



# 格子QCDによるヘビークォークを含む ハドロン間相互作用の研究

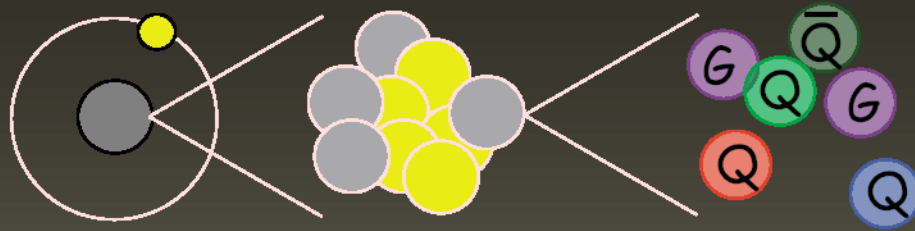
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In collaboration with M. Oka (TITech), G. Erkol, U. Can (Turkey)

# 1. Introduction



# QCD as the fundamental theory of strong interactions



Nuclear-Hadron system

Quark-Gluon system

Governed by **QCD**

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}(i\gamma^\mu D_\mu - m_f)\psi$$

- Color Confinement
- Chiral Symmetry Breaking

**Quark**

→ fermion  
Fundamental rep.

**Gluon**

→ gauge boson  
Adjoint rep.

**Global symmetry in 3-flavor QCD**

By spontaneous symmetry breaking

Massless NG bosons appear  
(Responsible for interactions)

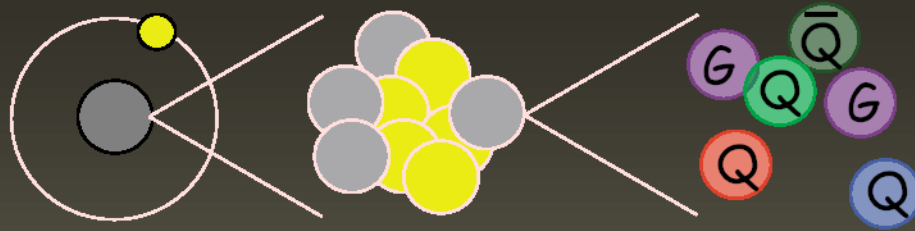
$$SU(3)_L \times SU(3)_R$$

$$SU(3)_V$$

$$\frac{SU(3)_L \times SU(3)_R}{SU(3)_V}$$



# QCD as the fundamental theory of strong interactions



Nuclear-Hadron system

Quark-Gluon system

Governed by **QCD**

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}(i\gamma^\mu D_\mu - m_f)\psi$$

- Color Confinement
- Chiral Symmetry Breaking

**Quark**

→ fermion  
Fundamental rep.

**Gluon**

→ gauge boson  
Adjoint rep.

Simple Lagrangian  
but  
Very difficult dynamics

Global symmetry in 3-flavor QCD

By spontaneous symmetry breaking

Massless NG bosons appear  
(Responsible for interactions)

$$\frac{SU(3)_L \times SU(3)_R}{SU(3)_V}$$



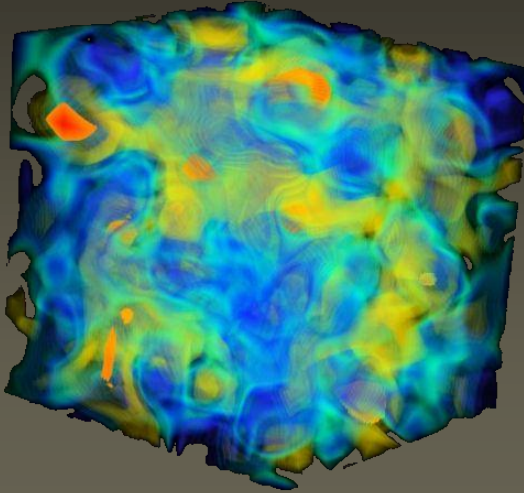


## 2. Lattice QCD

- Reliable nonperturbative method -

# Lattice QCD as one possible solution for QCD

## Continuum QCD

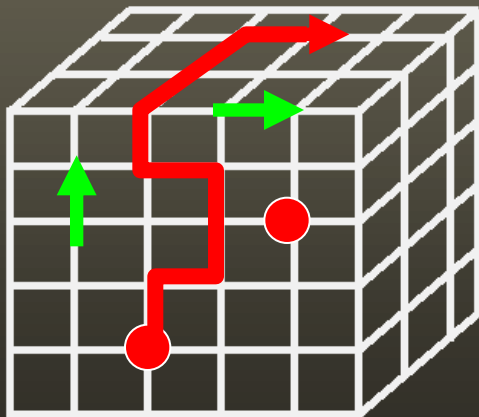


Gluon field :  $A_\mu(x)$

Quark field :  $q(x)$

Field strength :  $F_{\mu\nu}(x)$

## Lattice QCD



Continuum  
limit

Discretization

↑ Gluon field :  $U_\mu(n) \rightarrow$  lives on links

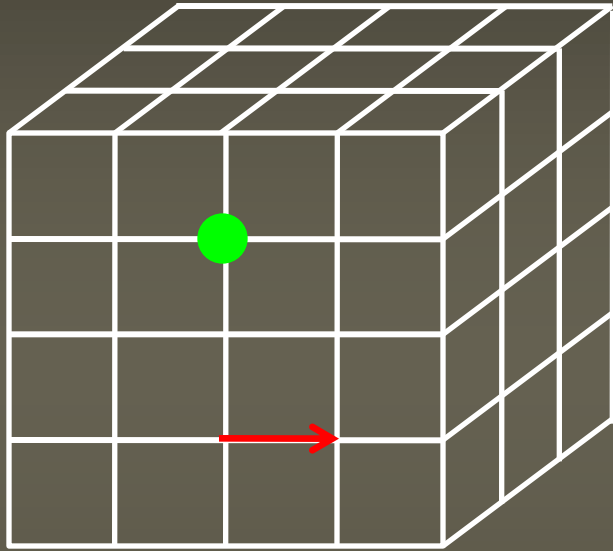
● Quark field :  $q(n) \rightarrow$  lives on sites

↻ Field strength : Plaquette (loop)

# Lattice QCD as one possible solution for QCD

## Compact formalism of QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{tr}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}(i\gamma^\mu D_\mu - m_f)\psi$$



Lattice + Euclid space

● Quark field  $\psi(s)$

→ Gauge field  $U_\mu(s) = e^{ig \int A_\mu dx_\mu}$   
 $U_\mu(s) \in \text{SU}(3)$

Nonperturbative evaluation  
of PATH-INTEGRAL



computers



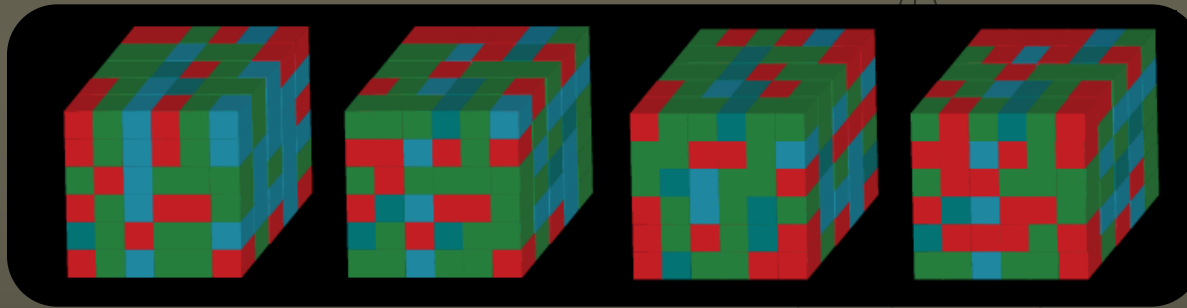
by hand

# Lattice QCD as one possible solution for QCD

Path integral  $\mathcal{Z} = \int \mathcal{D}A \mathcal{D}\psi e^{-S_{\text{QCD}}[A,\psi]}$

All about QCD

path-integral = weighted sum  
over all the possible quantum states



$$e^{-S_{\text{QCD}}[A,\psi]}$$

Summing up all the possible gauge/quark configurations  
with the **statistical weight**.

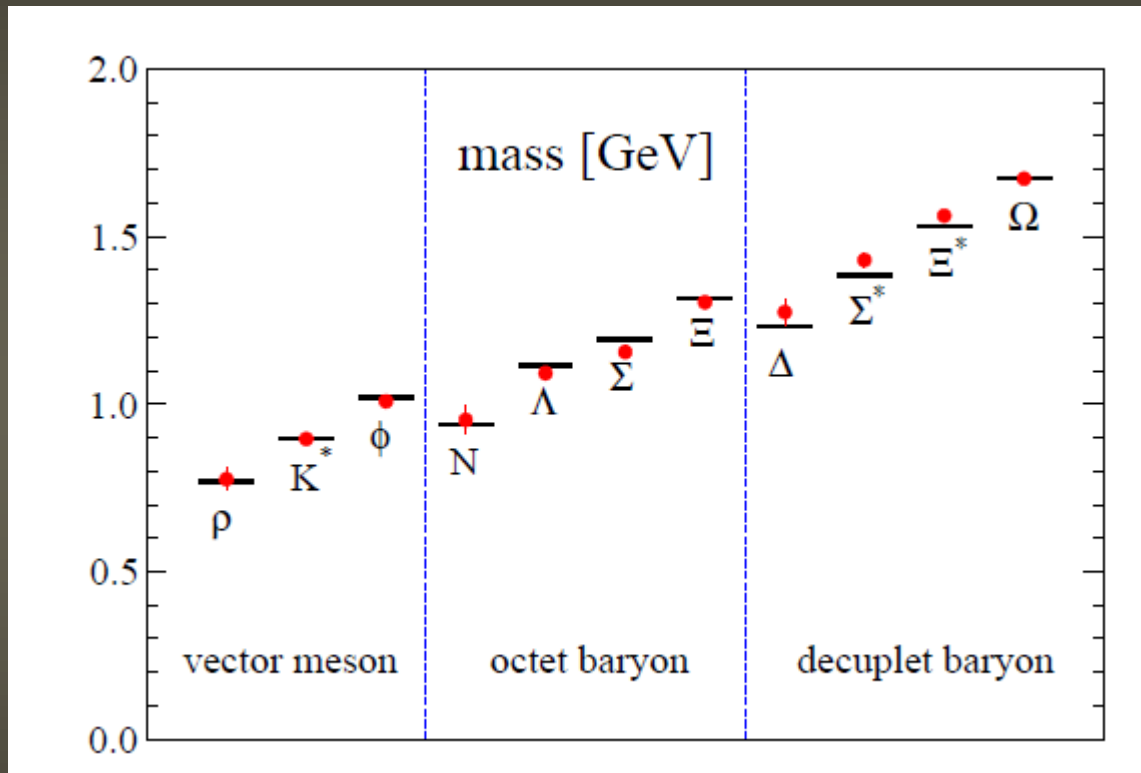
→ 離散による近似はあるが、QCD ダイナミクスを非摂動的に評価できる！



# Hadron spectrum from lattice QCD

*Now, physical point has been achieved*

NUMERICAL EXPERIMENTS !!



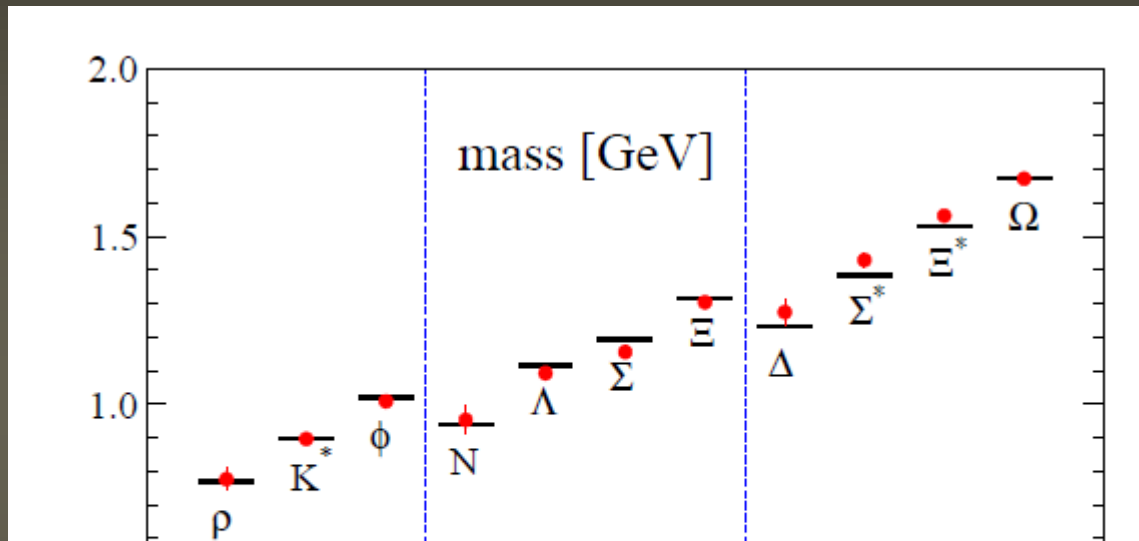
PACS-CS collaboration, arXiv:0807.1661

↑ Almost unique 1<sup>st</sup> principle calculations  
which can be compared with experiments

# Hadron spectrum from lattice QCD

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NUMERICAL EXPERIMENTS !!



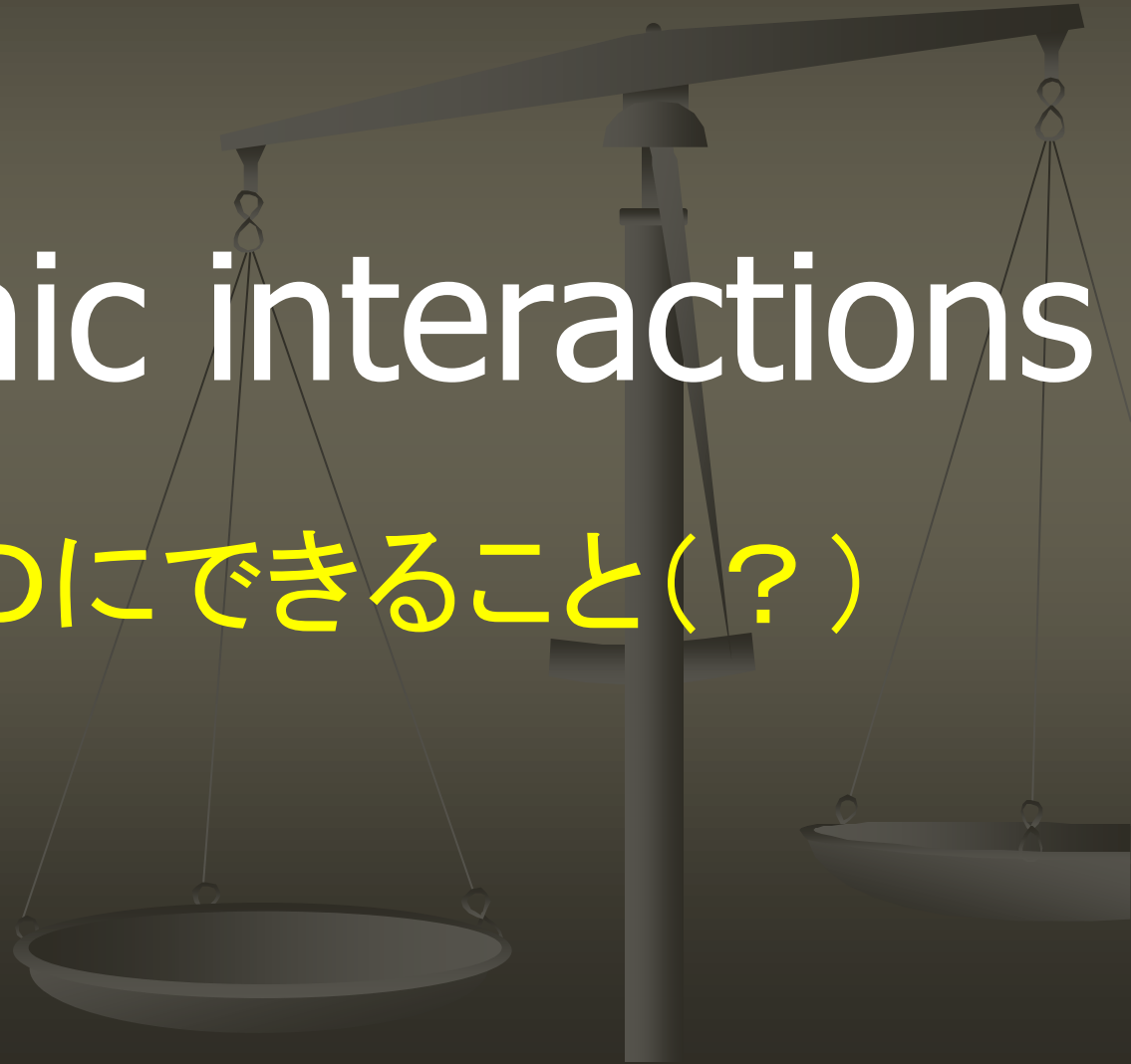
格子QCDは「数値実験」としても有用！

PACS-CS collaboration, arXiv:0807.1661

↑ Almost unique 1<sup>st</sup> principle calculations  
which can be compared with experiments

# 3. Hadronic interactions

格子QCDにできること(?)



