QCD Potential in PNRQCD

Yuichiro Kiyo, Juntendo Univ.
(collaboration with Y. Sumino)

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PNRQCD FRAMEWORK

PNRQCD

Brambilla-Pineda-Soto-Vairo

Lagrangian for color singlet/octet bi-linear field

$$S_{ab}(t;\mathbf{r}) = \left(\frac{1}{\sqrt{N_c}}\right)_{ab} \psi^\dagger(t,\frac{\mathbf{r}}{2})\chi(t,-\frac{\mathbf{r}}{2}) \qquad O_{ab}(t;\mathbf{r}) = \left(\sqrt{2}T^a\right)_{ab} \psi^\dagger(t,\frac{\mathbf{r}}{2})T^a\chi(t,-\frac{\mathbf{r}}{2})$$

$$V_s(r) = -\frac{C_F\alpha_s}{r} \qquad V_o(r) = \frac{(C_A/2 - C_F)\alpha_s}{r}$$

$$\begin{aligned}\mathcal{L}_{pNRQCD} = &\,\mathrm{Tr}\left\{S^\dagger\left(i\partial_0 + \frac{\nabla^2}{m} - V_s(r)\right)S\right\} \\ &+ \mathrm{Tr}\left\{O^\dagger\left(iD_0 + \frac{\nabla^2}{m} - V_o(r)\right)O\right\} \\ &- g_s\mathrm{Tr}\left\{O^\dagger\mathbf{r}\cdot\mathbf{E}_{us}(t)S + S^\dagger\mathbf{r}\cdot\mathbf{E}_{us}(t)O\right\}\end{aligned}$$

dipole interaction between gluon and bilinaer fields

STATIC LIMIT

$$\mathcal{L}_{PNRQCD} = S^\dagger (i\partial_0 - V_s) S + O^\dagger (i\partial_0 - g_s A_0) O - g_s O^\dagger \mathbf{r}\cdot\mathbf{E} S - g_s S^\dagger \mathbf{r}\cdot\mathbf{E} O$$

$$\langle \mathrm{T} S(t) S^\dagger(0) \rangle_0 = \theta(t) e^{-itV_s(r)}$$

$\langle \mathrm{T} S(t) S^\dagger(0) \rangle =$  $+$

$$[\text{exponent}] = V_s(r)t + \langle \mathrm{T}[\,\mathbf{r}\cdot\mathbf{E}(t) O(t) O^\dagger(0) \mathbf{r}\cdot\mathbf{E}(0)] \rangle + \cdots$$

corr. due to ultrasoft-gluon

- heavy quark limit=static limit
- quark-antiquark propagation (in Eucledian time) is exponentially suppressed by Vs(r) and ultrasoft gluon corrections

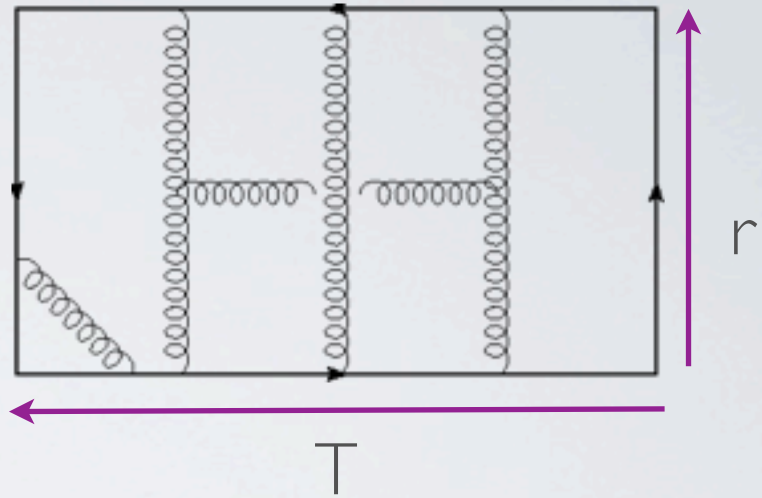
static energy

Extract the exponential factor (Vs+us corr) from Lattice QCD data in rectangle Wilson loop

STATIC ENERGY IN QCD

STATIC ENERGY& WILSON LOOP

$$W[C] = \langle \mathrm{Tr P} e^{ig \int_C dx \cdot A(x)} \rangle \stackrel{T\to\infty}{\longrightarrow} e^{-iT E_s(r)}$$



