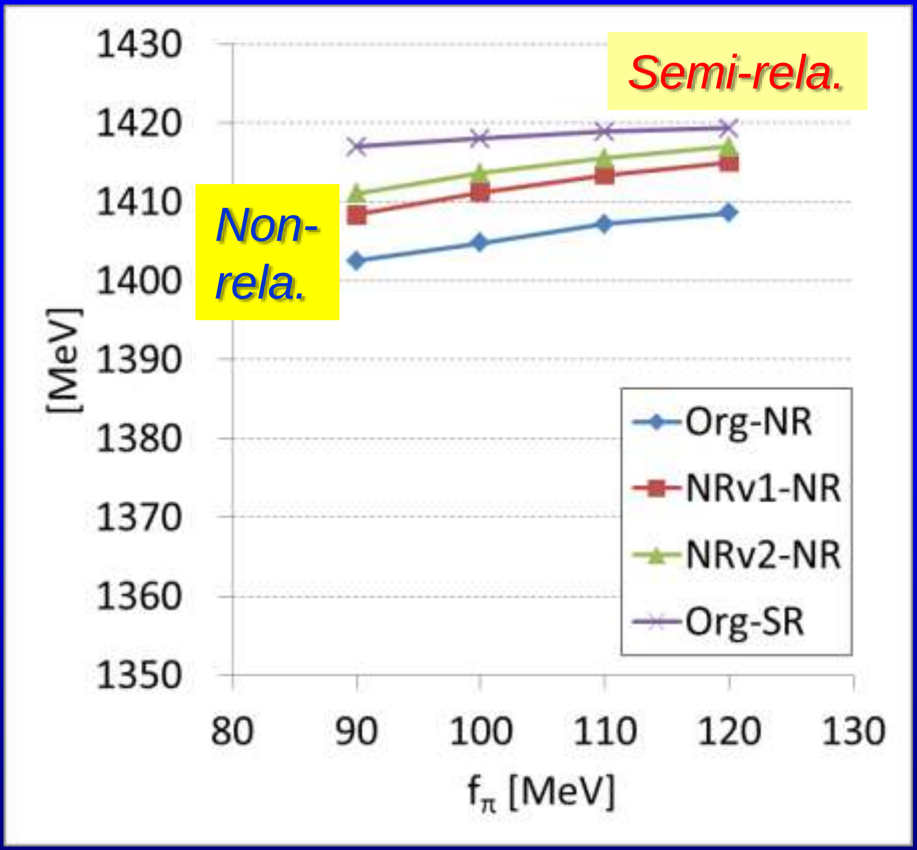
- Phase shift (πΣ)