## HADRONS

baryon

Baryons : nucleon, hyperon, ....  
**building blocks** of our world

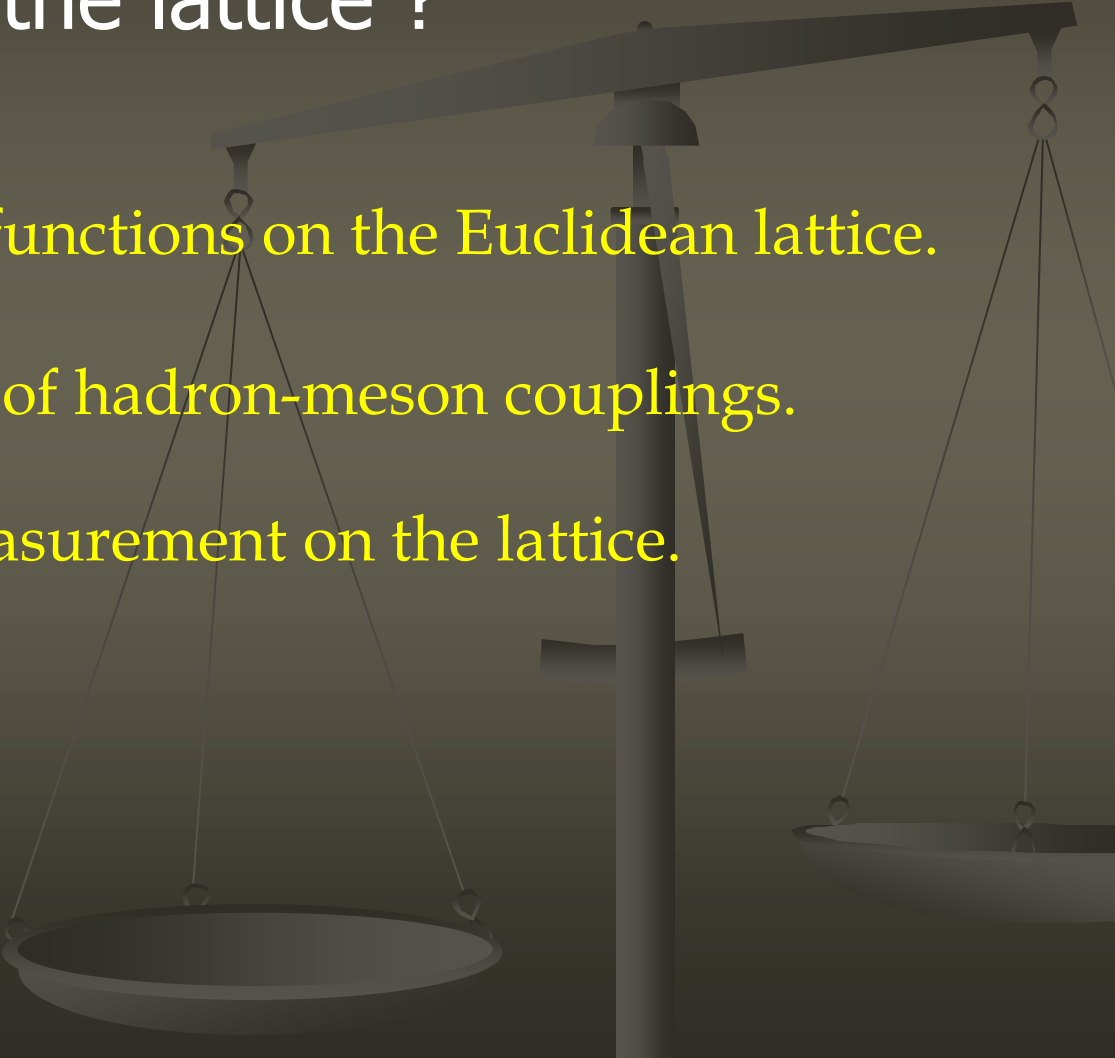
meson

Mesons : pion, kaon, ....  
exchanged between hadrons  
**generate hadronic interactions**  
**via One-Boson-Exchange-Potential**



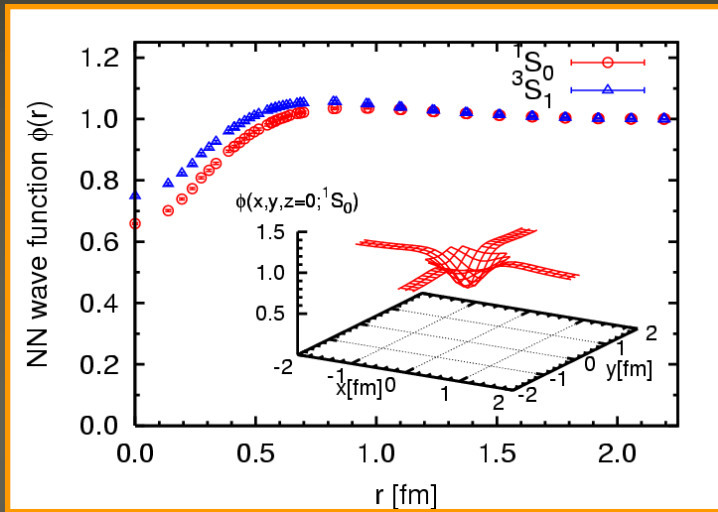
**Hadronic interactions are keys to understand our world**

## How to evaluate **hadronic interactions** on the lattice ?

- 
- 1. From BS wavefunctions on the Euclidean lattice.
  - 2. Determination of hadron-meson couplings.
  - 3. Phase-shift measurement on the lattice.

# Lattice QCD evaluation of hadronic interactions

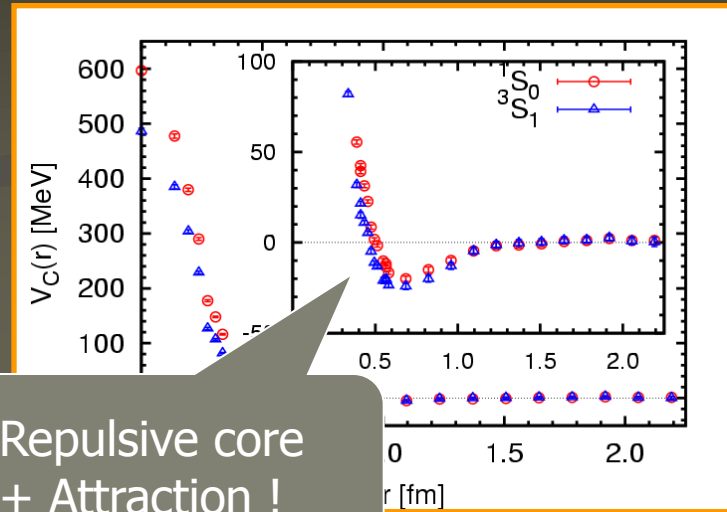
## 1. From BS wavefunctions on the Euclidean lattice.



Ishii, Aoki, Hatsuda, PRL99,022001('07).

Wave functions

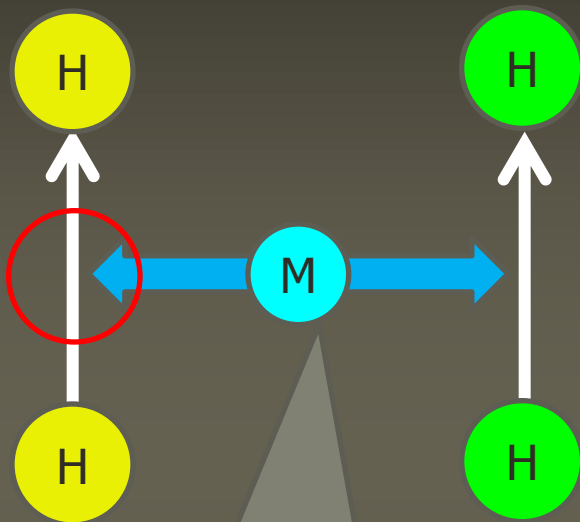
- useful input for model calculations
- we can draw nice figures



Repulsive core  
+ Attraction !

Hadronic potentials

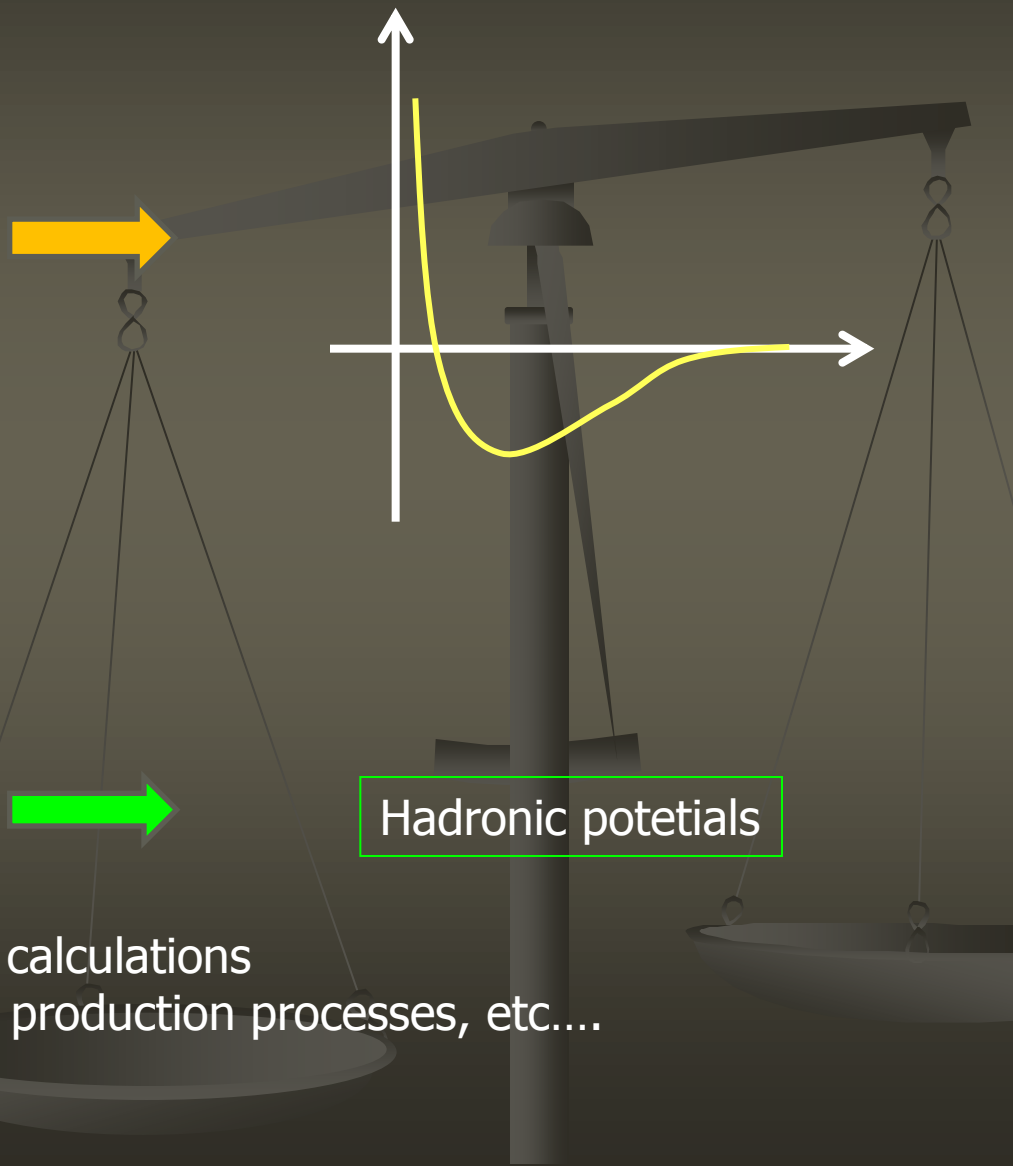
## 2. Determination of hadron-meson couplings.



Meson exchange  
→ Hadronic potential

Coupling constants

- useful input for model calculations
- information for meson production processes, etc....



Hadronic potentials

# OUR PROJECT

## Systematic study of Hadron-meson couplings with lattice QCD



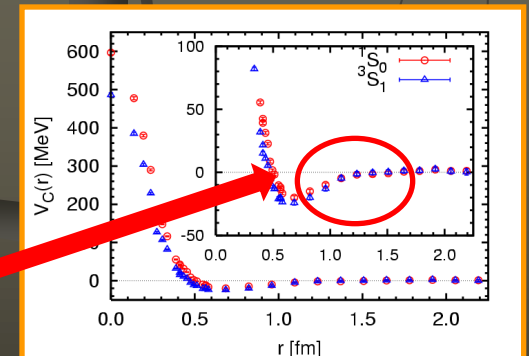
NG boson exchange

Building blocks of our world

- YUKAWA potential
- Responsible for hadronic matters
- Important

NG boson exchange

Leads to Yukawa potential  
But couplings constants are (in principle) undetermined  
← 1<sup>st</sup> principle calculations can do it  
Lattice QCD, QCD sum rules....





## 4. Measurement using Lattice QCD



# How to compute form factors on the lattice ?

On the lattice, we compute VEVs of operators.

$q_i$  : quark operator       $\Gamma$  : gamma matrix

Examples of hadronic operators

$B(x) = q_1 (q_2 \Gamma q_3)$       Baryons  $\rightarrow$  3 quark states

$M(x) = \bar{q}_1 \Gamma q_2$       Mesons  $\rightarrow$  2 quark states

$J(x) = \bar{q}_1 \Gamma q_2$       Currents

# How to compute form factors on the lattice ?

In lattice QCD, vacuum expectation values can be computed.

$$\langle B(p') \bar{\psi} i \gamma_5 \psi \bar{B}(p) \rangle$$

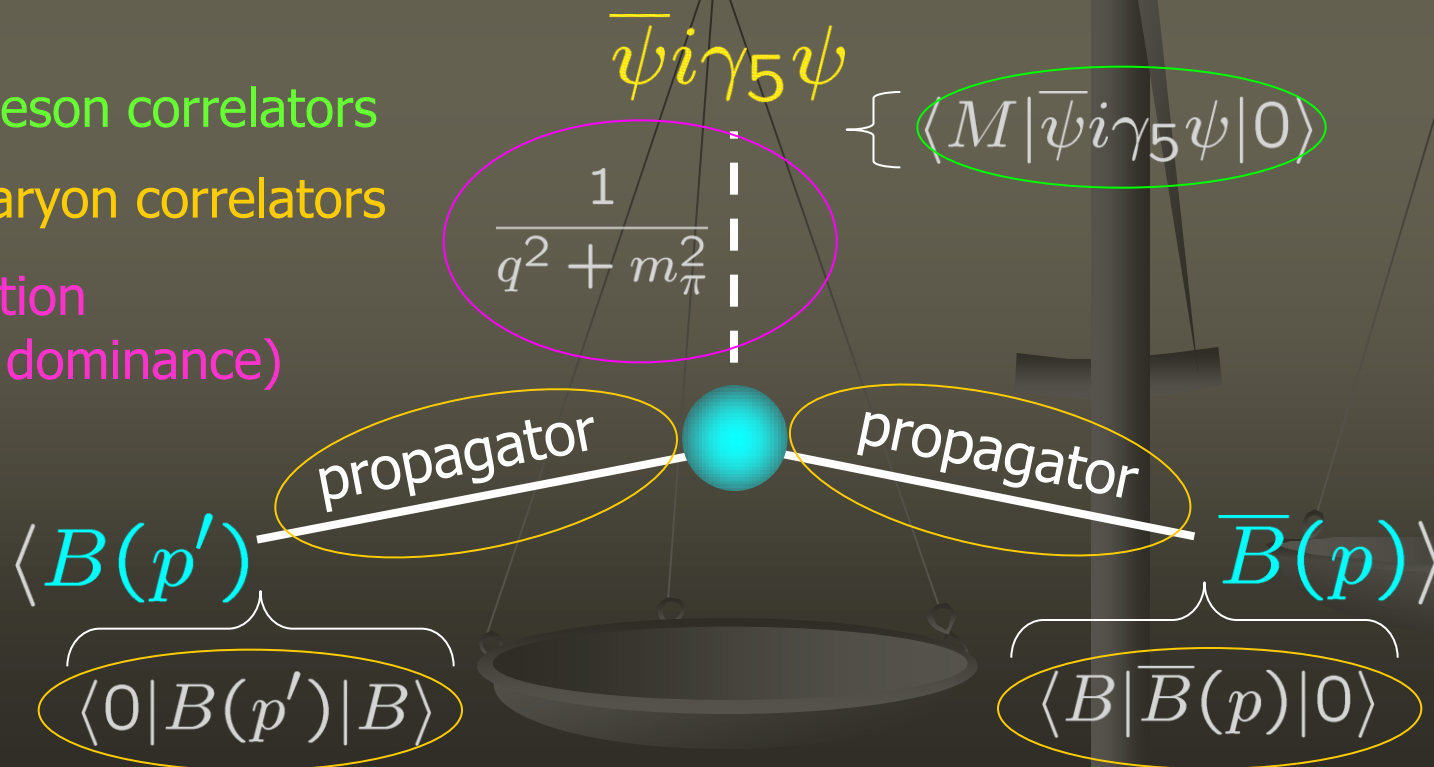
Baryon interpolation fields

Pseudo-scalar density

We compute 3-point functions, which can be expressed as follows.

