

テンソル最適化殻模型による 軽い核での核力の役割

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Outline

- **Role of V_{tensor}** in light nuclei
 - He & Li isotopes, ^8Be with V_{bare}
- Tensor Optimized Shell Model (**TOSM**) to describe tensor correlation.
- Unitary Correlation Operator Method (**UCOM**) to describe short-range correlation.

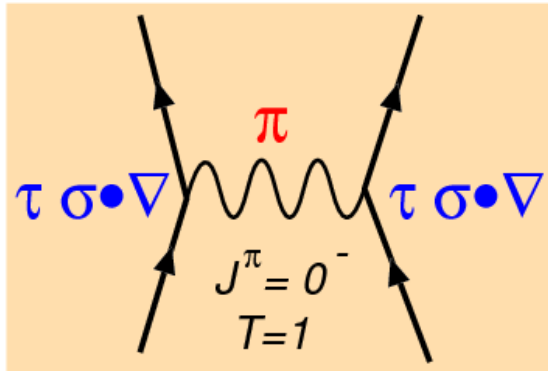
He and Li isotopes

TM, A. Umeya, H. Toki, K. Ikeda PRC84 (2011) 034315

TM, A. Umeya, H. Toki, K. Ikeda PRC86 (2012) 024318

Pion exchange interaction vs. V_{tensor}

$$3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) \frac{q^2}{m^2 + q^2} = (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \frac{q^2}{m^2 + q^2} + S_{12} \frac{q^2}{m^2 + q^2}$$



$$= (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left[\frac{m^2 + q^2}{m^2 + q^2} - \frac{m^2}{m^2 + q^2} \right] + S_{12} \frac{q^2}{m^2 + q^2}$$

δ interaction

Yukawa interaction

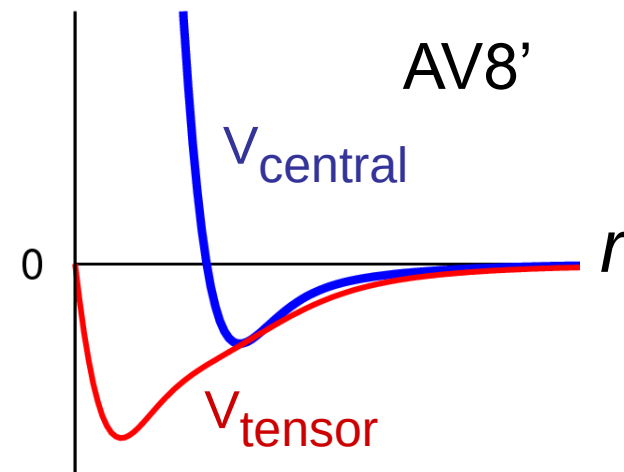
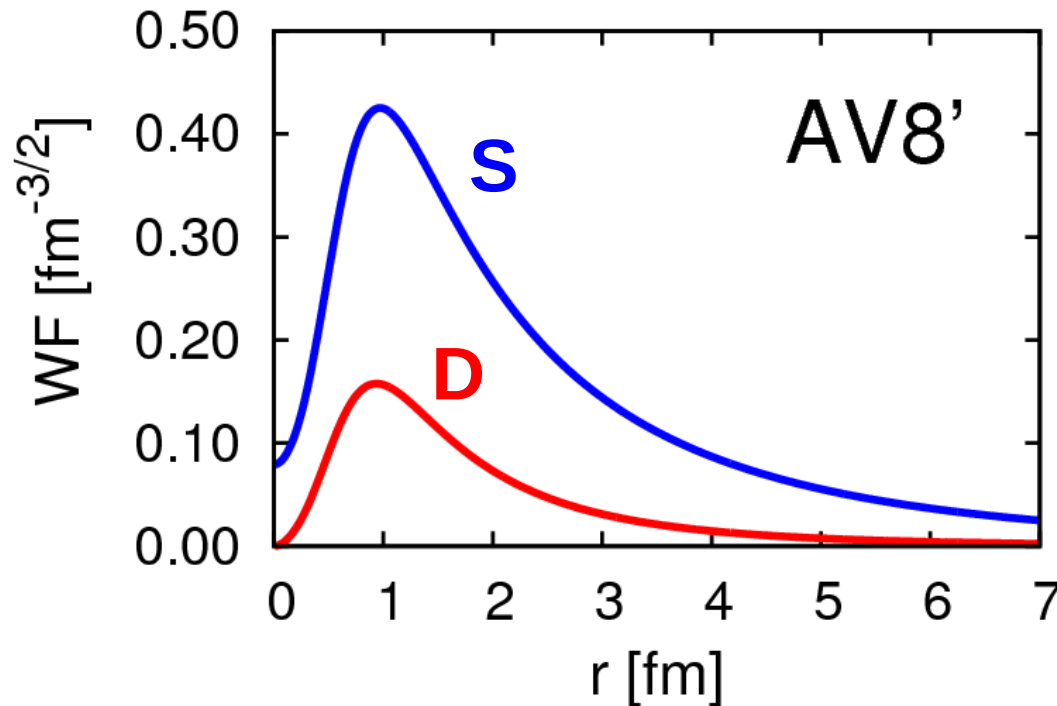
Involve large momentum

Tensor operator

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

- V_{tensor} produces the high momentum component.

Deuteron properties & tensor force



$$R_m(s) = 2.00 \text{ fm}$$

$$R_m(d) = 1.22 \text{ fm}$$

Energy	-2.24 MeV
Kinetic	19.88
Central	-4.46
Tensor	-16.64
LS	-1.02
P(L=2)	5.77%
Radius	1.96 fm

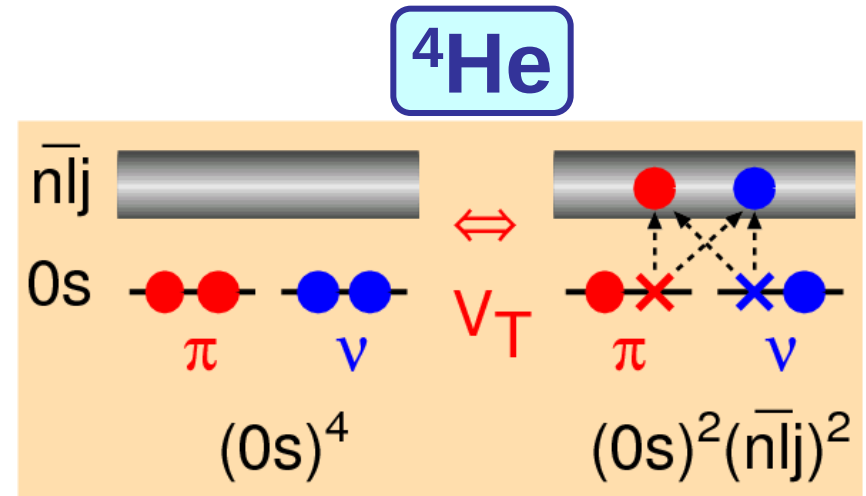
d-wave is
“spatially compact”
 (high momentum)

Tensor-optimized shell model (TOSM)

TM, Sugimoto, Kato, Toki, Ikeda PTP117(2007)257

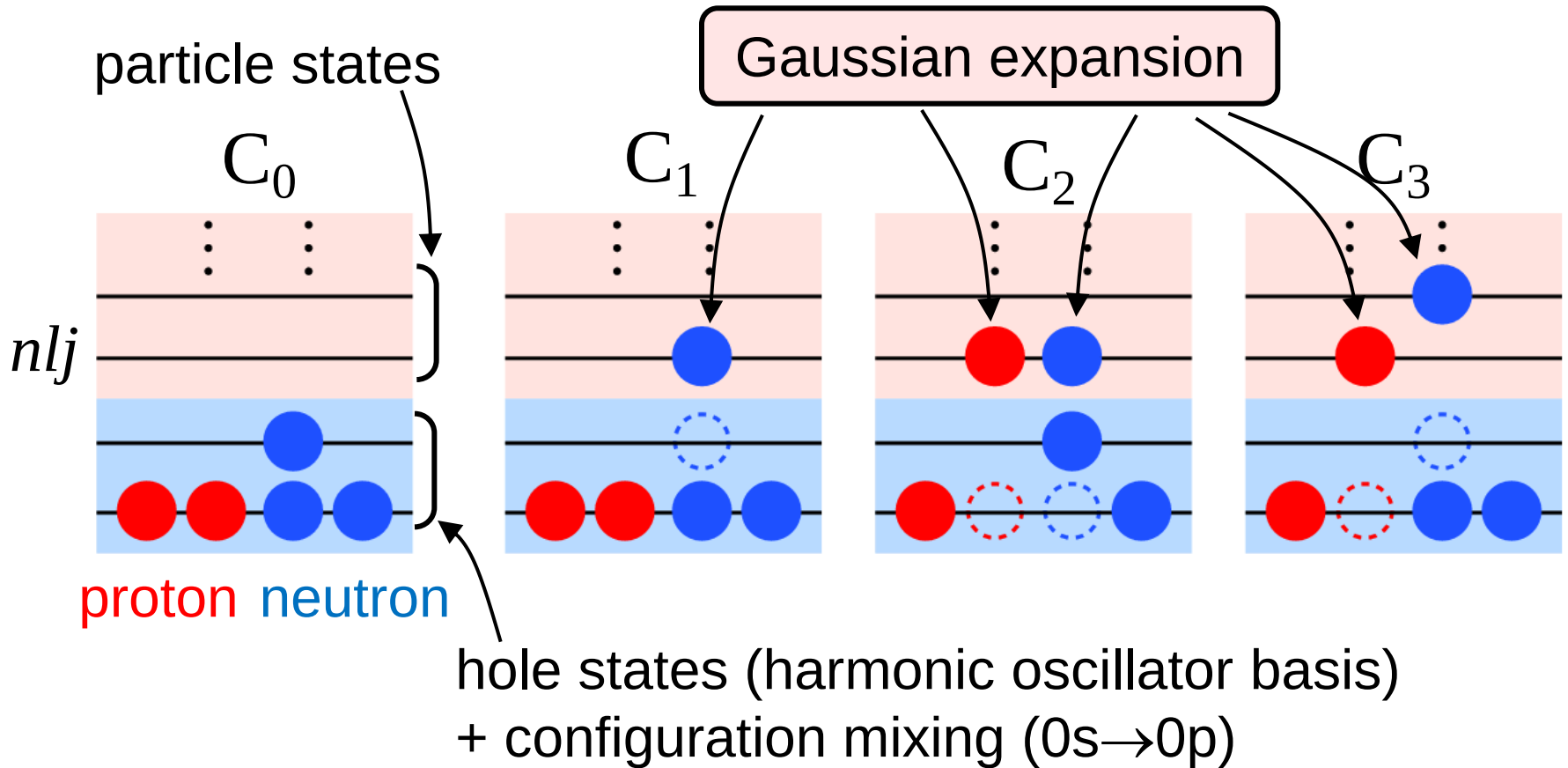
- 2p2h excitations
with high- L orbits.
- V_{tensor} is **NOT** treated as
residual interactions

cf. $\frac{V_{\pi}}{V_{NN}} \sim 80\%$ in GFMC



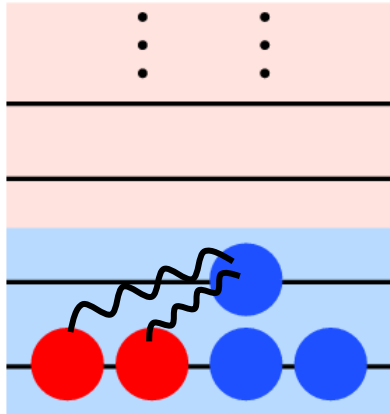
- Length parameters such as b_{0s} , b_{0p} , ... are optimized
independently, or **superposed by many Gaussian bases**.
 - **Spatial shrinkage** of **D-wave** as seen in deuteron.
HF (Sugimoto, NPA740) , RMF (Ogawa, PRC73), AMD (Dote et al., PTP115)
- Satisfy few-body results with Minnesota central force ($^4, ^6\text{He}$)

