Structure of near-threshold s-wave resonances

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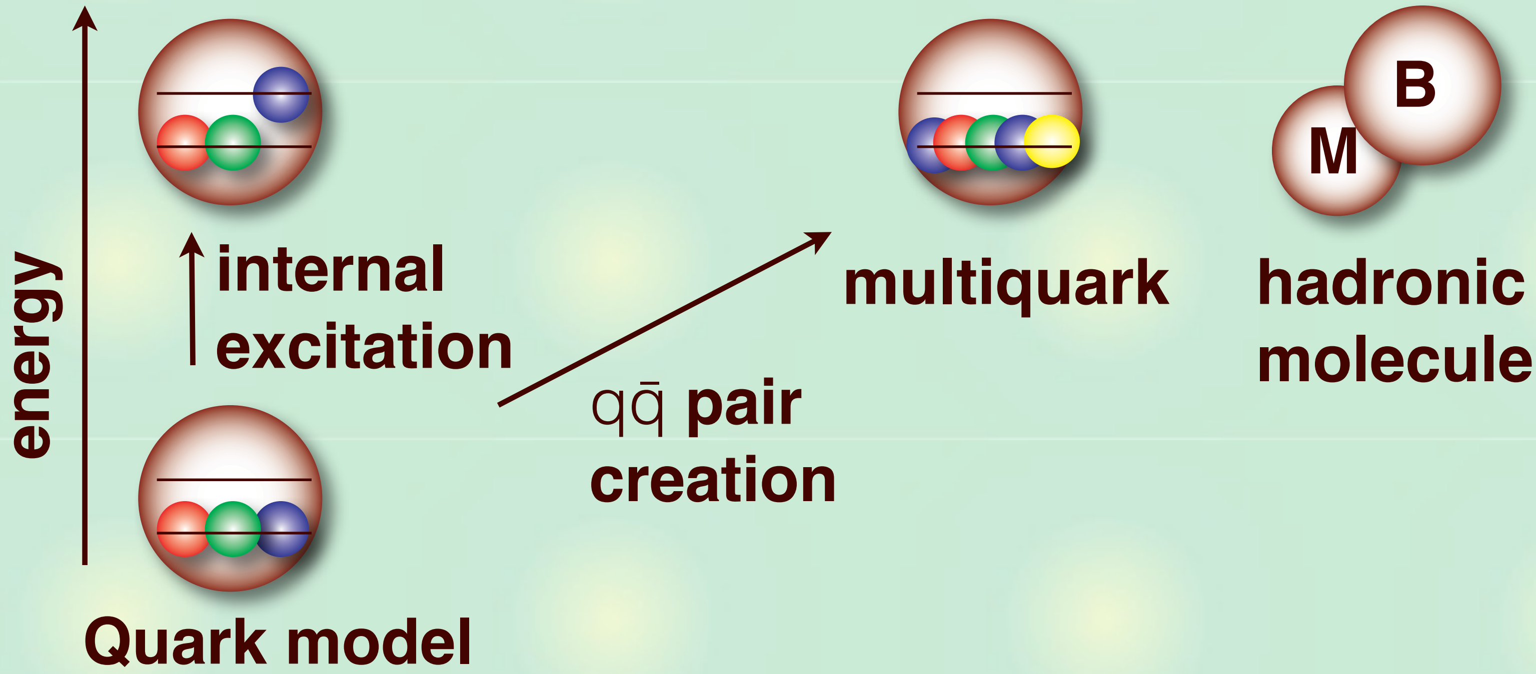
Structure of hadron excited states

Various excitations of baryons



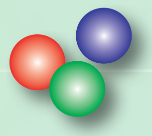
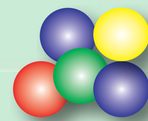
What are 3q state, 5q state, MB state, ...?

- Comparison of data (spectrum, width,...) with quark **models**
- Analysis of scattering data by dynamical **models**

Clear (model-independent) definition of the structure?

