ミニ滞在型研究検討会「チャームバリオンの構造と生成」
2013年9月10日(火)−9月13日(金)

Composite and elementary nature of a resonance in the sigma model

"複合成分と素成分の混合"

Hideko Nagahiro[A,B], Atsushi Hosaka[B]

[A] Nara Women's University
[B] RCNP, Osaka University

References:

H. Nagahiro, and A. Hosaka, arXiv:1307.2031

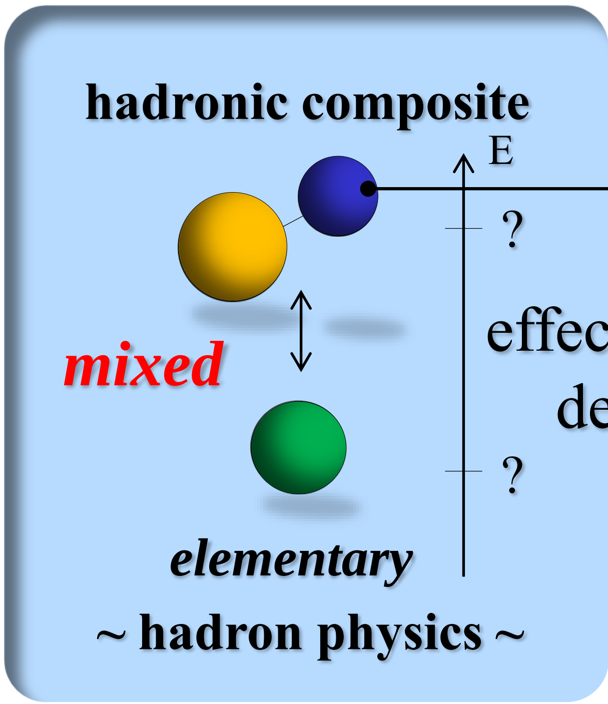
H. Nagahiro, and A. Hosaka, in progress

H. Nagahiro, K. Nawa, S. Ozaki, D. Jido, and A. Hosaka, PRD83(2011)111504(R)

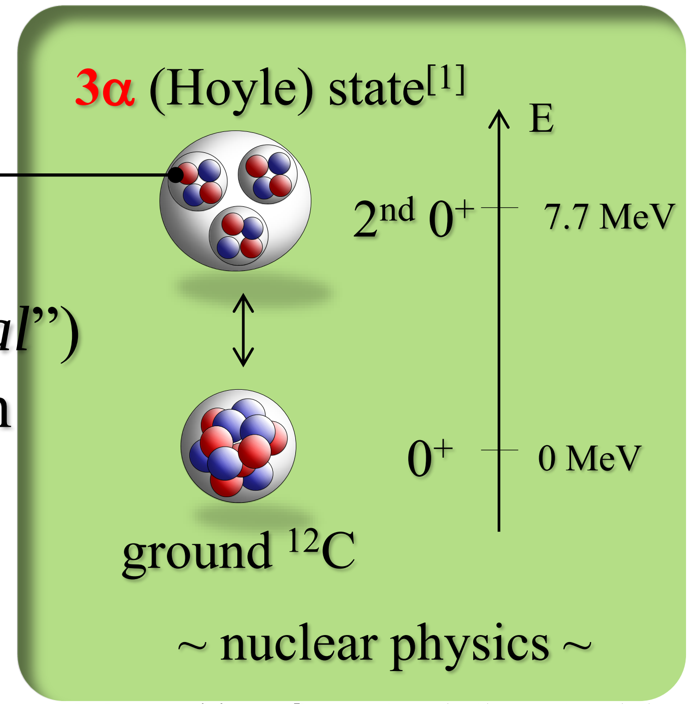
Introduction & motivation

various hadron structure

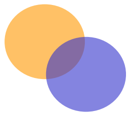
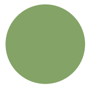
Complex mass spectrum



effective ("*economical*") degrees of freedom



[1] Funaki *et al.*, PRC67(03)051306(R)

| physical state ⟩ = C_1 |  ⟩ + C_2 |  ⟩ + …

Simple question : "How and how much they are mixed ? Can we estimate it?"

contents

First Half

» Composite and elementary nature of a resonance in the sigma model
arXiv:1307.2031 : within two-level problem

$$|\sigma\rangle_{\rm phys} = C_1 \,|\, \pi\pi \,\rangle + C_2 \,|\, \sigma \,\rangle + \dots$$

Latter Half

» "compositeness condition $Z = 0$" [1-3]
⇔ "elementary component z"

» Representation **Dependence**

» Which (what) is an "economical" picture ?