What we want is

- From meson correlators
- From Baryon correlators
- Assumption (meson dominance)



# How to compute form factors on the lattice ?

$$C_{\mu\nu}^{(3)bc}(t_x, \vec{q}; t_y, \vec{p}) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{q} \cdot \vec{x}} e^{-i\vec{p} \cdot \vec{y}} \langle P^c(y) A^\nu(x) V_\mu^b(0) \rangle$$

← 3-point function on the lattice

$$C_{PP}^{(2)bb}(t_y; \vec{p}) = \sum_{\vec{y}} e^{-i\vec{p} \cdot \vec{y}} \langle P^b(y) P^b(0) \rangle$$

← 2-point function (PS)

$$C_{V_\mu V_\nu}^{(2)cc}(t_y; \vec{p}) = \sum_{\vec{y}} e^{-i\vec{p} \cdot \vec{y}} \langle V_\mu^c(y) V_\nu^c(0) \rangle$$

← 2-point function (VT)

$$C_{PP}^{(2)}(t; \vec{p}) \simeq Z_P \frac{e^{-E_P t}}{2E_P}, \quad C_{V_\mu V_\nu}^{(2)}(t; \vec{p}) \simeq Z_V \frac{e^{-E_V t}}{2E_V} (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})$$

2- and 3- point functions contains field renormalization (unwanted)



$$R_1(t) = \frac{C_{ii}^{(3)SW}(t) \sqrt{Z_V} \sqrt{Z_P}}{C_{V_i V_i}^{(2)SS}(t) C_{PP}^{(2)WW}(t_y - t)} \sqrt{V}$$

In their ratios, we can eliminate such factors....

$$\begin{aligned} \langle P(p') | A^\mu | V(p, \lambda) \rangle &= (m_P + m_V) F_1(q^2) \epsilon^{\lambda\mu} \\ &+ (2p'^\mu + q^\mu) F_2(q^2) \frac{\epsilon^\lambda \cdot q}{m_P + m_V} \\ &+ \frac{\epsilon^\lambda \cdot q}{q^2} q^\mu [2m_V F_0(q^2) - (m_P + m_V) F_1(q^2) - (m_V - m_P) F_2(q^2)] \end{aligned}$$

Matrix  
Element !  
(form factors)



# 5. Our previous study

-- Octet Meson-Baryon couplings --

# Our previous study

Pseudoscalar-meson--octet-baryon coupling constants in two-flavor lattice QCD

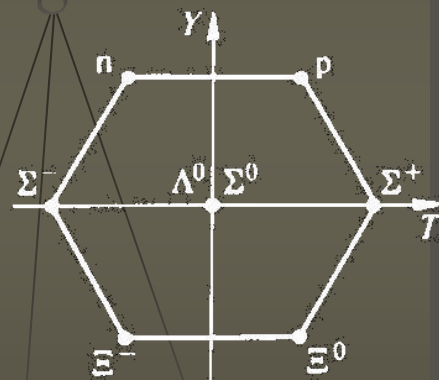
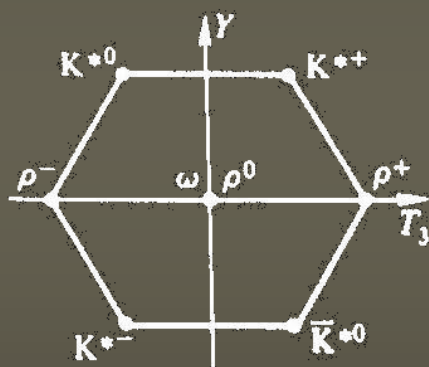
Physical Review D79 (2009) 074509 arXiv:0805.3068

Guray Erkol, Makoto Oka and Toru T. Takahashi

Axial Charges of Octet Baryons in Two-flavor Lattice QCD

Physics letters B686 (2010) 36 arXiv:0911.2447

Guray Erkol, Makoto Oka and Toru T. Takahashi



We systematically studied octet meson-baryon couplings with 2-flavor lattice QCD

→ 2-flavor QCD, quark are not very light (not very chiral)

Our next goal is the extension of these works

# Our previous study

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \quad M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

Baryon fields

Meson fields

In terms of QCD's symmetry,  
effective interactions can be constructed

(Eg.)  $\text{tr}[[\bar{B}, B]M] \quad \text{tr}[\{\bar{B}, B\}M] \quad , \dots$

- Overall coefficients remain undetermined, which are determined by QCD dynamics.
- SU(3) symmetry is actually broken.

# Our previous study

$$\mathcal{L}_{\text{BBM}} = F \text{tr}[[\bar{B}, B]M] + D \text{tr}[\{\bar{B}, B\}M]$$

F and D cannot be determined

→ Two unknown parameters

## SU(3) relations

$$g_{NN\pi} = g$$

$$g_{\Sigma\Sigma\pi} = 2g\alpha, \quad g_{\Lambda\Sigma\pi} = \frac{2}{\sqrt{3}}g(1 - \alpha), \quad g_{\Xi\Xi\pi} = g(2\alpha - 1)$$

$$g_{\Sigma NK} = g(1 - 2\alpha), \quad g_{\Lambda NK} = -\frac{1}{\sqrt{3}}g(1 + 2\alpha)$$

$$g_{NN\eta_8} = \frac{1}{\sqrt{3}}g(4\alpha - 1), \quad g_{\Sigma\Sigma\eta_8} = \frac{2}{\sqrt{3}}g(1 - \alpha)$$

$$g_{\Lambda\Lambda\eta_8} = -\frac{2}{\sqrt{3}}g(1 - \alpha), \quad g_{\Xi\Xi\eta_8} = -\frac{1}{\sqrt{3}}g(1 + 2\alpha)$$

$$g_{\pi NN}$$

$$\alpha \equiv \frac{F}{F + D}$$

Two parameters  
(cannot be determined  
by the symmetry)

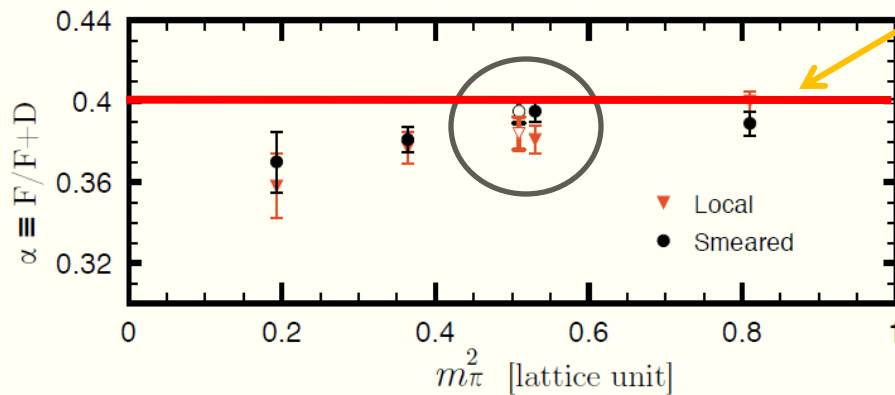
- Equations of motion
- Blackhole formation
- SU(3) symmetry is good ?
- How the symmetry broken ?

→ Lattice QCD calculations



# $\alpha = F/(F+D)$ and SU(3) breaking parameters

$\alpha = F/(F+D)$  (obtained by global fit)



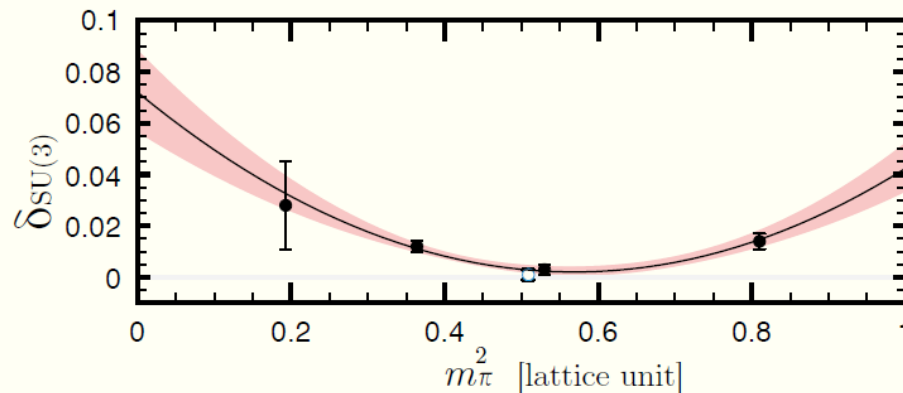
0.4 — exact SU(6)

SU(3) limit :  $\alpha = 0.395(6)$

c.f )  $\alpha = 0.4$  under SU(6) symmetry

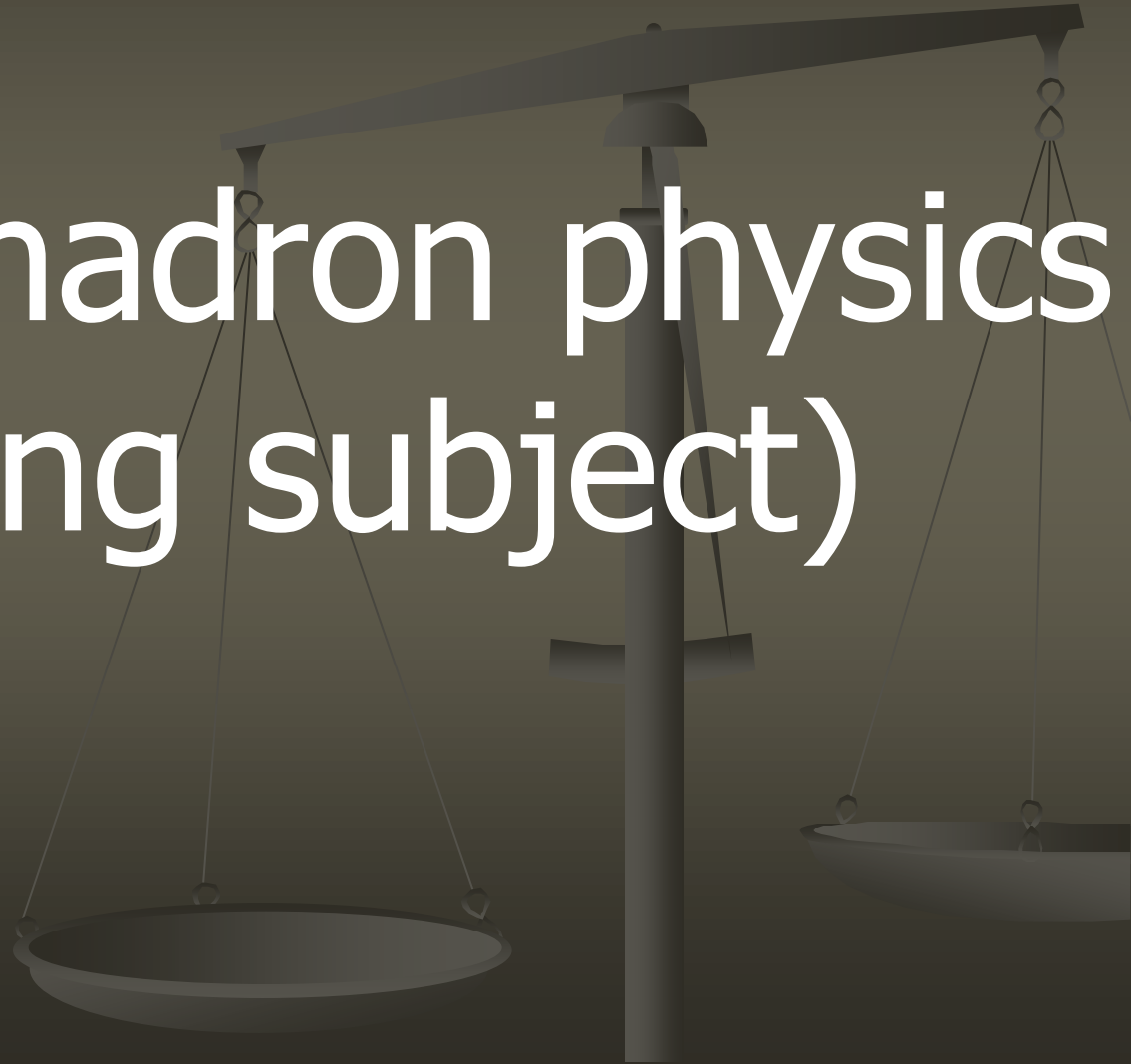
It decreases towards chiral limit

SU(3) breaking parameter  $\delta$



Breaking in SU(3) relations  
remains small (a few %)

# 6. Heavy hadron physics (Ongoing subject)



# Heavy hadron form factors

Charmed (or bottomed) hadron couplings or form factors are also important.

## HH $\chi$ PT

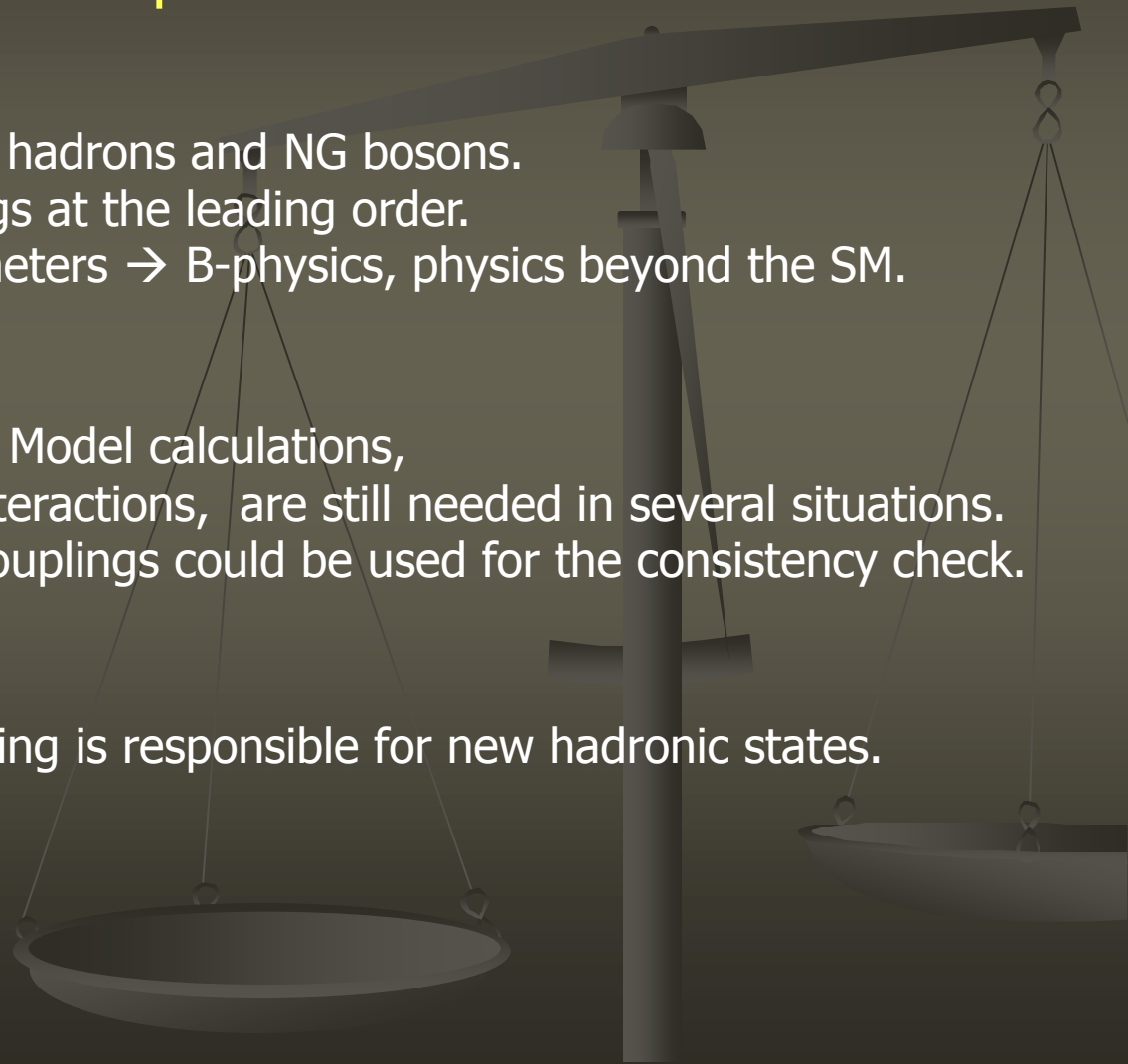
describes int. of heavy-light hadrons and NG bosons.  
contains three axial couplings at the leading order.  
precise knowledge of parameters  $\rightarrow$  B-physics, physics beyond the SM.

