格子QCDからダイクォーク相関を探る

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in collaboration with

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and

HAL QCD Collaboration

ミニ研究会「チャームバリオンの構造と生成」@J-PARC，9/11, 2013.

Diquarks in hadrons

Attractive color-magnetic interactions in constituent quark models

$$V_{\mathrm{CMI}} = A \sum_{i<j} \frac{(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))}{M_i M_j}\delta^3(\vec{r}_i - \vec{r}_j)$$

- Color-spin matrix elements : $\langle v_{ij}\rangle = \langle(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))\rangle$

$<v_{ij}>$	C=1	C=8	C=3*	C=6
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

attractive

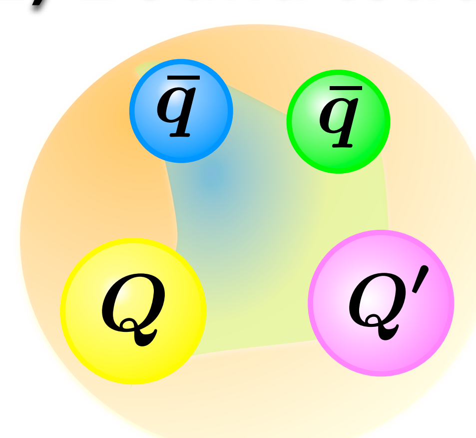
- C=3*, S=0 (I=0) : -8
- C=6, S=1 (I=0) : -4/3
- C=3*, S=1 (I=1) : 8/3
- C=6, S=0 (I=1) : 4

repulsive

This talk : indirect measurements of "diquark" correlations from LQCD

1) Inter-quark interactions in baryons

2) Bound tetraquarks T_{cc}?



H. J. Lipkin, PLB172 (1986).

(1) Interquark potentials from LQCD

☑ **Potential defined from Nambu-Bethe-Salpeter (NBS) amplitude :**

Inter-quark potential with finite quark mass --> input of phenomenological model

Y. Ikeda, H. Iida, POS. LATTICE 2010, 143 (2010).
Y. Ikeda, H. Iida, Prog. Theor. Phys. 128 (2012) 941 [arXiv:1102.2097[hep-lat](2011)].

☑ **Kinetic quark mass :**

Self-consistent determination of “kinetic” quark mass

T. Kawanai and S. Sasaki, Phys. Rev. Lett. 107, 091601 (2011).

☑ **Effective QQ potentials in baryons :**

Diquark correlations in baryons??

T. Abe, H. Iida, Y. Ikeda, T. T. Takahashi, M. Oka, (N. Yamanaka), in progress

☑ **Application to finite temperature :**

imaginary-time potential at finite temperature

H. Iida, Y. Ikeda, POS. LATTICE 2011, 195 (2011).
P. Evans, C. Allton, J. Skullerud, arXiv:1303.5331 [hep-lat].

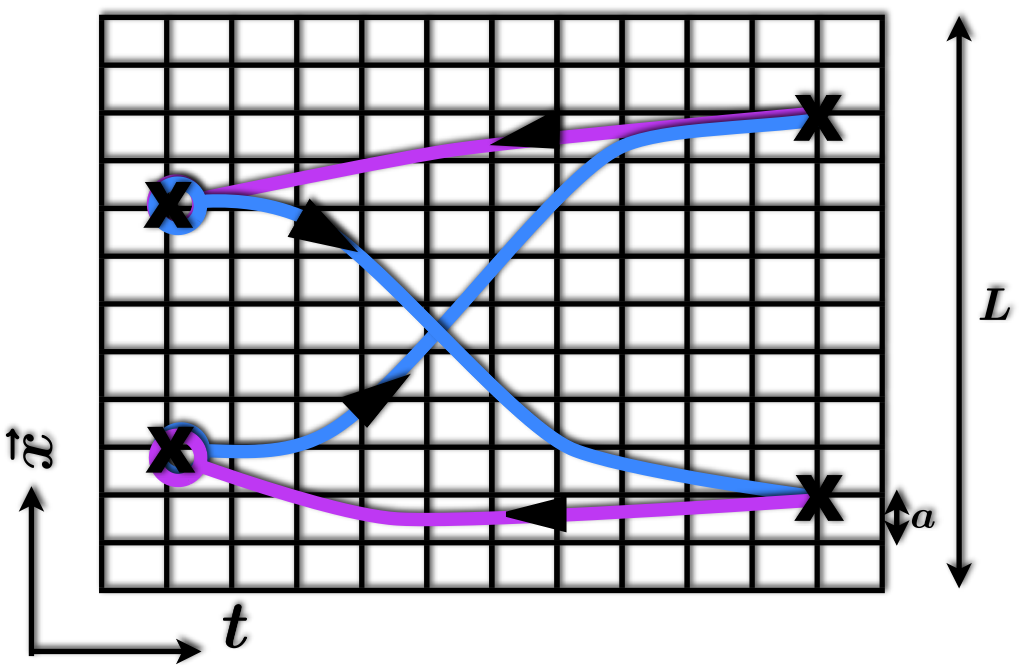
Hadron scattering on the lattice

Key quantity : Equal-time Nambu-Bethe-Salpeter amplitude

$$\psi(\vec{r},t) = \sum_{\vec{x},\vec{X},\vec{Y}} \langle 0|\phi_1(\vec{x}+\vec{r},t)\phi_2(\vec{x},t)\phi_1(\vec{X},t=0)^\dagger\phi_2(\vec{Y},t=0)^\dagger|0\rangle$$

$$= \sum_{W(\vec{k})} A_{W(\vec{k})}\exp\left[-W(\vec{k})t\right]\psi_{W(\vec{k})}(\vec{r})$$

$$\psi_{W(\vec{k})}(\vec{r}) \equiv \sum_{\vec{x}}\langle 0|\phi_1(\vec{x}+\vec{r})\phi_2(\vec{x})|W(\vec{k}),B,J^P,F\rangle$$



- **Helmholtz eq. of NBS wave func.:**

$$(\nabla^2+\vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) = 0 \quad (|\vec{r}|>R)$$

- **NBS wave func. ~ wave func. in Q.M.**

  information on phase shift

$$\psi^{(l)}_{W(\vec{k})}(r) \sim \frac{e^{i\delta_l(k)}}{kr}\sin(kr+\delta_l(k)-l\pi/2)$$

- **Temporal correlation, W(k) : phase shift (Luscher's formula)** M. Lüscher, NPB354, 531 (1991).
- **Spacial correlation, ψ(r) : potential --> observable** CP-PACS Coll., PRD71, 094504(2005). Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Lattice QCD potential -- HAL QCD --

Full details, see, Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Helmholtz equation of NBS wave function:

$$(\nabla_r^2 + \vec{k}^2)\psi_W(\vec{r}) = 0 \quad (r > R)$$

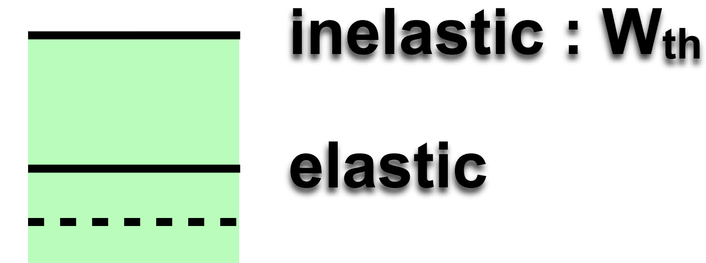
Define half off-shell T-matrix in interacting region:

$$(\nabla_r^2 + \vec{k}^2)\psi_W(\vec{r}) = 2\mu\mathcal{K}_W(\vec{r}) \quad (r < R)$$

Plane wave components are projected out

Derive energy-independent potentials

$$U(\vec{r}, \vec{r}') = \int^{W_{\rm th}} \frac{dW}{2\pi} \mathcal{K}_W(\vec{r})\psi^*_W(\vec{r}')$$



Schrödinger-type equations:

$$(E - H_0)\psi_W(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}')\psi_W(\vec{r}')$$

$$E = \frac{\vec{k}^2}{2\mu}, \quad H_0 = -\frac{\nabla_r^2}{2\mu}$$

Point :

Hadron-hadron potentials defined from NBS wave functions become faithful to scattering phase shift

How to define interquark potential

Y. Ikeda, H. Iida, Prog. Theor. Phys. 128 (2012) 941.

Homogenous Nambu-Bethe-Salpeter equation :

$$\tilde{\chi}_W(p^0, \mathrm{p}) = G(p;W) \int d^4p' K(p,p';W) \tilde{\chi}_W(p'^0, \mathrm{p}')$$

W : meson energy $W = M_{meson}$ at meson-rest frame
p, p' : relative 4-momentum of q^{bar}-q system
K(p,p';P) : irreducible kernel
G(p;P) : quark propagators

3-dimensional realization of interaction kernel :

➡ Replacement of G(p;W) with $Q^{bar}Q$ non-relativistic $G_{N.R.}(p;W)$ leads to rearrangement of interaction kernel of original NBS equation

$$\tilde{\chi}_W(p^0, \mathrm{p}) = \mathcal{F}(p^0) G_{\mathrm{N.R.}}(\mathrm{p};W) \int d^4p' I(p,p';W) \tilde{\chi}_W(p'^0, \mathrm{p}')$$

I(p,p';P) is "new" kernel : $I = K + K(G - \mathcal{F}G_{\mathrm{N.R.}})I$

For $Q^{bar}Q$ systems, we choose $\mathcal{F}(p^0) \equiv \frac{1}{L_{p^0}}$ (constant)

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For $Q^{bar}Q$ systems, we choose $\mathcal{F}(p^0) \equiv \frac{1}{L_{p^0}}$ (constant)

Reviewed in Klein, Lee, PRD 10 (1974).

Note :

Instantaneous approximation for NBS kernel K(p,p';P) is NOT required

How to define interquark potential

Aoki, Hatsuda, Ishii, PTP 123 (2010).
Y. Ikeda, H. Iida, Prog. Theor. Phys. 128 (2012).

- **Equal-time choice of NBS amplitude :**

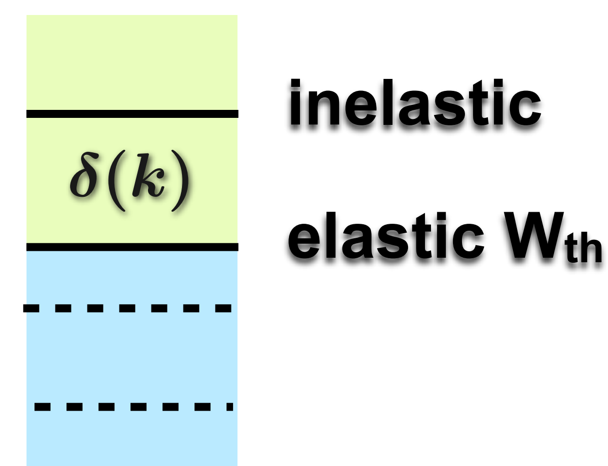
$$\tilde{\phi}_W(\mathrm{p}, t_Q = t_{\bar{Q}}) = \frac{1}{L_{p^0}} \int dp^0 \tilde{\chi}_W(p^0, \mathrm{p}) e^{-ip^0(t_Q - t_{\bar{Q}})}|_{t_Q = t_{\bar{Q}}}$$

- **3-dimensional amplitude from NBS wave function**

$$(W - 2m_Q + \nabla_r^2/m_Q)\phi_W(\vec{r}) = \mathcal{K}_W(\vec{r})$$

Energy-independent potential :

$$U(\vec{r}, \vec{r}') = \sum_{W < W_{\mathrm{th}}} \mathcal{K}_W(\vec{r})\phi^*_W(\vec{r}')$$



Energy-independent potential --> Schrödinger-type equation

$$(E - H_0)\phi_W(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}')\phi_W(\vec{r}')$$

$$E = W - 2m_Q, \ H_0 = -\nabla_r^2/m_Q$$

Interquark potential constructed from NBS wave function :

Information on mass spectra: pole position & residue

$$\left[G^{(2)}(p, p'; W)\right]_{\mathrm{pole}} = i \sum_r \frac{\tilde{\chi}_{W,r}(p)\bar{\tilde{\chi}}_{W,r}(p')}{P^2 - W^2 + i\epsilon}$$

Q^{bar}-Q potential from LQCD

Aoki, Hatsuda, Ishii, PTP 123 (2010).
Y. Ikeda, H. Iida, Prog. Theor. Phys. 128 (2012).