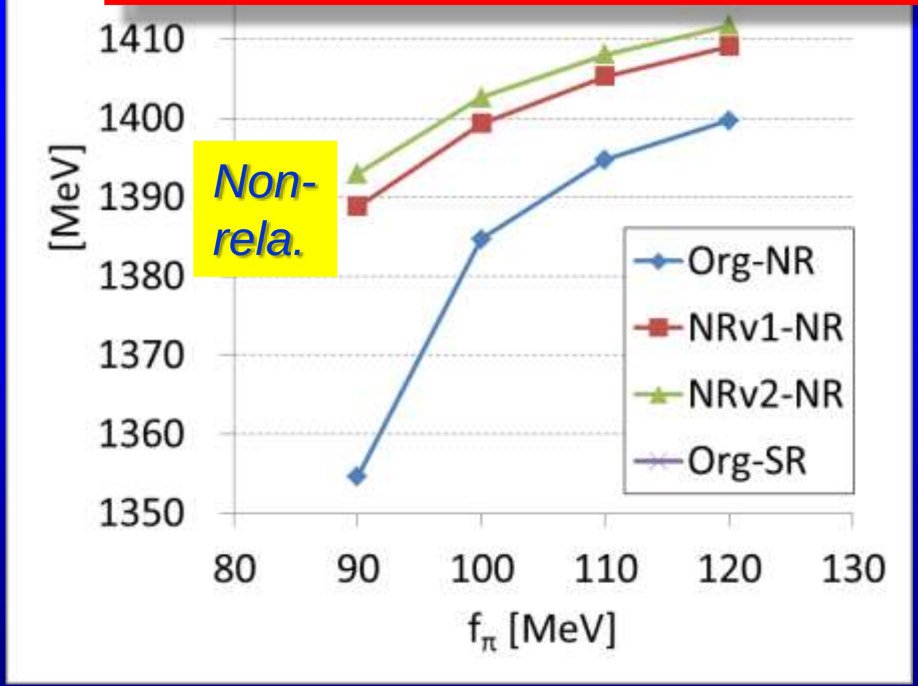


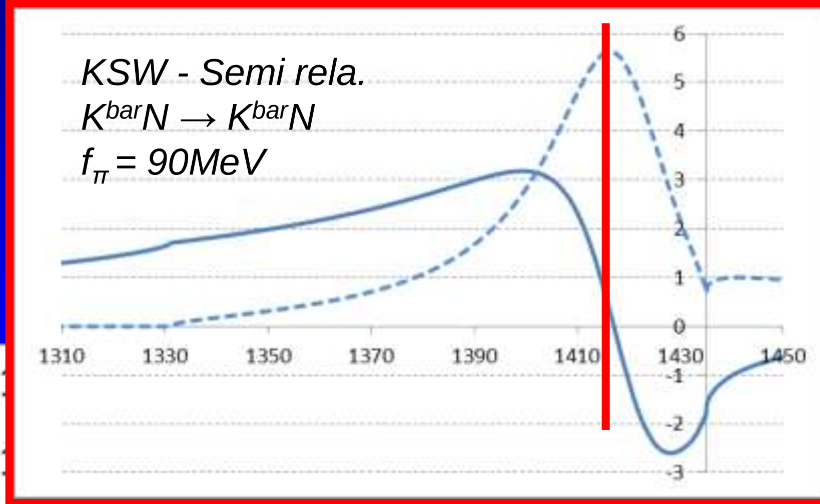
Position of resonant structure in the scattering amplitude

f_π=90 - 120MeV

Energy @ Re $f_{KbarN \rightarrow KbarN}$ = 0