Configurations in TOSM



Application to Hypernuclei to investigate ΛN - ΣN coupling
by **Umeya** (NIT), **Hiyama** (RIKEN)

Tensor force matrix elements



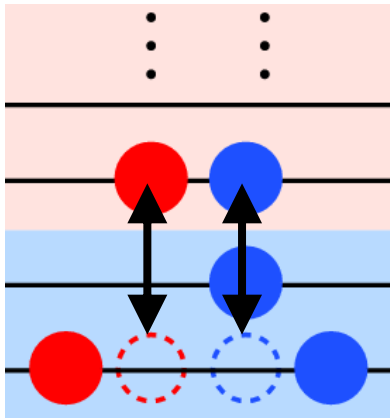
$$b_D \sim b_S$$

$$V_T = V_{\text{residual}}$$

1st order

$$M_{SD}(r) = r^2 \phi_S(r, b_S) \cdot V_T \cdot \phi_D(r, b_D)$$

: Integrand of Tensor ME

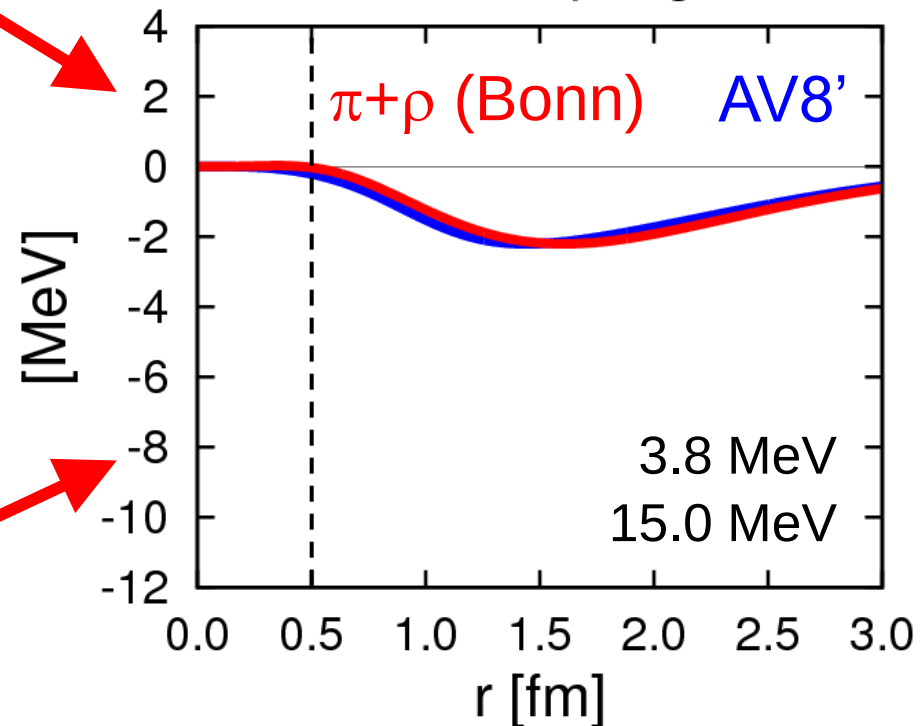


$$b_D \sim b_S \times 0.5$$

$$V_T \neq V_{\text{residual}}$$

0p0h-2p2h

SD coupling



- Centrifugal potential (1GeV@0.5fm) pushes away D-wave.

Hamiltonian and variational equations in TOSM

$$H = \sum_{i=1}^A t_i - T_G + \sum_{i<j}^A v_{ij},$$

(0p0h+1p1h+2p2h)

$$\Phi(A) = \sum_k C_k \cdot \psi_k(A)$$

Shell model type configuration
with mass number A

Particle state : Gaussian expansion for each orbit

$$\phi_{lj}^{n'}(\mathbf{r}) = \sum_{n=1}^N C_{lj,n}^{n'} \cdot \phi_{lj,n}(\mathbf{r}) \quad \phi_{lj,n}(\mathbf{r}) \propto r^l \exp\left[-\frac{1}{2}\left(\frac{r}{b_{lj,n}}\right)^2\right] [Y_l(\hat{\mathbf{r}}), \chi_{1/2}^\sigma]_j$$

$$\langle \phi_{lj}^{n'} | \phi_{lj}^{n''} \rangle = \delta_{n',n''}$$

Gaussian basis function

Hiyama, Kino, Kamimura
PPNP51(2003)223

$$\frac{\partial \langle H - E \rangle}{\partial C_k} = 0, \quad \frac{\partial \langle H - E \rangle}{\partial b_{lj,n}} = 0$$

c.m. excitation is excluded
by Lawson's method

Unitary Correlation Operator Method

(short-range part)

$$\Psi_{\text{corr.}} = \textcolor{red}{C} \cdot \Phi_{\text{uncorr.}}$$

TOSM

short-range correlator

$$C^\dagger = C^{-1} \quad (\text{Unitary trans.})$$

$$H\Psi = E\Psi \rightarrow C^\dagger H C \Phi \equiv \hat{H}\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance

$$C = \exp(-i \sum_{i < j} g_{ij}), \quad g_{ij} = \frac{1}{2} \{ \underline{p_r s(r_{ij})} + \underline{s(r_{ij}) p_r} \} \quad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

Amount of shift, variationally determined.

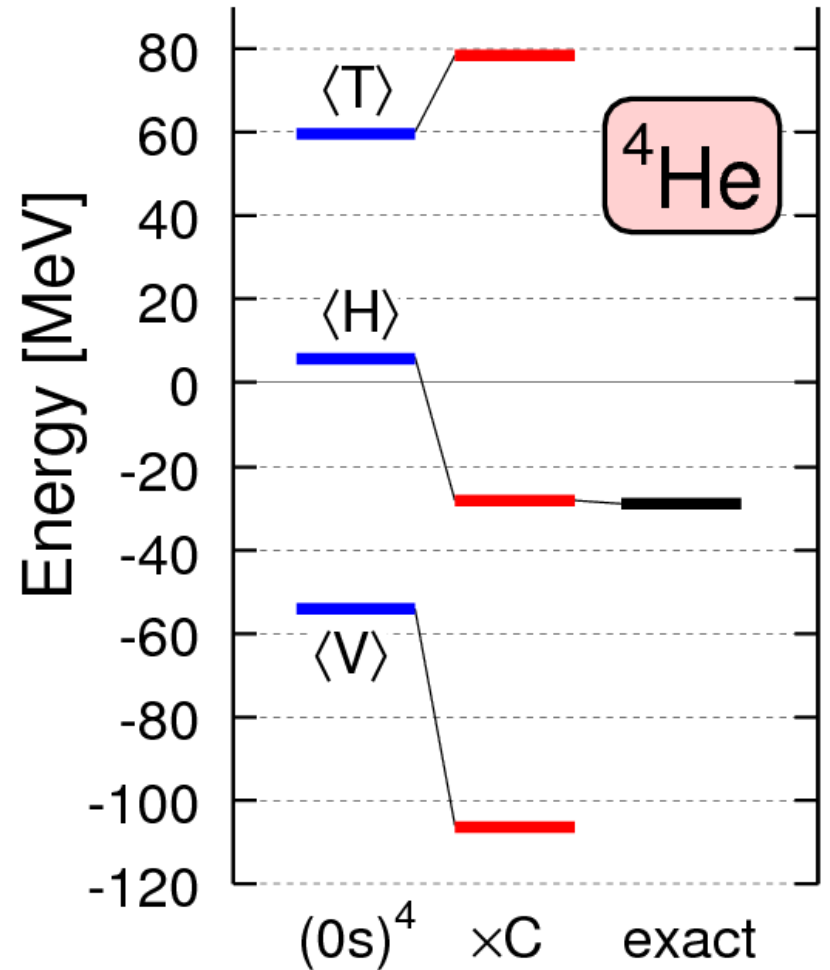
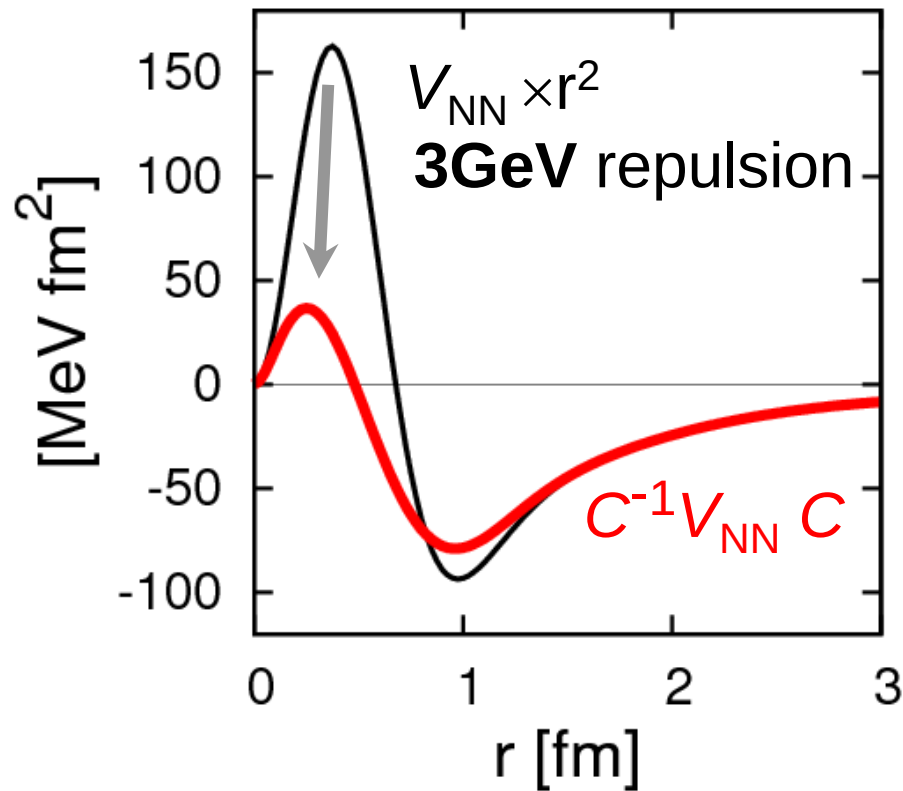
$$C^\dagger r C \simeq r + s(r) + \frac{1}{2} s(r) s'(r) \dots$$