## Validity check of models

Lattice QCD is not almighty. Model calculations, which describes hadronic interactions, are still needed in several situations. Lattice QCD estimation of couplings could be used for the consistency check.

## Possible new hadronic state?

Charmed hadron-pion coupling is responsible for new hadronic states.  
(Yasui-san's work)

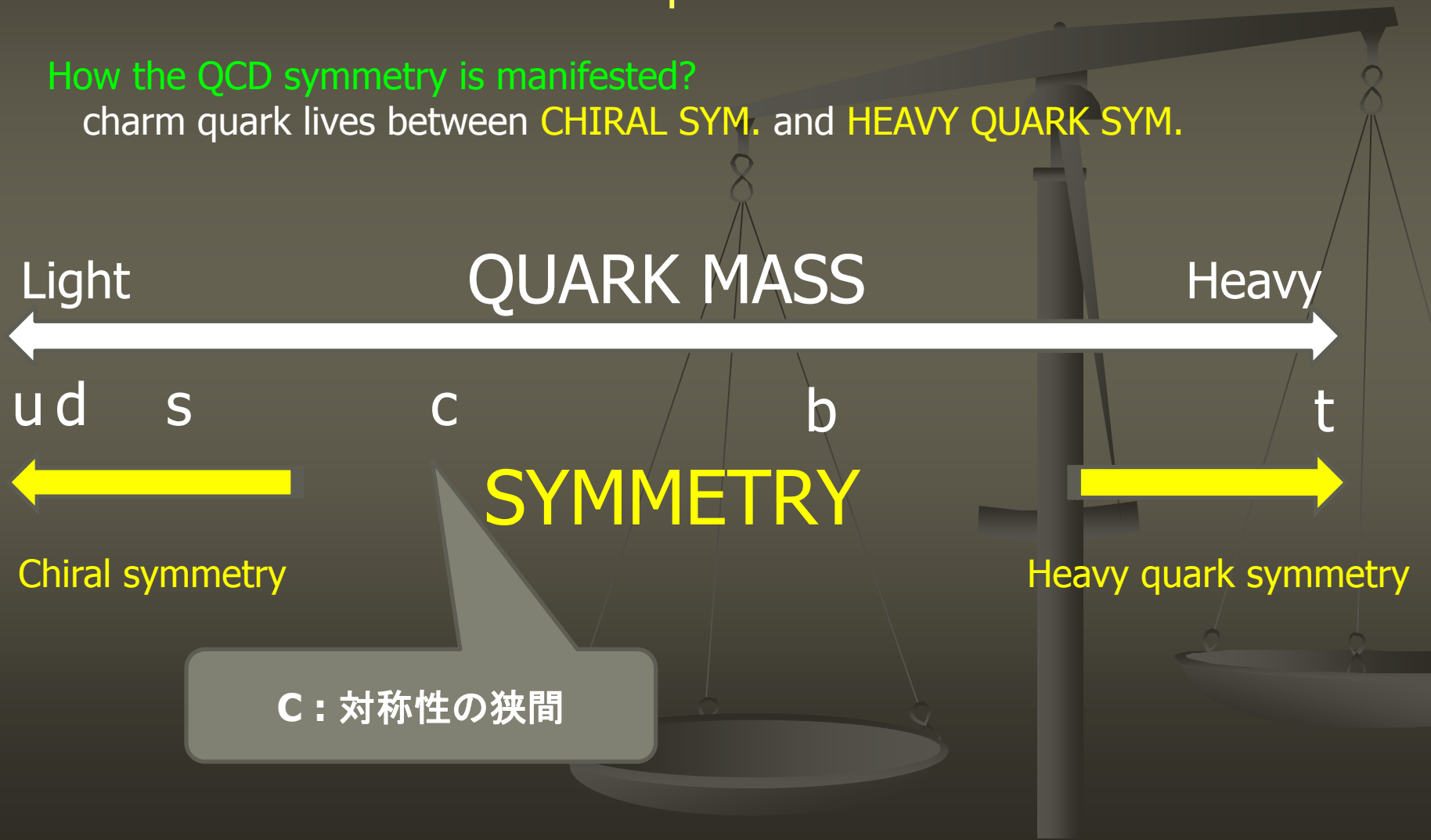


# Heavy hadron form factors

Charmed (or bottomed) hadron couplings or form factors are also important.

How the QCD symmetry is manifested?

charm quark lives between CHIRAL SYM. and HEAVY QUARK SYM.

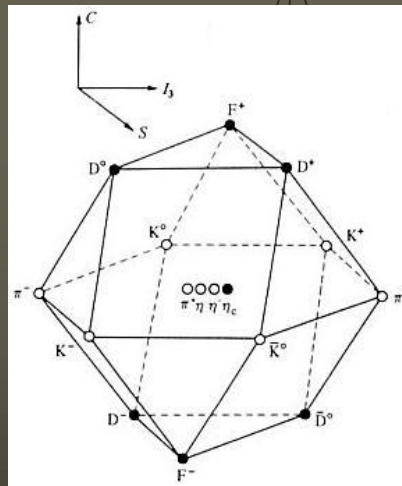


# Heavy hadron form factors

Charmed (or bottomed) hadron couplings or form factors are also important.

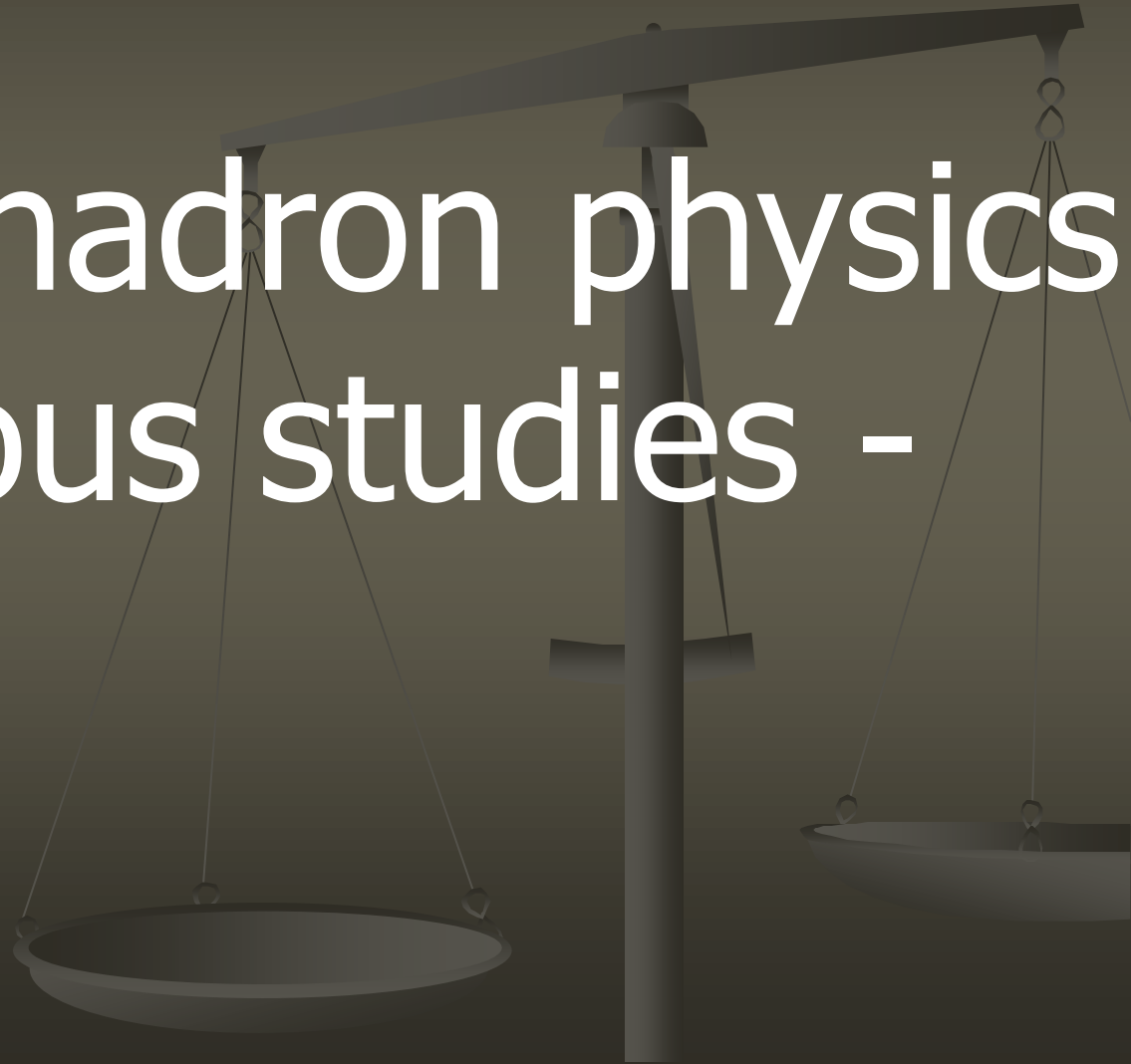
How the symmetry is broken?

In flavor-SU(3) sector, the breaking in couplings were found to be small. But, SU(4) should be largely broken. How large?



# 6. Heavy hadron physics

- Previous studies -



# Non-Lattice QCD determination of form factors

Charm couplings and form factors in QCD sum rules.

M.E. Bracco, M. Chiapparini, F.S. Navarra, M. Nielsen

e-Print: arXiv:1104.2864 [hep-ph]

LQCD 含む様々な計算結果 (LQCDの結果はまだ少ない)

Approach	$g_{D^* D \pi}$	$g_{B^* B \pi}$
QCDSR [41]	$9 \pm 2$	$20 \pm 4$
QCDSR [41]	$7 \pm 2$	$15 \pm 4$
LCSR [42]	$11 \pm 2$	$28 \pm 6$
QCDSR [43]	$6.3 \pm 1.9$	$14 \pm 4$
LCSR [44]	$10.5 \pm 3$	$22 \pm 9$
QCDSR [19]	$14.0 \pm 1.5$	$42.5 \pm 2.6$
QCDSR plus meson loops [37]	$17.5 \pm 1.5$	$44.7 \pm 1.0$
LQCD [45]	$20 \pm 2$	
LQCD [46]	$18.8^{+2.5}_{-3.0}$	
dispersive quark model [47]	$18 \pm 3$	$32 \pm 5$
Dyson-Schwinger equations [48]	$15.8^{+2.1}_{-1.0}$	$30.0^{+3.2}_{-1.4}$

# Non-Lattice QCD determination of form factors

Charm couplings and form factors in QCD sum rules.

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QCDSR の利点(?)を生かして、様々なチャネルが調べられている

Coupling	QCDSR	VMD	Other models
$g_{\rho DD}$	$3.0 \pm 0.2$ [17]	2.52 [32]	
$g_{\rho D^* D} \text{ (GeV}^{-1}\text{)}$	$4.3 \pm 0.9$ [24]	2.82 [33]	$4.17 \pm 1.04$ [54]
$g_{\rho D^* D^*}$	$4.7 \pm 0.2$ [23]	2.52 [32]	$1.8 \pm 0.5$ [52]
$g_{\omega DD}$	$-2.9$ [53]	-2.84 [32]	
$g_{J/\psi DD}$	$5.8 \pm 0.9$ [22]	7.64 [32]	$8.0 \pm 0.5$ [40]
$g_{J/\psi D^* D} \text{ (GeV}^{-1}\text{)}$	$4.0 \pm 0.6$ [22]	$8.0 \pm 0.6$ [33]	$4.05 \pm 0.25$ [40]
$g_{J/\psi D^* D^*}$	$6.2 \pm 0.9$ [20]	7.64 [32]	$8.0 \pm 0.5$ [40]

しかし、格子QCDでは、それほど詳細に調べられてはいない



# Non-Lattice QCD determination of form factors

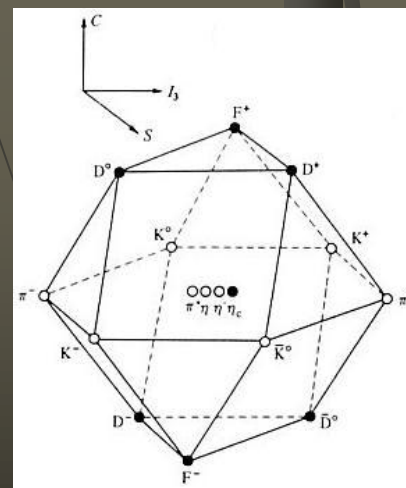
Charm couplings and form factors in QCD sum rules.

M.E. Bracco, M. Chiapparini, F.S. Navarra, M. Nielsen

e-Print: arXiv:1104.2864 [hep-ph]

QCDSR による、フレーバーSU(4)対称性の破れの検証

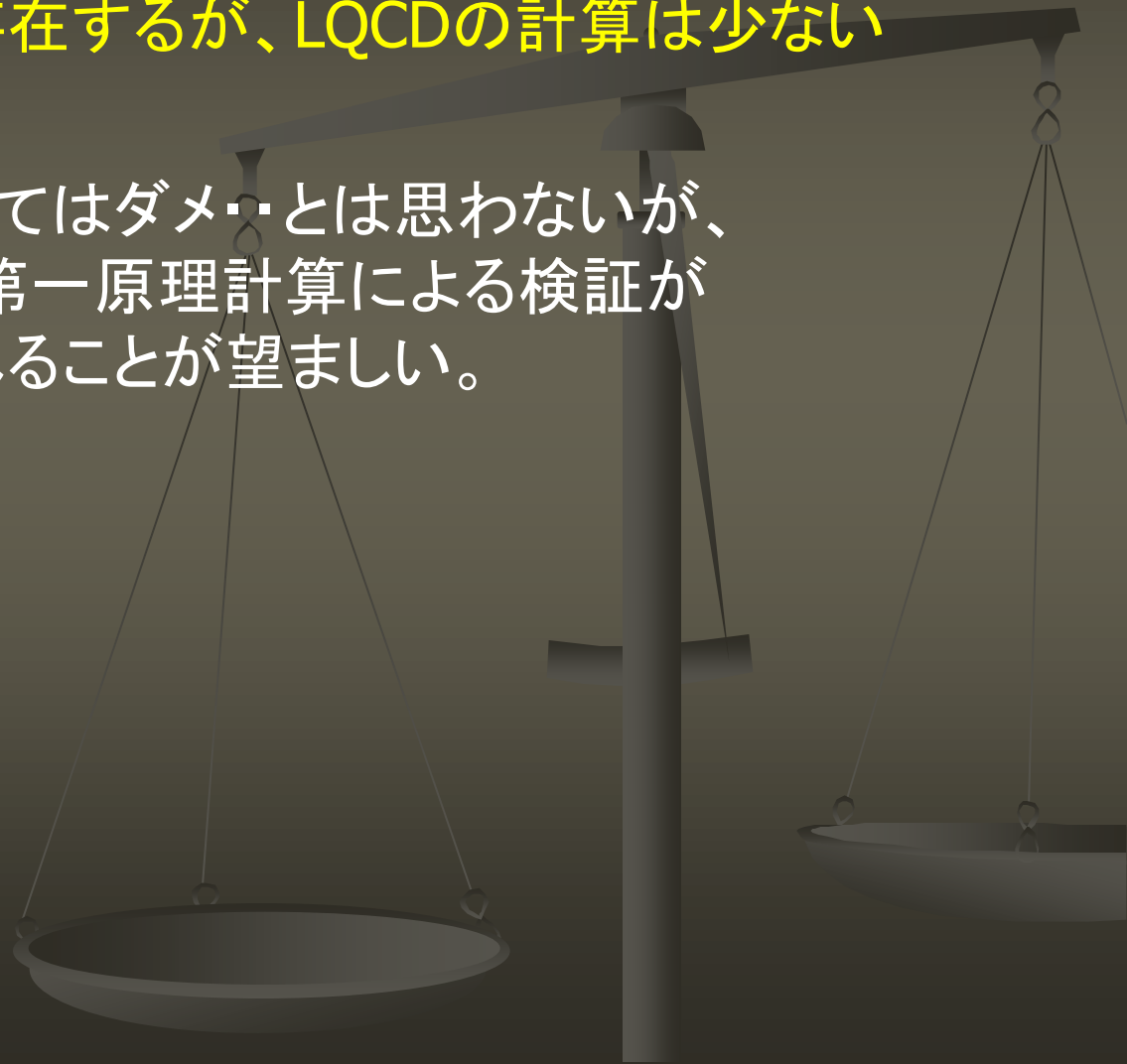
SU(4) Relation	Violation
$g_{J/\psi DD} = g_{J/\psi D^* D^*}$	(7%)
$g_{\rho DD^*} = \frac{\sqrt{6}}{2} g_{J/\psi DD^*}$	(12%)
$g_{\rho DD} = \frac{\sqrt{6}}{4} g_{J/\psi DD}$	(17%)
$g_{\pi D^* D^*} = \frac{\sqrt{6}}{2} g_{J/\psi DD^*}$	(20%)
$g_{D^* D^* \rho} = \frac{\sqrt{6}}{4} g_{J/\psi D^* D^*}$	(20%)
$g_{DD\rho} = \frac{\sqrt{6}}{4} g_{J/\psi D^* D^*}$	(21%)
$g_{\rho D^* D^*} = \frac{\sqrt{6}}{4} g_{J/\psi DD}$	(25%)
$g_{\pi D^* D^*} = g_{\rho DD^*}$	(29%)
$g_{\rho DD} = g_{\rho D^* D^*}$	(36%)
$g_{D^* D \pi} = g_{D^* D^* \rho}$	(52%)
$g_{D^* D \pi} = \frac{\sqrt{6}}{4} g_{J/\psi D^* D^*}$	(62%)
$g_{D^* D \pi} = \frac{\sqrt{6}}{4} g_{J/\psi DD}$	(64%)
$g_{D^* D \pi} = g_{DD\rho}$	(70%)