Position of resonant structure in the scattering amplitude

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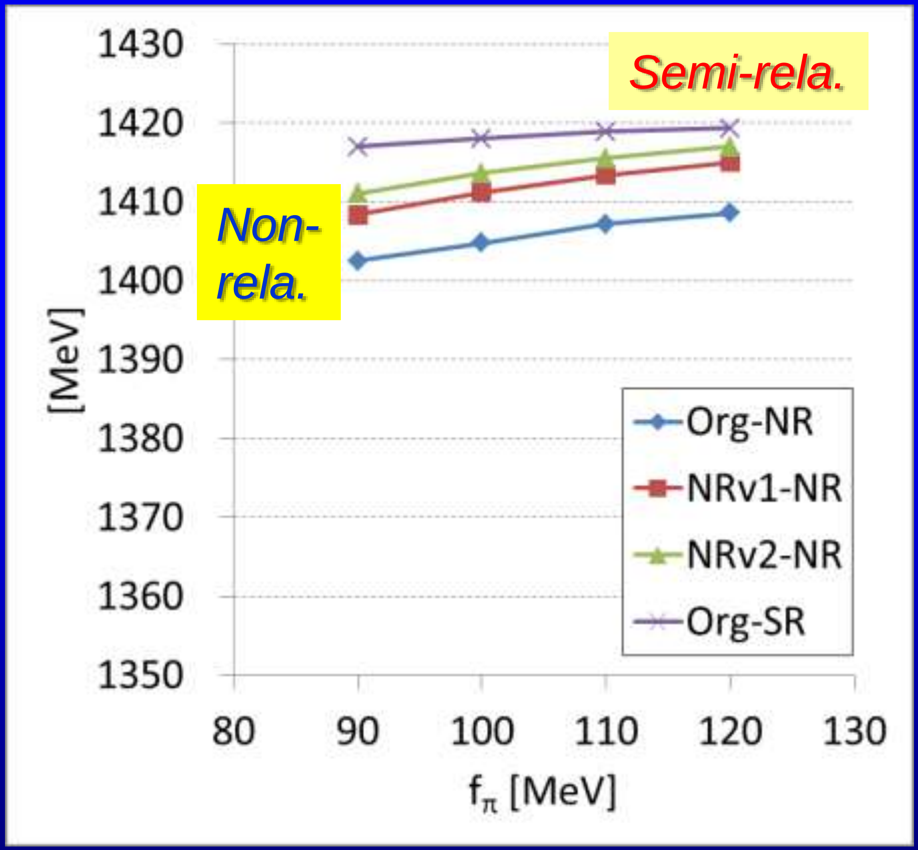
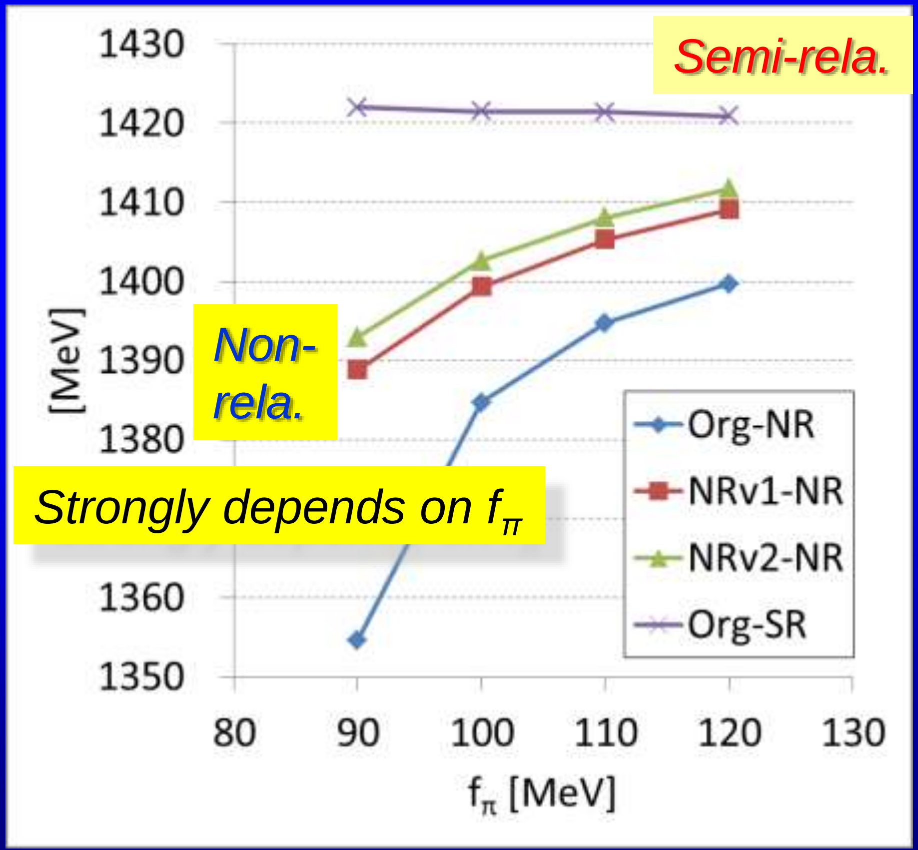
Energy @ Re $f_{KbarN \to KbarN}$ = 0