2-body cluster expansion

Unitary Correlation Operator Method

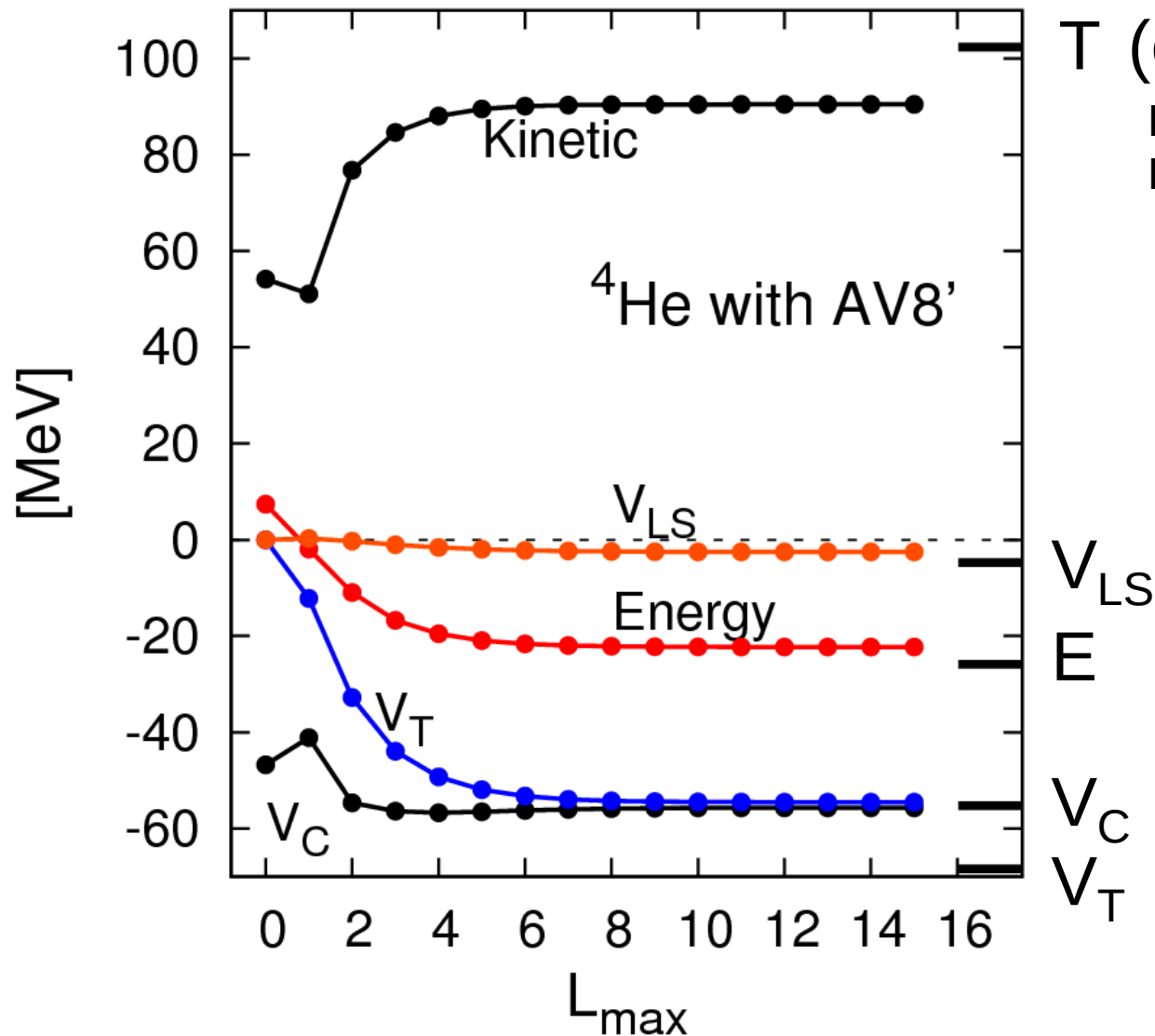
(short-range part)

C : UCOM operator



${}^4\text{He}$ in UCOM (Afnan-Tang, Central only)

^4He in TOSM + short-range UCOM



T (exact)

Kamada et al.
PRC64 (Jacobi)

TM, H. Toki, K. Ikeda
PTP121(2009)511

- variational calculation

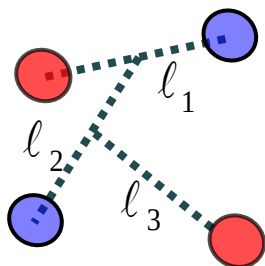
- Gaussian expansion with 9 bases

good convergence

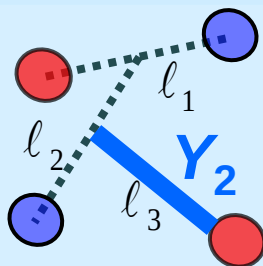
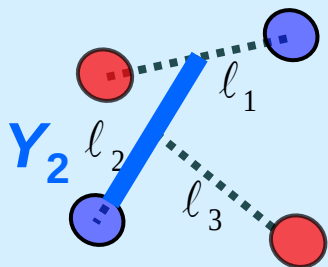
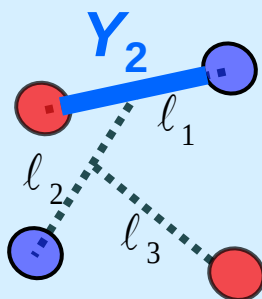
Tensor Optimized Few-body Model (TOFM)

- Same as TOSM concept
- No use of UCOM
- Correlated Gaussian basis + Global vector in SVM

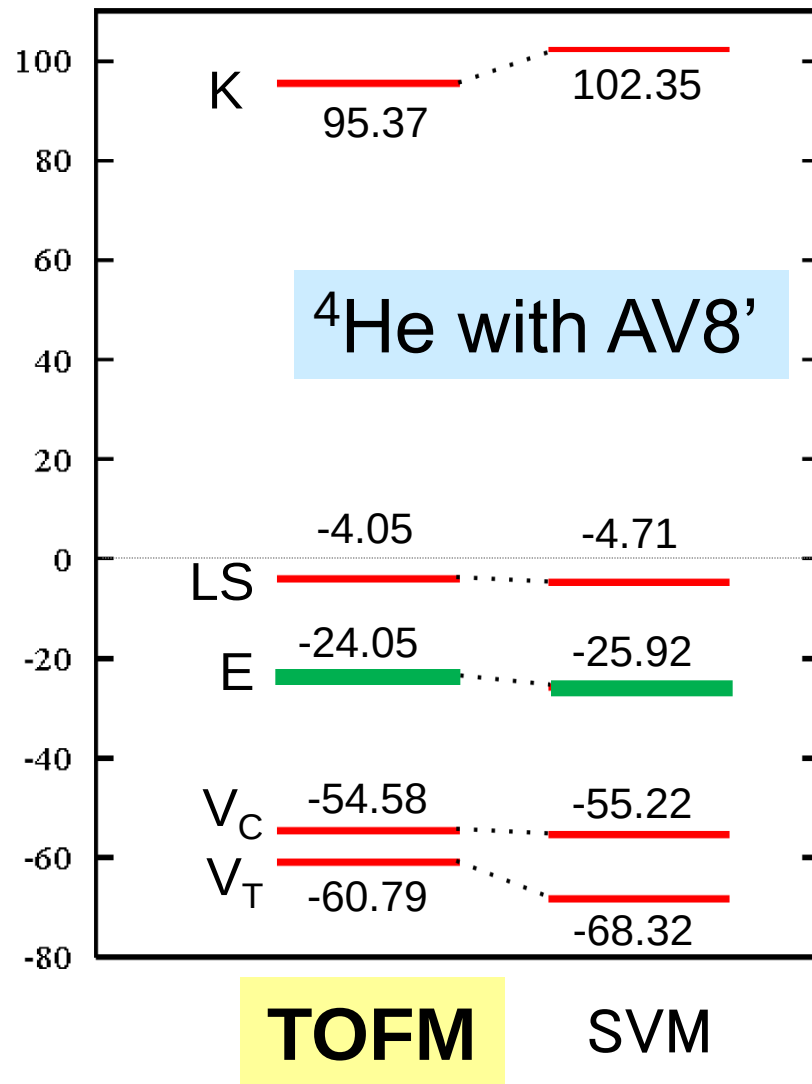
S-wave ($L=0$)



D-wave ($L=2$)



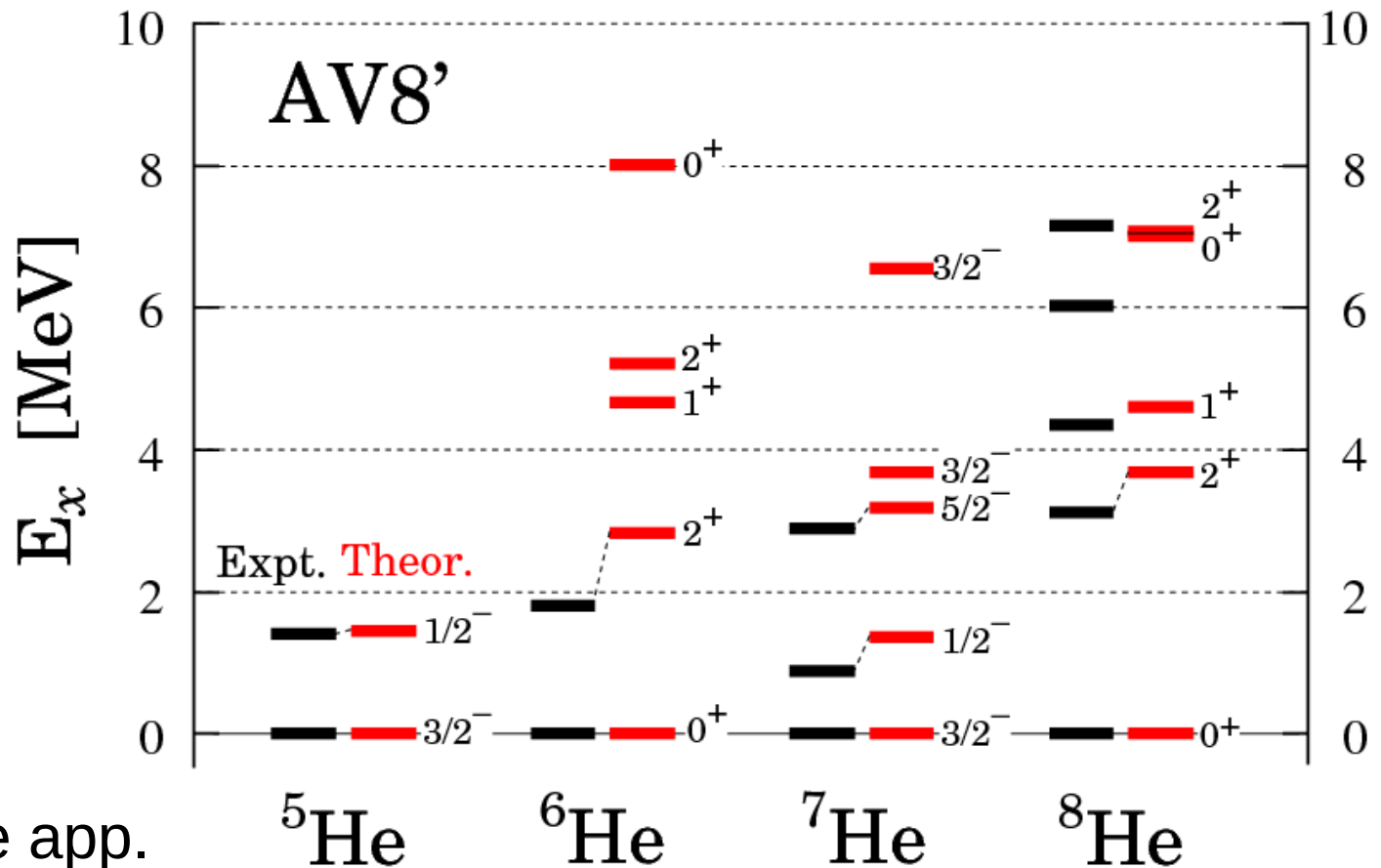
Energy [MeV]



$5\text{-}^8\text{He}$ with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda
PRC84 (2011) 034315