1. Measure Nambu-Bethe-Salpeter wave function

$$\phi_W(\mathrm{r}) = \sum_{\mathrm{x}} \langle 0|\bar{q}(\mathrm{x})\Gamma q(\mathrm{x}+\mathrm{r})|W; J^P\rangle$$

Spacial correlation of 4-point function

$$\begin{aligned} G^{(2)}(\mathrm{r}, t-t_{\mathrm{src}}) &= \sum_{\mathrm{x},\mathrm{X},\mathrm{Y}} \langle 0|\bar{q}(\mathrm{x},t)\Gamma q(\mathrm{x}+\mathrm{r},t)\Big(\bar{q}(\mathrm{X},t_{\mathrm{src}})\Gamma q(\mathrm{Y},t_{\mathrm{src}})\Big)^\dagger|0\rangle \\ &= \sum_{\mathrm{x}}\sum_{n} A_n \langle 0|\bar{q}(\mathrm{x})\Gamma q(\mathrm{x}+\mathrm{r})|n\rangle e^{-W_n(t-t_{\mathrm{src}})} \\ &\to A_0\phi_{W_0}(\mathrm{r})e^{-W_0(t-t_{\mathrm{src}})} \qquad (W_0 = M, \quad t \gg t_{\mathrm{src}}) \end{aligned}$$

2. Define potential from Schrödinger-type equation

$$(E - H_0)\phi_W(\mathrm{r}) = \int d\mathrm{r}' U(\mathrm{r},\mathrm{r}')\phi_W(\mathrm{r}')$$

3. Velocity expansion of non-local potential

$$U(\mathrm{r},\mathrm{r}') = \left(V_{\mathrm{C}}(r) + V_{\mathrm{spin}}(r)\vec{S}_{\bar{\mathrm{Q}}}\cdot\vec{S}_{\mathrm{Q}} + V_{\mathrm{T}}(r)\hat{S}_{12} + V_{\mathrm{LS}}(r)\vec{L}\cdot\vec{S} + \cdots\right)\delta(\mathrm{r}-\mathrm{r}')$$

Leading order: $V_{\mathrm{C}}(r) + V_{\mathrm{spin}}(r)\vec{S}_{\bar{\mathrm{Q}}}\cdot\vec{S}_{\mathrm{Q}} + V_{\mathrm{T}}(r)\hat{S}_{12}$

NLO: $V_{\mathrm{LS}}(r)\vec{L}\cdot\vec{S}$

this talk : S-wave "effective LO potentials" in pseudo-scalar and vector channels

LQCD setup

◆ Quench QCD simulation

Plaquette gauge action & Standard Wilson quark action

β=6.0 (a=0.104 fm, a^{-1}=1.9GeV)

Box size : 32^3 x 48 -> L=3.3 (fm)

◆ Hopping parameters (κ=0.1320, 0.1420, 0.1480, 0.1520)

--> M_{PS} = 2.53, 1.77, 1.27, 0.94 (GeV)

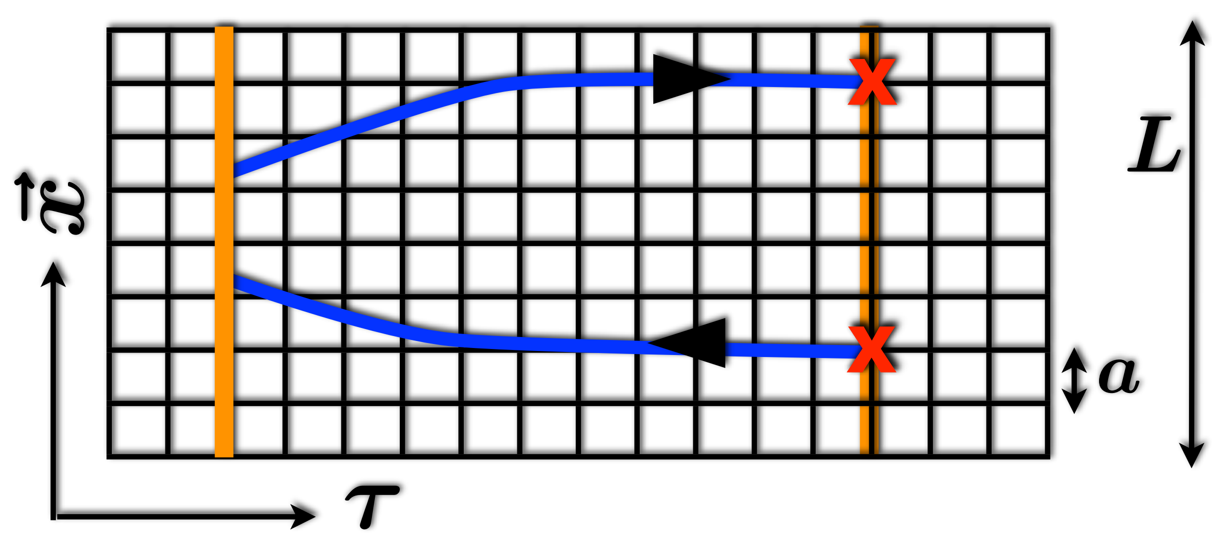
--> M_V = 2.55, 1.81,1.35, 1.04 (GeV)

(between charm and strange quarks region)

$N_{conf.}$=100

Wall source

◆ Coulomb gauge

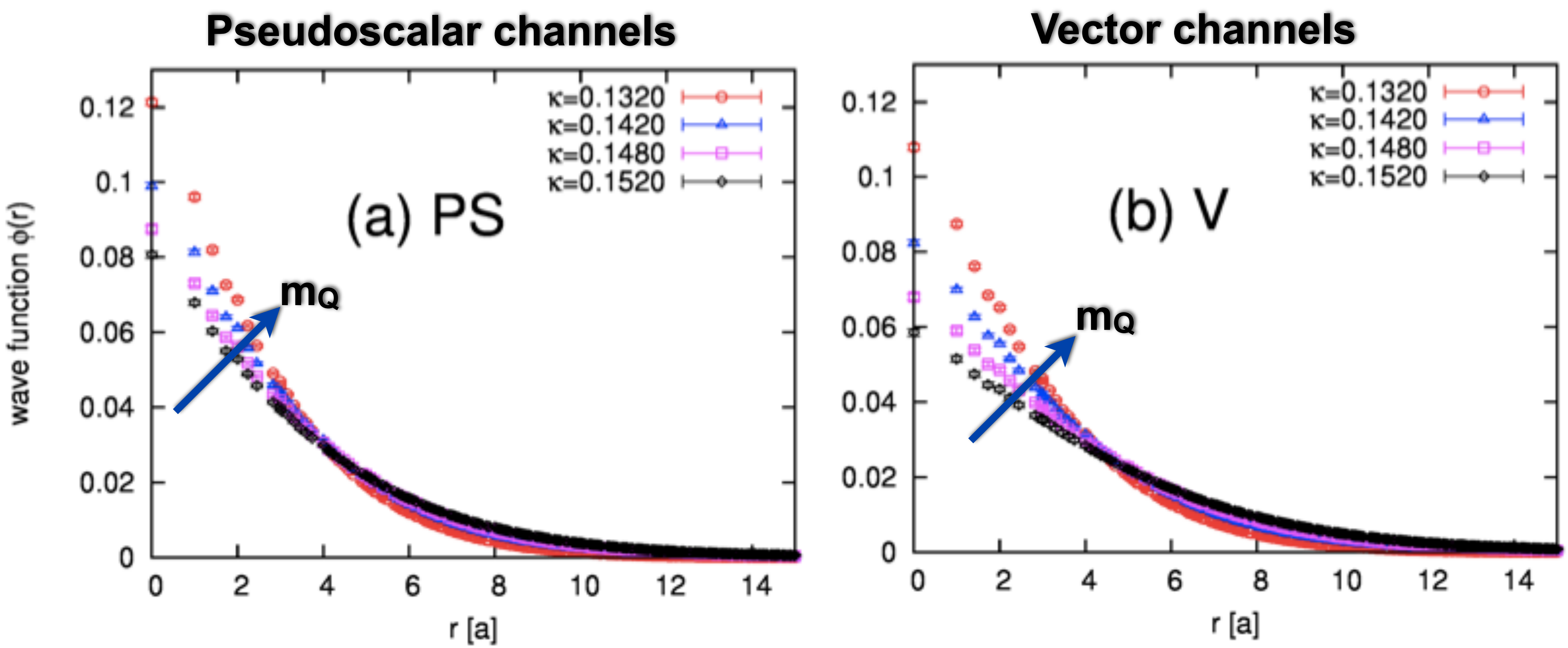


Q^{bar}-Q wave functions

NBS wave functions (channel & quark mass dependence)

M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)

$$\phi_W(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|W;J^P\rangle$$



- **Channel dependence appears in light quark sector**
  - **-> spin-spin interaction is enhanced at light quark mass (one-gluon exchange predicts V_{spin} proportional to $1/m_q^2$)**
- **Size of wave function becomes smaller as increasing m_q**

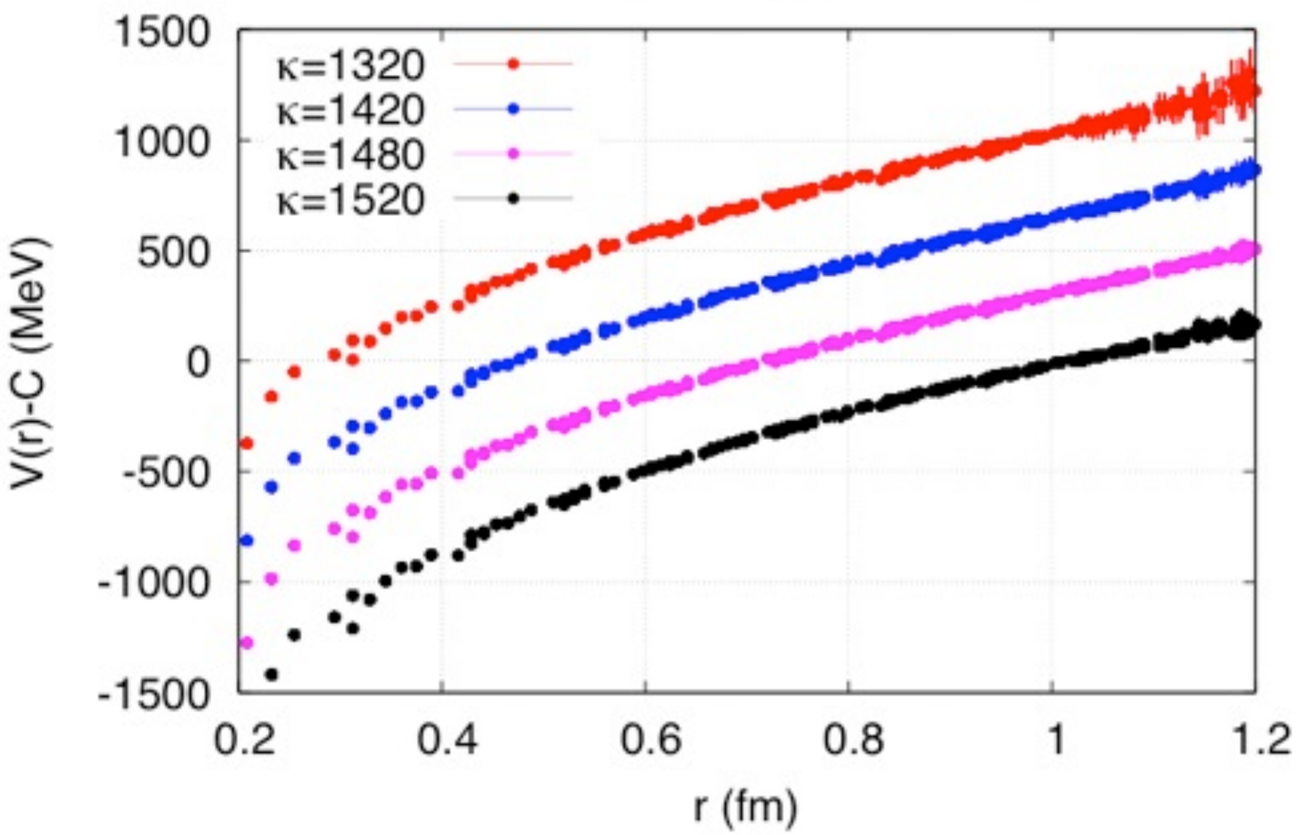
$\bar{Q}$-Q potentials

Effective spin-independent & dependent forces are constructed by linear combination of PS & V channel potentials