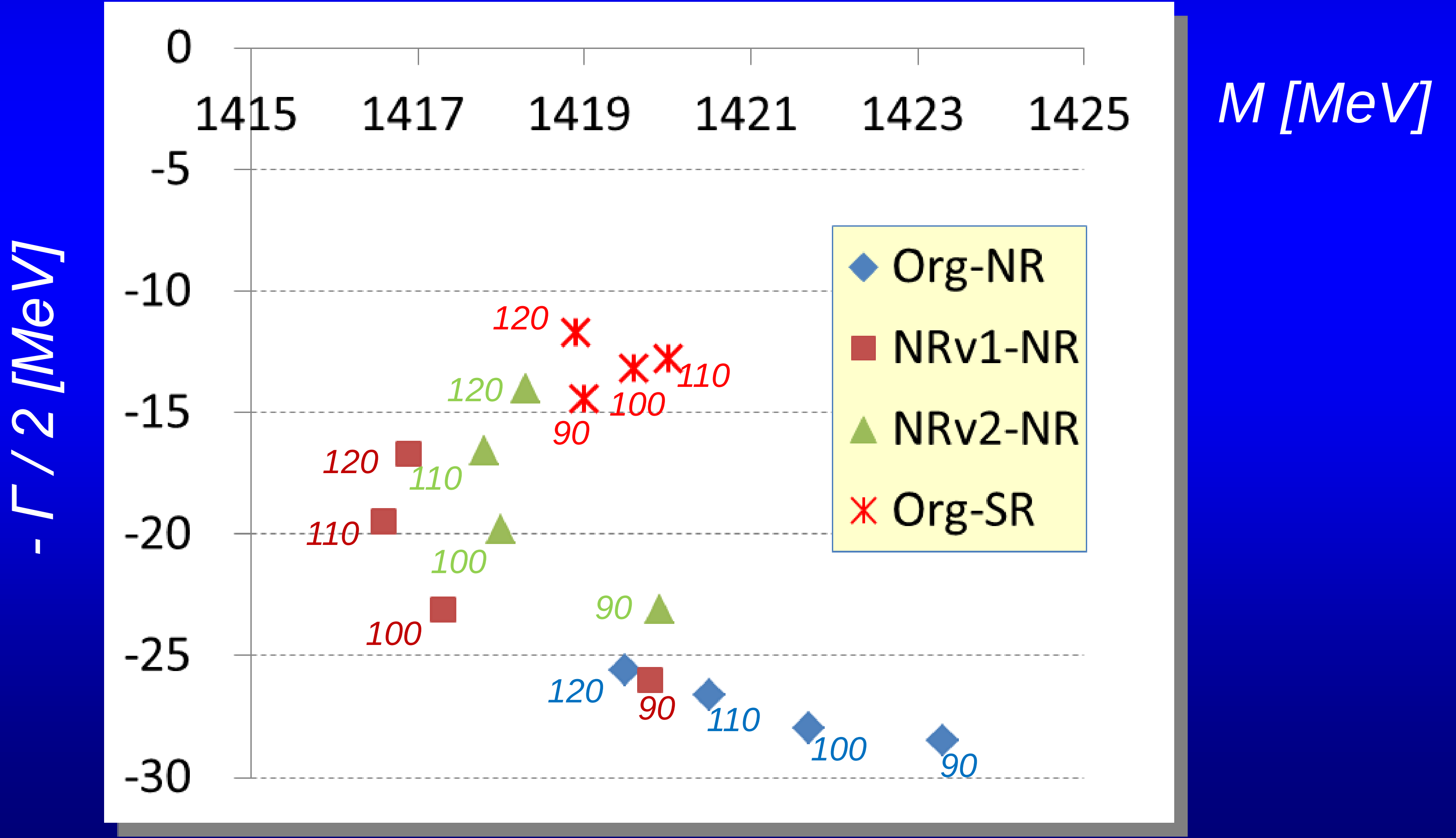
Energy @ Re $f_{\pi\Sigma \to \pi\Sigma}$ = 0