Wilson loop in large T limit defines the static energy

- W[C] is gauge invariant, and well-defined quantity; e.g. comparison possible with Lattice QCD
- Important quantity in heavy quark physics, J/ψ,ϒ,...; Quarkonia energy spectrum

PERT. PART OF STATIC ENERGY

$$\tilde{V}_s(q) = -\frac{C_F \alpha_s(q)}{\vec{q}^2}\left[1 + \frac{\alpha_s(q)}{4\pi}a_1 + \left(\frac{\alpha_s(q)}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s(q)}{4\pi}\right)^3 \left(a_3 + \frac{8\pi^2 C_A^3}{3}\left(\frac{1}{\epsilon} + 6\ln\frac{\mu}{q}\right)\right)\right]$$

$$a_1 = \frac{31}{3}C_A - \frac{20}{9}T_F n_l$$

W.Fischler '77; A.Billoire '80

$$a_2 = \left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^4}{4} + \frac{22\zeta_3}{3}\right)C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\right)C_A T_F n_l - \left(\frac{55}{3} - 16\zeta_3\right)C_F T_F n_l + \left(\frac{20}{9}T_F n_l\right)^2$$

M.Peter '97; Y.Schroeder '99

$$a_3 = 502.22(12)C_A^3 - 136.8(14)\frac{d_F^{abcd}d_A^{abcd}}{N_A}$$

Anzai-YK-Sumino(09)
Smirnov-Smirnov-Steinhauser(09)

$$- 709.717 C_A^2 T_F n_l - 56.83(1)\frac{d_F^{abcd}d_F^{abcd}}{N_A} + \left(-\frac{71281}{162} + 264\zeta_3 + 80\zeta_5\right)C_A C_F T_F n_l$$

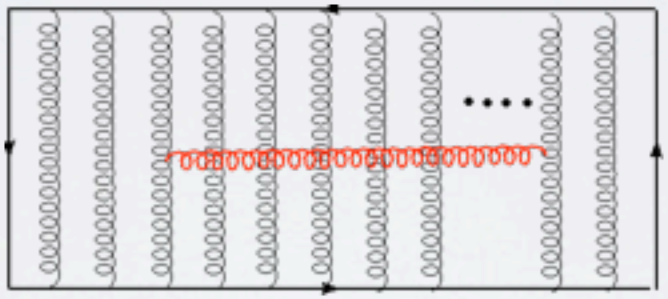
$$+ \left(\frac{286}{9} + \frac{296\zeta_3}{3} - 160\zeta_5\right)C_F^2 T_F n_l + \left(\frac{12541}{243} + \frac{368\zeta_3}{3} + \frac{64\pi^2}{135}\right)C_A\left(T_F n_l\right)^2$$

$$+ \left(\frac{14002}{81} - \frac{416\zeta_3}{3}\right)C_F\left(T_F n_l\right)^2 - \left(\frac{20}{9}T_F n_l\right)^3$$

Smirnov-Smirnov-Steinhause(08)

ULTRA-SOFT CORRECTION

- There are non-perturbative corrections for Wilson loop, which at leading order is known as ultra-soft correction. In the following we will combine this with our perturbative corrections.



$$\delta E_{us}(r) = -ig_s^2 \frac{T_R}{N_R} \int_0^\infty e^{-it(V_{ad}-V_s)} \langle 0 | \mathrm{T}\left[\vec{r} \cdot \vec{E}^a(t) \left(\mathrm{P} e^{ig_s \int_0^t dt' A_0^c T_A^c} \right)_{ab} \vec{r} \cdot \vec{E}^b(0) \right] | 0 \rangle$$

$$= \frac{C_R C_A^3 \alpha_s(\mu)^4}{24\pi r} \left[\frac{1}{\epsilon} + 4 \ln(\mu^2 r^2) - 2 \ln(C_A \alpha_s(\mu)) + \frac{5}{3} + 6\gamma_E \right] + \mathrm{h.o.}$$

Brambilla-Pineda-Soto-Vairo('99),
Kniehl-Penin('99),
Anzai-YK-Sumino (2010)