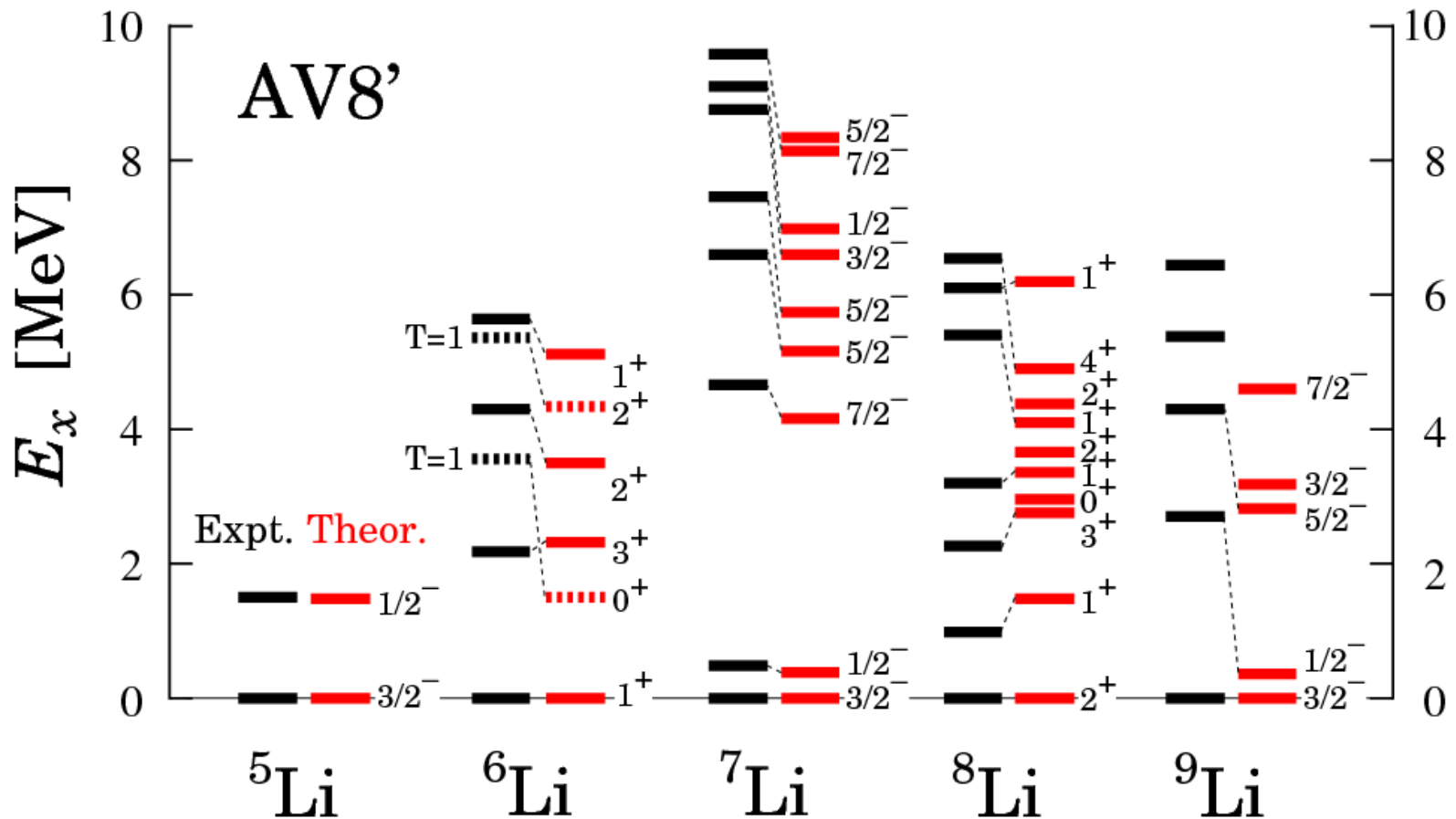
- Bound state app.
- No continuum
- No V_{NNN}

Excitation energy spectra are reproduced well

^{5-9}Li with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda
PRC86(2012) 024318

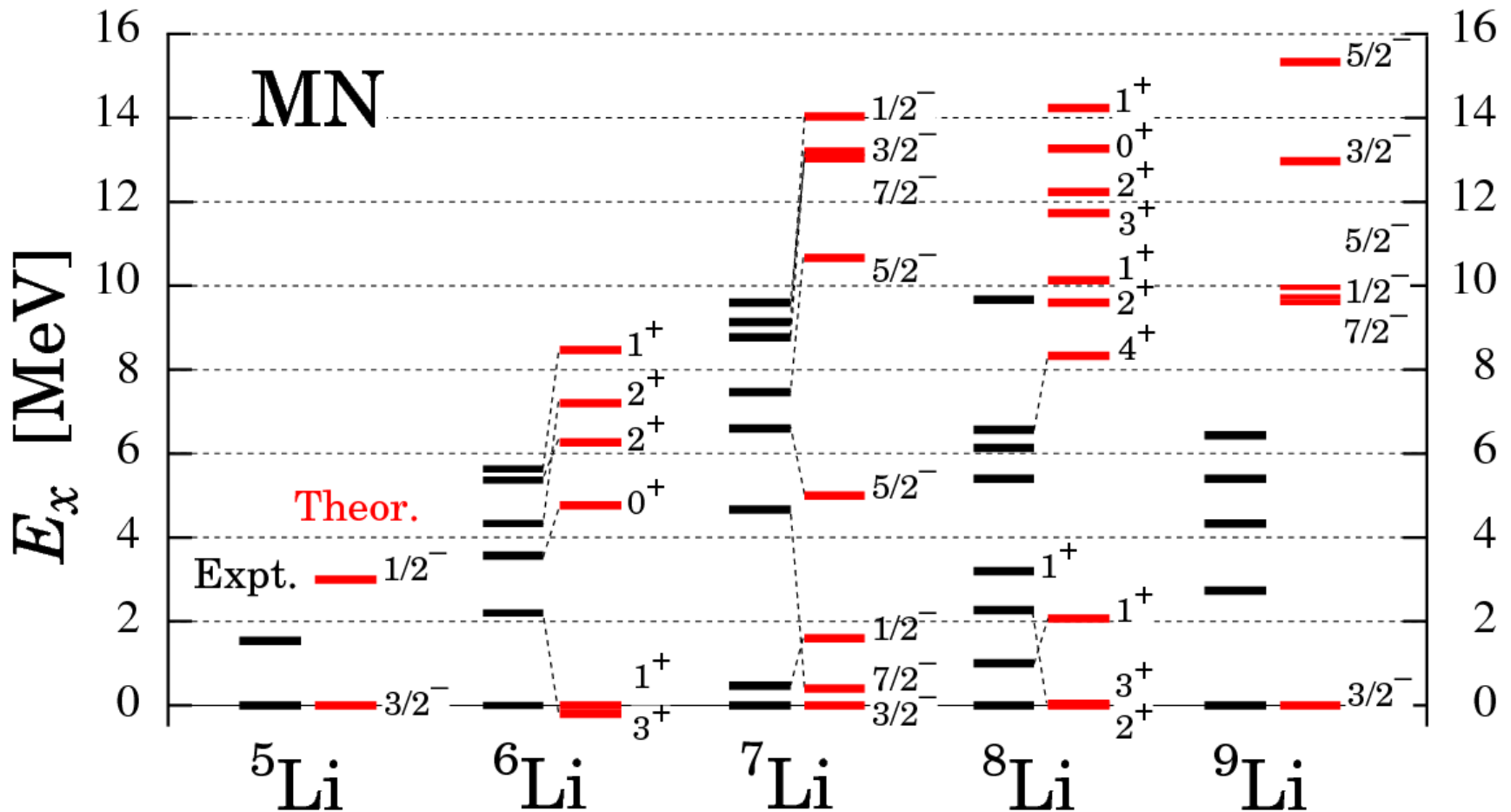


- Excitation energy spectra are reproduced well

^{5-9}Li with TOSM

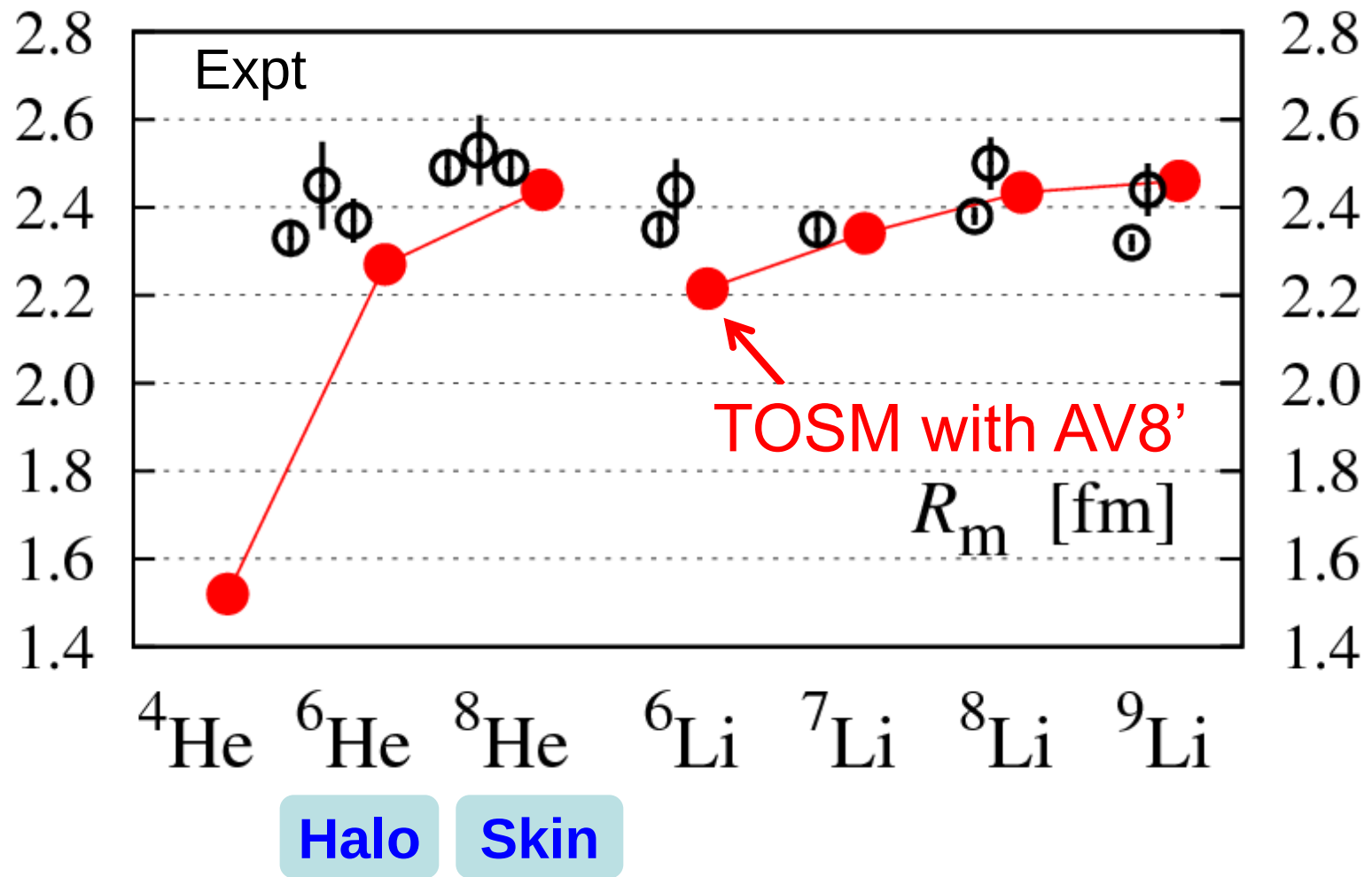
Minnesota force
NO tensor

- Excitation energies in MeV



- Too large excitation energy

Matter radius of He & Li isotopes



A. Dobrovolsky, NPA 766(2006)1
 G. D. Alkhazov et al., PRL78('97)2313
 P. Mueller et al., PRL99(2007)252501

I. Tanihata et al., PLB289('92)261
 O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.

Configurations of ${}^4\text{He}$ with $AV8'$

TM, H. Toki, K. Ikeda
PTP121(2009)511

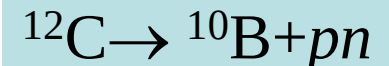
$(0s_{1/2})^4$	83.0 %
$(0s_{1/2})^{-2}_{JT}(p_{1/2})^2_{JT}$ $JT=10$	2.6 ←
$JT=01$	0.1
$(0s_{1/2})^{-2}_{10}(1s_{1/2})(d_{3/2})_{10}$	2.3
$(0s_{1/2})^{-2}_{10}(p_{3/2})(f_{5/2})_{10}$	1.9
Radius [fm]	1.54

- deuteron correlation
with $(J,T)=(1,0)$

Cf. R.Schiavilla et al. (VMC)
PRL98(2007)132501
R. Subedi et al. (JLab)
Science320(2008)1476