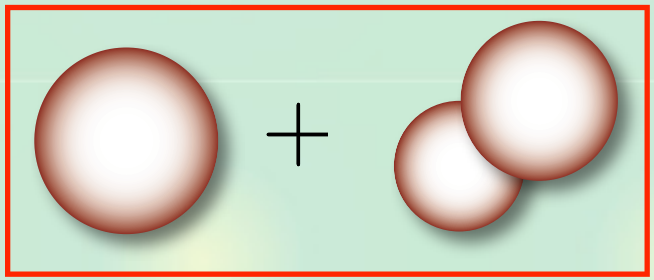
Difficulty 1 : definition and model space

Number of quarks + antiquarks (≠ quark number) ?

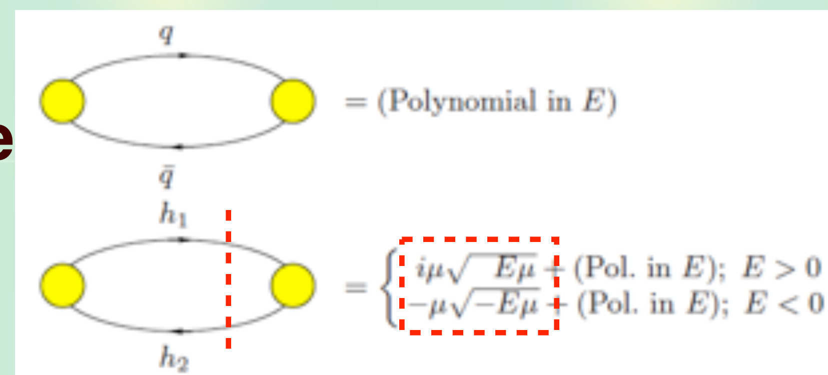
$|\Lambda(1405)\rangle =$  $+$  $+ \dots$

This may not be a good classification scheme.

Number of hadrons

$|\Lambda(1405)\rangle =$  $+ \dots$

Hadrons are asymptotic states.
--> different kinematical structure



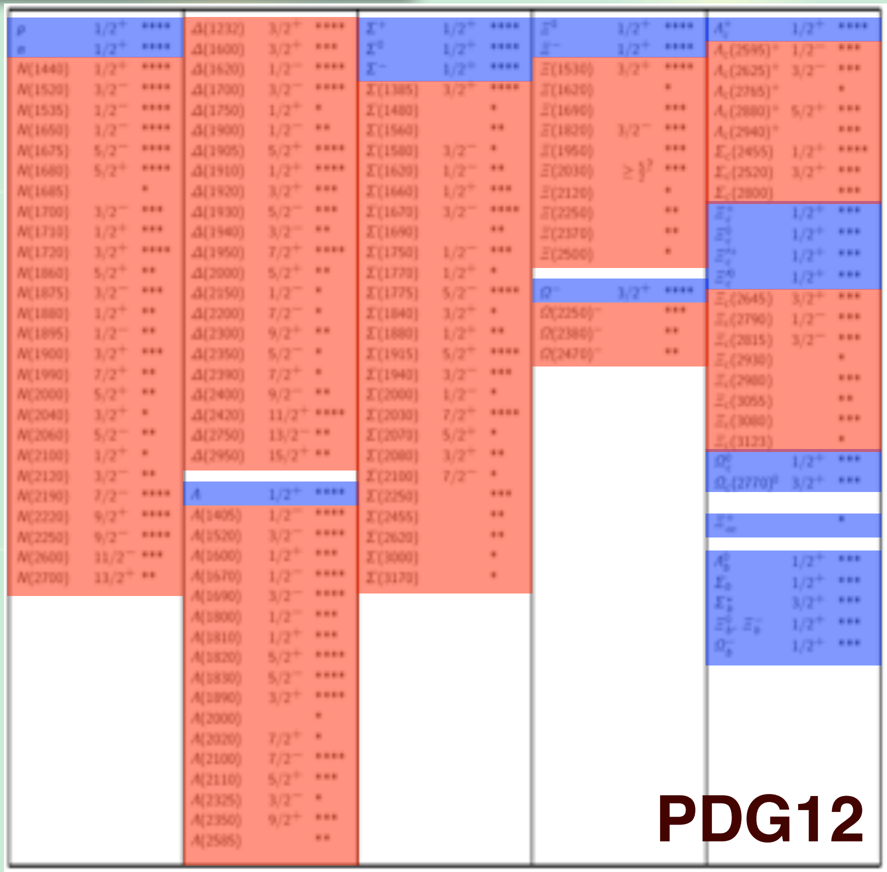
C. Hanhart, Eur. Phys. J. A 35, 271 (2008)