[1] S.Weinberg, PR137(65)B672
[2] D. Lurie, A.J.Macfarlane, PR136(64)B816
[3] T.Hyodo, D.Jido, A. Hosaka, PRC85(12)015201

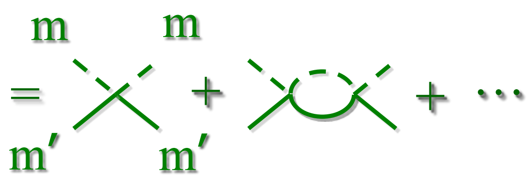
Nagahiro, Nawa, Ozaki, Jido, Hosaka, PRD83(2011)111504(R)

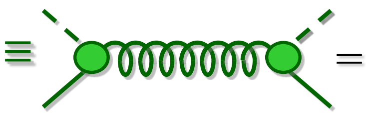
Two level problem

composite meson

dynamically generated s-wave resonance
in chiral unitary approach

$$t_{WT} = \frac{v_{WT}}{1 - v_{WT} G} =$$

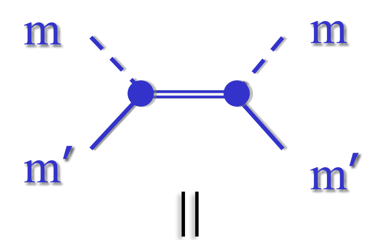




$$g_R(s) \frac{1}{s - s_p} g_R(s)$$

elementary meson

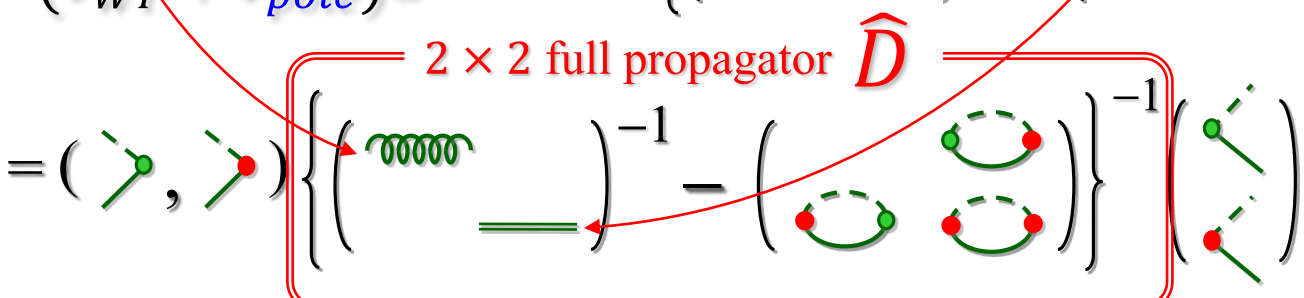
elementary field in Lagrangian



$$v_{pole} = g \frac{1}{s - M^2} g$$

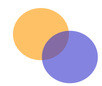
full scattering amplitude: mixture of two basis state

$$T = \frac{v_{WT} + v_{pole}}{1 - (v_{WT} + v_{pole}) G} = (g_R, g) \left\{ \begin{pmatrix} s - s_p & \\ & s - M^2 \end{pmatrix} - \begin{pmatrix} & g_R G g \\ g G g_R & g G g \end{pmatrix} \right\}^{-1} \begin{pmatrix} g_R \\ g \end{pmatrix}$$



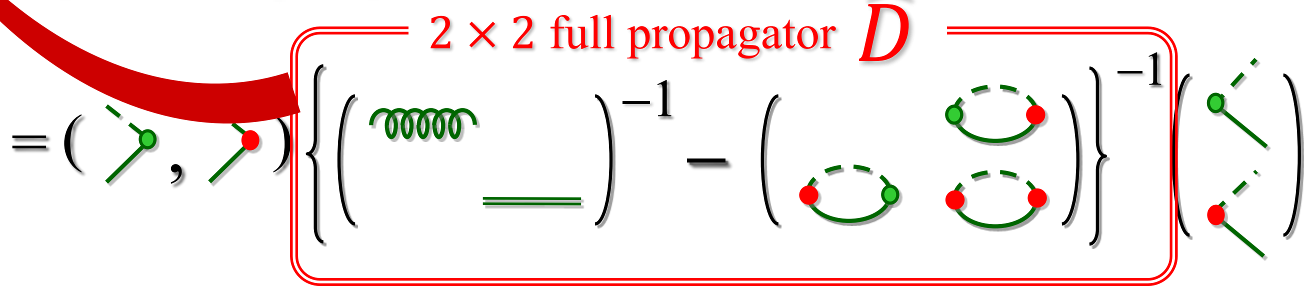
Two level problem

$$D^{11} = \frac{z_a^{11}}{s - M_a^2} + \frac{z_b^{11}}{s - M_b^2}\ , \quad D^{22} = \frac{z_a^{22}}{s - M_a^2} + \frac{z_b^{22}}{s - M_b^2}$$