S.C.Simpson, J.A.Tostevin
PRC83(2011)014605

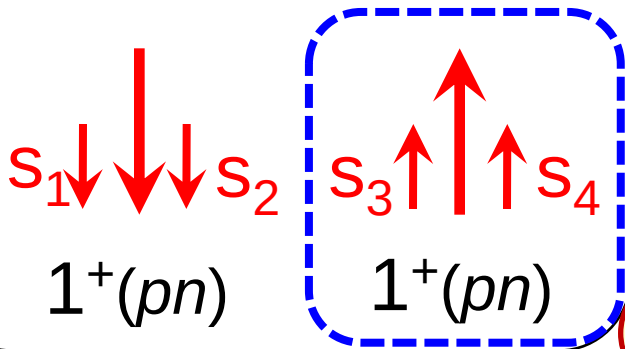


- ^4He contains $p_{1/2}$ of “ pn -pair”
 - Same feature in ^5He - ^8He ground state

Selectivity of the tensor coupling in ^4He

$$0p0h : (0s)_{00}^4 \\ \supset (0s)_{10}^2 (0s)_{10}^2$$

$$\ell_1 = \ell_2 = \ell_3 = \ell_4 = 0$$



V_T

V_T

Selectivity of S_{12}

$$\Delta L=2, \Delta S=-2$$

$$2p2h : (0s)_{10}^2 (0p_{1/2})_{10}^2$$

$$\ell_1 = \ell_2 = 0$$

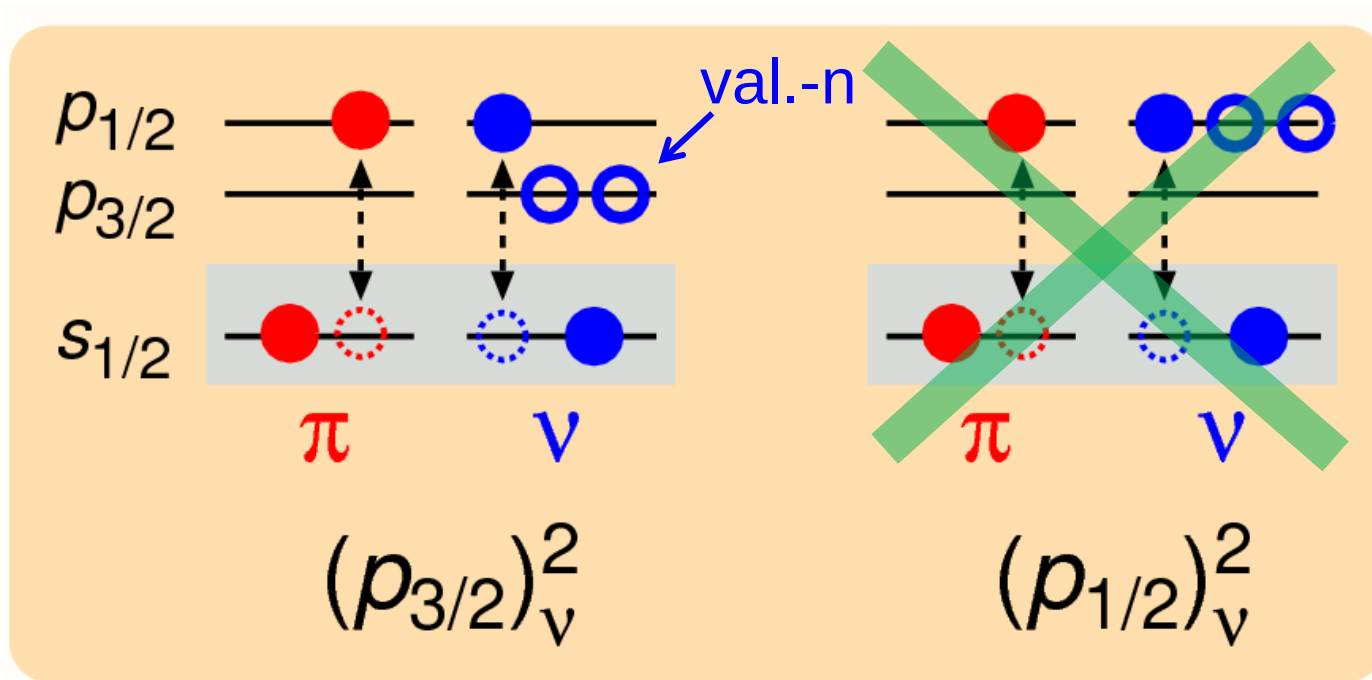


$$2p2h : (0s)_{10}^2 [(1s)(0d_{3/2})]_{10}$$

$$\ell_1 = \ell_2 = 0$$



Tensor correlation in ${}^6\text{He}$



Ground state

halo state (0^+)

Excited state

↑
Tensor correlation is **suppressed**
due to Pauli-Blocking

${}^6\text{He}$: Hamiltonian components

- Difference from ${}^4\text{He}$ in MeV

${}^6\text{He}$	0^+_1	0^+_2
n^2 config	$(p_{3/2})^2$	$(p_{1/2})^2$
$\Delta\text{Kin.}$	<u>53.0</u>	<u>34.3</u>
$\Delta\text{Central}$	-27.8	-14.1
ΔTensor	<u>-12.0</u>	<u>-0.2</u>
ΔLS	-4.0	2.1

$$b_{\text{hole}}=1.5 \text{ fm}$$

$$\hbar\omega=18.4 \text{ MeV}$$

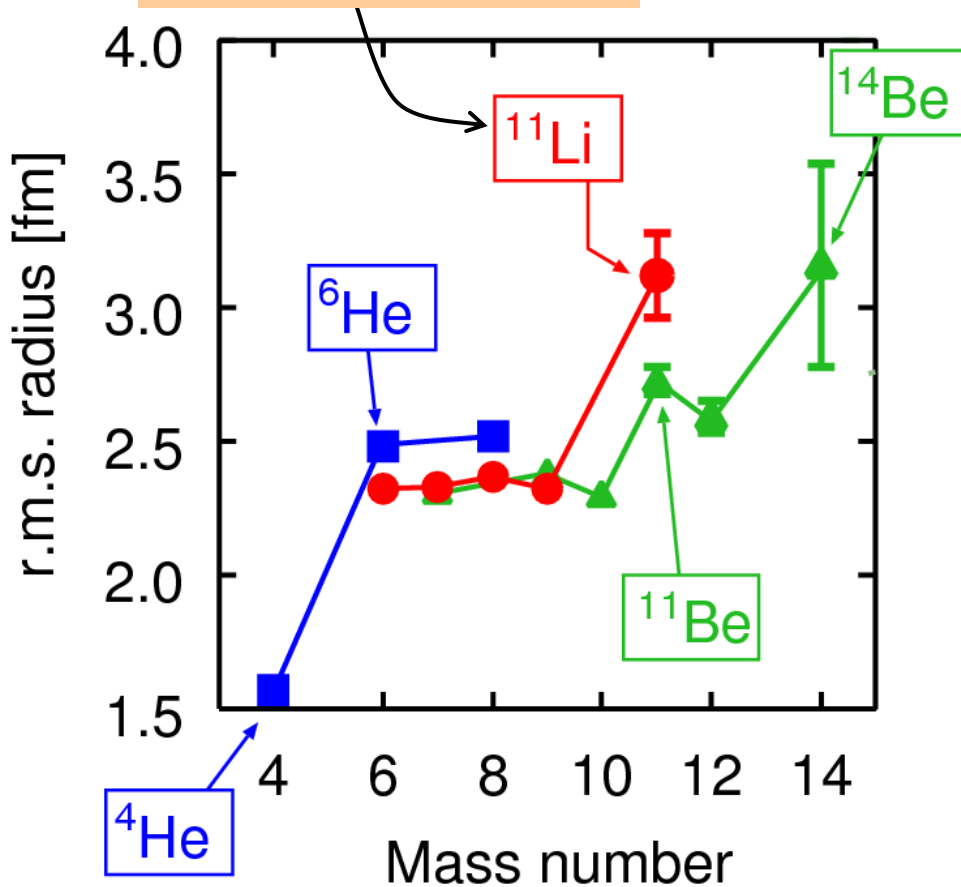
LS splitting
energy in ${}^6\text{He}$

same trend
in ${}^5\text{-}{}^8\text{He}$, ${}^{10,11}\text{Li}$

Halo formation with
large s-wave in ${}^{11}\text{Li}$

Characteristics of Li-isotopes

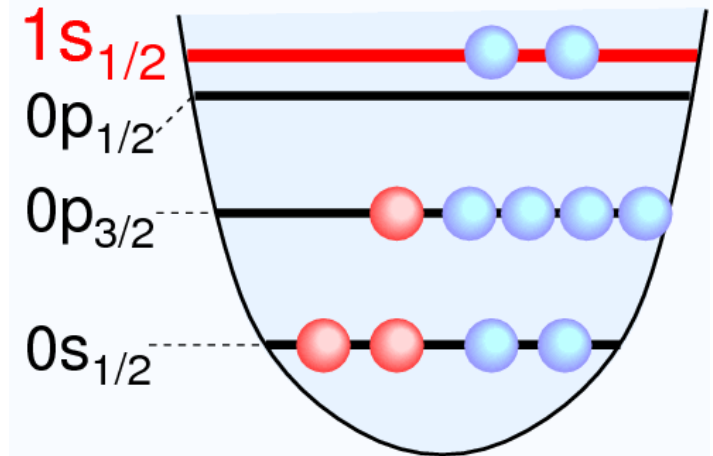
Halo structure



Tanihata et al., PRL55(1985)2676.
PLB206(1998)592.

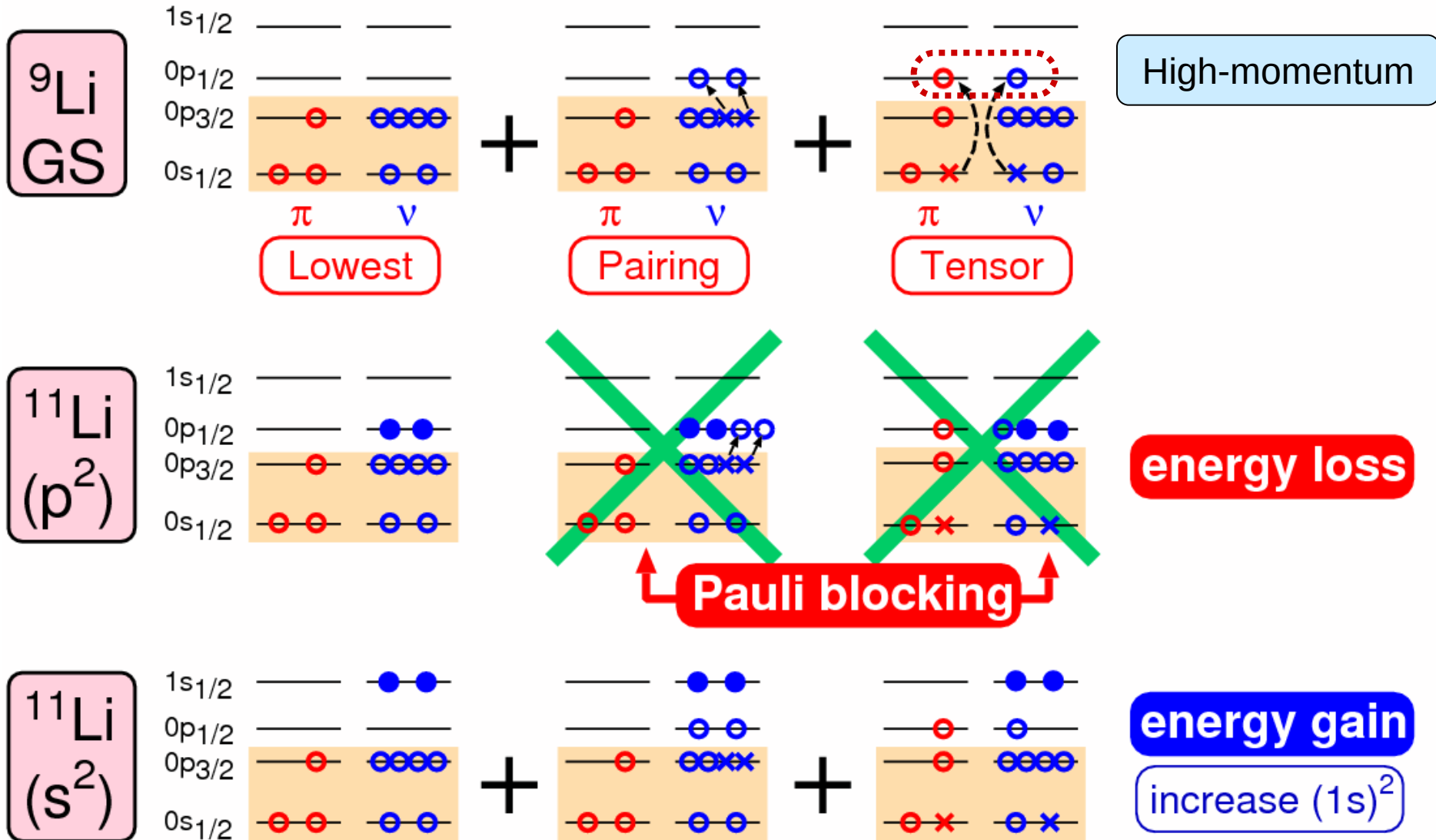
✓ Breaking of magicity $N=8$

- $^{10-11}\text{Li}$, $^{11-12}\text{Be}$
- $^{11}\text{Li} \dots (1s)^2 \sim 50\%$.
(Expt by Simon et al., PRL83)
- **Mechanism is unclear**



^{11}Li

Expected effects of pairing and tensor correlations in ^{11}Li



Pairing-blocking :

K.Kato,T.Yamada,K.Ikeda,PTP101('99)119, Masui,S.Aoyama,TM,K.Kato,K.Ikeda,NPA673('00)207. TM,S.Aoyama,K.Kato,K.Ikeda,PTP108('02)133, H.Sagawa,B.A.Brown,H.Esbensen,PLB309('93)1.