--> compositeness in terms of hadronic degrees of freedom

Difficulty 2 : resonances

Excited states : finite width (unstable against strong decay)

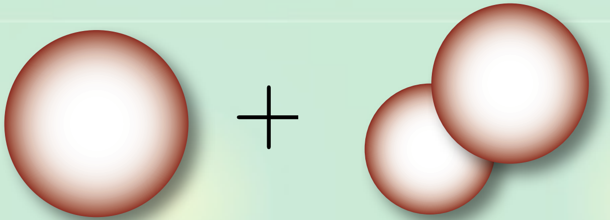
- **stable (ground) states**
- **unstable states**



Mostly resonances!

"Wave function" of resonance?

$$|\,\Lambda(1405)\,\rangle = \quad + \quad + \ldots$$



↑?

--> First consider stable states, then extend it to resonances.

Contents

- Introduction: ideal strategy
  - Model independent approach
  - Hadronic degrees of freedom
  - Extension to resonances
- Field renormalization constant Z

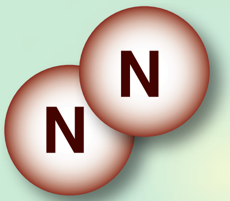
  S. Weinberg, Phys. Rev. 137, B672 (1965)
- Application to near-threshold resonances

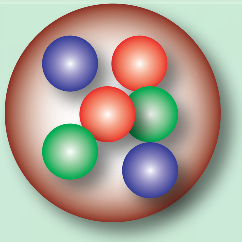
  T. Hyodo, arXiv:1305.1999 [hep-ph]
- Summary

Compositeness of the deuteron

Z: probability of finding deuteron in a bare elementary state

S. Weinberg, Phys. Rev. 137, B672 (1965)

$|\mathrm{deuteron}\rangle =$ 

or 

$\notin$ NN **model space**

~ elementary particle

Z = 0 ⟷ Z = 1

Model-independent relation for a shallow bound state

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

$a_s \sim 5.41$ [fm] **: scattering length**

$r_e \sim 1.75$ [fm] **: effective range**

$R \sim (2\mu B)^{-1/2} \sim 4.31$ [fm] **: deuteron radius (binding energy)**

--> $Z \lesssim 0.2$: Deuteron is almost composite!

Compositeness in quantum mechanics

Hamiltonian of a single channel scattering system

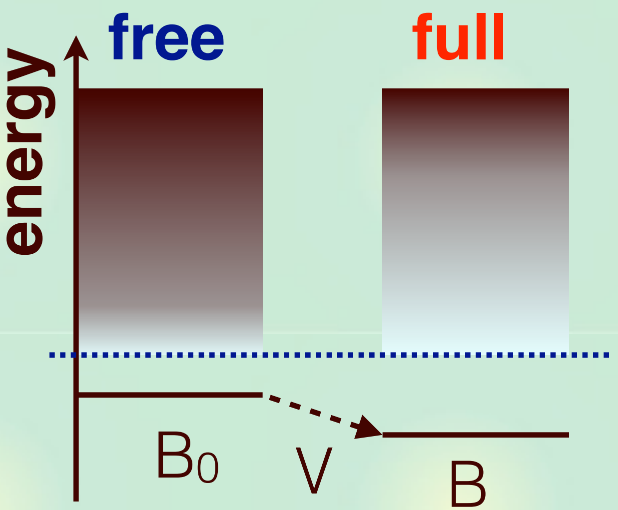
$$\mathcal{H} = \boxed{\mathcal{H}_0} + V$$

Complete set for free Hamiltonian: bare |B0 > + continuum

$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

Physical bound state |B> with binding energy B

$$(\mathcal{H}_0 + V)|\, B \,\rangle = -B|\, B \,\rangle$$



Z : overlap of B and B0

$$Z \equiv |\langle\, B_0 \,|\, B \,\rangle|^2$$

$$0 \leq Z \leq 1$$

For small B, Z is related to observables

$$a = \left[\frac{2(1-Z)}{2-Z}\right] R, \quad r_e = \left[\frac{-Z}{1-Z}\right] R$$