# Non-Lattice QCD determination of form factors

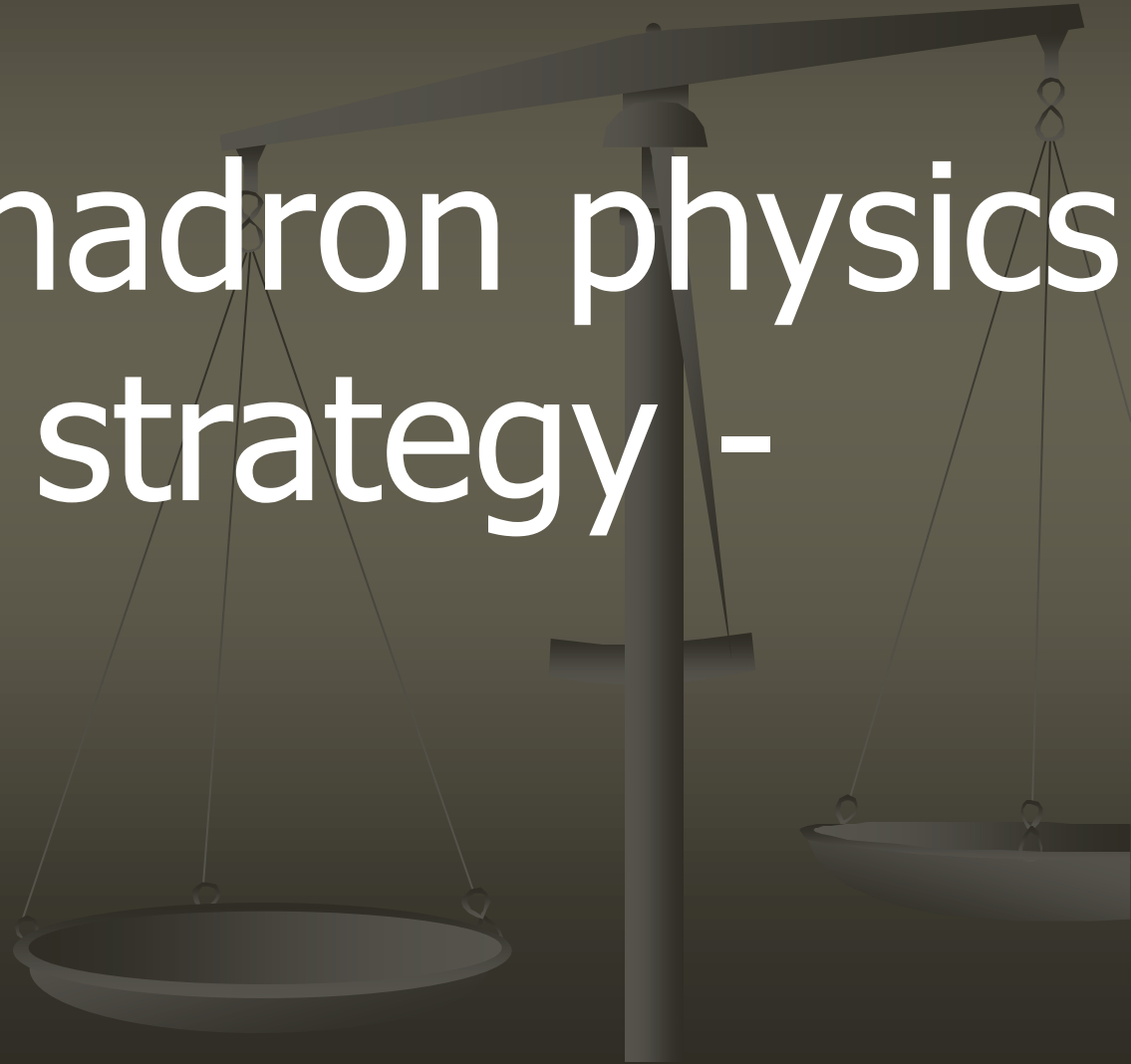
様々な先行研究が存在するが、LQCDの計算は少ない

格子QCDでなくてはダメ...とは思わないが、  
やはり非摂動第一原理計算による検証が  
行われることが望ましい。



# 6. Heavy hadron physics

## - Our strategy -

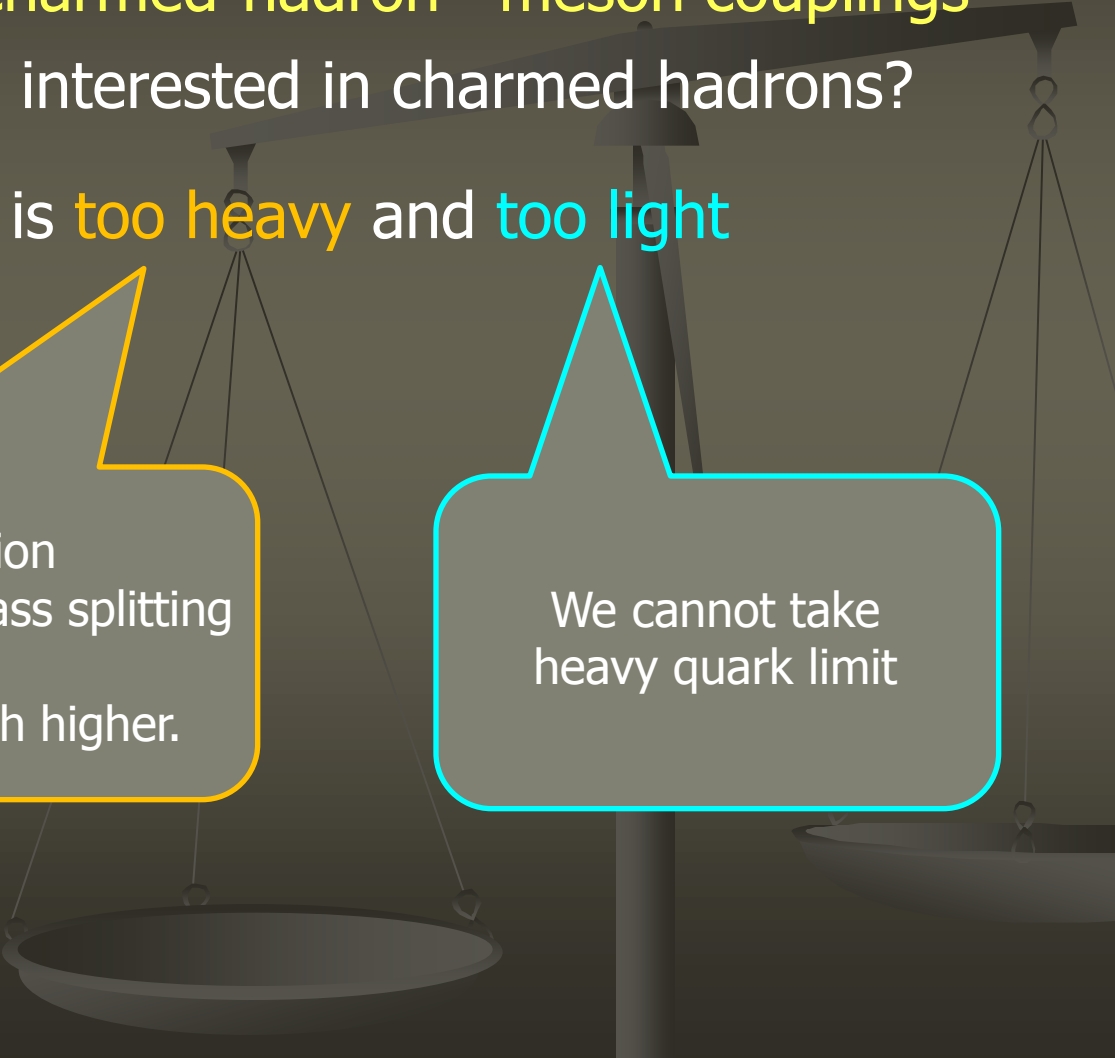


# Lattice QCD determination of form factors

Few lattice study for charmed-hadron—meson couplings

Few lattice people are interested in charmed hadrons?

Charm quark is **too heavy** and **too light**

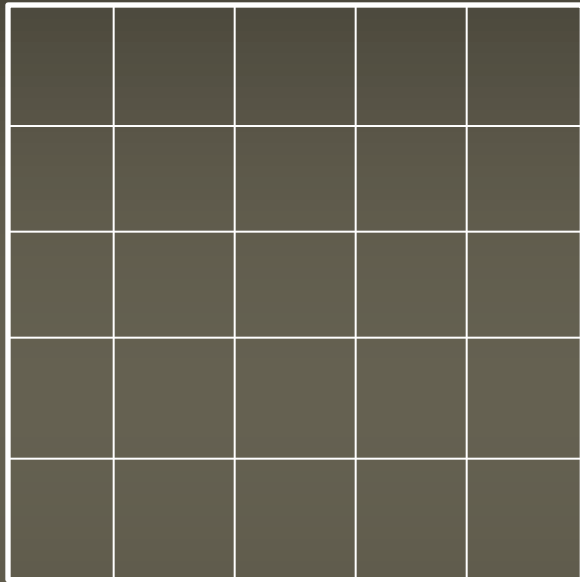


Naïve Wilson quark action  
cannot reproduce hyper fine mass splitting  
in charmonia.  
Lattice cut-off should be much higher.

We cannot take  
heavy quark limit

# Lattice QCD determination of form factors

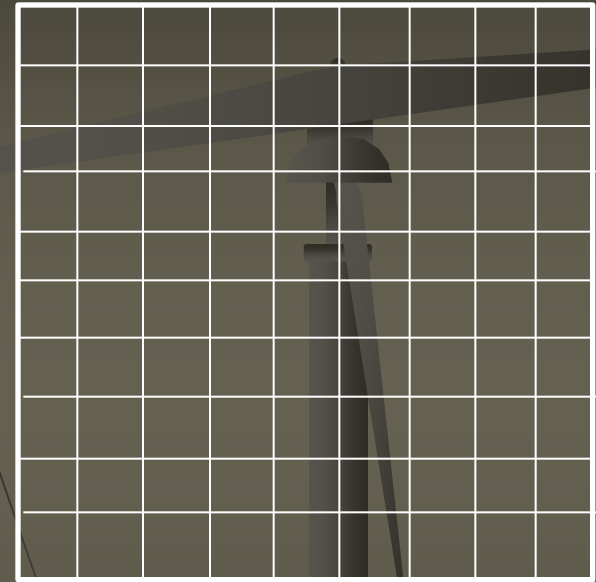
↔ Lattice spacing [ $a$ ]  
Lattice cutoff [ $1/a$ ]



Too coarse.

Small numerical cost.

Cutoff is insufficient  
for charm quarks ( $\sim 1.3$  GeV).



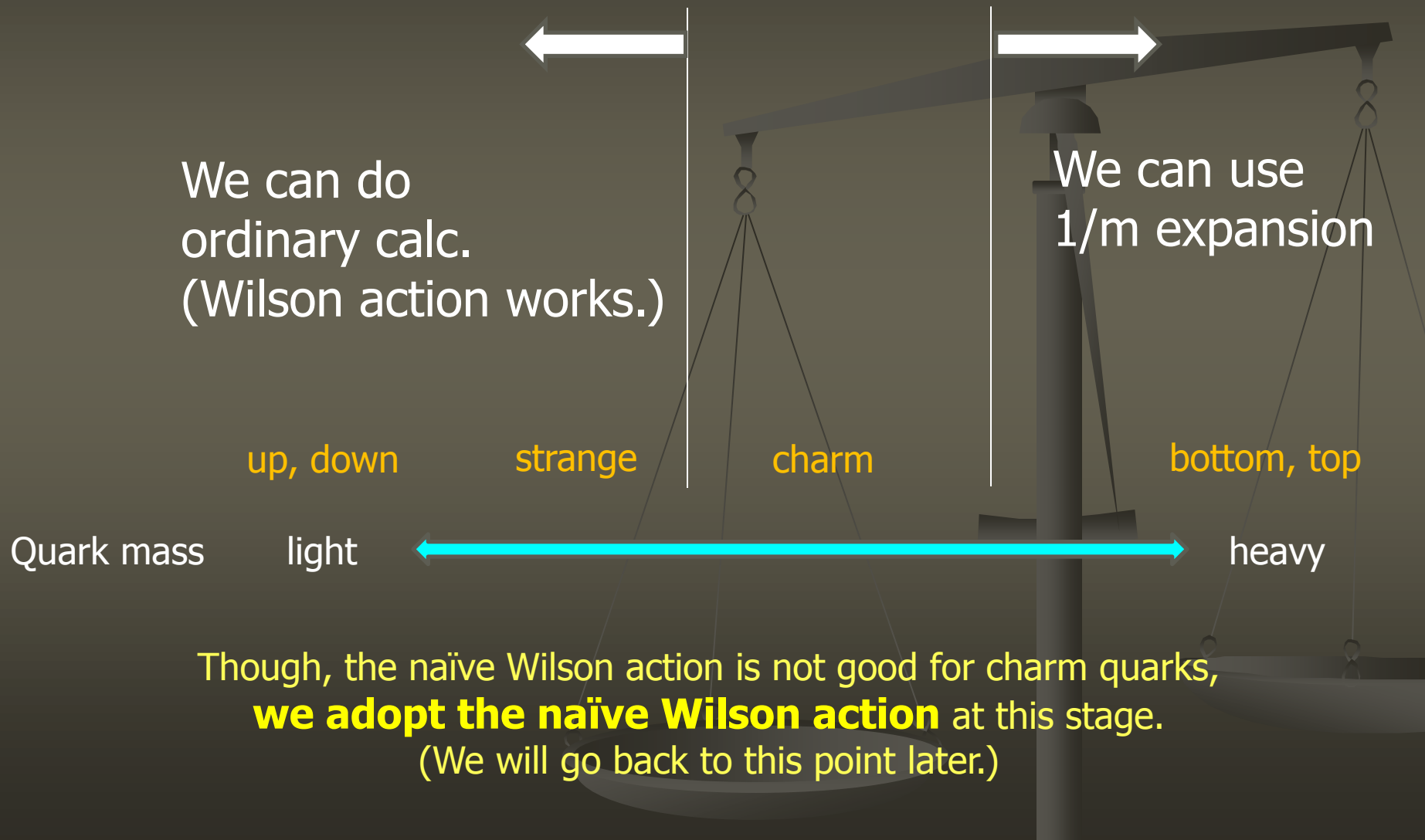
Fine.

Cutoff is sufficient.

Huge numerical cost.

# Lattice QCD determination of form factors

Charm quark is **too heavy** and **too light**



# Our strategy

2+1 flavor gauge configurations (generated by PACS-CS)  
Iwasaki gauge action and the Wilson quark action  
 $32^3 \times 64$ ,  $a \sim 0.1$  fm (spatial volume is large)

We give up reproducing HF mass splitting.