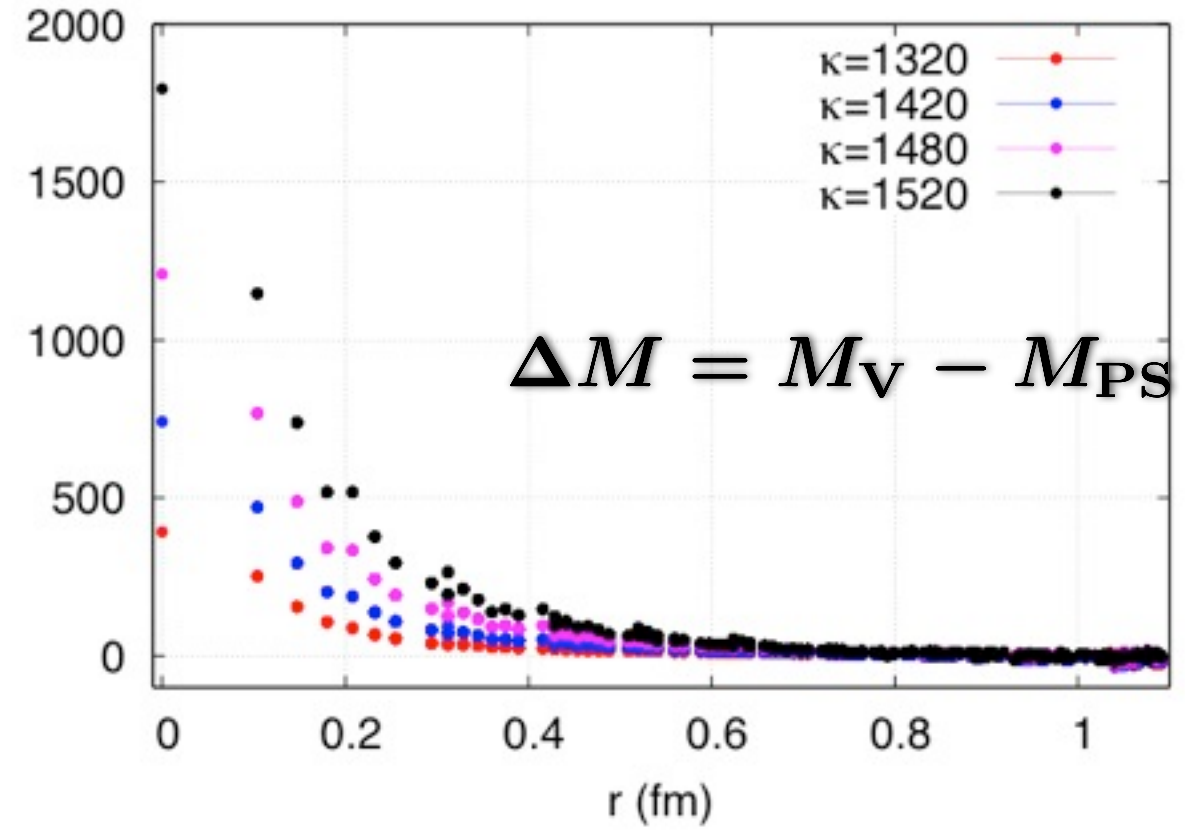
$$V^{\text{eff}}_{\text{spin-indep.}}(\mathbf{r}) - E = \frac{1}{m_q}\left[\frac{1}{4}\frac{\nabla^2\phi_{\text{PS}}(\mathbf{r})}{\phi_{\text{PS}}(\mathbf{r})} + \frac{3}{4}\frac{\nabla^2\phi_{\text{V}}(\mathbf{r})}{\phi_{\text{V}}(\mathbf{r})}\right]$$

$$V^{\text{eff}}_{\text{spin-dep.}}(\mathbf{r}) - E = \frac{1}{m_q}\left(-\frac{\nabla^2\phi_{\text{PS}}(\mathbf{r})}{\phi_{\text{PS}}(\mathbf{r})} + \frac{\nabla^2\phi_{\text{V}}(\mathbf{r})}{\phi_{\text{V}}(\mathbf{r})}\right)$$

Effective spin-independent force



Effective spin-dependent force

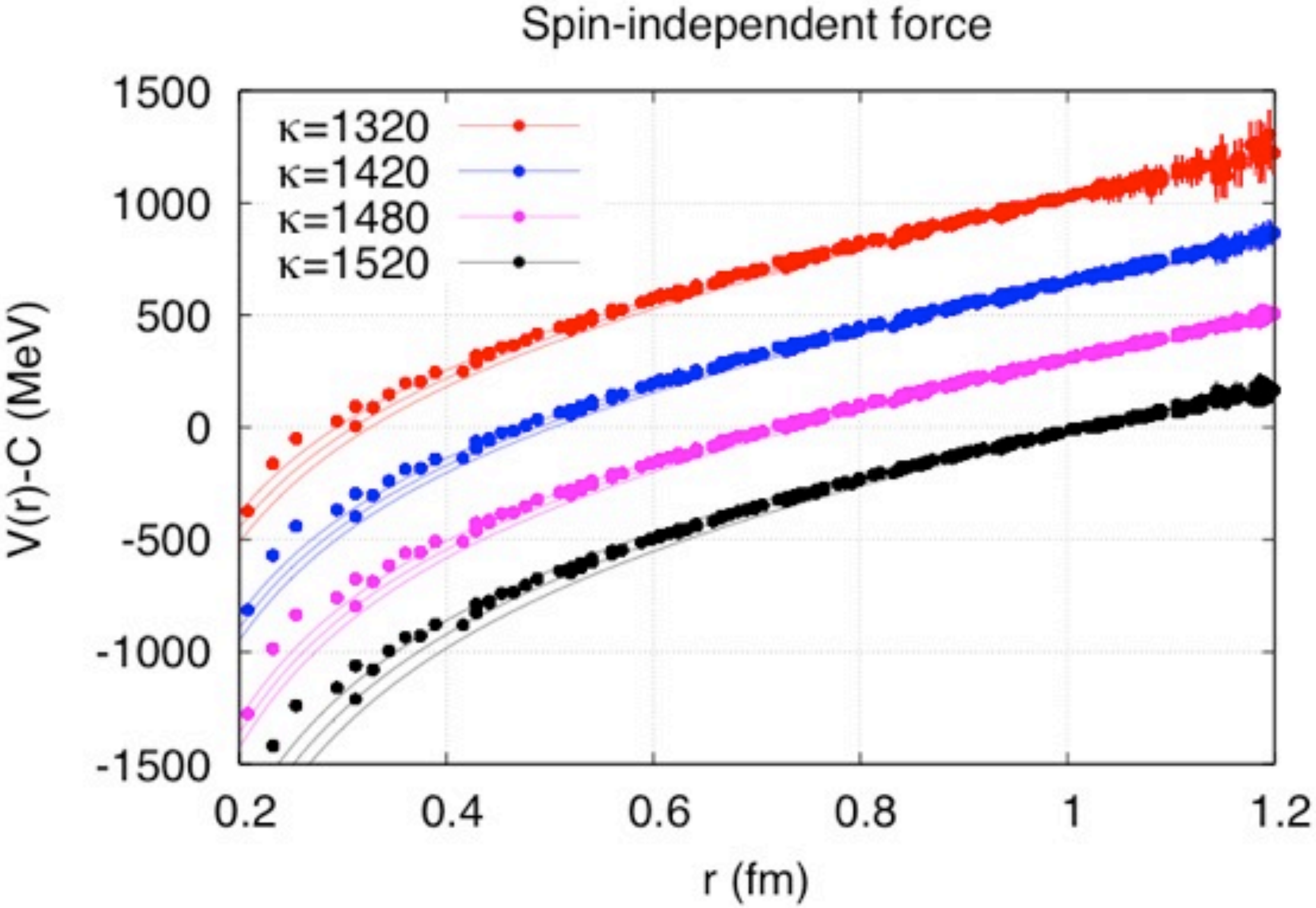


$$\Delta M = M_{\text{V}} - M_{\text{PS}}$$

- Spin-independent forces reveal Coulomb + linear behaviors
- Spin-dependent forces have strong quark mass dependence
- Repulsive spin-dependent forces as expected by mass spectrum

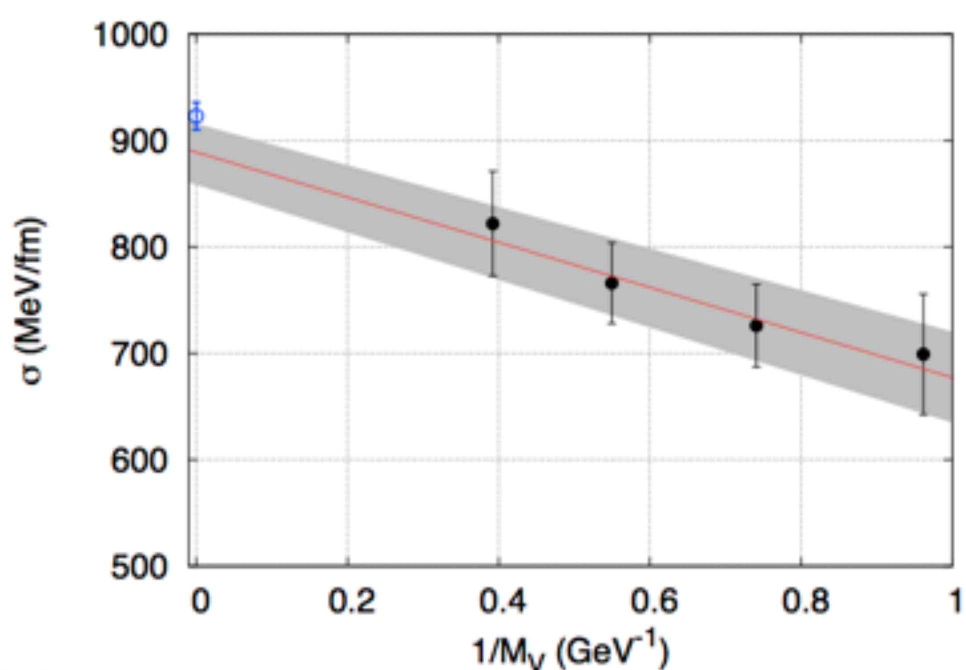
Fit results of Q^{bar}-Q potentials

$$V^{\rm eff}_{\rm spin-indep.}(\mathbf{r}) - E = \frac{1}{m_q}\left[\frac{1}{4}\frac{\nabla^2\phi_{\rm PS}(\mathbf{r})}{\phi_{\rm PS}(\mathbf{r})} + \frac{3}{4}\frac{\nabla^2\phi_{\rm V}(\mathbf{r})}{\phi_{\rm V}(\mathbf{r})}\right]$$



fit function: $V(\mathbf{r}) = \sigma r - \frac{A}{r} + C$

M_V (GeV)	σ (MeV/fm)	A (MeV fm)
2.55	822 (49)	200 (7)
1.87	766 (38)	228 (6)
1.35	726 (39)	269 (7)
1.04	699 (57)	324 (12)



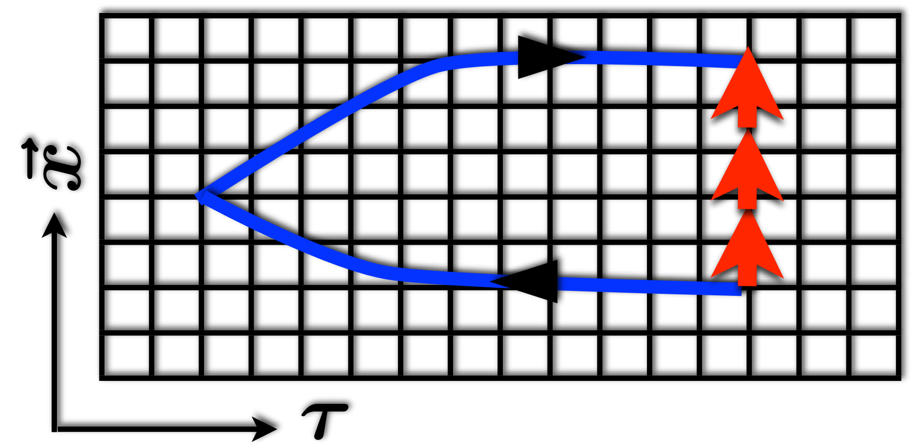
- "String tension" has moderate m_Q dependences
- Naive extrapolation to infinite mass gives comparable value from Wilson loop
- Coulomb coefficients increase as decreasing m_Q

see also, Kawanai and Sasaki, PRL 107 (2011).