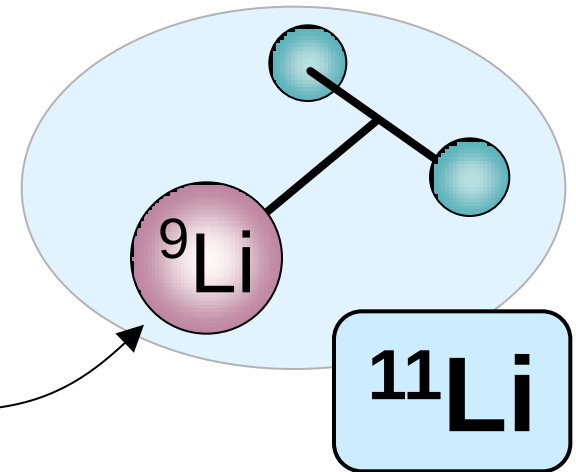
^{11}Li with $^9\text{Li}_{\text{TOSM}} + n + n$ cluster model

- System is solved based on RGM

$$H(^{11}\text{Li}) = H(^9\text{Li}) + H_{\text{rel}}(^9\text{Li}-n-n)$$

$$\Phi(^{11}\text{Li}) = \mathcal{A} \left\{ \sum_{i=1}^N \underline{\psi_i(^9\text{Li})} \cdot \chi_i(nn) \right\}$$

TOSM basis



- Orthogonality Condition Model (OCM) is applied.

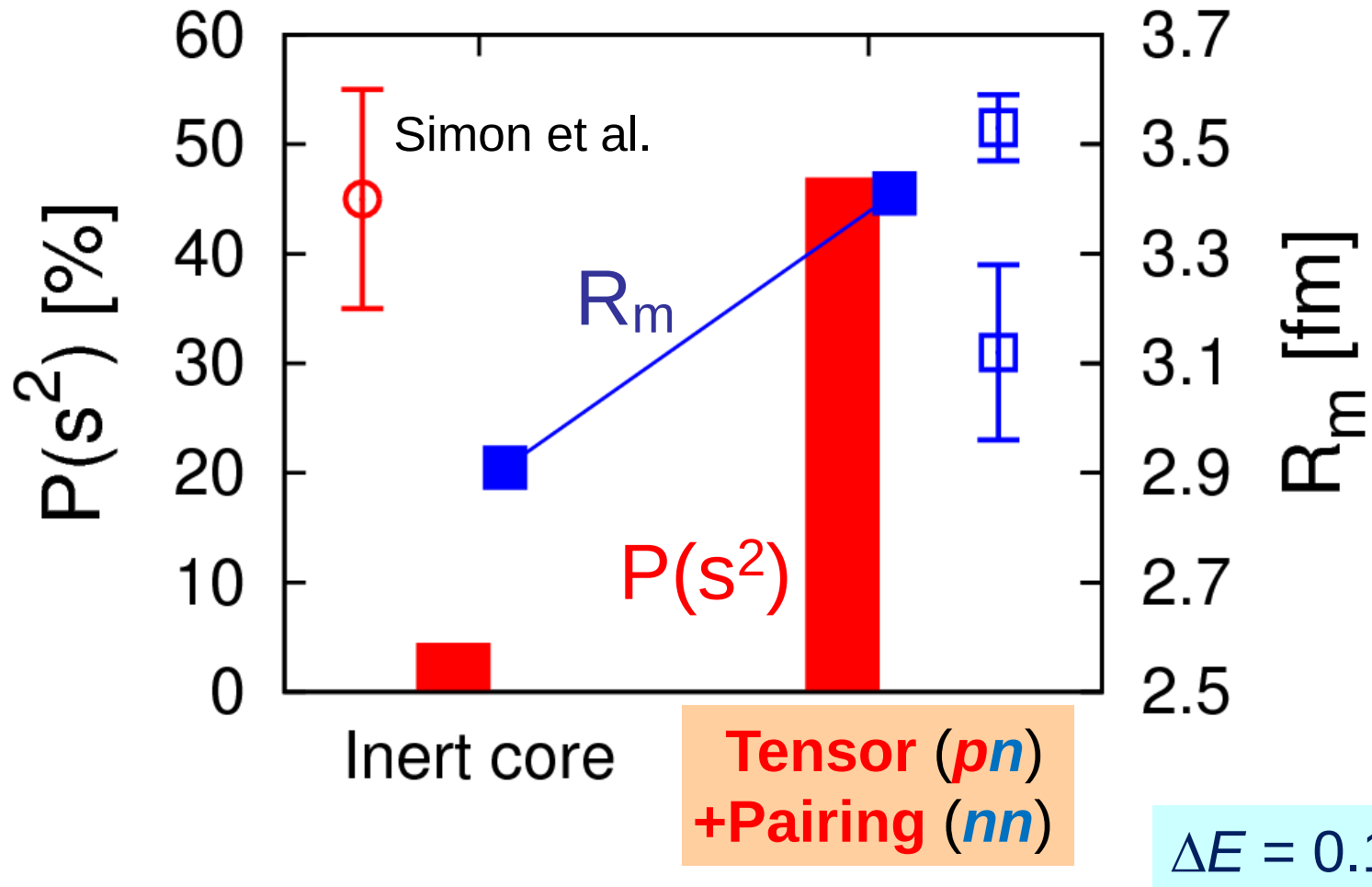
$$\sum_{i=1}^N \left[H_{ij} (^9\text{Li}) + (T_1 + T_2 + V_{c1} + V_{c2} + V_{12}) \cdot \delta_{ij} \right] \chi_j(nn) = E \chi_i(nn)$$

$$H_{ij} (^9\text{Li}) = \langle \psi_i | H(^9\text{Li}) | \psi_j \rangle : \text{Hamiltonian for } ^9\text{Li with TOSM}$$

$$\chi(nn) = \mathcal{A} \{ \phi_1 \phi_2 \} : \text{few-body method with Gaussian expansion}$$

$$\langle \phi_i | \phi_\alpha \rangle = 0, \quad \{ \phi_\alpha \in ^9\text{Li} \} : \text{Orthogonality to the Pauli-forbidden states}$$

^{11}Li properties ($S_{2n}=0.31$ MeV)

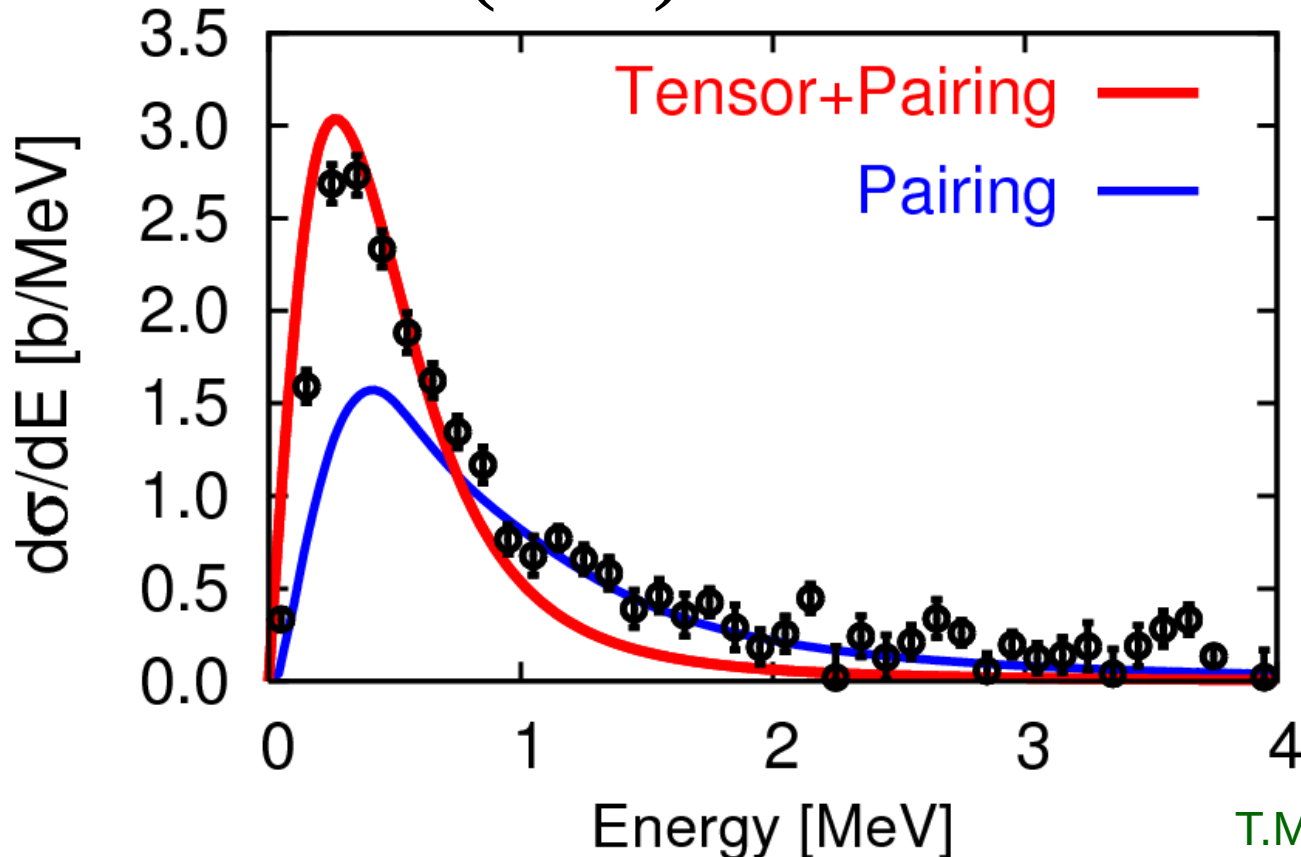
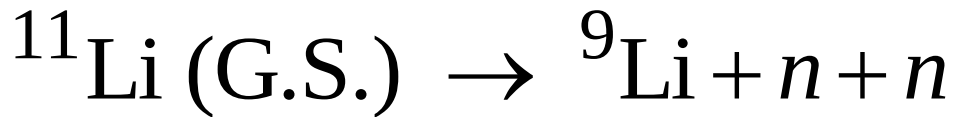