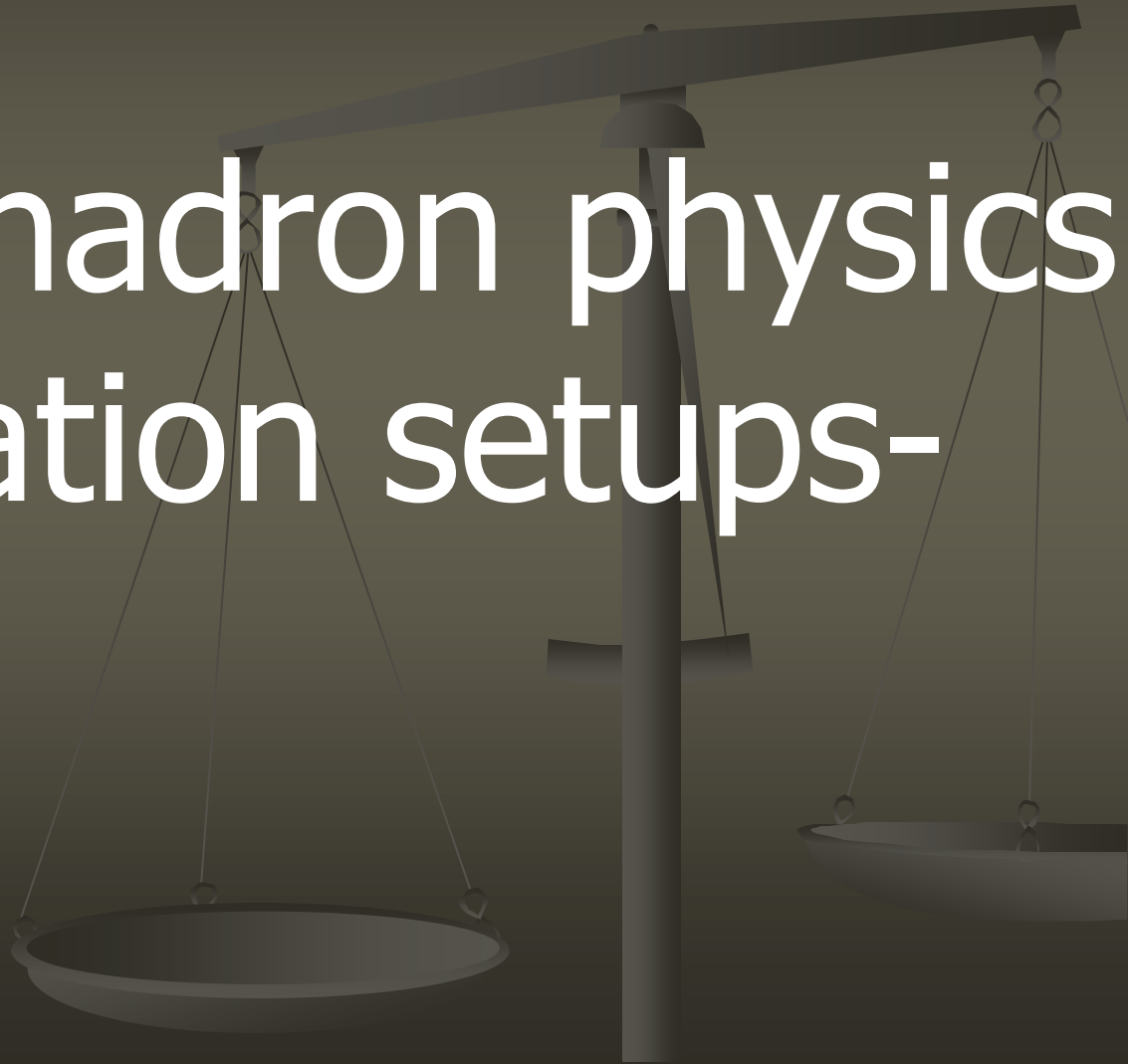
WALL-type sink operators (to avoid many matrix inversions)

TARGET (our hope, our desire)

- All the possible MB or MM couplings including flavor-SU(3) sector
- We aim at systematic study.

# 6. Heavy hadron physics

- Simulation setups-





# Lattice QCD results

## Simulation conditions

2+1 flavor gauge configurations (generated by PACS-CS)

Iwasaki gauge action and the Wilson quark action

$32^3 \times 64$ , cutoff  $\sim 2.2$  GeV,  $a \sim 0.1$  fm (spatial volume is large)

WALL-type sink operators (to avoid many matrix inversions)

heavy  light

Kappa\_ud  $\rightarrow$  0.13700, 0.13727, 0.13754, 0.13700

Pion mass  $\rightarrow$  700, 569, 411, 295 (MeV)

Current renormalization factors are estimated in a perturbative way  
(nonperturbative determination will improve the results.)

# Lattice QCD results (hyperfine mass splitting)

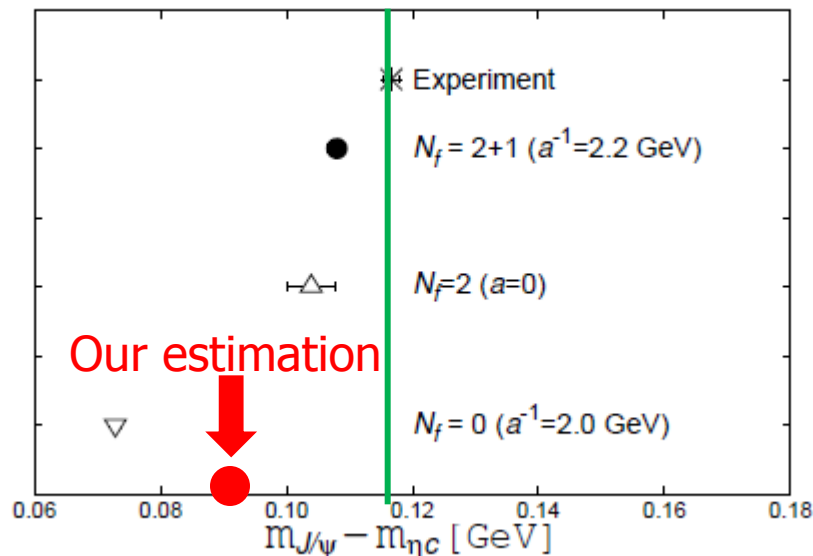
$J/\psi$ ,  $\eta_c$  mass **CHECK** (chiral-extrapolated)

$$\kappa_c = 0.12241 \text{ (} J/\psi\text{- input)}$$

$$m_{\eta_c} = 3.00478, \quad m_{J\psi} = 3.097 \text{ (GeV)}$$



$$m_{\eta_c} - m_{J\psi} = 0.0922 \text{ (GeV)}$$



Charm quark system at the physical point of 2+1 flavor lattice QCD.

PACS-CS Collaboration  
(Y. Namekawa et al.)

Phys.Rev. D84 (2011) 074505

# Lattice QCD results (hyperfine mass splitting)

$J/\psi$ ,  $\eta_c$  mass **CHECK** (chiral-extrapolated)

$$\kappa_c = 0.12241 \text{ (} J/\psi\text{- input)}$$

$$m_{\eta_c} = 3.00478, \quad m_{J\psi} = 3.097 \text{ (GeV)}$$



$$m_{\eta_c} - m_{J\psi} = 0.0922 \text{ (GeV)}$$



Mass splitting is clearly underestimated with the Wilson quark action.

On the other hand,



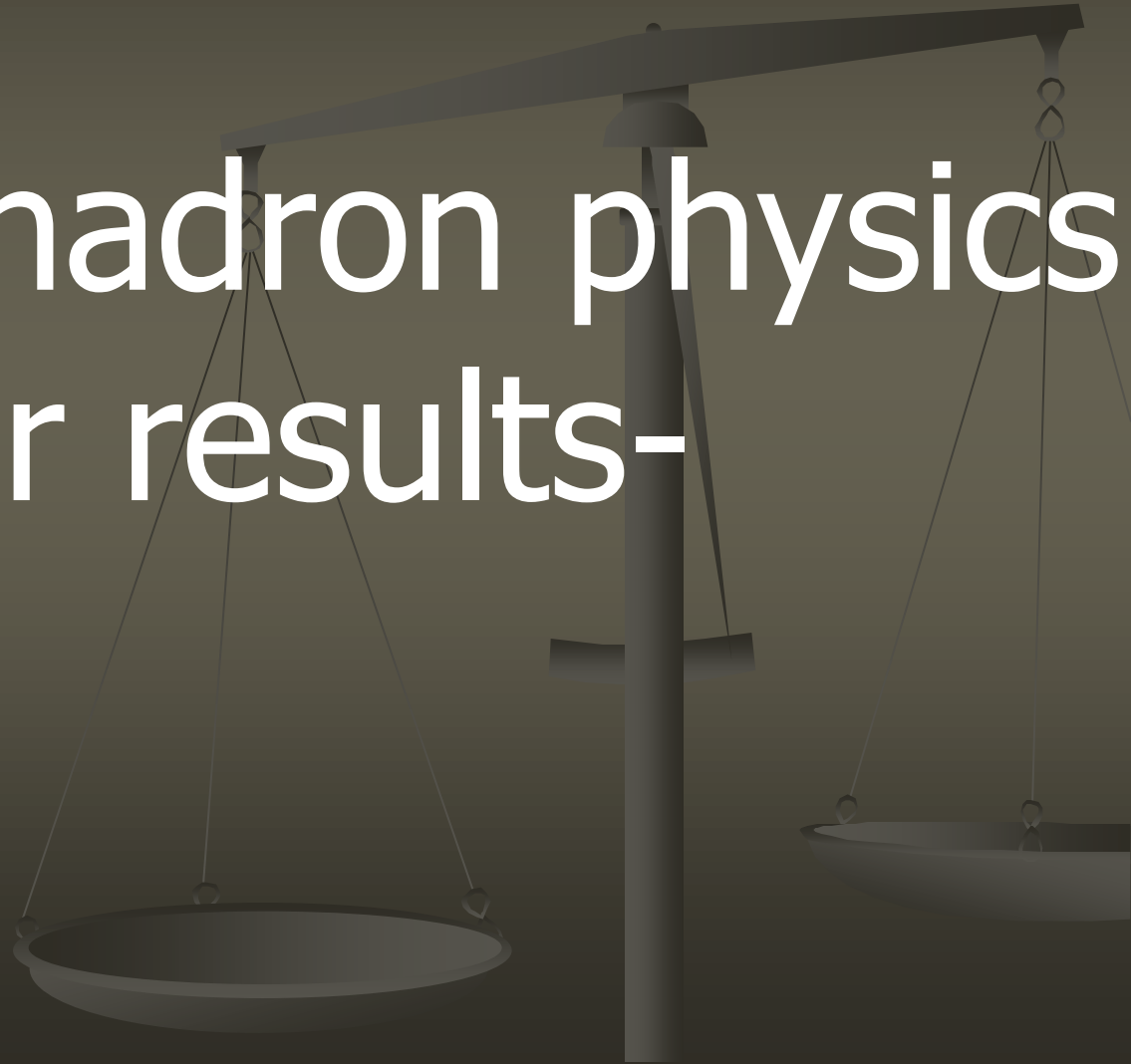
Coupling constants seem insensitive to charm-quark mass variation.

→ We adopt the Wilson quark action at this stage.

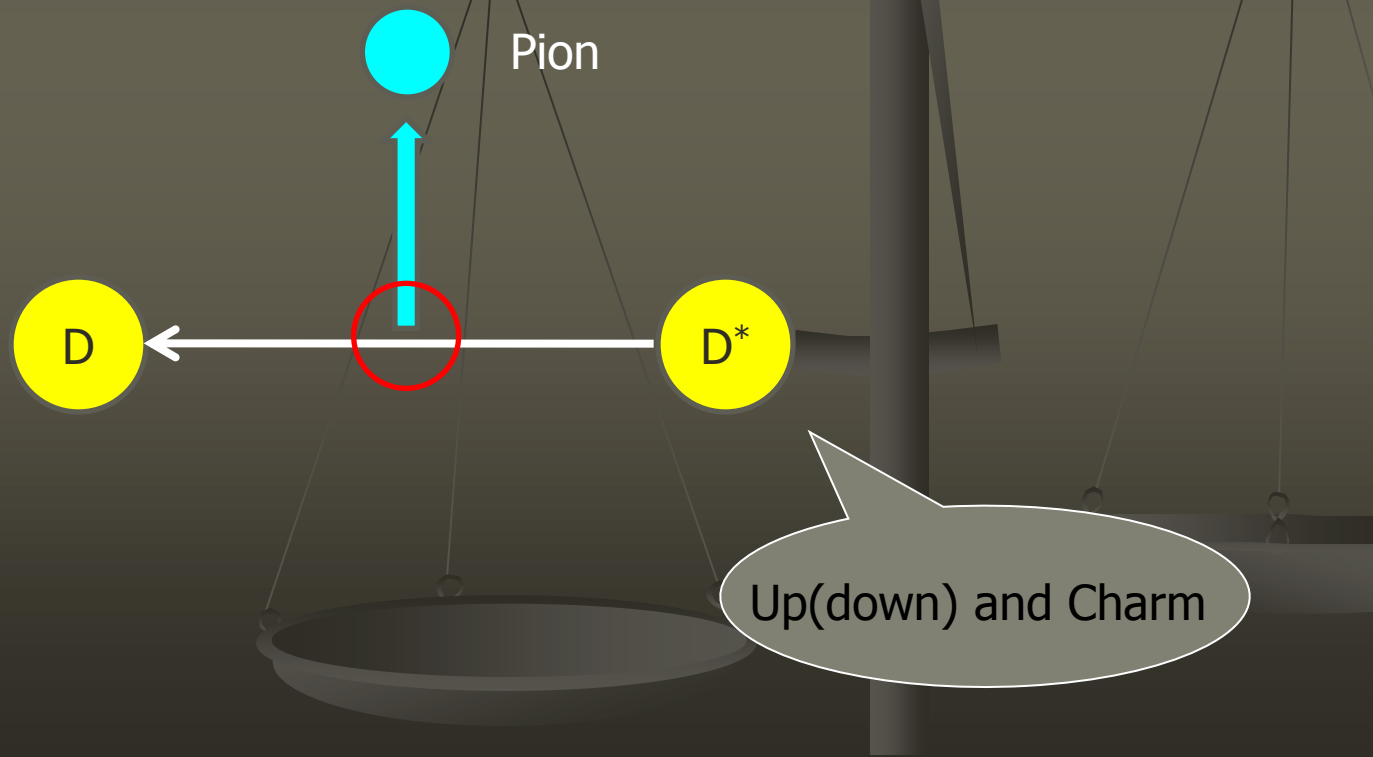
(we are planning to adopt heavy quark actions)

# 6. Heavy hadron physics

## - Our results-



# $D^*D\pi$ couplings from the lattice QCD (consistency check)



# Lattice QCD results (couplings)

$D^*D\pi$  – coupling (chiral – extrapolated)

Pion mass

$\kappa_{ud}$	$G_1(q^2 = 0)$	$G_2/G_1$	$g_{D^*D\pi}$
700 MeV	0.13700	14.15(1.58) 0.09(2)	15.45(1.78)
569 MeV	0.13727	13.63(1.57) 0.12(4)	15.24(1.81)
411 MeV	0.13754	12.76(1.43) 0.15(7)	15.54(2.08)
295 MeV	0.13770	15.46(2.17) 0.07(6)	16.44(2.41)
0 MeV	Lin. Fit		16.23(1.71)
	Quad. Fit		17.09(3.23)

Our chiral-fit ->  $16.23 \pm 1.7$  (exp:  $17.9 \pm 2.2$ )