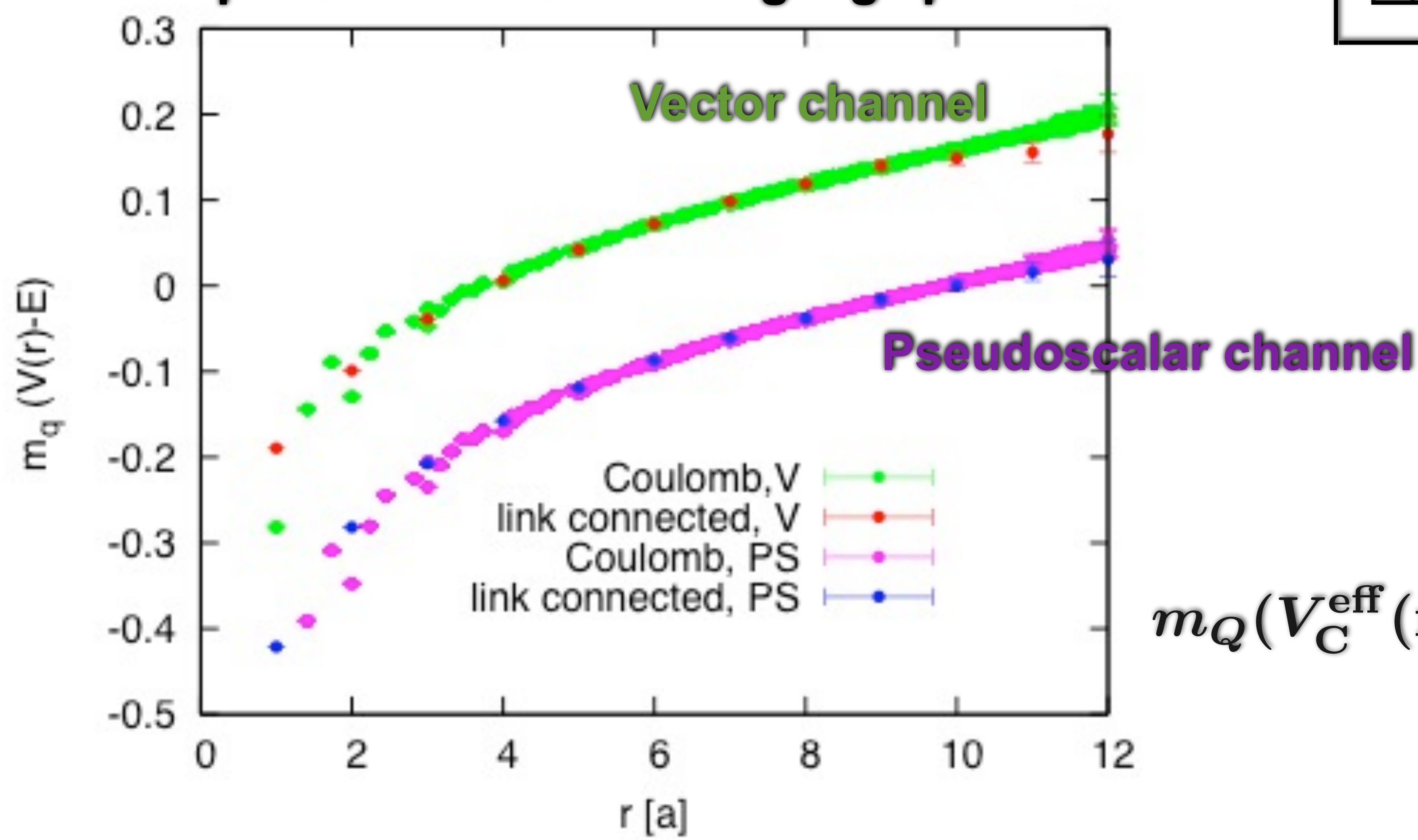
Operator dependence of Q^{bar}-Q potentials

Operator dependence of inter-quark potential is studied
by using gauge invariant smearing operator

$$\phi_W^{\mathrm{smr.}}(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0 | \bar{q}(\mathbf{x}) L(\mathbf{x}, \mathbf{r}) \Gamma q(\mathbf{x}+\mathbf{r}) | W; J^P \rangle$$



Comparison with Coulomb gauge potentials



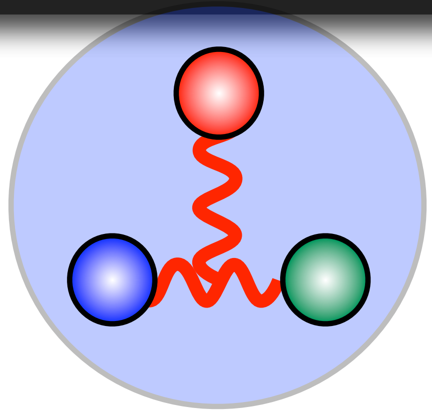
$$m_Q(V_C^{\mathrm{eff}}(\mathbf{r}) - E) = \frac{\nabla^2 \phi_W^{\mathrm{smr.}}(\mathbf{r})}{\phi_W^{\mathrm{smr.}}(\mathbf{r})}$$

The potentials obtained from gauge invariant smearing operators are consistent with the Coulomb gauge potentials

Interquark potentials for baryons

- **Three-body Schrödinger-type equation :**

$$\left(-\frac{\nabla_{\mathbf{r}}^2}{2\mu}-\frac{\nabla_{\rho}^2}{2\eta}+V(\mathbf{r},\rho)\right)\psi_E(\mathbf{r},\rho)=E\psi_E(\mathbf{r},\rho)$$



- **Effective QQ potentials : integrate out spectator particle**

Doi et al. (HAL QCD Coll.), arXiv:1106.2276 [hep-lat] (2011).

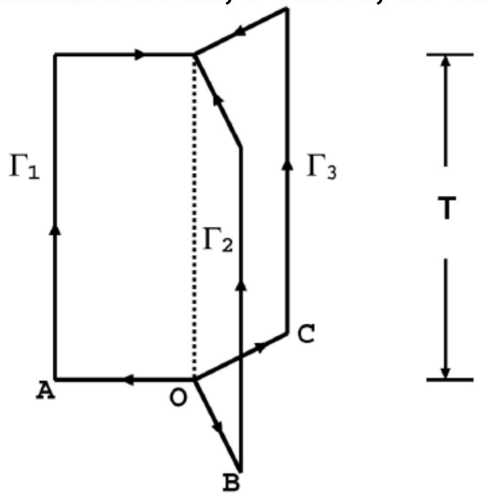
$$\psi_E^{2Q}(\mathbf{r})\equiv\int d\rho\,\psi_E(\mathbf{r},\rho) \quad\Rightarrow\quad \left(-\frac{\nabla_{\mathbf{r}}^2}{2\mu}+V_{\mathrm{eff.}}^{2Q}(\mathbf{r})\right)\psi_E^{2Q}(\mathbf{r})=E\psi_E^{2Q}(\mathbf{r})$$

- **Effective QQ potential v.s. Q^{bar}-Q potential (spin independent parts)**

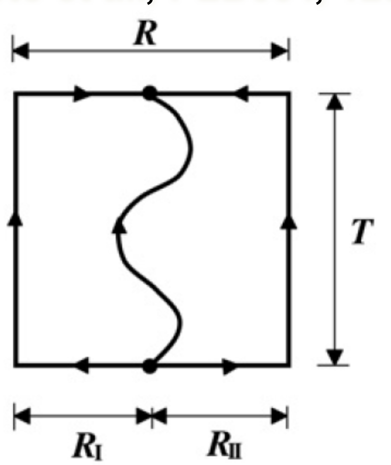
$$V_{2Q}(r)=\sigma r-\frac{2}{3}\frac{\alpha_s}{r}+\cdots \qquad V_{\bar{Q}Q}(r)=\sigma r-\frac{4}{3}\frac{\alpha_s}{r}+\cdots$$

➡ **String tension would be same <-- Polyakov line analysis**
➡ **Coulomb part is different by factor two within one-gluon exchange mechanism**

Three-quark potential
T.T.Takahashi et al., PRD70, 074506 (2004)



Valence light quark effect for 2Q
A.Yamamoto et al., PLB664, 129 (2008)

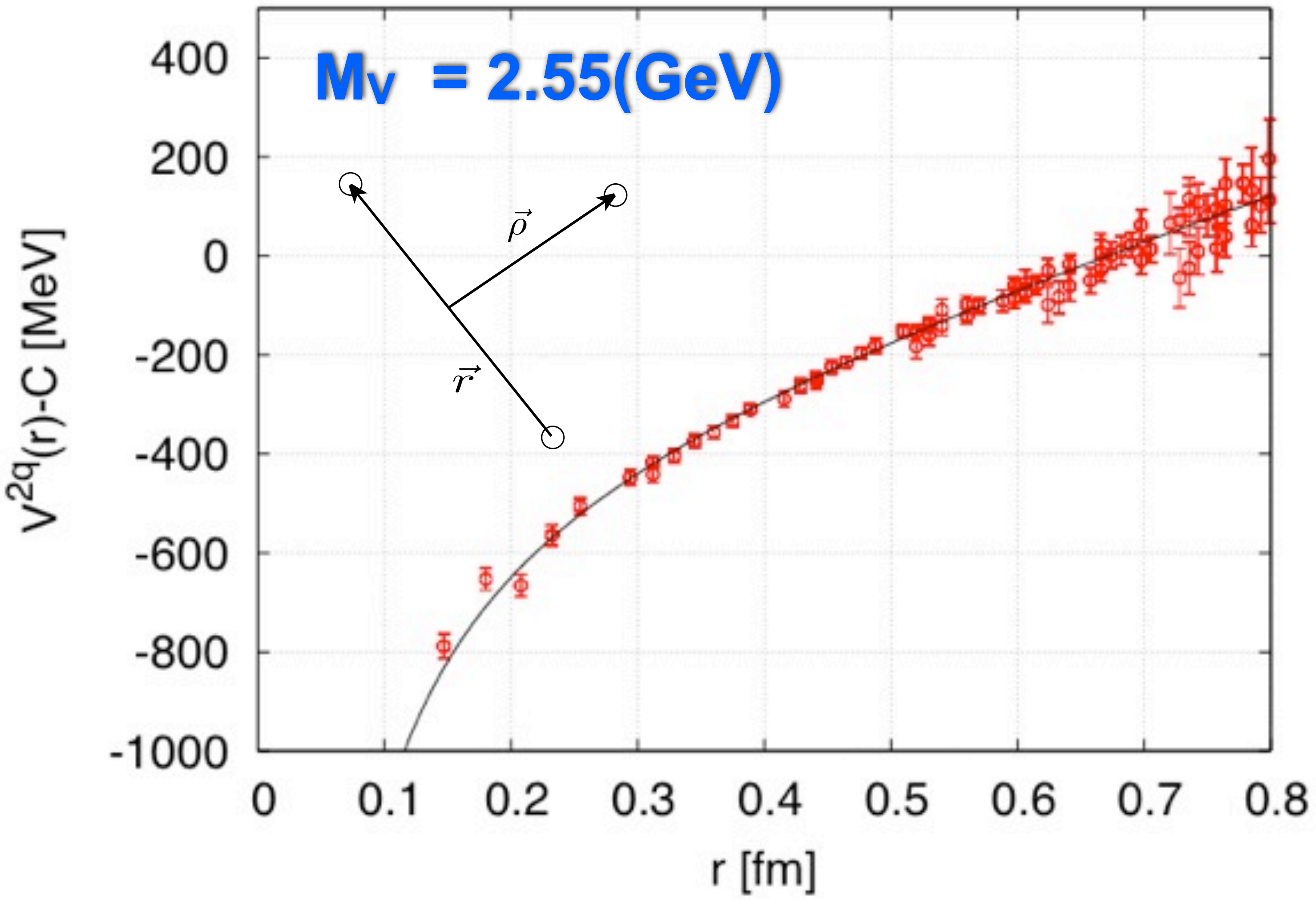


Interquark potentials for baryons

- **Lattice QCD result of effective 2Q potential (Coulomb gauge):**

$$V^{2Q}_{\mathrm{eff.}}(\mathbf{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi^{2Q}_E(\mathbf{r})}{\psi^{2Q}_E(\mathbf{r})} + E$$

$$\psi^{2Q}_E(\mathbf{r}) = \sum_{\rho}\langle 0|\epsilon_{abc}[q^T_a(\mathbf{r}/2)C\gamma_5 q_b(-\mathbf{r}/2)]q_{c,\alpha}(\rho)|B=1, J^P=1/2^+\rangle$$



Effective 2Q potential shows Coulomb + linear form

$$V^{\mathrm{fit}}_{2q}(r) = \sigma r - \frac{A}{r} + C$$

χ^2/N_{dof} = 0.6 [0.2 < r < 0.8 fm]

σ = 870 (209) [MeV / fm]

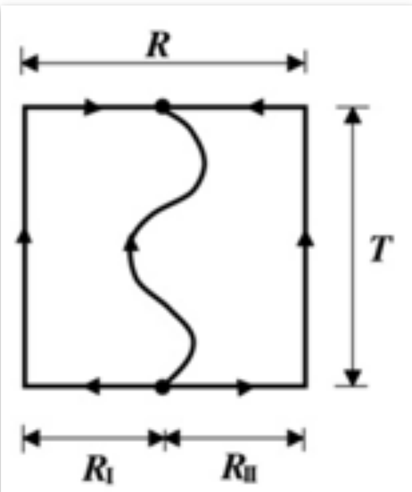
A = 101 (24) [MeV fm]