Pole position of the resonance

f_π=90 - 120MeV

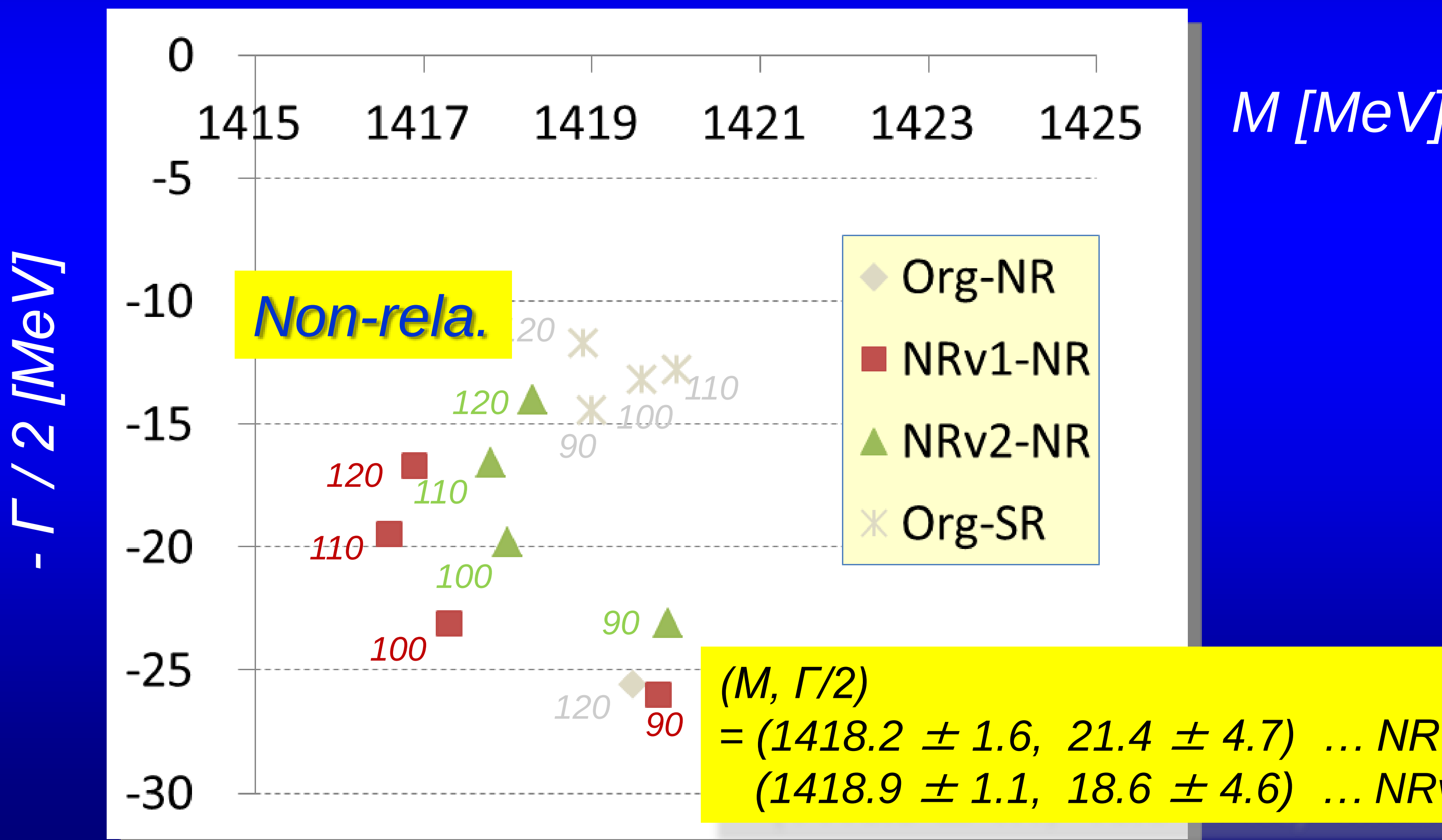
fpi	Org-NR		NRv1-NR		NRv2-NR		Org-SR	
	M	−Γ /2	M	−Γ /2	M	−Γ /2	M	−Γ /2
90	1423.3	−28.5	1419.8	−26.0	1419.9	−23.1	1419.0	−14.4
100	1421.7	−28.0	1417.3	−23.1	1418.0	−19.8	1419.6	−13.2
110	1420.5	−26.6	1416.6	−19.5	1417.8	−16.6	1420.0	−12.8
120	1419.5	−25.6	1416.9	−16.7	1418.3	−14.0	1418.9	−11.7