比較

Abada et al. ->  $18.8 \pm 2.3$

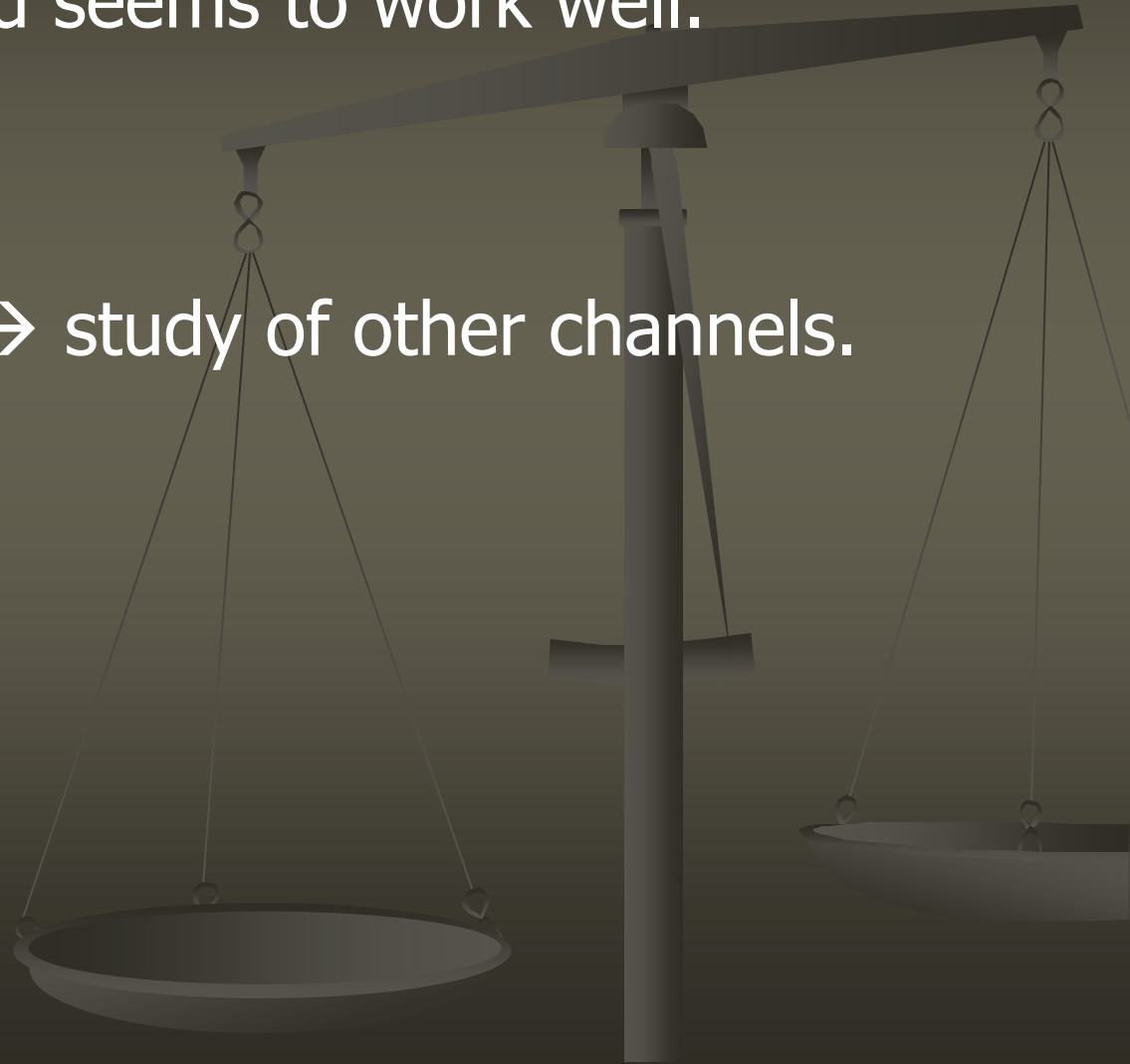
Becirevic et al. (2009) ->  $20 \pm 2$

Becirevic et al. (2012) ->  $15.9 \pm 0.7$

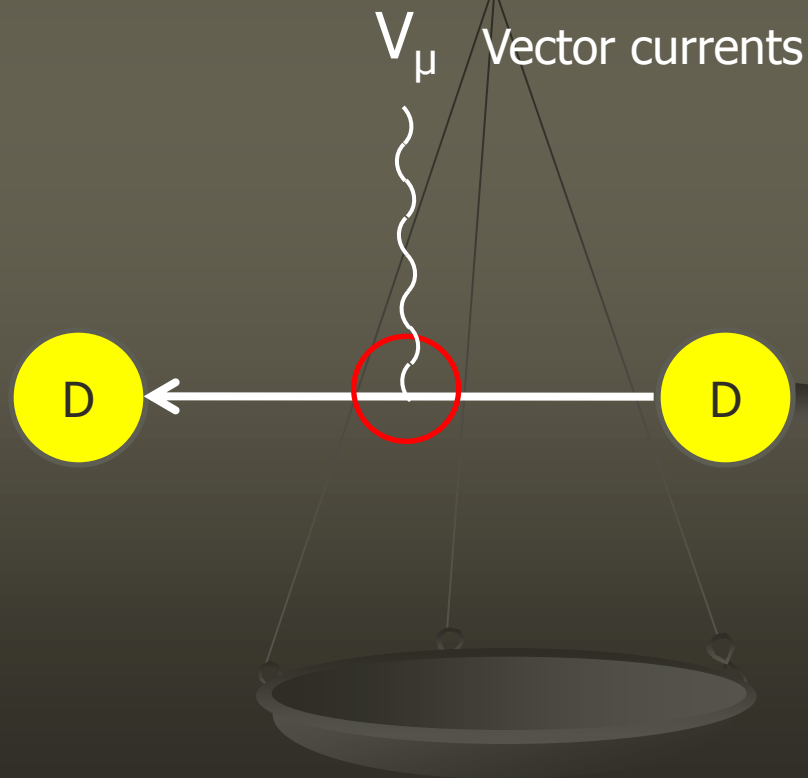
# Lattice QCD results (couplings)

Consistent with previous studies.  
Wall-method seems to work well.

Our next task → study of other channels.



# D meson form factors from the lattice QCD

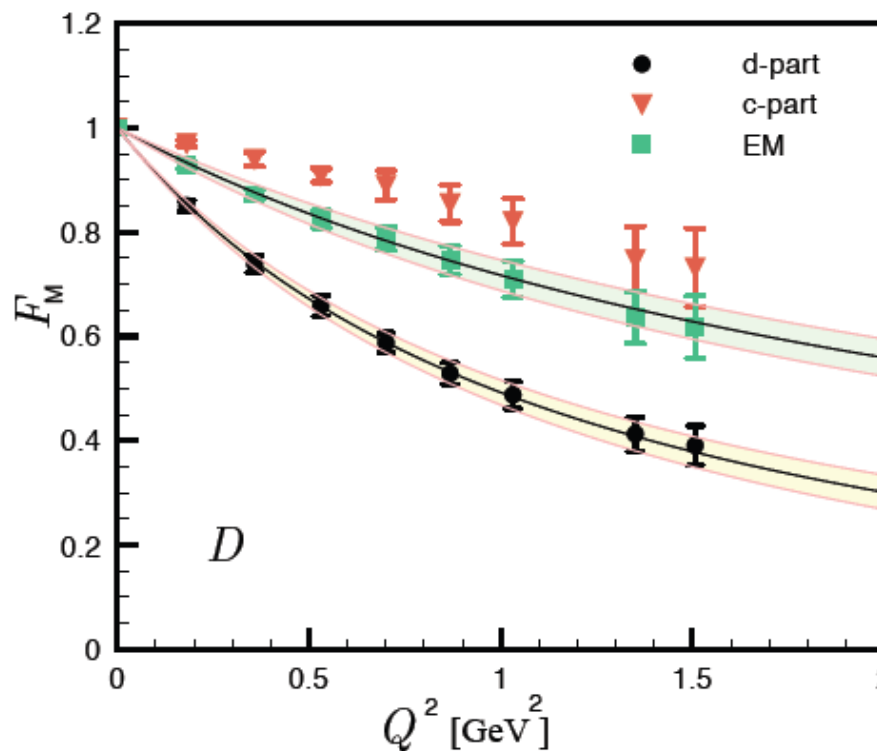




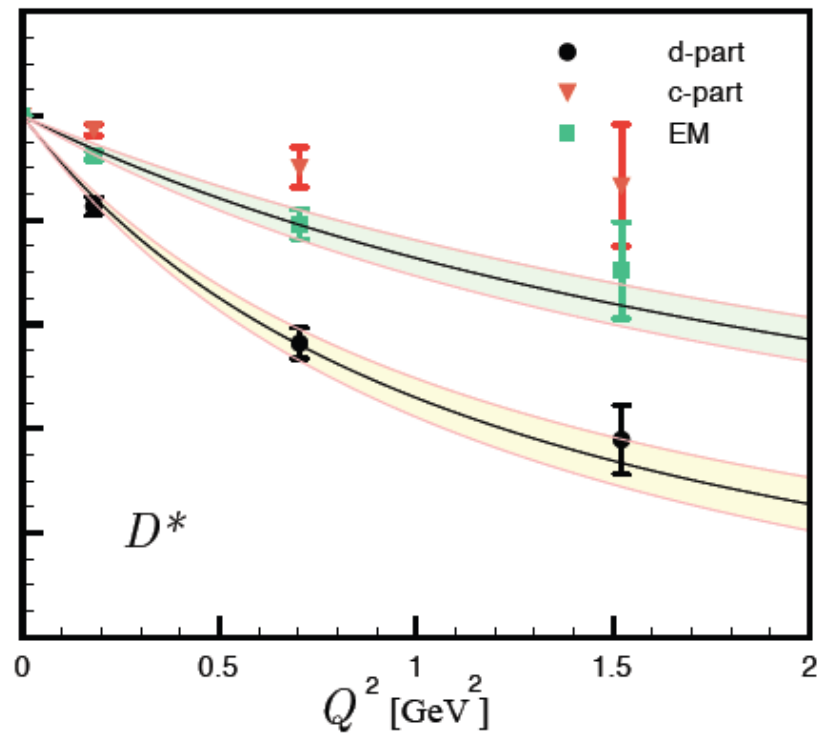
# Lattice QCD results (couplings)

## Form factors of D-meson

$$\langle \mathcal{D}(p') | V_\mu(q) | \mathcal{D}(p) \rangle = \frac{(p + p')_\mu}{2\sqrt{E_D E_{D'}}} [2/3 F_D^c(q^2) + 1/3 F_D^d(q^2)]$$



$D$  – meson



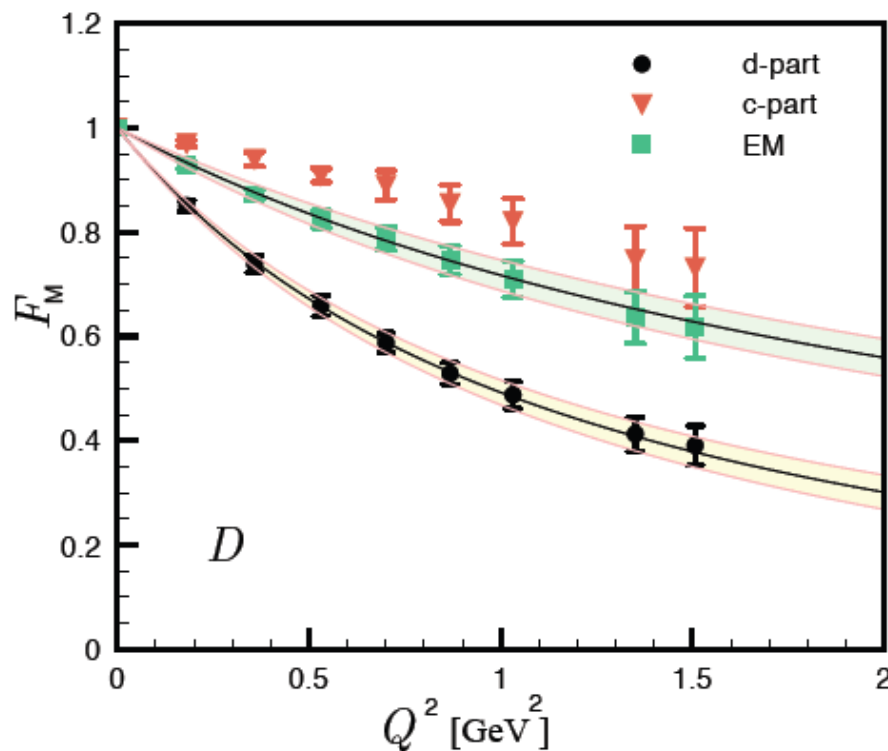
$D^*$  – meson

# Lattice QCD results (couplings)

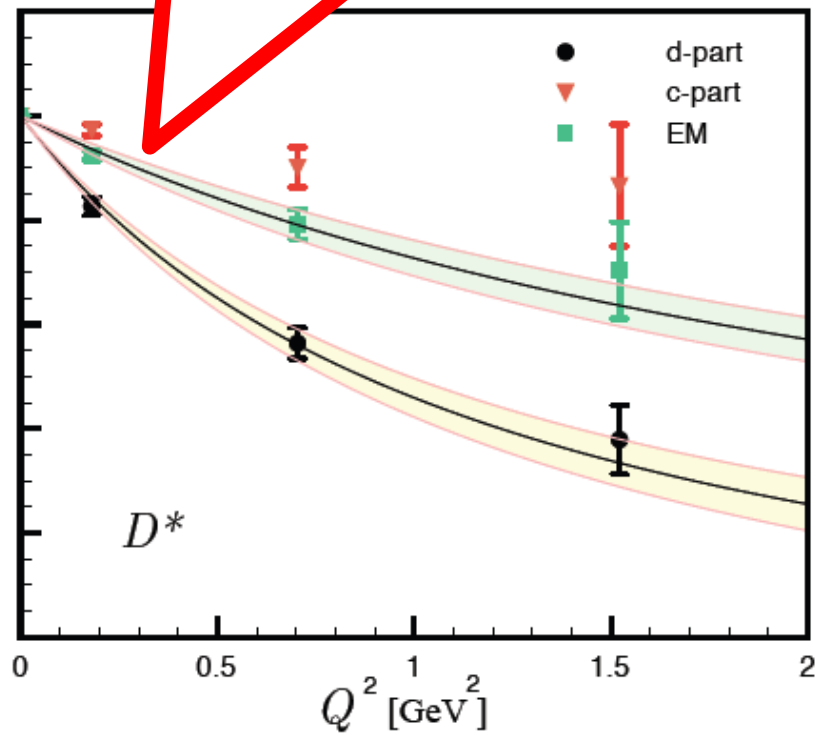
## Form factors of D-meson

VMD-ansatz reproduces data well.

$$F_V(Q^2) = \left[ 1 - \frac{Q^2}{m_\rho^2 + Q^2} \frac{g_{DD\rho}}{g_\rho} \right].$$



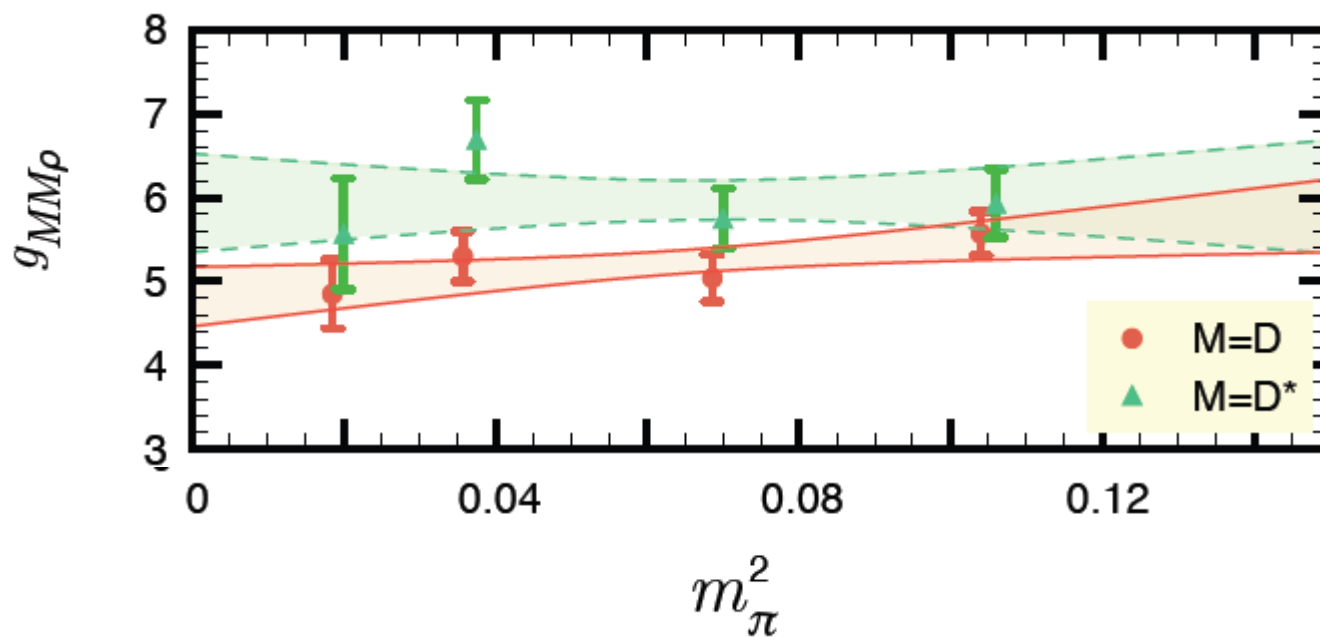
$D$  – meson



$D^*$  – meson

# Lattice QCD results (couplings)

Quark-mass dependences of  $DD\rho$  and  $D^*D^*\rho$  couplings  
 *$g_{\{D^*D^*\rho\}}$  is systematically smaller than  $g_{DD\rho}$*



Our chiral-fit ->  $4.48 \pm 0.34$  ,  $5.94 \pm 0.56$

# Lattice QCD results (couplings)

Quark-mass dependences of  $DD\rho$  and  $D^*D^*\rho$  couplings

Our chiral-fit ->  $4.48 \pm 0.34$  ( $DD\rho$ )  
 $5.94 \pm 0.56$  ( $D^*D^*\rho$ )

QCD Sum Rule ->  $2.9 \pm 0.4$  ( $DD\rho$ )  
 $5.2 \pm 0.3$  ( $D^*D^*\rho$ )

DS equation -> 5.05 ( $DD\rho$ )

他計算とは、おおむねコンシステント  
(第一原理計算とはいえ、他の結果が多少気になるところです)