"String tension" of effective 2Q potential is comparable with that of Q^{bar}-Q potential (c.f., $\sigma_{Qbar\text{-}Q}$ =822 (49) [MeV / fm], $A_{Qbar\text{-}Q}$ =200 (7) [MeV fm])

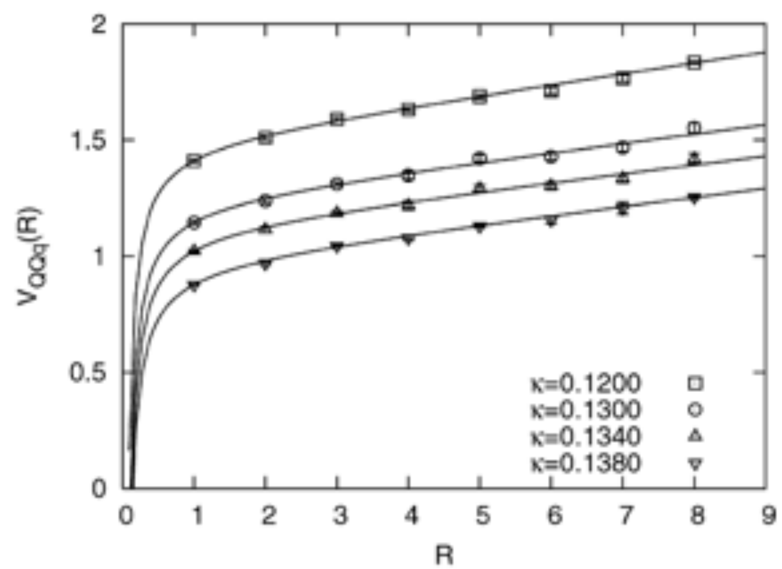
Interquark potentials for baryons

Valence light quark effect for 2Q interaction

A.Yamamoto et al., PLB664, 129 (2008).



$$V_{QQq}(R) = \sigma_{\rm eff} R - \frac{A_{\rm eff}}{R} + C_{\rm eff}$$



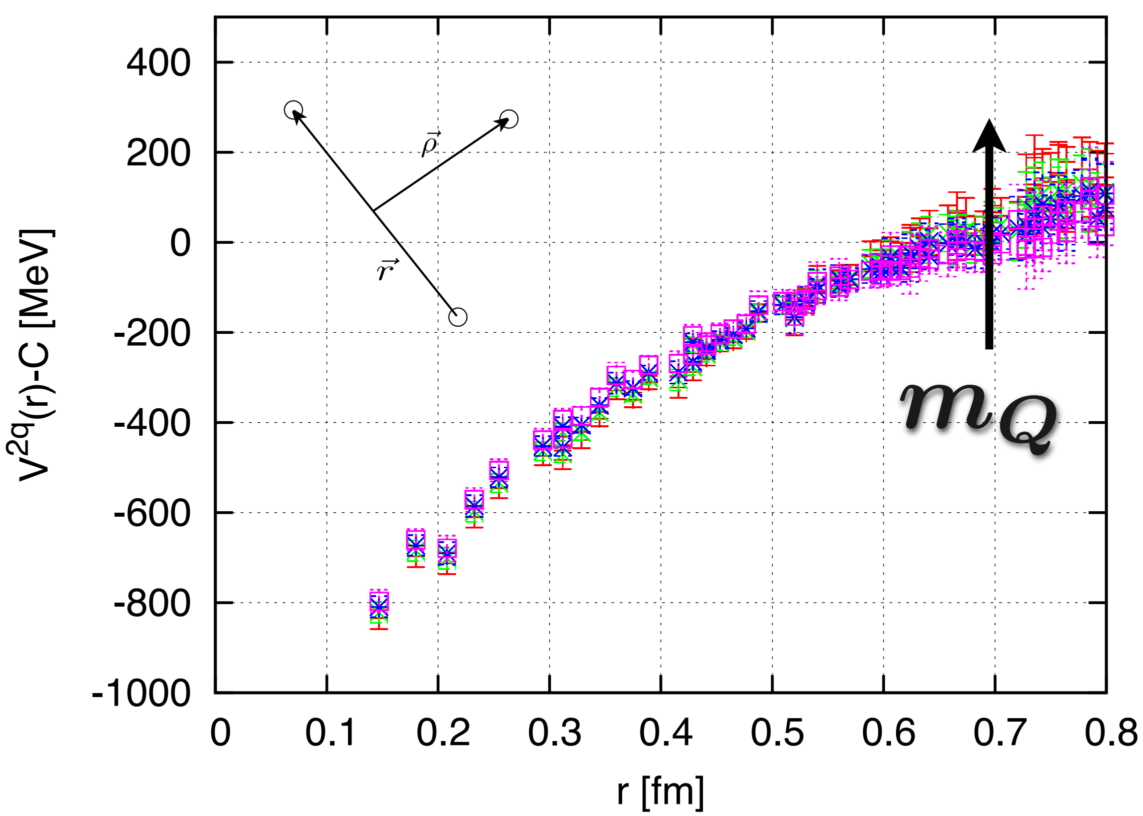
$$\sigma_{\rm eff} \simeq \sigma_{Q\bar{Q}} \quad A_{\rm eff} \simeq \frac{1}{2} A_{Q\bar{Q}}$$

for heavy spectator quark

$$\sigma_{\rm eff.}(m_c) > \sigma_{\rm eff.}(m_Q), \quad m_Q < m_c$$

Screening effect due to light spectator quark

NBS wave function approach



$$\sigma_{\rm eff.}(m_c) \sim \sigma_{\bar{Q}Q}, \quad A_{\rm eff.}(m_c) \sim \frac{1}{2} A_{\bar{Q}Q}$$

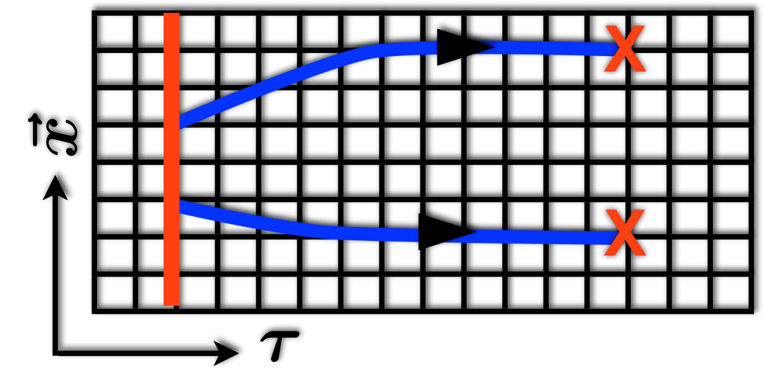
spectator m_Q decreasing...

- **Smaller effective 2Q "string tension"**
- **Larger Coulomb coefficient**

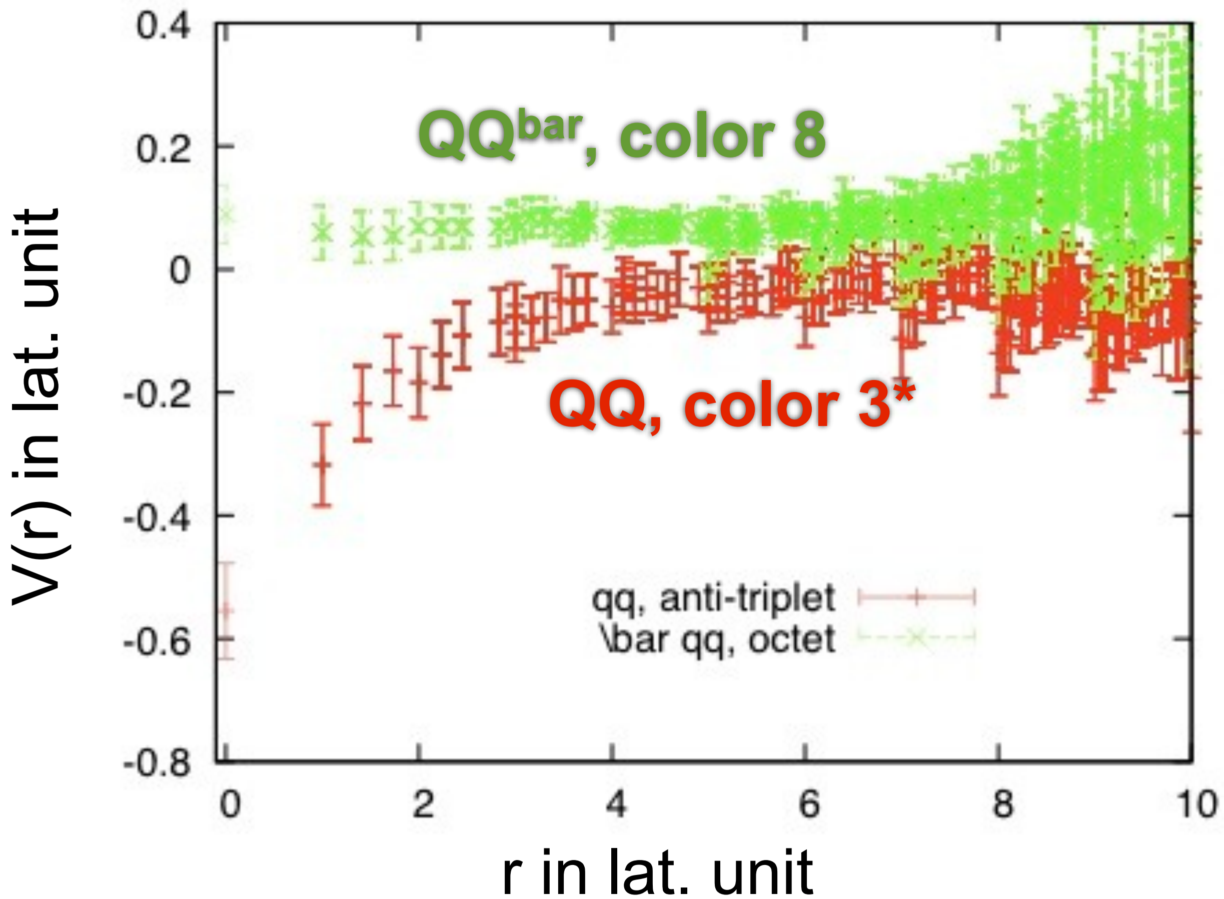
For fun : Color non-singlet potential???

QQbar, color 8: $\bar{q}^a \gamma_5 q^b - \frac{1}{3}\delta^{ab}\bar{q}^c\gamma_5 q^c$

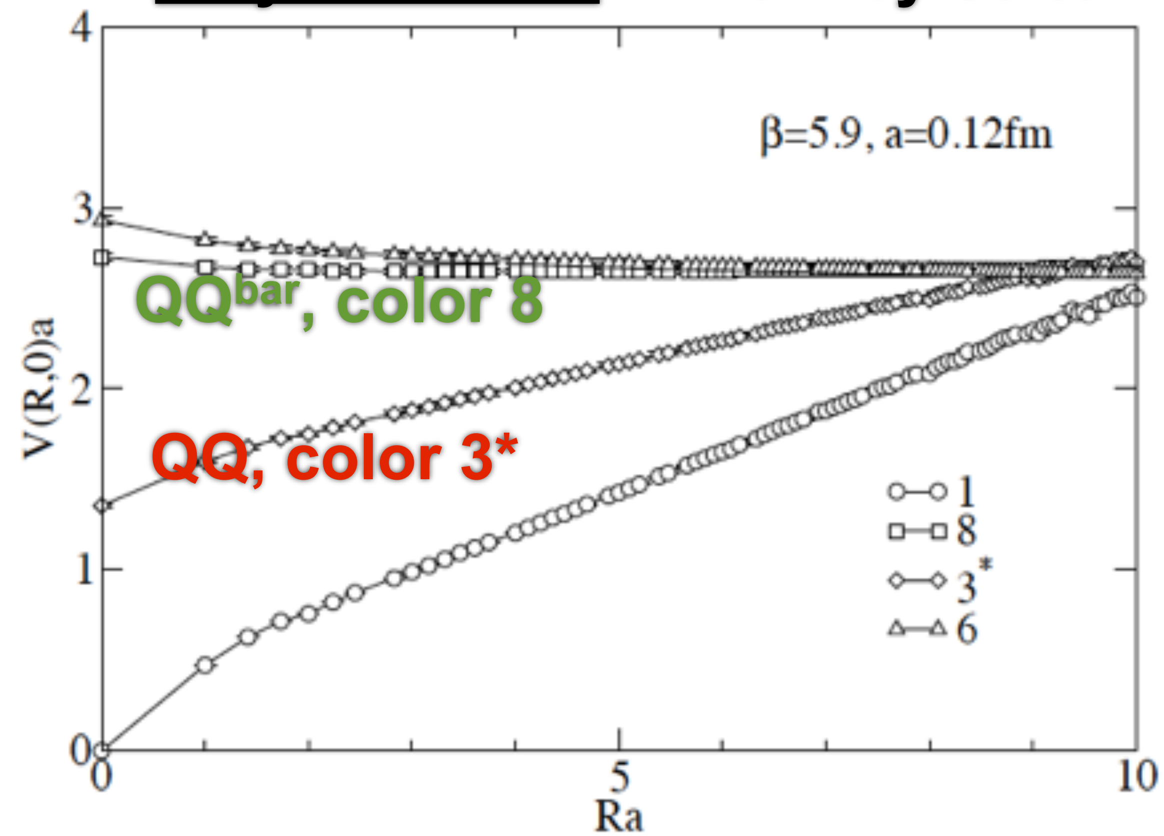
QQ, color 3*: $\epsilon_{abc} q_a^T (C\gamma_5) q_b$



NBS wave function approach???