$|phys\rangle = \sqrt{z_a^{11}}\ |$  $\rangle + \sqrt{z_a^{22}}\ |$  $\rangle$

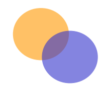
<u>**full scattering amplitude:**</u> <u>**mixture of two basis state**</u>

$$T = \frac{v_{WT} + v_{pole}}{1 - (v_{WT} + v_{pole})G} = (g_R, g)\left\{\begin{pmatrix} s - s_p & \\ & s - M^2 \end{pmatrix} - \begin{pmatrix} & g_R G g \\ g G g_R & g G g \end{pmatrix}\right\}^{-1}\begin{pmatrix} g_R \\ g \end{pmatrix}$$



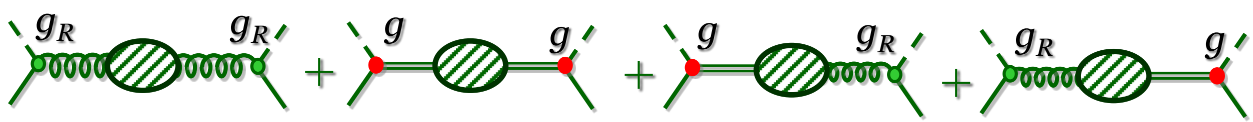
Two level problem

$$\color{red}{D^{11}} = \frac{\color{red}{z_a^{11}}}{s - M_a^2} + \frac{\color{red}{z_b^{11}}}{s - M_b^2} \;, \quad \color{blue}{D^{22}} = \frac{\color{blue}{z_a^{22}}}{s - M_a^2} + \frac{\color{blue}{z_b^{22}}}{s - M_b^2}$$

$|phys\rangle = \sqrt{z_a^{11}}\,|$  $\rangle + \sqrt{z_a^{22}}\,|$  $\rangle$

<u>Full scattering amplitude around a pole M_a</u>

$$T = g_R \frac{z_a^{11}}{s - M_a^2} g_R + g \frac{z_a^{22}}{s - M_a^2} g + g \frac{z_a^{21}}{s - M_a^2} g_R + g_R \frac{z_a^{12}}{s - M_a^2} g + \cdots$$



$$= \left(g_R \sqrt{z_a^{11}} + g \sqrt{z_a^{22}} \right)^2 \frac{1}{s - M_a^2} + \cdots \qquad (\because\ (z^{12})^2 = z^{11} z^{22}\ )$$

cf. in a simple Yukawa model $\quad T_{\text{Yukawa}} = \left(g_0 Z^{1/2}\right)^2 \dfrac{1}{s - M^{*2}}$