2013年9月12日木曜日

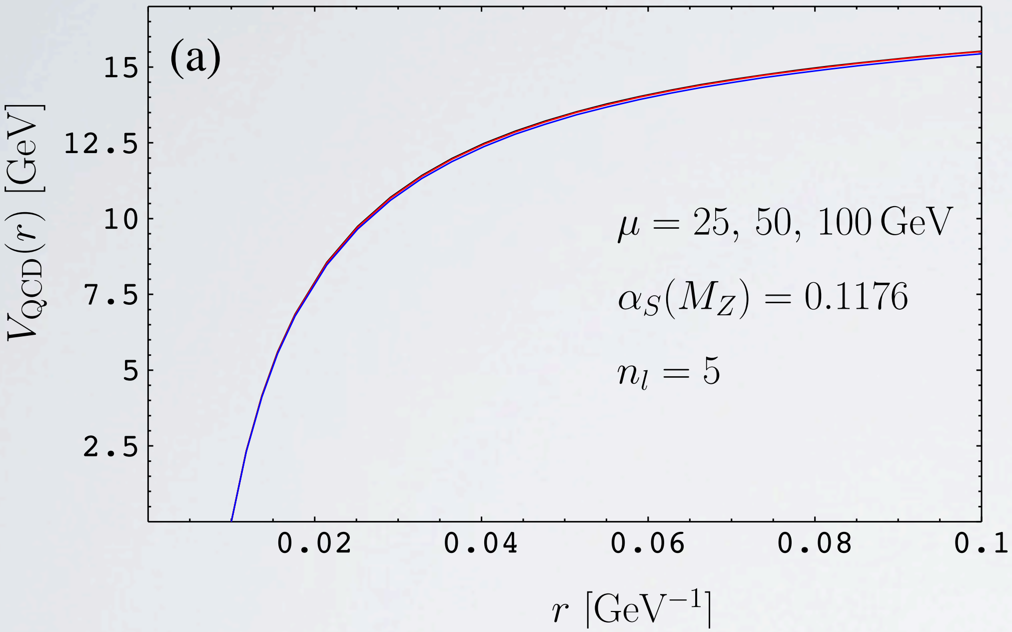
Static energy defined using Wilson loop contains two parts; one purely perturbative and the other non-perturbative (ultra-soft cont.)

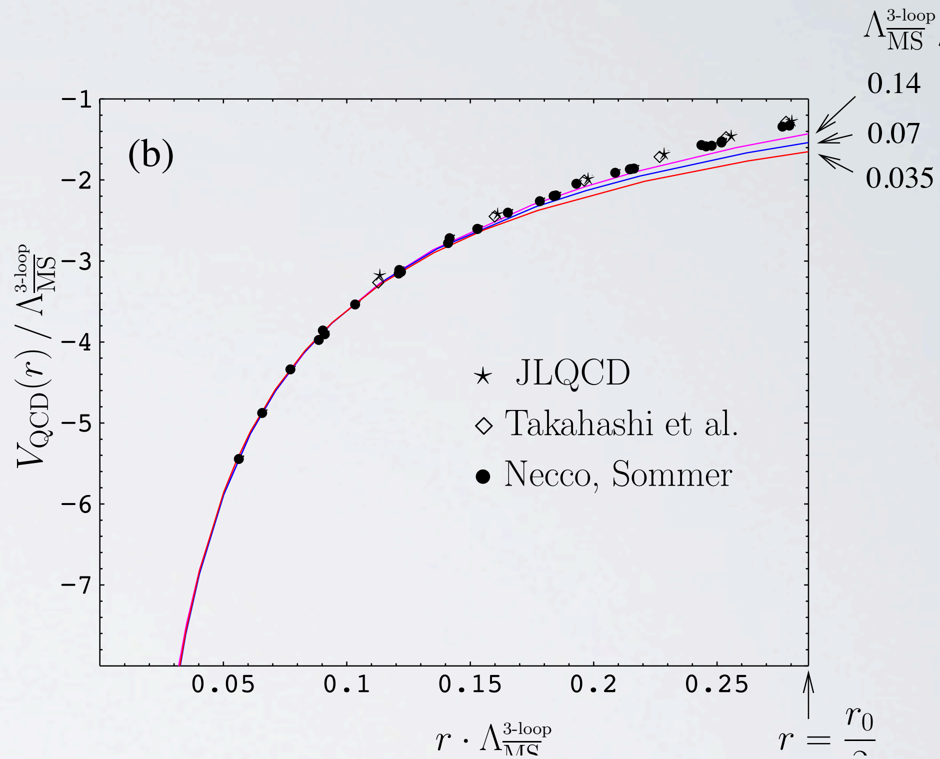
$$E_s(r) \equiv V_s(r) + \delta E_{us}(r)$$

static energy | QCD potential | ultra-soft corrections

STATIC ENERGY AT THREE-

N.B. The static energy is denoted as $V_{QCD}(r)$ in the plots below





QCD Potential (PT+US) at three-loop for mu=25, 50, 100 [GeV] for top quark. The three lines coincide each other and one sees no difference