Pole position of the resonance

f_π=90 - 120MeV

fpi	Org−NR		NRv1−NR		NRv2−NR		Org−SR	
	M	−Γ /2	M	−Γ /2	M	−Γ /2	M	−Γ /2
90	1423.3	−28.5	1419.8	−26.0	1419.9	−23.1	1419.0	−14.4
100	1421.7	−28.0	1417.3	−23.1	1418.0	−19.8	1419.6	−13.2
110	1420.5	−26.6	1416.6	−19.5	1417.8	−16.6	1420.0	−12.8
120	1419.5	−25.6	1416.9	−16.7	1418.3	−14.0	1418.9	−11.7

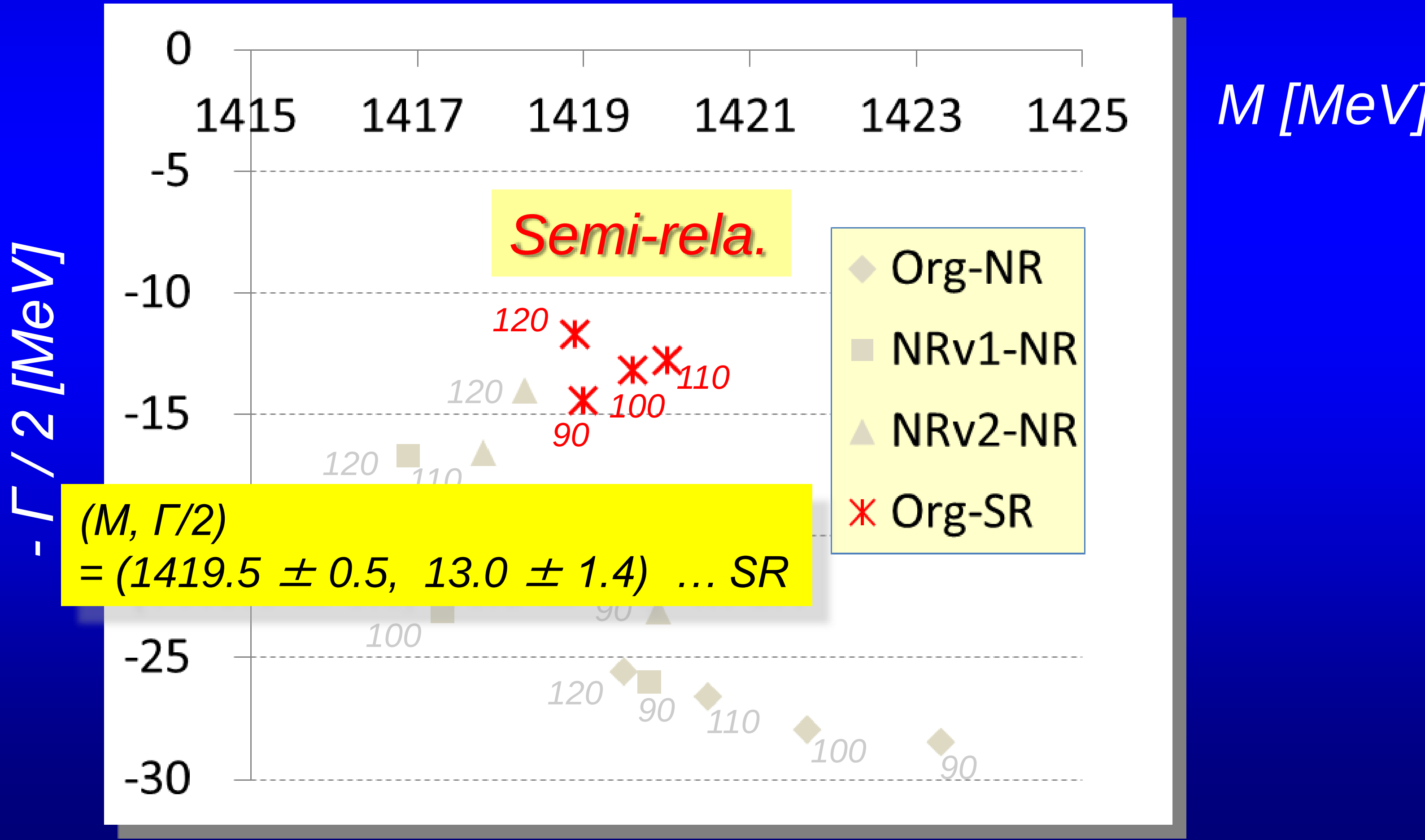


(M, Γ/2)
= (1418.2 ± 1.6, 21.4 ± 4.7) … NRv1
(1418.9 ± 1.1, 18.6 ± 4.6) … NRv2

Pole position of the resonance

f_π=90 - 120MeV

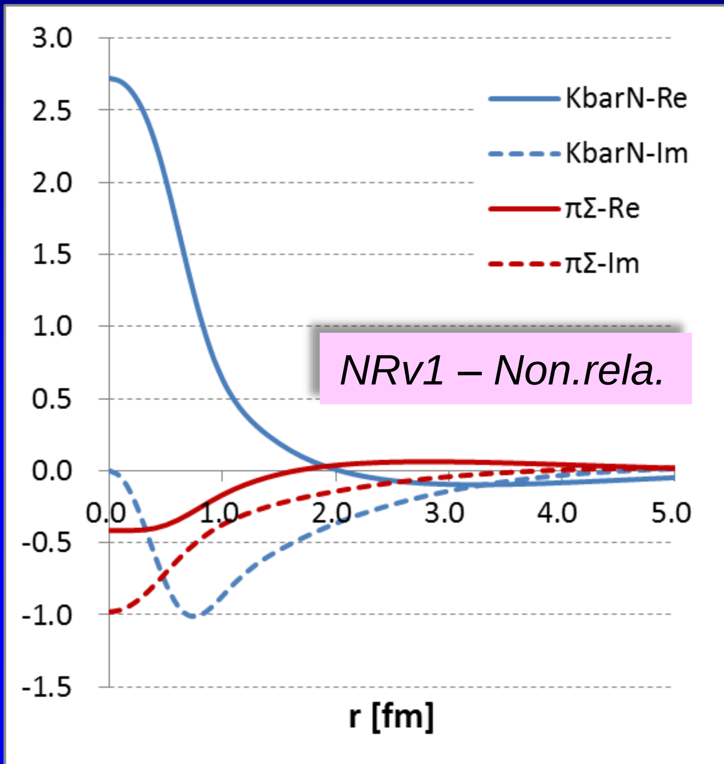
fpi	Org−NR		NRv1−NR		NRv2−NR		Org−SR	
	M	−Γ /2	M	−Γ /2	M	−Γ /2	M	−Γ /2
90	1423.3	−28.5	1419.8	−26.0	1419.9	−23.1	1419.0	−14.4
100	1421.7	−28.0	1417.3	−23.1	1418.0	−19.8	1419.6	−13.2
110	1420.5	−26.6	1416.6	−19.5	1417.8	−16.6	1420.0	−12.8
120	1419.5	−25.6	1416.9	−16.7	1418.3	−14.0	1418.9	−11.7