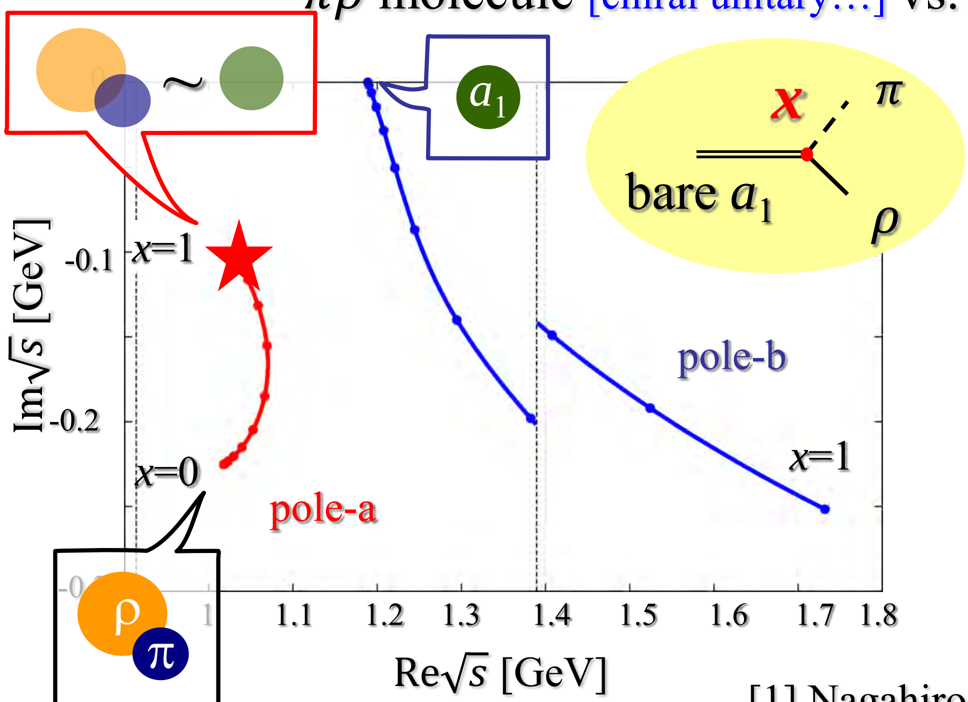
Application to $a_1(1260)$ axial meson

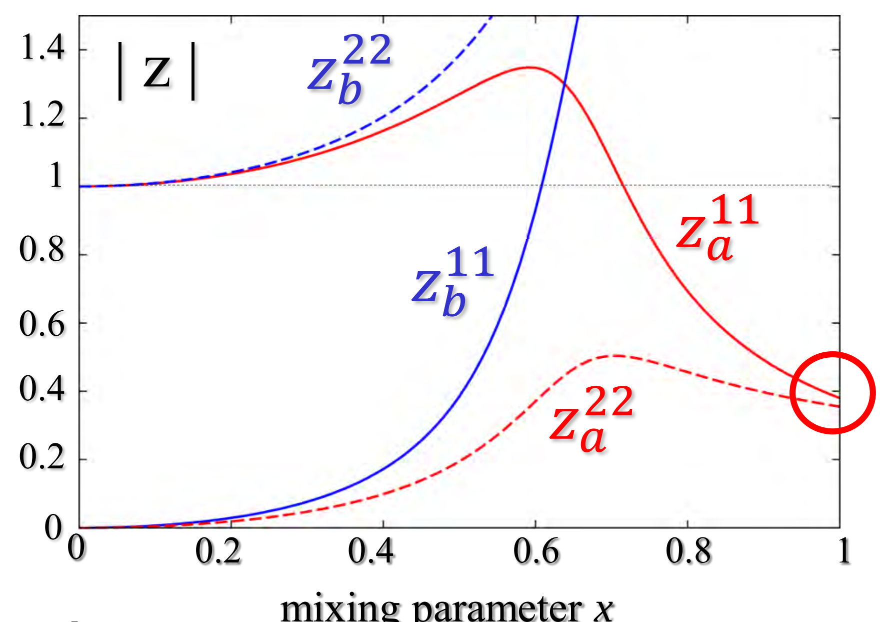
$$D^{11} = \frac{z_a^{11}}{s - M_a^2} + \frac{z_b^{11}}{s - M_b^2}\ , \quad D^{22} = \frac{z_a^{22}}{s - M_a^2} + \frac{z_b^{22}}{s - M_b^2}$$

$$|phys\rangle = \sqrt{z_a^{11}}\ |\quad\rangle + \sqrt{z_a^{22}}\ |\quad\rangle$$

<u>Hidden local symmetry (or Holographic) model</u>

$\pi\rho$ molecule [chiral unitary…] vs. elementary a_1 [NJL, Lattice…]





[1] Nagahiro *et al.*, '11

Application to sigma meson

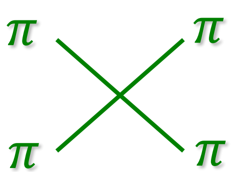
Nagahiro-Hosaka, arXiv:1307.2031

sigma model in non-linear representation

$$\mathcal{L} = \frac{1}{4}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \frac{\mu^2}{4}\,\mathrm{tr}\left(\Sigma^\dagger \Sigma\right) - \frac{\lambda}{16}\left(tr\left(\Sigma^\dagger \Sigma\right)\right)^2 + a\,\mathrm{tr}\left(\Sigma^\dagger \Sigma\right)$$

$$\Sigma = (f_\pi + \sigma)U, \quad U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi\,)$$

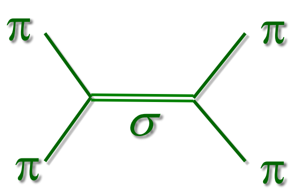
Interaction kernel (potential v) in Bethe-Salpeter eq.