Comparison with (quench) lattice QCD, where the region roughly corresponds to Upsilon 1S.
Agreement is very good at short distance, while a small deviation starts to show up at long distance region

Anzai-YK-Sumino, PRL104, 112003

SUMMARY: STATIC ENERGY (STATIC POTENTIAL)

- QCD static potential at three loop is completed after 32 (10) years from 1-loop (2-loop) computation.(Anzai-YK-Sumino, Smirnov-Smirnov-Steinhauser 2009)
- Agreement between PT and Lattice QCD is good
- Further improvement should be possible using running potential.

Nowadays, direct comparison of static energy between perturbative computation and lattice QCD gives accurate determination of α_s.

e.g. Brambilla-Tormo-Soto-Vairo, 1205.6155[hep-ph]

QCD POTENTIAL AND CASIMIR SCALING

Anazai-YK-Sumino(2010)

STATIC POTENTIAL FOR GENERAL COLOR REPRESENTATION

$$V_R^{\mathrm{PT}}(r) = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \int \frac{d^d\vec{q}}{(2\pi)^d} e^{i\vec{q}\cdot\vec{r}} \left[-4\pi C_R \frac{\alpha_{V_R}^{\mathrm{PT}}(q)}{q^2}\right],$$

The perturbative expansion of $\alpha_{V_R}^{\mathrm{PT}}(q)$ is expressed as

$$\alpha_{V_R}^{\mathrm{PT}}(q) = \alpha_s(\mu) \sum_{n=0}^{\infty} P_n(L) \left(\frac{\alpha_s(\mu)}{4\pi}\right)^n$$

with

$$L = \log\left(\frac{\mu^2}{q^2}\right).$$

$$P_0 = a_0, \qquad P_1 = a_1 + a_0\beta_0 L, \qquad P_2 = a_2 + (2a_1\beta_0 + a_0\beta_1)L + a_0{\beta_0}^2 L^2$$

$$P_3 = a_3 + (3a_2\beta_0 + 2a_1\beta_1 + a_0\beta_2)L + \left(3a_1{\beta_0}^2 + \frac{5}{2}a_0\beta_0\beta_1\right)L^2 + a_0{\beta_0}^3 L^3,$$

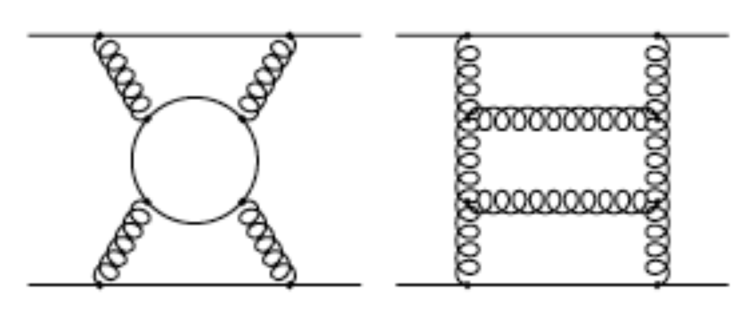
CASIMIR SCALING

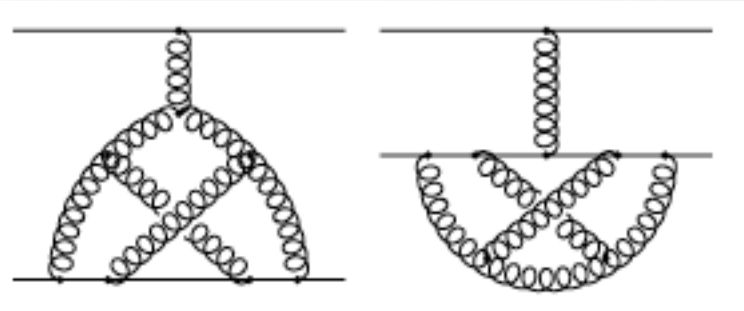
$$a_0 = 1, \qquad a_1 = \frac{31}{9}C_A - \frac{20}{9}T_F n_l,$$

$$a_2 = \left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^2}{4} + \frac{22}{3}\zeta_3\right)C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta_3\right)C_A T_F n_l - \left(\frac{55}{3} - 16\zeta_3\right)C_F T_F n_l + \left(\frac{20}{9}T_F n_l\right)^2,$$