Application to resonances

Features of the Weinberg's argument:

- Model-independent approach (no potential, wave-fn, ...)
- Relation with experimental observables
- Only for bound states with small binding

Application to resonances by analytic continuation

$$1 - Z = \int d\boldsymbol{q} \frac{|\langle \boldsymbol{q}|V|B\rangle|^2}{[E(q)+B]^2} \sim -g^2 \left.\frac{dG(W)}{dW}\right|_{W=M_B}$$

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)
F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012)

- Z can be complex. Interpretation?
- $|Z|$ can be larger than unity. Normalization?

What about near-threshold resonances (~ small binding) ?

Effective range expansion

S-wave scattering amplitude at low momentum

$$f(k) = \frac{1}{k\cot\delta - ki} \to \left(\frac{1}{a} - ki + \frac{r_e}{2}k^2\right)^{-1}$$

Truncation is valid only at small k**.**

Scattering length a

- **strength of the interaction**
- **cross section at zero momentum :** $4\pi a^2$

Effective range r_e

- **typical length scale of the interaction**
- **can be negative**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

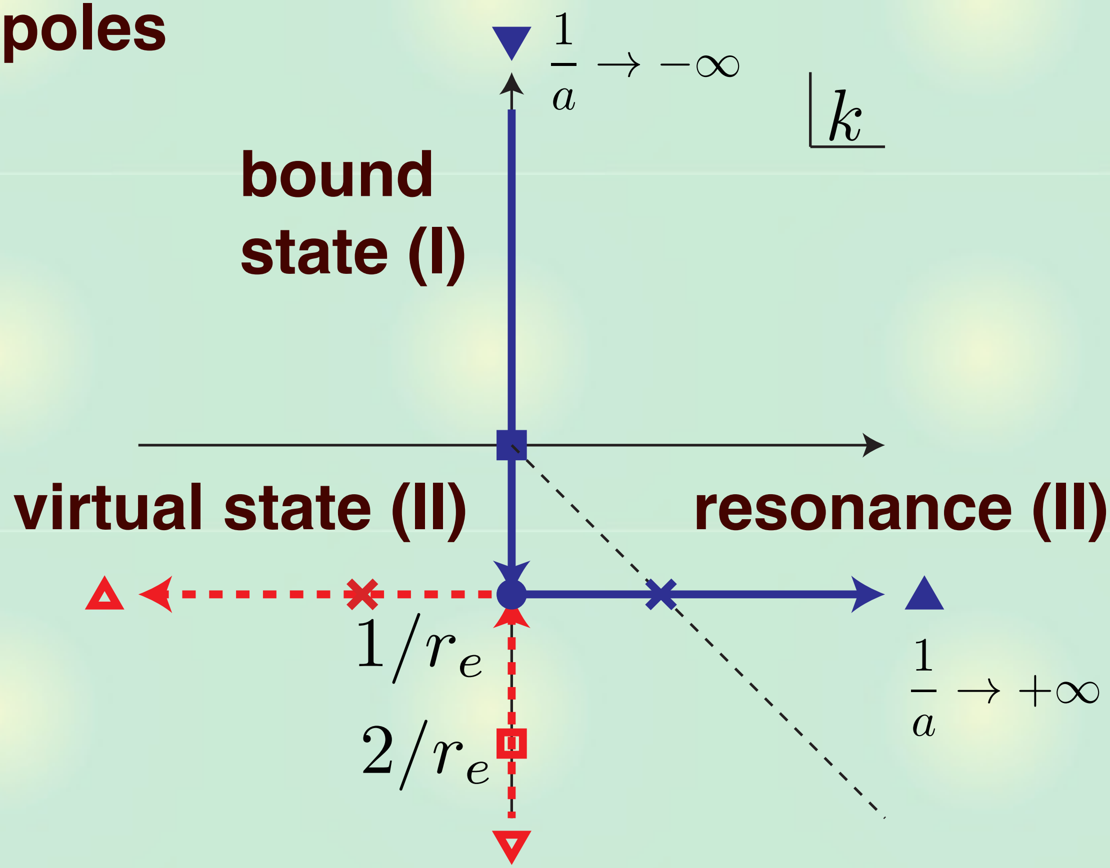
Poles of the amplitude

The amplitude has two poles

$$f(k) = \left(\frac{1}{a} - ki + \frac{r_e}{2}k^2\right)^{-1}$$

$$k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{-\frac{2r_e}{a} - 1}$$

Pole trajectories with a fixed $r_e < 0$



Positions of poles <--> scattering length + effective range

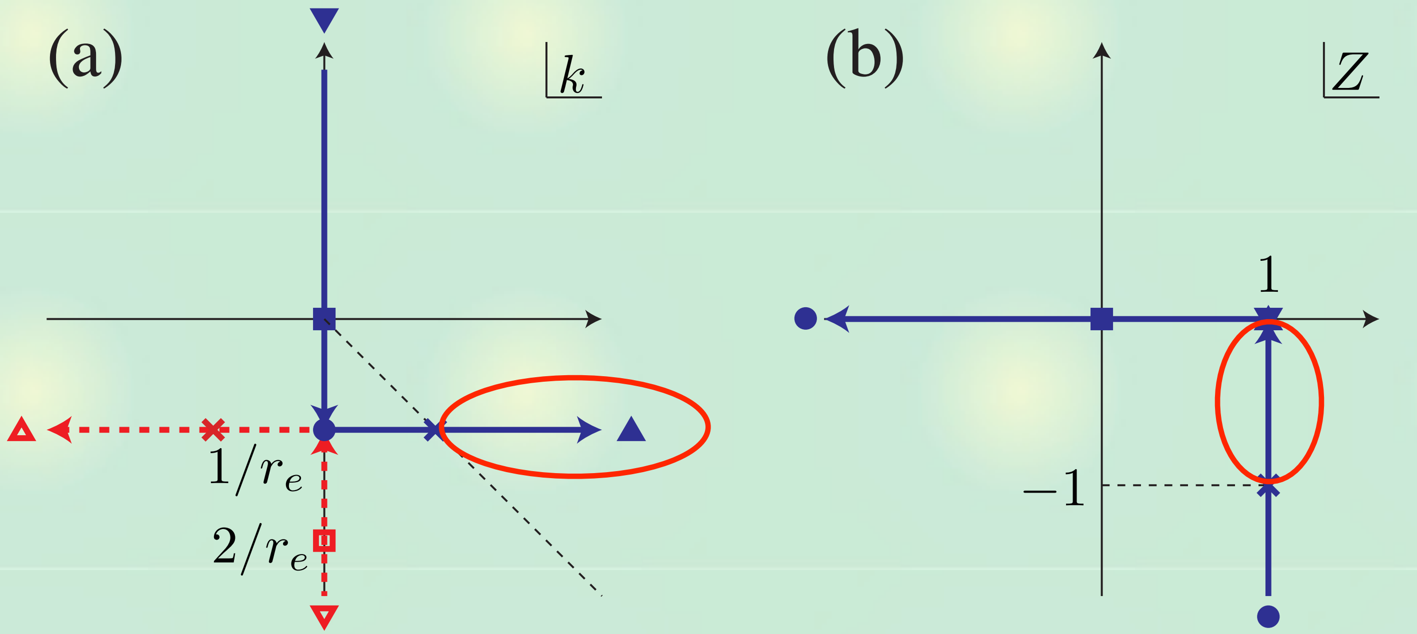
$$a = \frac{k^+ + k^-}{ik^+k^-}, \quad r_e = \frac{2i}{k^+ + k^-}$$