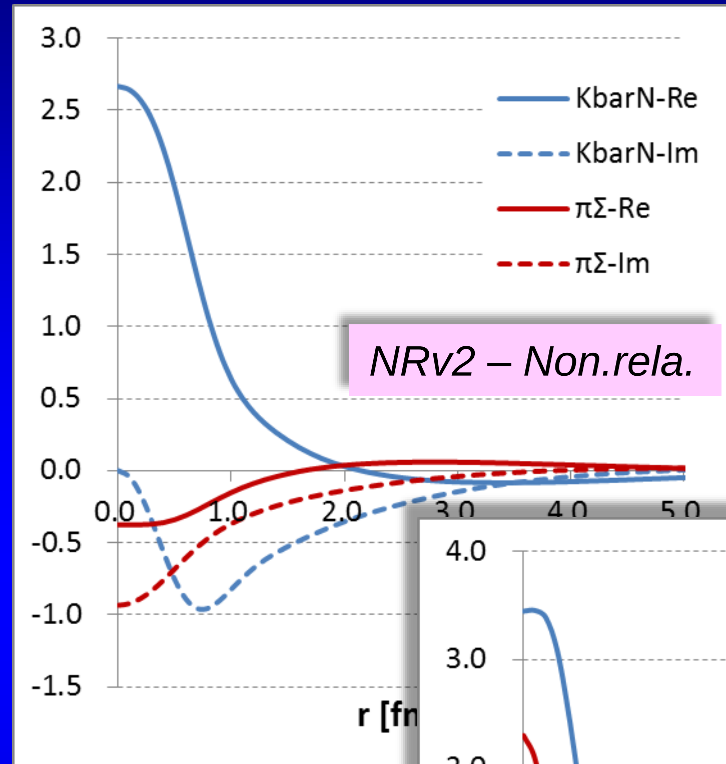


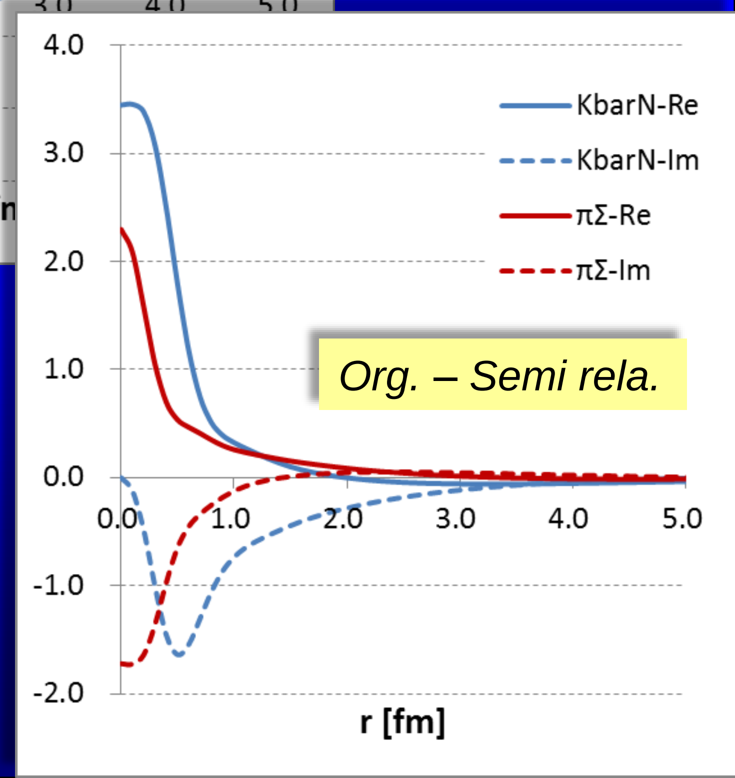
(M, Γ/2)
= (1419.5 ± 0.5, 13.0 ± 1.4) … SR