Instantaneous potential from Polyakov lines --> Talk by Saito



Y.Nakagawa et al., Phys. Rev.D. 77 (2008).

Note :

➡ **Color non-singlet “states” are not QCD eigenstates**

➡ **Impossible to define NBS wave functions**

Summary

- Interquark potentials from NBS wave function

➡ Quench QCD simulation

➡ QbarQ potential: linear + Coulomb force
heavy mass extrapolation of potential --> consistent with Wilson loop

➡ Effective QQ potential: linear + Coulomb force
Casimir factor of color 3* ($A_{3^*c} = 1/2\ A_{1c}$) is observed

- Future plan

➡ Full QCD simulations
➡ Isospin dependence of effective 2Q potentials (Λc & Σc)
➡ Quantitative comparison with quark models??

(2) Tetraquark search on the lattice

Y. Ikeda et al. [HAL QCD Coll.], in preparation.

HAL QCD Collaboration

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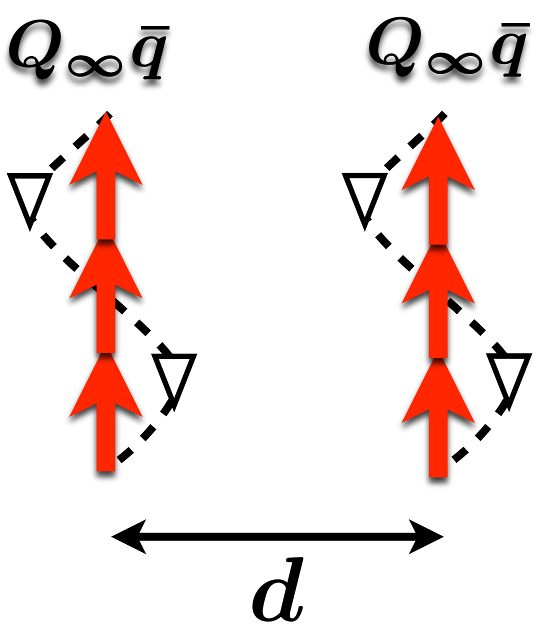
Hadrons to Atomic nuclei HAL from Lattice QCD

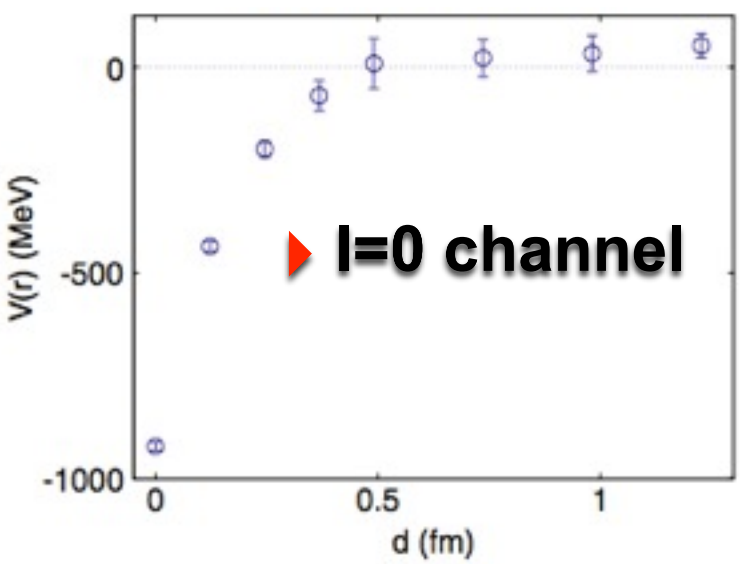
(Hadrons to Atomic nuclei from Lattice QCD)

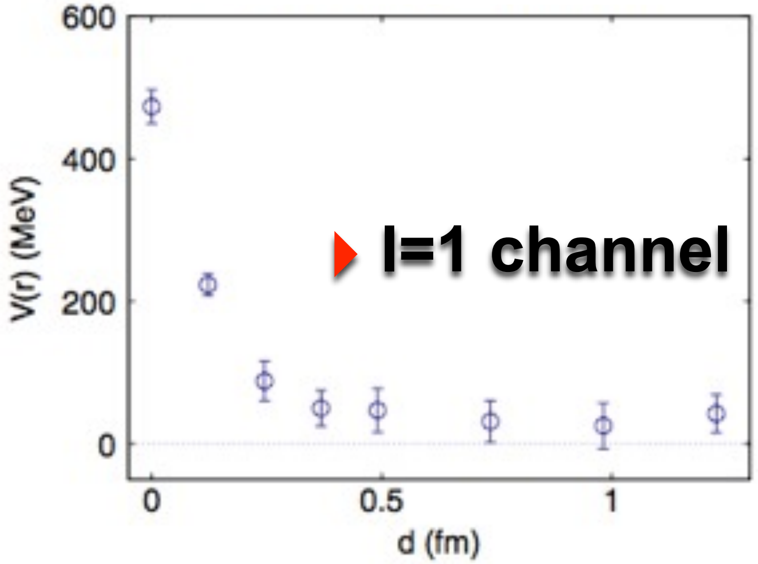
Search for bound T_{QQ} from lattice QCD

- **Static meson-meson interactions** Z. Brown, K Orginos, PRD86, 114506 (2012).

--> Interaction energy in static limit ($Q\bar{q}$) -- ($Q\bar{q}$)

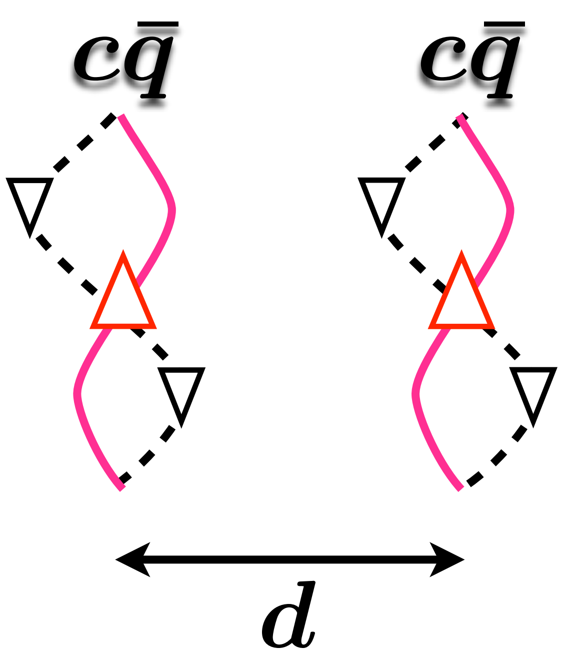






➡ used for T_{bb} search

- **HAL QCD method:** derive potentials to search for bound Tcc



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Advantage

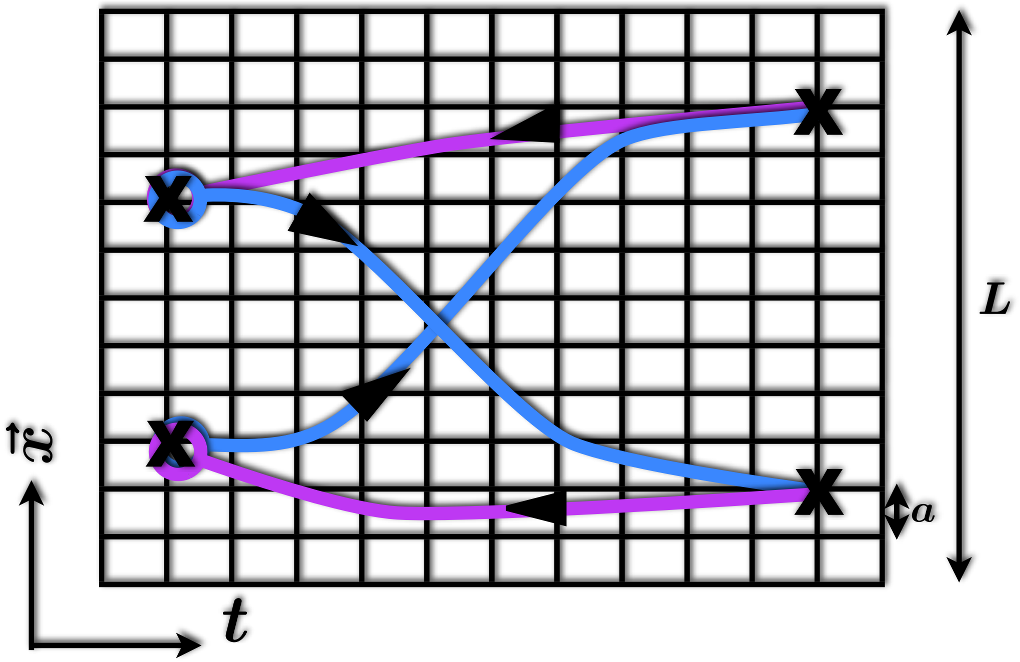
➡ dynamics of charm quarks are automatically taken into account

Scattering on the lattice

Key quantity : Equal-time Nambu-Bethe-Salpeter amplitude

$$\psi(\vec{r},t) = \sum_{\vec{x},\vec{X},\vec{Y}} \langle 0|\phi_1(\vec{x}+\vec{r},t)\phi_2(\vec{x},t)\phi_1(\vec{X},t=0)^\dagger\phi_2(\vec{Y},t=0)^\dagger|0\rangle$$

$$= \sum_{W(\vec{k})} A_{W(\vec{k})} \exp\left[-W(\vec{k})t\right]\psi_{W(\vec{k})}(\vec{r}) \qquad \psi_{W(\vec{k})}(\vec{r}) \equiv \sum_{\vec{x}} \langle 0|\phi_1(\vec{x}+\vec{r})\phi_2(\vec{x})|W(\vec{k}),B,J^P,F\rangle$$



- Helmholtz eq. of NBS wave func.:

$$(\nabla^2 + \vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

- NBS wave func. ~ wave func. in Q.M.

  information on phase shift

$$\psi^{(l)}_{W(\vec{k})}(r) \sim \frac{e^{i\delta_l(k)}}{kr}\sin(kr+\delta_l(k)-l\pi/2)$$

- Temporal correlation, W(k) : phase shift (Luscher's formula) M. Lüscher, NPB354, 531 (1991).
- Spacial correlation, ψ(r) : potential --> observable CP-PACS Coll., PRD71, 094504(2005). Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Lattice QCD potential

Full details, see, Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Helmholtz equation of NBS wave function:

$$(\nabla_r^2 + \vec{k}^2)\psi_W(\vec{r}) = 0 \quad (r > R)$$

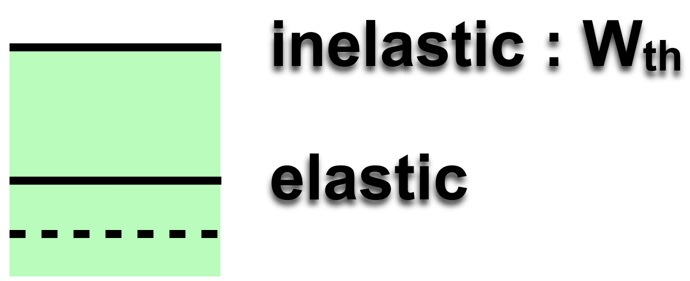
Define half off-shell T-matrix in interacting region:

$$(\nabla_r^2 + \vec{k}^2)\psi_W(\vec{r}) = 2\mu\mathcal{K}_W(\vec{r}) \quad (r < R)$$

Plane wave components are projected out

Derive energy-independent potentials

$$U(\vec{r},\vec{r}') = \int^{W_{\rm th}} \frac{dW}{2\pi}\, \mathcal{K}_W(\vec{r})\psi_W^*(\vec{r}')$$



Energy-independent potentials satisfy time-independent Schrödinger-type equations:

$$(E - H_0)\psi_W(\vec{r}) = \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi_W(\vec{r}')$$

$$E = \frac{\vec{k}^2}{2\mu}, \quad H_0 = -\frac{\nabla_r^2}{2\mu}$$

Further improvement :

Potentials derived HAL QCD method is energy-independent, so that we can extract signal of potentials even with excited state contaminations

Lattice QCD potential

✓ Define energy-independent potential below inelastic threshold:

$$\psi_{W(\vec{k})}(\vec{r}) = \langle 0|\phi_1(\vec{r}+\vec{x})\phi_2(\vec{x})|W(\vec{k})\rangle$$

$$(\vec{k}^2 + \nabla^2)\psi_{W(\vec{k})}(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r},\vec{r}')\psi_{W(\vec{k})}(\vec{r}')$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$W(\vec{k}) = \sqrt{m_1^2 + \vec{k}^2} + \sqrt{m_2^2 + \vec{k}^2}$$

✓ Extract energy-independent potential from time-dependent Schrodinger eq.