(a, r_e) are real for resonances

Field renormalization constant

Eliminate R from the Weinberg's relations

$$Z = 1 - \sqrt{1 - \frac{1}{1 + a/(2r_e)}} = \frac{2k^-}{k^- - k^+}$$



Z (residue) is determined by the pole position

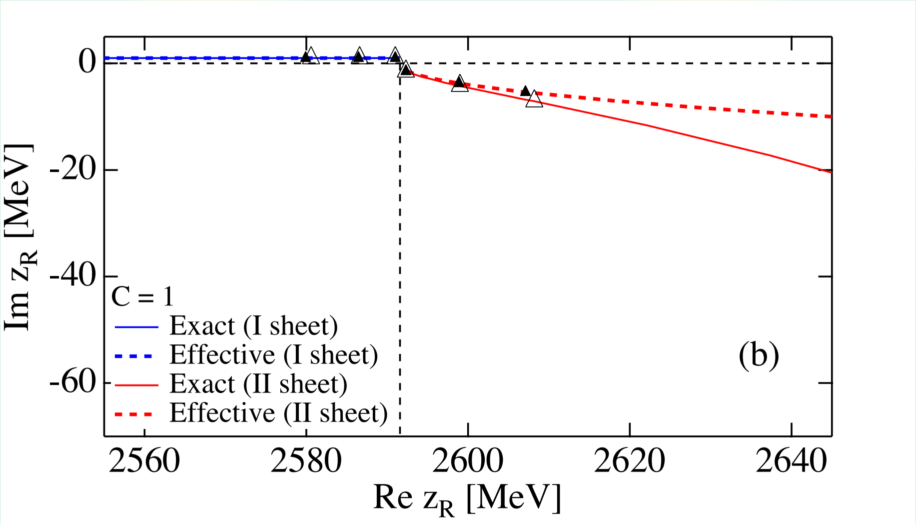
<-- Amplitude is given by two parameters.

1-Z **is pure imaginary and** 0≤|1-Z|≤1

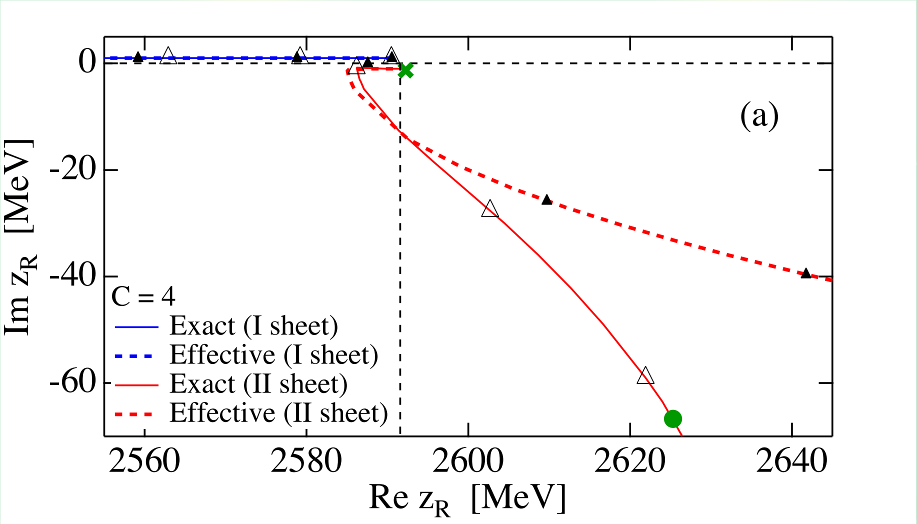
Validity of the effective range expansion

A model calculation

- solid lines: pole position in a scattering model
- dashed lines: position deduced from (a, r_e)



large r_e



small r_e

If the effective range is large, the expansion works well.

Example: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ with $\pi\Sigma_c$ threshold in PDG

E [MeV]	Γ [MeV]	a [fm]	r_e [fm]
0.67	2.59	10.5	-19.5

- Isospin symmetry is assumed.
- $\pi\pi\Lambda$ channel is not taken into account.

$|1-Z| \sim 0.6$ **Interpretation ?**

Larger effective range than typical hadronic scale
Chiral interaction gives $r_e \sim -4.6$ fm

--> $\Lambda_c(2595)$ is not likely a $\pi\Sigma_c$ molecule

Summary

Near-threshold s-wave resonances

- **Effective range expansion :**

  pole position <--> observables (a, r_e)

- **Compositeness** $1-Z$ **:**

  pure imaginary and normalized

- **Application to** $\Lambda_c(2595)$

  Large r_e **--> not likely a molecule**

T. Hyodo, arXiv:1305.1999 [hep-ph]