Form factor は比較的 $Q^2$  が大きいところまで、VMD-ansatz でよく合う。

比較

# Lattice QCD results (couplings)

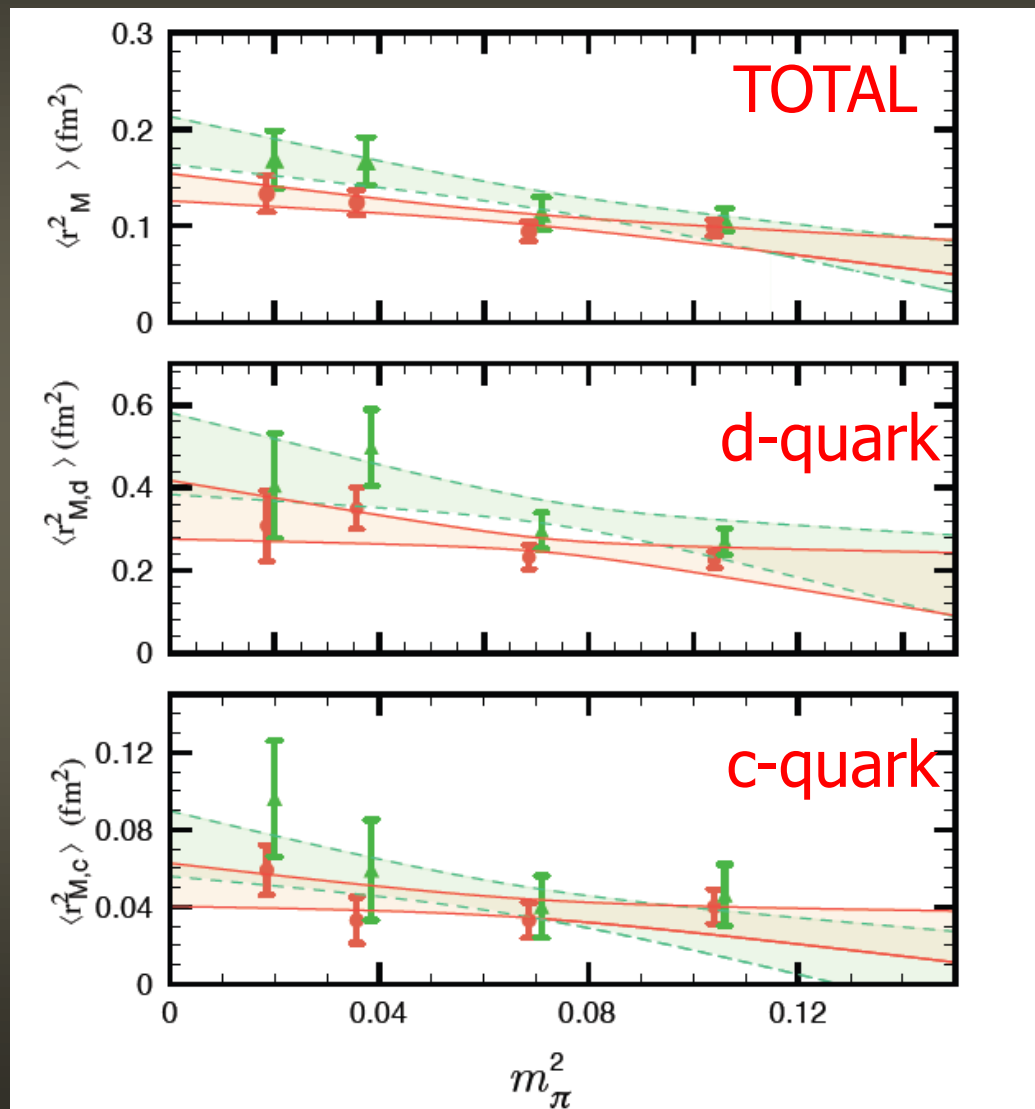
Charge radii each quark contributes

D\* is systematically larger

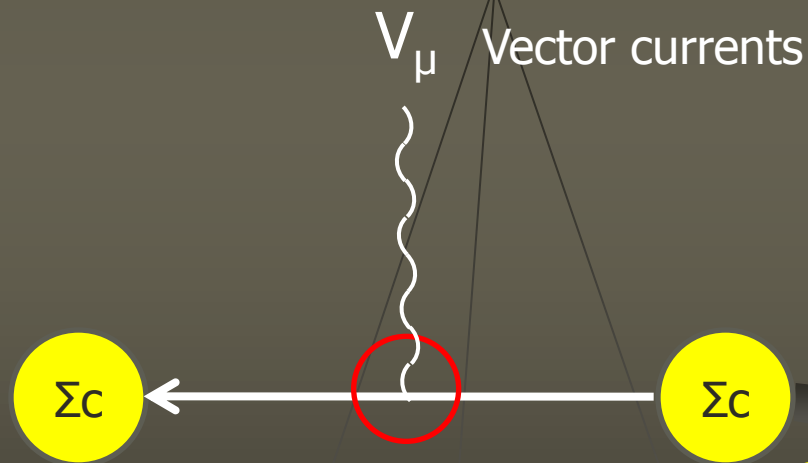
When quarks are heavy,  
They degenerate.

← CM interaction

中間子中の  
Charm quarkの広がり  
Down quarkの広がりより小さい。  
← 質量が違うため



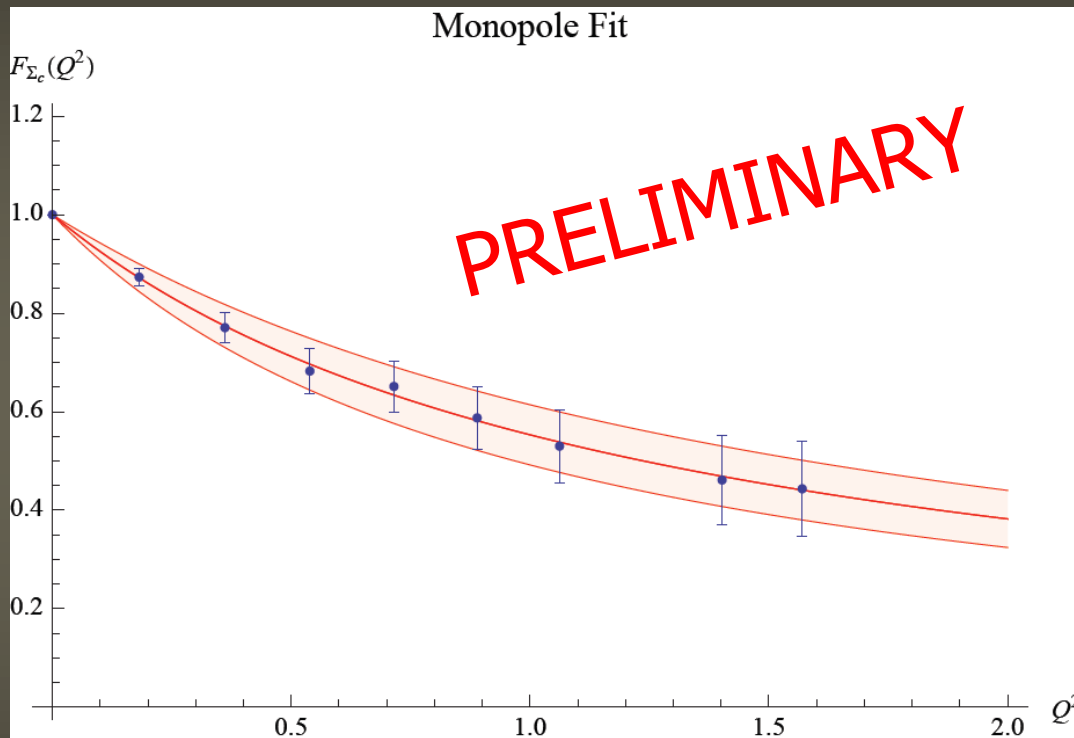
# $\Sigma_c$ form factors from the lattice QCD



PRELIMINARY

# Lattice QCD results (couplings)

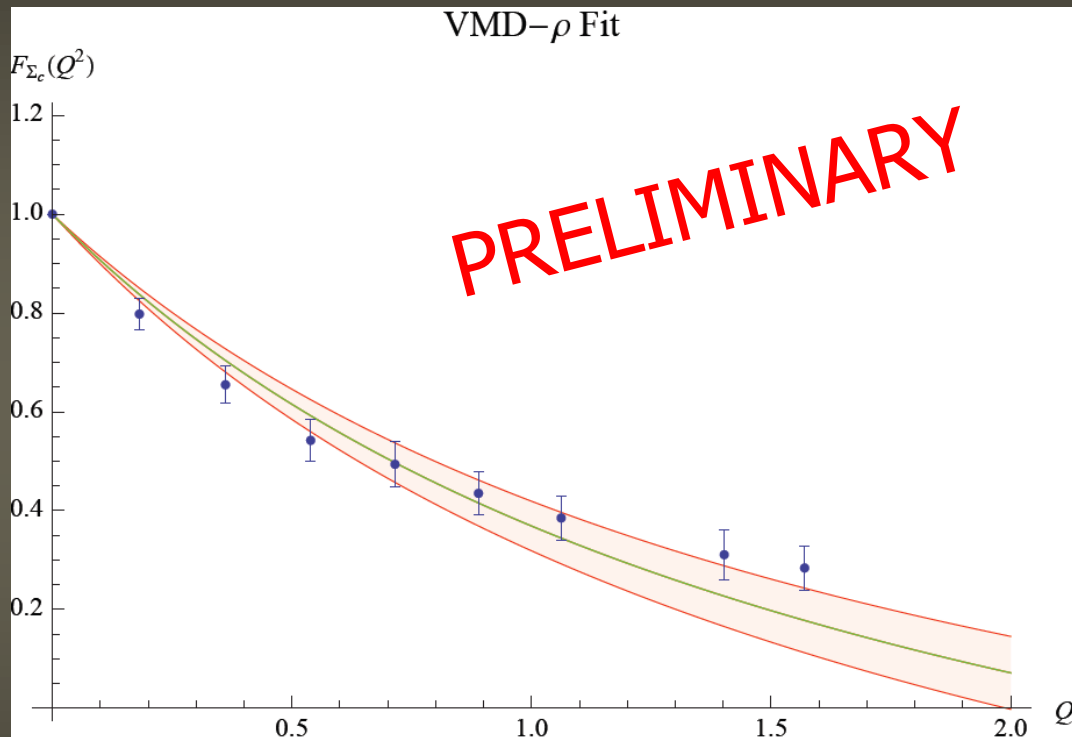
$\Sigma_c \Sigma_c$  – form factors



EM-form factors can be reproduced by monopole-ansatz well.

# Lattice QCD results (couplings)

$\Sigma_c \Sigma_c$  – form factors

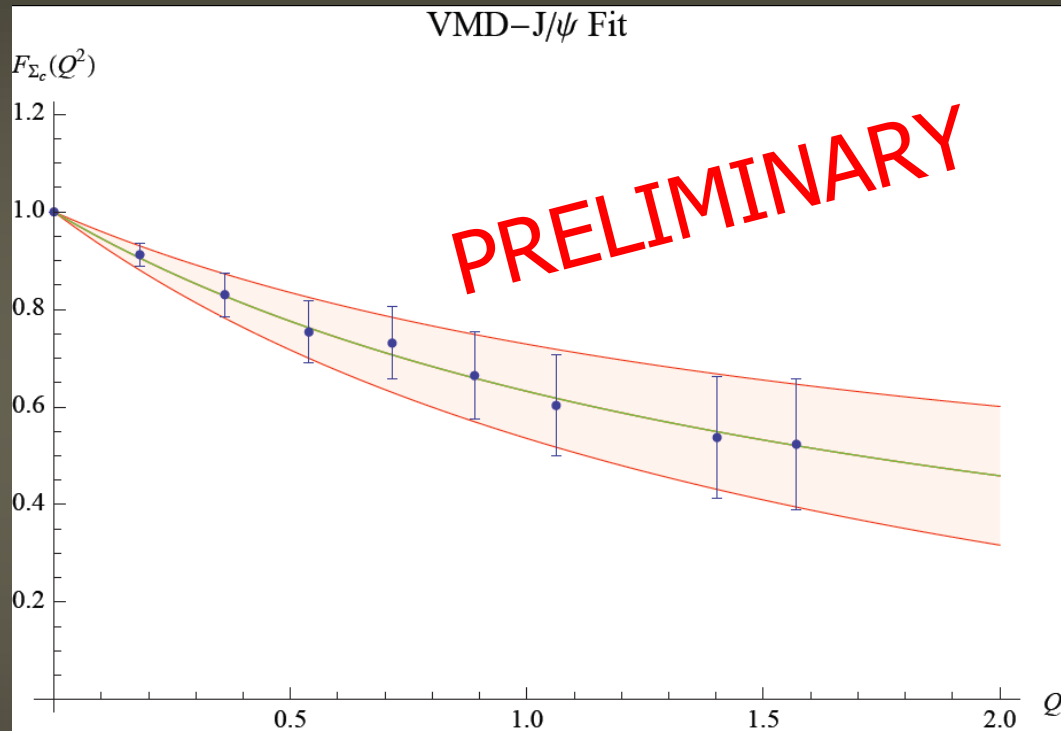


Ud-part can be reproduced by VMD-ansatz,  
but we can see small discrepancy.



# Lattice QCD results (couplings)

$\Sigma_c \Sigma_c$  – *form factors*

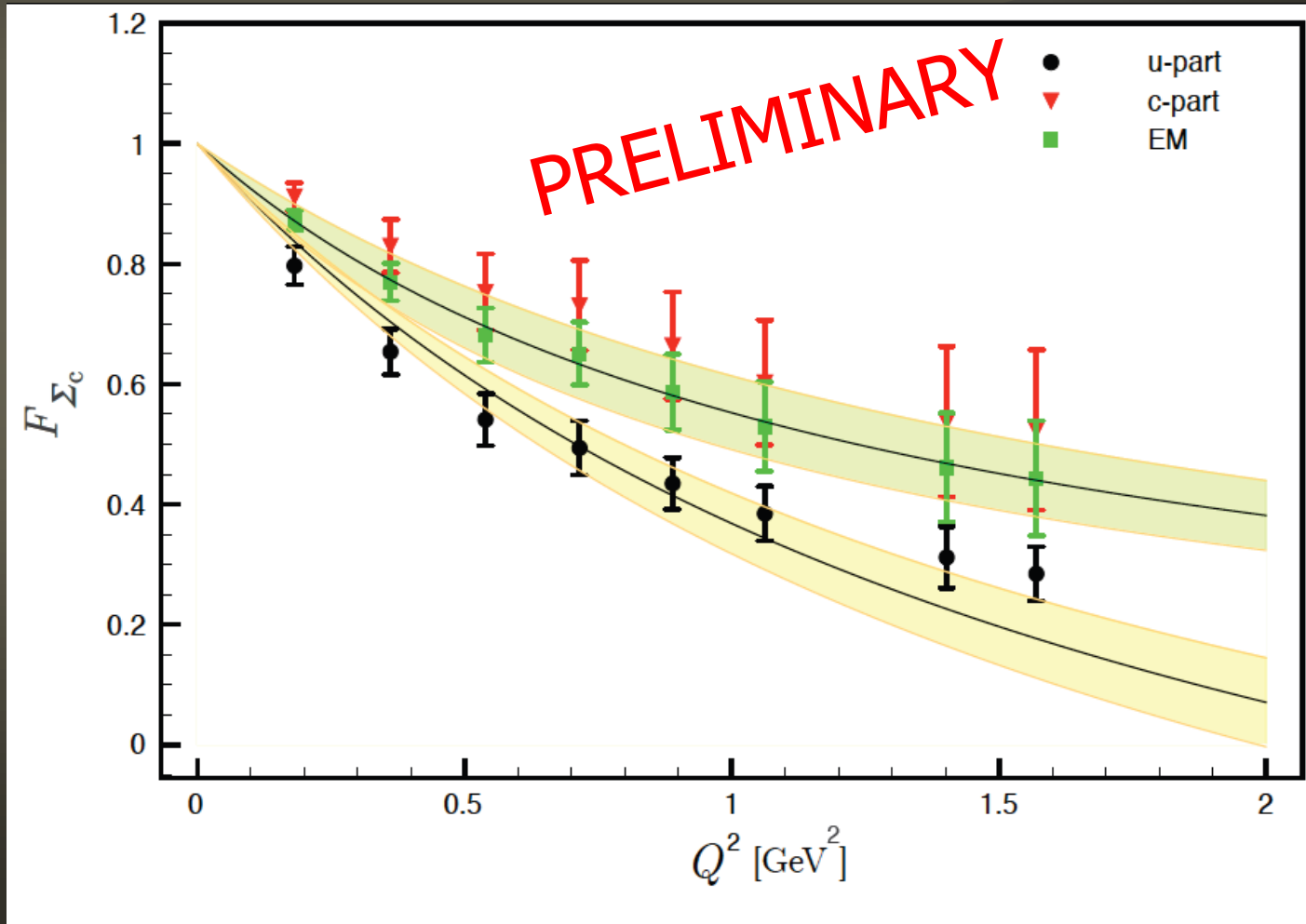


C-quark part can be reproduced by VMD-ansatz.

# Lattice QCD results (couplings)

$\Sigma_c \Sigma_c$  – form factors

All-in-one figure



# Summary

## 格子QCDでできること(?)

- ・ハドロン質量の計算
- ・散乱における波動関数 (Bethe-Salpeter amplitude) の計算
- ・ハドロン間の結合定数の計算
- ・form factorの計算
- ・ハドロン内部のクォーク分布の計算 (しかし、これはゲージやオペレータに依存)
- ・phase shift の計算 (しかし、これは少々骨が折れるかもしれない)

## チャレンジ(?)

- ・格子QCDでヘリウムの計算が行われたのなら、  
いっそ、チャーム原子核などの計算も？