Ishii et al.(HAL QCD Coll.), PLB712,437(2012).

$$R(\vec{r},t) = \sum_{\vec{k} \le \vec{k}_{th}} \psi_{W(\vec{k})}(\vec{r}) e^{-\Delta W(\vec{k})t}$$

$$(-\partial_t - H_0 + \mathcal{O}(\partial_t^2))R(\vec{r},t) = \int d\vec{r}' U(\vec{r},\vec{r}') R(\vec{r}',t)$$

$$\Delta W(\vec{k}) = W(\vec{k}) - (m_1 + m_2)$$

$$H_0 = -\frac{\nabla^2}{2\mu}$$

✓ Velocity expansion: leading order central potential

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t} \log R(\vec{r},t)$$

✓ Calculate observable: phase shift, binding energy, ...

Advantage: We can obtain potentials w/o ground/single state saturation

Lattice QCD Setup

N_f=2+1 full QCD configurations generated by PACS-CS Coll.

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- a=0.0907(13) fm --> L~2.9 fm (32^3 x 64)

Light meson mass [conf.1, conf.2, conf.3] (MeV)

M_π=699(1), 572(2), 411(2) [PDG:135 (π^0)]

M_K=787(1), 714(1), 635(2) [PDG:498 (K^0)]

Tsukuba-type Relativistic Heavy Quark (RHQ) action for charm quark

S. Aoki et al., PTP109, 383 (2003)

➡ remove leading cutoff errors $O(m_c a)$, $O(\Lambda_{QCD} a)$, ...

- We are left with $O((a\Lambda_{QCD})^2)$ error (~ a few %)
- We employ RHQ parameters tuned by Namekawa et al.

Y. Namekawa et al., PRD84, 074505 (2011)

Charmed meson mass [conf.1, conf.2, conf.3] (MeV)

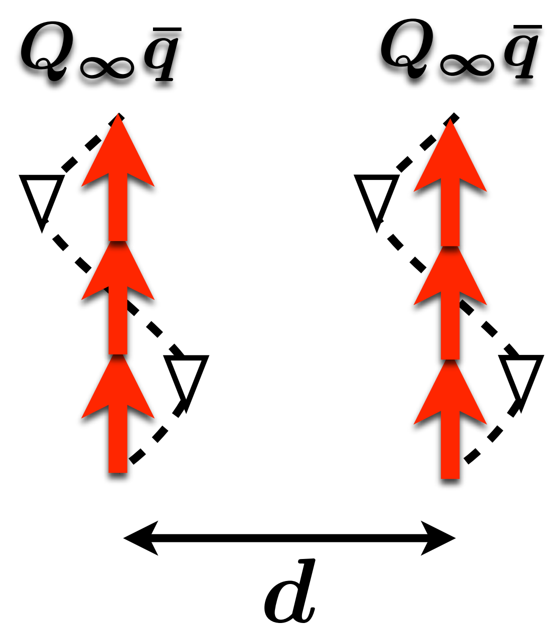
$M_{\eta c}$=3024(1), 3005(1), 2988(2) [PDG:2981]

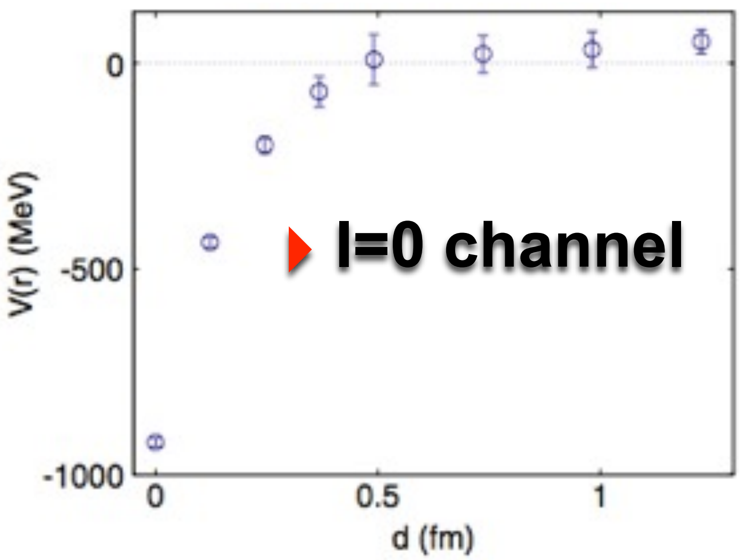
$M_{J/\psi}$=3142(1), 3118(1), 3097(2) [PDG:3097]

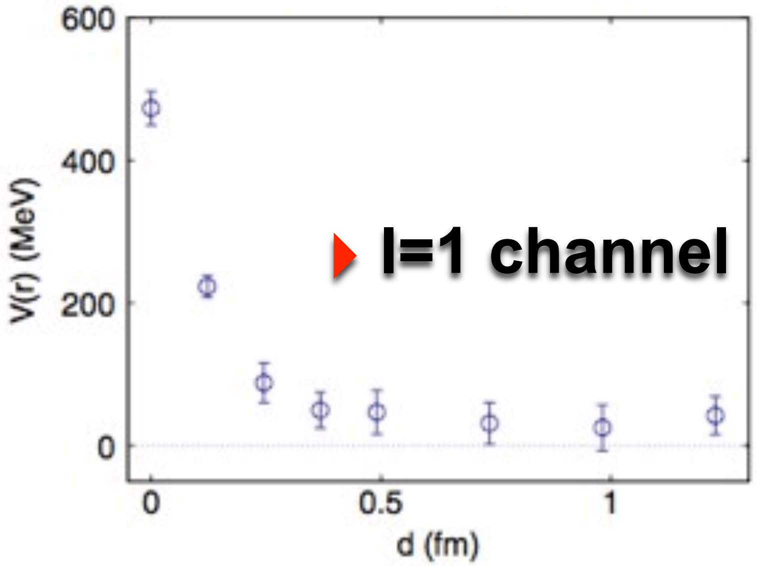
M_D=1999(1), 1946(1), 1912(1) [PDG:1865 (D^0)]

M_{D^*}=2159(4), 2099(6), 2059(8) [PDG:2007 (D^{*0})]

Results : isospin 1 channels



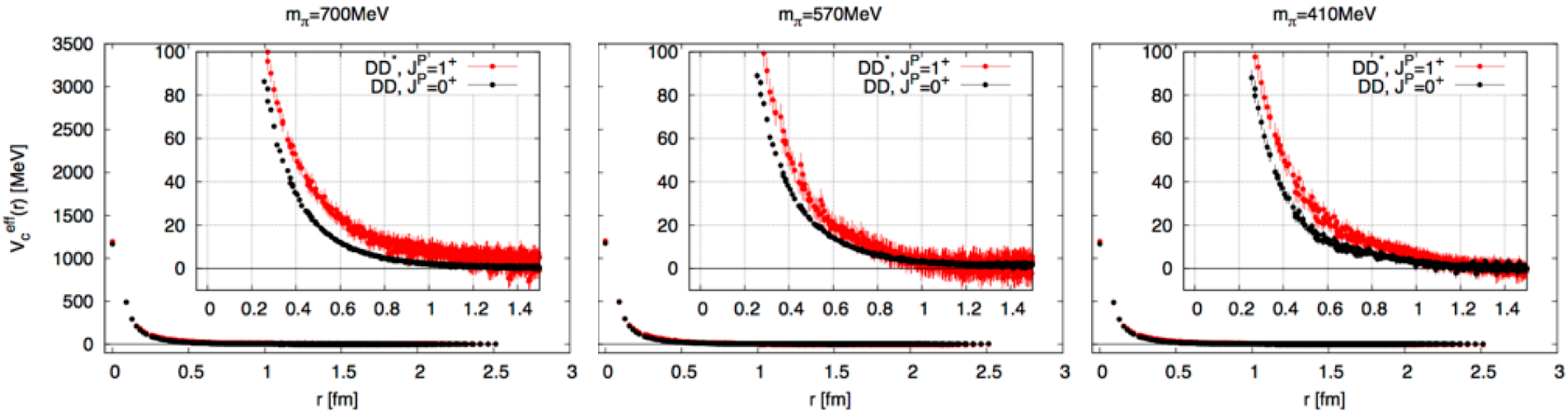




S-wave $DD^{(*)}$ potentials: $T_{cc}(1(0^+, 1^+))$

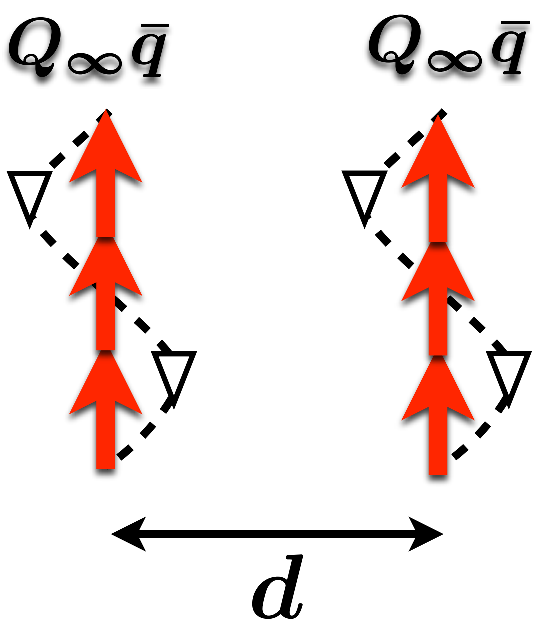
Y. Ikeda et al. [HAL QCD Coll.], in preparation.

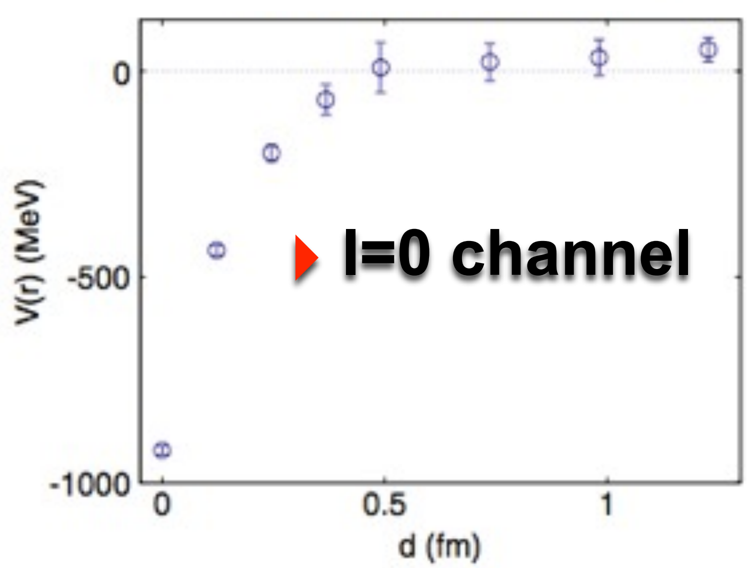
$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t} \log R(\vec{r},t)$$