"Wave function" of the resonance pole

f_{π}=90 MeV
θ=30°







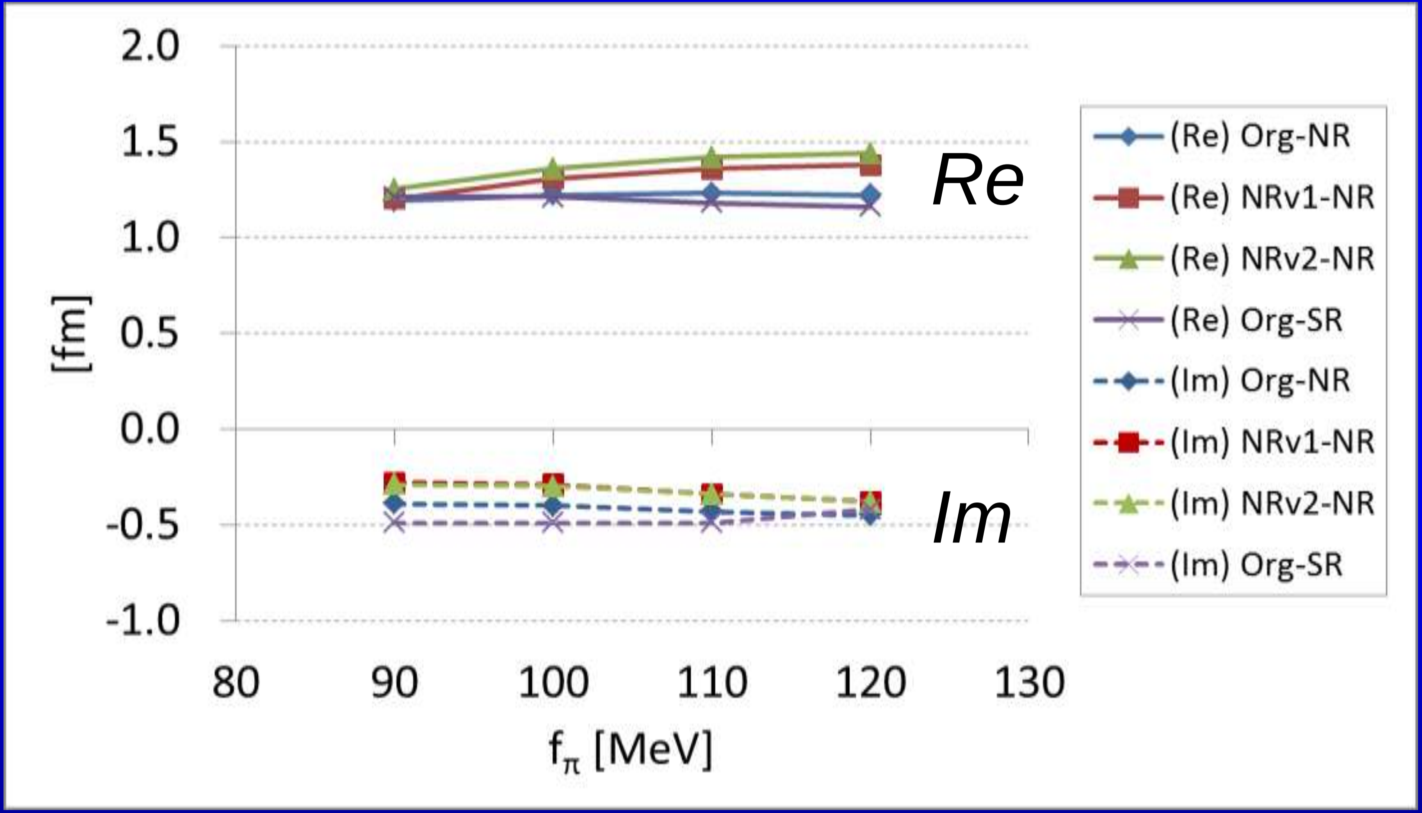
πΣ component also localized due to Complex scaling.

“Size” of the resonance pole state

f_π=90 - 120MeV
θ=30°

Mean distance between meson and baryon

$$\sqrt{\left\langle \hat{r}_{MB}^{2} \right\rangle_\theta}\ [fm]$$



NR: ～1.3 - 0.3 i [fm]
SR: ～1.2 - 0.5 i [fm]

5. Summary
and
Future plan

5. Summary

Scattering and resonant states of I=0 $K^{bar}N$-πΣ system is studied with a coupled-channel Complex Scaling Method using a chiral SU(3) potential

- Coupled Channel problem = $K^{bar}N + \pi\Sigma$
- Solved with *Gaussian base*
- A Chiral SU(3) potential (KSW)
  … r-space, Gaussian form, energy dependence

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}\left(\omega_i + \omega_j\right)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\; g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2} a_{ij}^3} \exp\left[-\left(r/a_{ij}\right)^2\right]$$

- Calculated scattering amplitude with help of CSM

Non-rela. / Semi-rela. kinematics and two types of non-rela. approximation of KSW potential are tried.

- The KSW potential can be determined by the experimental value of $I=0$ $K^{bar}N$ scattering length.
  … use independent range parameters for $K^{bar}N$ and πΣ channels

- Mismatch between kinematics and potential causes singularity near the πΣ threshold.
  - Original KSW potential + (Semi) Rela. kinematics
  - Non-rela. reduced KSW potential + Non-rela. kinematics

5. Summary and future plans

Non-rela. / Semi-rela. kinematics and two types of non-rela. approximation of KSW potential are tried.

(cont'd)

- f_π dependence ($f_\pi = 90 \sim 120$MeV) … Small for Semi-Rela. case
- Constrained by I=0 $K^{bar}N$ scattering length …

• Pole position

(M, Γ/2)

NRv1 :	*(1418.2 ± 1.6, 21.4 ± 4.7)*
NRv2 :	*(1418.9 ± 1.1, 18.6 ± 4.6)*
SR :	*(1419.5 ± 0.5, 13.0 ± 1.4)*

? ccCSM finds the upper pole of
the two poles of Λ(1405),
if the present potential has the double pole.

• "Size" of the I=0 pole state

NR: ∼ 1.3 – 0.3i fm
SR: ∼ 1.2 – 0.5i fm

? Somehow small.
cf) 1.9 fm @ M = 1423 MeV†
(B = 12 MeV)

- Difference of binding energy?
- Different definition of pole?
Gamow state or bound state

† A. D., T. Hyodo, W. Weise,
PRC79, 014003 (2009)

Future plans

- *I=1 $K^{bar}N$* potential
- Three-body system ($K^{bar}NN$-πYN)