$$\overset{C.S.}{\alpha_{V_R} =? = \alpha_{V_F}}$$

$$\bar{a}_3 = -\left(\frac{20}{9}n_l T_F\right)^3 + \left[C_A\left(\frac{12\,541}{243} + \frac{64\pi^4}{135} + \frac{368}{3}\zeta_3\right) + C_F\left(\frac{14\,002}{81} - \frac{416}{3}\zeta_3\right)\right](n_l T_F)^2 + \left[2c_1 C_A^2 + \left(-\frac{71\,281}{162} + 264\zeta_3 + 80\zeta_5\right)C_A C_F + \left(\frac{286}{9} + \frac{296}{3}\zeta_3 - 160\zeta_5\right)C_F^2\right]n_l T_F + \frac{1}{2}c_2 n_l\left(\frac{d_F^{abcd}d_R^{abcd}}{N_A T_R}\right) + \left[c_3 C_A^3 + \frac{1}{2}c_4\left(\frac{d_A^{abcd}d_R^{abcd}}{N_A T_R}\right)\right],$$





~~Casimir Scaling~~

GROUP THEORY FACTORS

Table 1

Some of the color factors related to the fundamental (vector) and adjoint representations ($R = F$ and $R = A$, respectively) in the case $G = SU(N)$ and $G = SO(N)$. Our convention is $T_F = 1/2$.

	$SU(N)$	$SO(N)$
N_F	N	N
N_A	$N^2 - 1$	$N(N-1)/2$
C_F	$(N^2 - 1)/(2N)$	$(N-1)/4$
C_A	N	$(N-2)/2$
T_A	N	$(N-2)/2$
$d_F^{abcd} d_F^{abcd}/(N_A T_F)$	$(N^4 - 6N^2 + 18)/(48N^2)$	$(N^2 - N + 4)/192$
$d_F^{abcd} d_A^{abcd}/(N_A T_F)$	$N(N^2 + 6)/24$	$(N-2)(N^2 - 7N + 22)/192$
$d_F^{abcd} d_A^{abcd}/(N_A T_A)$	$(N^2 + 6)/48$	$(N^2 - 7N + 22)/192$
$d_A^{abcd} d_A^{abcd}/(N_A T_A)$	$N(N^2 + 36)/24$	$(N^3 - 15N^2 + 138N - 296)/192$

C.S. VIOLATION

$$\delta_{\rm CS}^{V_R} = \frac{[V_R(r) - V_R(r_1)]/C_R}{[V_F(r) - V_F(r_1)]/C_F} - 1,$$

$$\delta_{\rm CS}^{F_R} = \frac{V_R'(r)/C_R}{V_F'(r)/C_F} - 1.$$

renormalon free quantity

$$\delta_{\rm CS}^{V_R} = \delta_{\rm CS}^{F_R} = \left(\frac{\alpha_s(\mu)}{4\pi}\right)^3 \left[\frac{c_2 n_l}{2N_A}\left(\frac{d_F^{abcd} d_R^{abcd}}{T_R} - \frac{d_F^{abcd} d_F^{abcd}}{T_F}\right) + \frac{c_4}{2N_A}\left(\frac{d_A^{abcd} d_R^{abcd}}{T_R} - \frac{d_A^{abcd} d_F^{abcd}}{T_F}\right)\right] + \mathcal{O}(\alpha_s^4).$$

example for R=A

$$\delta_{\rm CS}^{V_A}, \delta_{\rm CS}^{F_A} \approx \alpha_s^3(-0.129 - 0.0030 n_l) \approx \begin{cases} -0.00032 & (n_l = 0,\ \mu = \mu_1 = \Lambda_{\overline{\rm MS}}^{\text{3-loop}}/0.035), \\ -0.0013 & (n_l = 0,\ \mu = \mu_2 = \Lambda_{\overline{\rm MS}}^{\text{3-loop}}/0.14). \end{cases}$$

CASIMIR SCALING

- G.S.Bali(2000) hep-lat/9911008 ovserved no C.S. violation withing 5% accuracy, for R=8,6,15a,10,27,24,15s.
- If take large representation we may observe C.S. violation by comparing lattice data, e.g. k th-rank completely symmetric rep. with k=30 , $\delta_{CS}^{V,F}$ becomes -6%, -26% for V and F case, respectively.