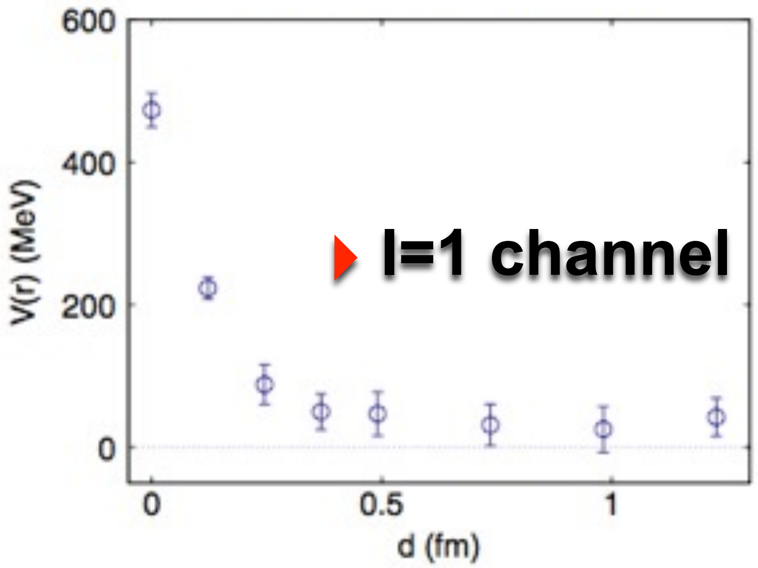


- black: DD s-wave potentials in I=1 [Tcc(0^+) channel]
- red: DD* s-wave potentials in I=1 [Tcc(1^+) channel]
- Weak quark mass dependence of repulsive potentials
- It is unlikely to form bound state even at physical point

Results : isospin 0 channel



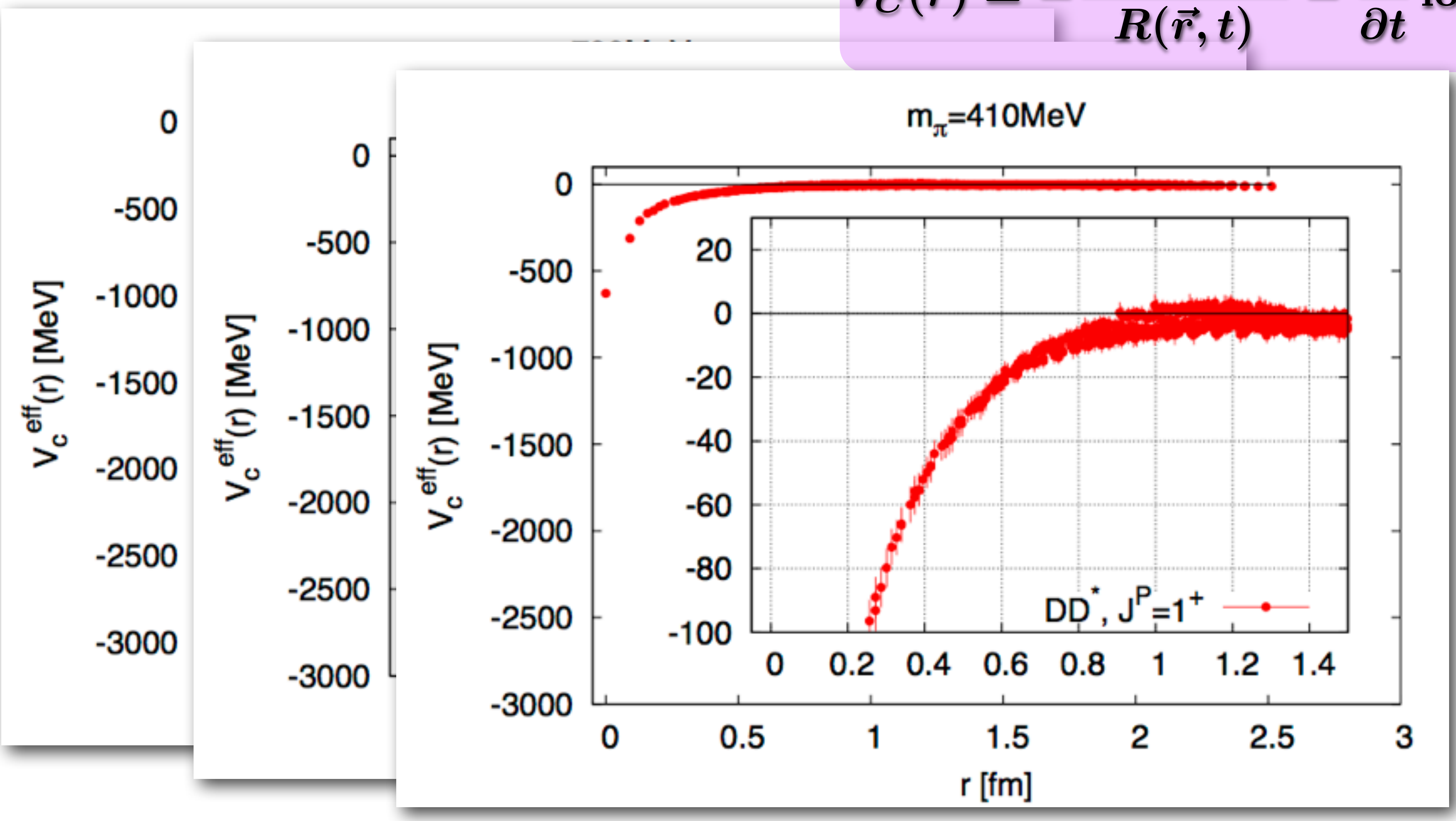




S-wave DD* potential : $T_{cc}(0(1^+))$

Y. Ikeda et al. [HAL QCD Coll.], in preparation.

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

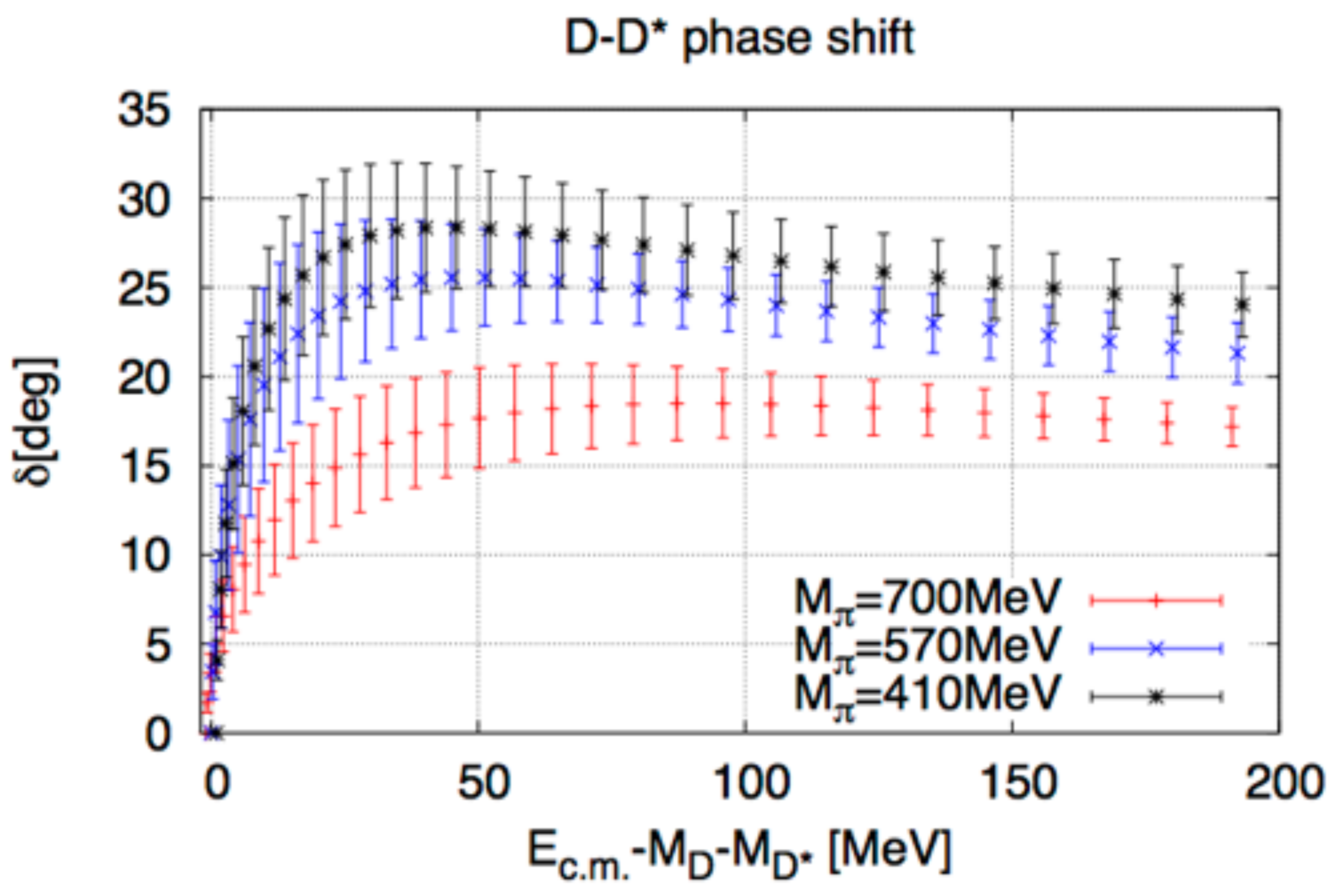


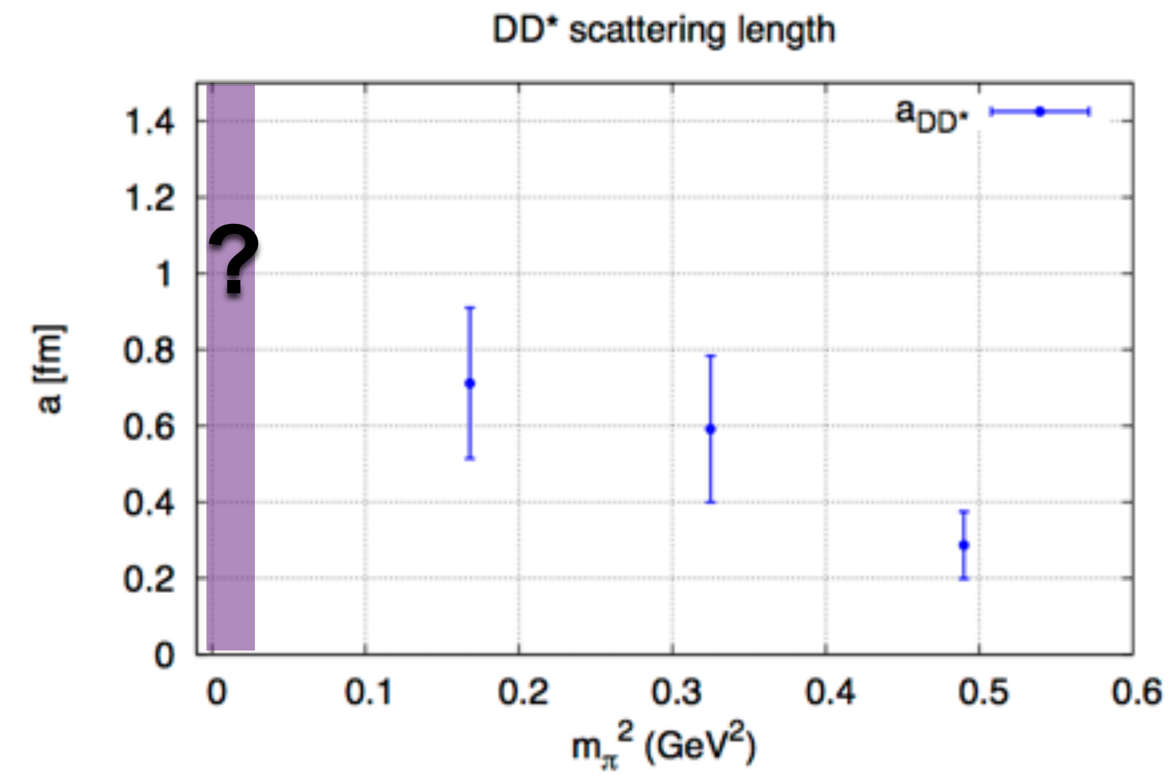
- Attractive DD* s-wave potential in I=0 is observed [Tcc(1+) channel]
- Check whether bound Tcc exist or not --> phase shift analysis

S-wave phase shift : $T_{cc}(0(1^+))$

- fit multi-range gaussian: $f(r) = \sum_i a_i e^{-\nu_i r^2}$

- solve Schrodinger equation in an infinite volume





- Attraction is not enough strong to generate bound state
- Attraction gets stronger as decreasing quark mass
- For definite conclusion, physical point simulations are necessary

Summary

- **Search for Tcc on the lattice@m_π=410, 570, 700MeV**

➡ N_f=2+1 full QCD simulation (PACS-CS configuration)
➡ Charm quarks: Relativistic Heavy Quark action

➡ Tcc ($J^P=0^+, 1^+$, I=1) : s-wave $DD^{(*)}$ potentials are repulsive
Bound states are unlikely...

➡ Tcc ($J^P=1^+$, I=0) : s-wave DD^* potential is attractive,
but not strong enough to form bound states@m_π=410, 570, 700 MeV

- **Future plan**

➡ Physical point simulation
➡ T_{bb} search
➡ Coupled-channel analysis (DD^*-D^*D^*,...)
= resonant-tetraquark search