Pairing correlation between halo neutrons couples $(0p)^2$ and $(1s)^2$

TM, K.Kato, H.Toki, K.Ikeda, PRC76(2007)024305

TM, Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119(2008)561

Coulomb breakup strength of ^{11}Li



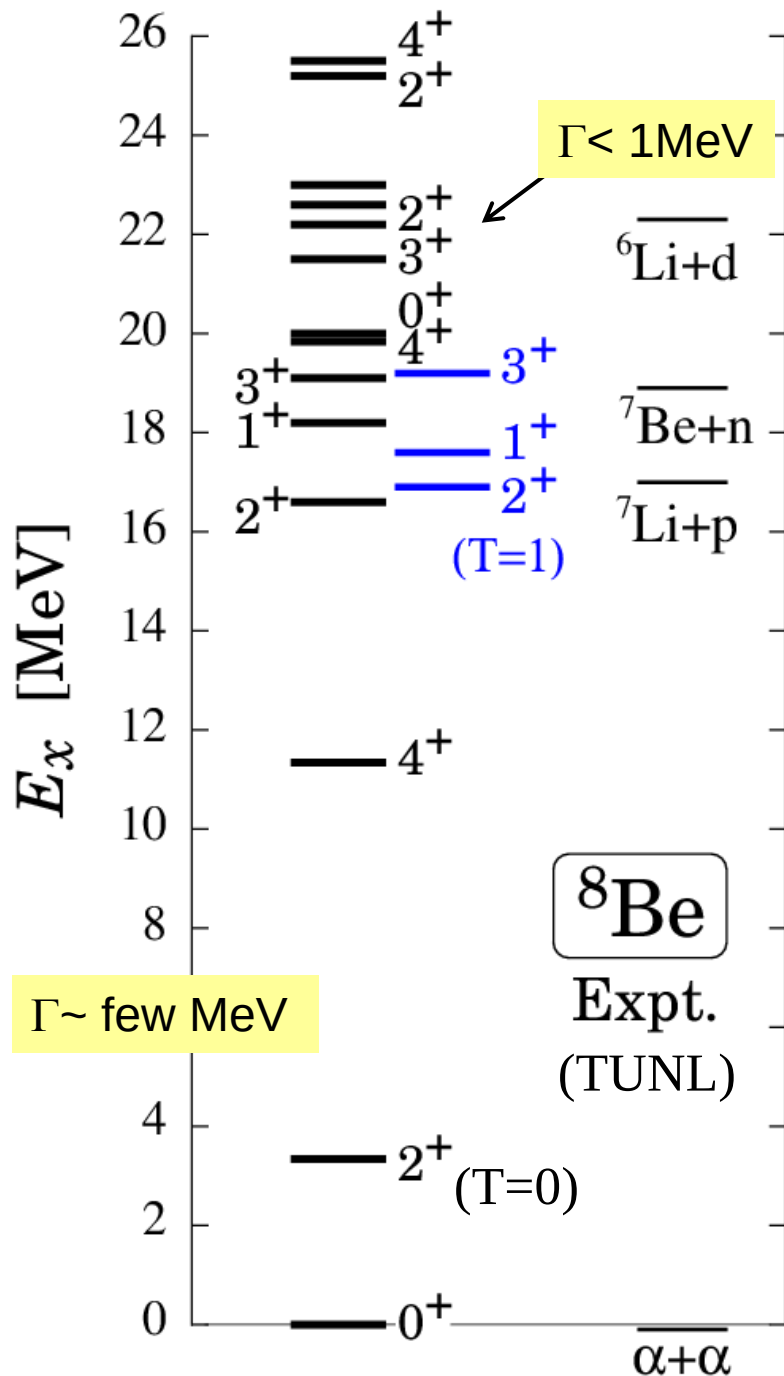
**No three-body
resonance**

E1 strength by using the
Green's function method
+Complex scaling method
+Equivalent photon method
(TM et al., PRC63('01))

T.Myo, K.Kato, H.Toki, K.Ikeda
PRC76(2007)024305

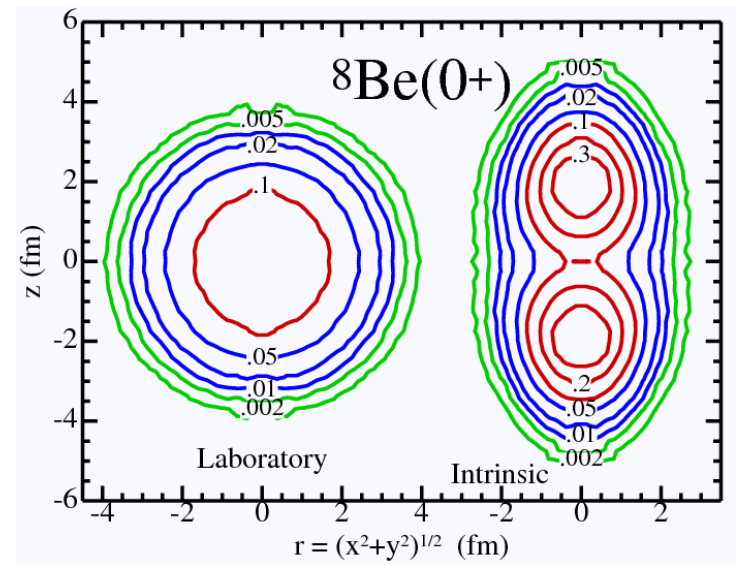
- Expt: T. Nakamura et al. , PRL96,252502(2006)
- Energy resolution with $\sqrt{E} = 0.17$ MeV.

^8Be spectrum



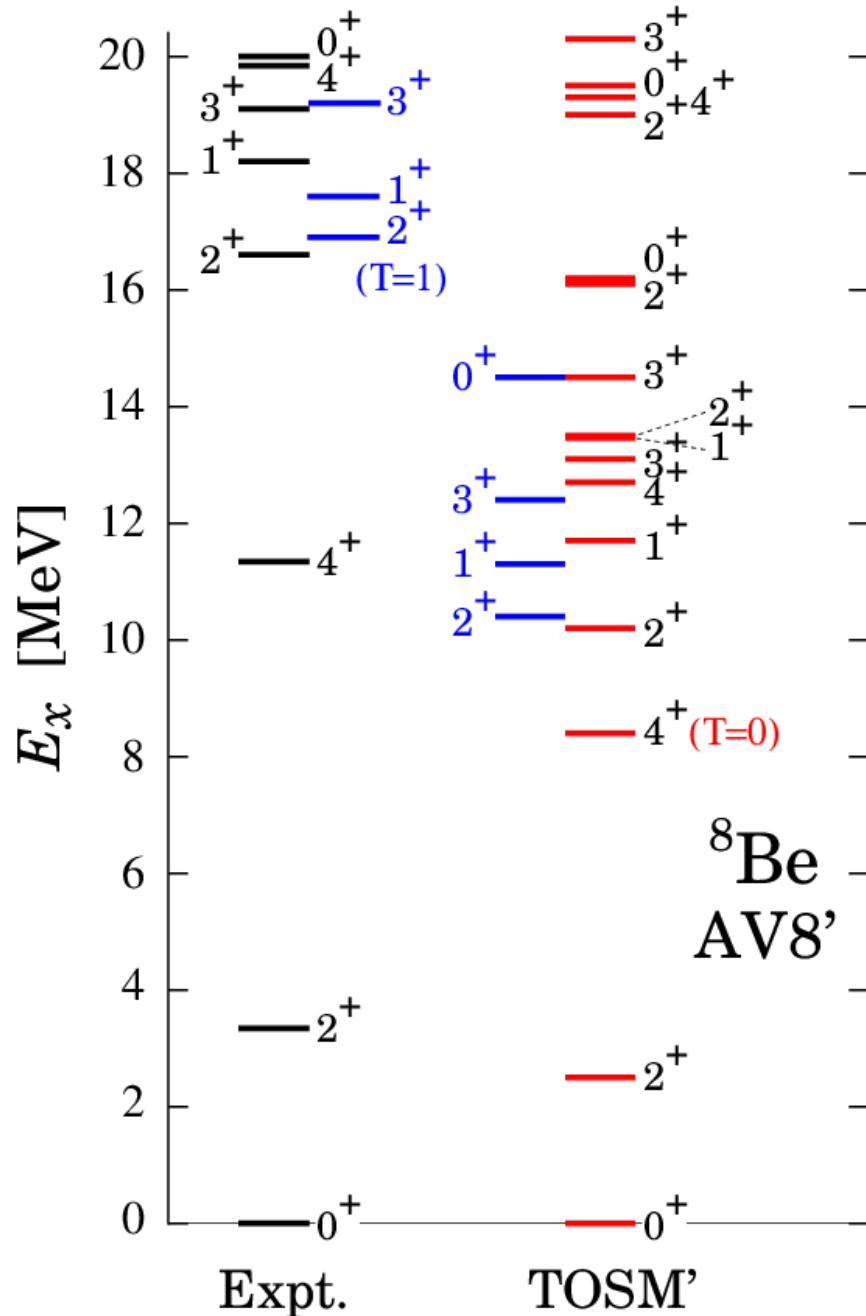
Argonne Group

- Green's function Monte Carlo
C. Pieper, R.B. Wiringa,
Annu. Rev. Nucl. Part. Sci. 51 (2001)



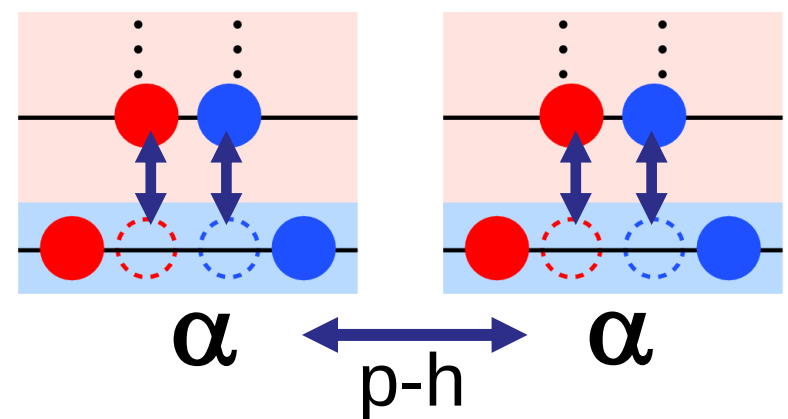
α - α structure

^8Be in TOSM – AV8' –



- $V_T \times 1.1, V_{LS} \times 1.4$
 - simulate ^4He benchmark (Kamada et al., PRC64)
- ground band
- highly excited states
 - small E_x
 - correct level order (T=0,1)
- $R_m(^8\text{Be})=2.21$ fm
 - Brink 2α model: 2.48 fm
 - ^4He :1.52 fm
 - ^{12}C :2.35 fm

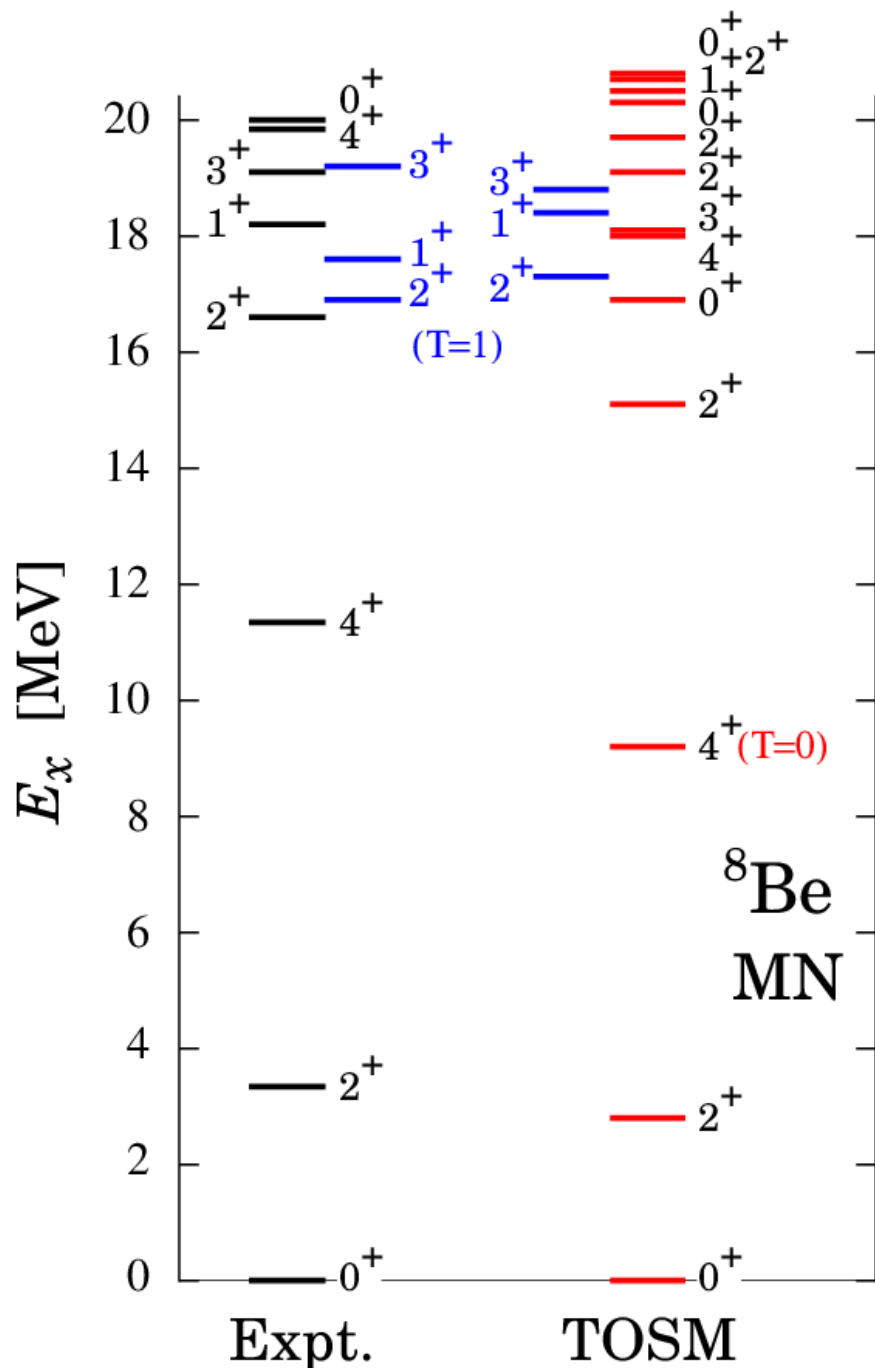
- $V_T \times 1.1, V_{LS} \times 1.4$
 - simulate ^4He benchmark (Kamada et al., PRC64)
- α : 0p0h+2p2h with high- k
 - naively 2α needs 4p4h.



Hamiltonian components in ^8Be

State		Kinetic	Central	Tensor
^4He		95	-56	-62
^8Be	0^+_1	192	-115	-97
	2^+_1	191	-112	-95
	2^+_2	185	-98	-92
	$2^+_{T=1}$	168	-94	-82

- Grand state
 - Kinetic & Central
~ twice of ^4He
 - Tensor ~ 1.6 of ^4He
 - larger $\langle H \rangle$ components than highly excited states.
- Kinetic & Tensor
 - $T=0$ states > $T=1$ states

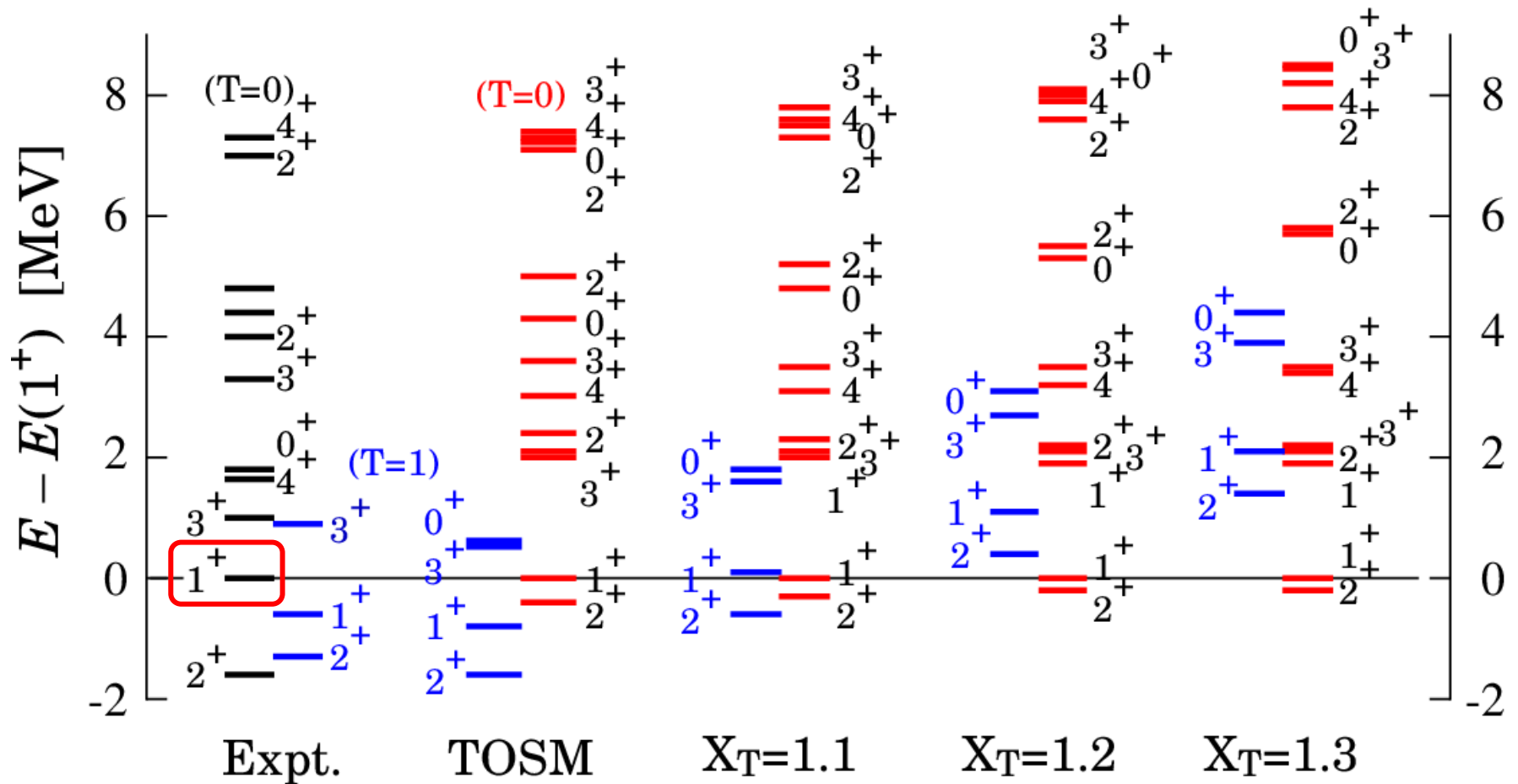


^8Be in TOSM

– Minnesota –

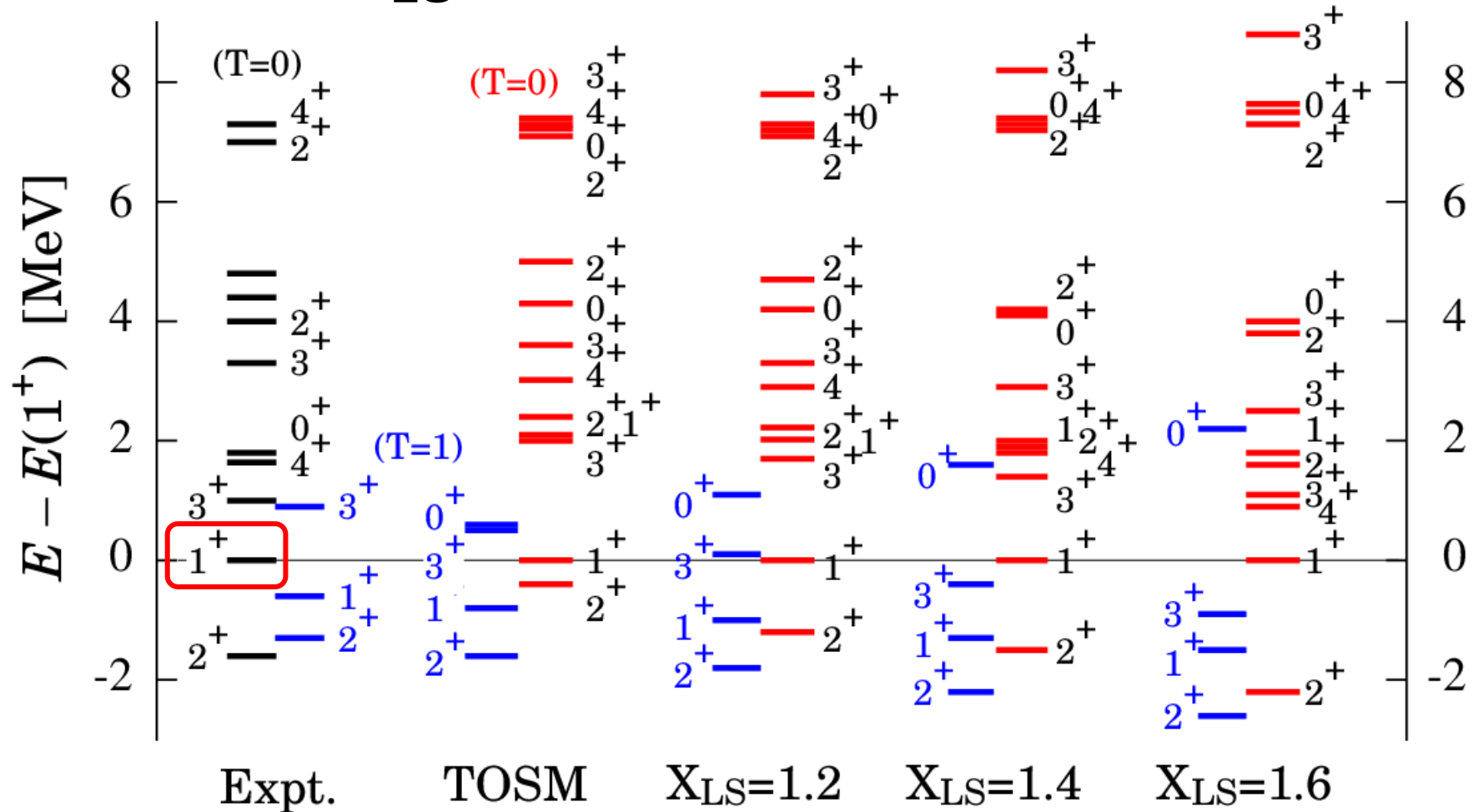
- ground band
- highly excited states
 - good E_x
 - incorrect level order ($0^+, 1^+, 3^+$)
 - $E_x(T=0) < E_x(T=1)$
- Radius (R_m) is small
 - ^4He 1.39 fm
 - ^8Be 1.89 fm
 - ^{12}C 1.85 fm

V_{tensor} dependence of ${}^8\text{Be}$



- S-wave UCOM can be simulated with $X_T \sim 1.1$ (PTP121)
- Stronger tensor correlation in **T=0 states** than T=1 states.

V_{LS} dependence of ${}^8\text{Be}$



- UCOM underestimates $\langle LS \rangle$ due to the lack of three-body term.
- Spectrum gets wider (2^+ , 1^+ , 3^+), same trend in $T=0$ & $T=1$.

Summary

- **TOSM+UCOM** using V_{bare} .
 - Strong tensor correlation from 0p0h-2p2h.
- He & Li isotopes
 - Energy spectra, Radius
 - ^4He contains “ **pn -pair of $p_{1/2}$** ” due to V_T .
- ^8Be , $T=0$ & $T=1$.
 - Two aspects : Grand band states & highly excited states.
 - Indication of more configurations such as 4p4h to describe 2α structure in the grand band states.