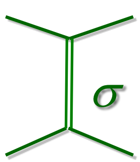
$$v_{con} = -\frac{1}{f_\pi^2}(2s - m_\pi^2)$$

→ Attractive enough to develop σ pole as $\pi\pi$ composite

Oller, Oset, NPA620(97)438



$$v_{pole} = -\frac{3}{f_\pi^2}(s - m_\pi^2)^2 \frac{1}{s - m_\sigma^2}$$



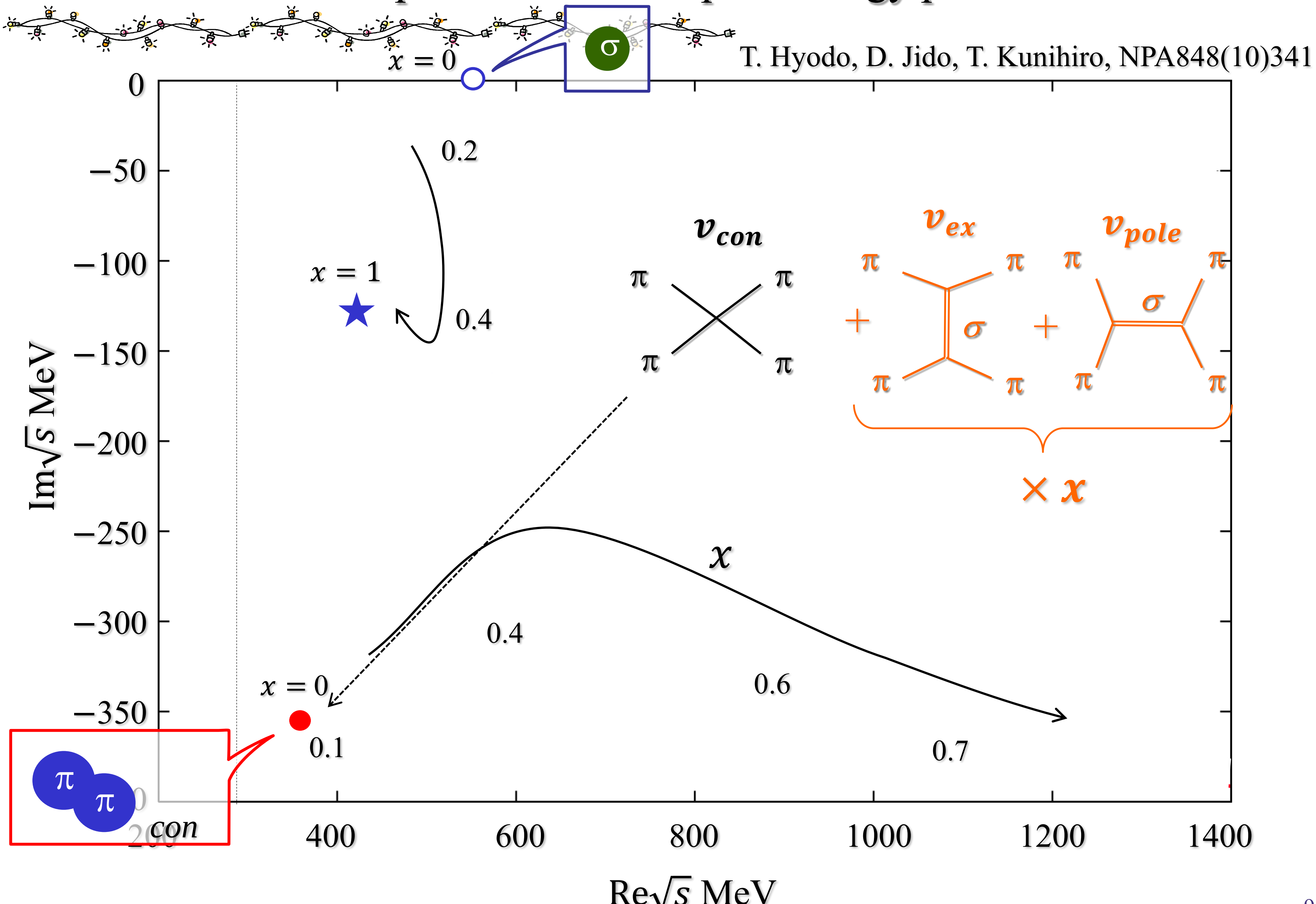
$$v_{ex} = -\frac{1}{f_\pi^2}\left\{-(s - 2m_\sigma^2) + \frac{2(m_\pi^2 - m_\sigma^2)}{s - 4m_\pi^2}\ln\left(\frac{m_\sigma^2}{m_\sigma^2 - 4m_\pi^2 + s}\right)\right\}$$

BS eq. for s-wave $\pi\pi$ scattering amplitude

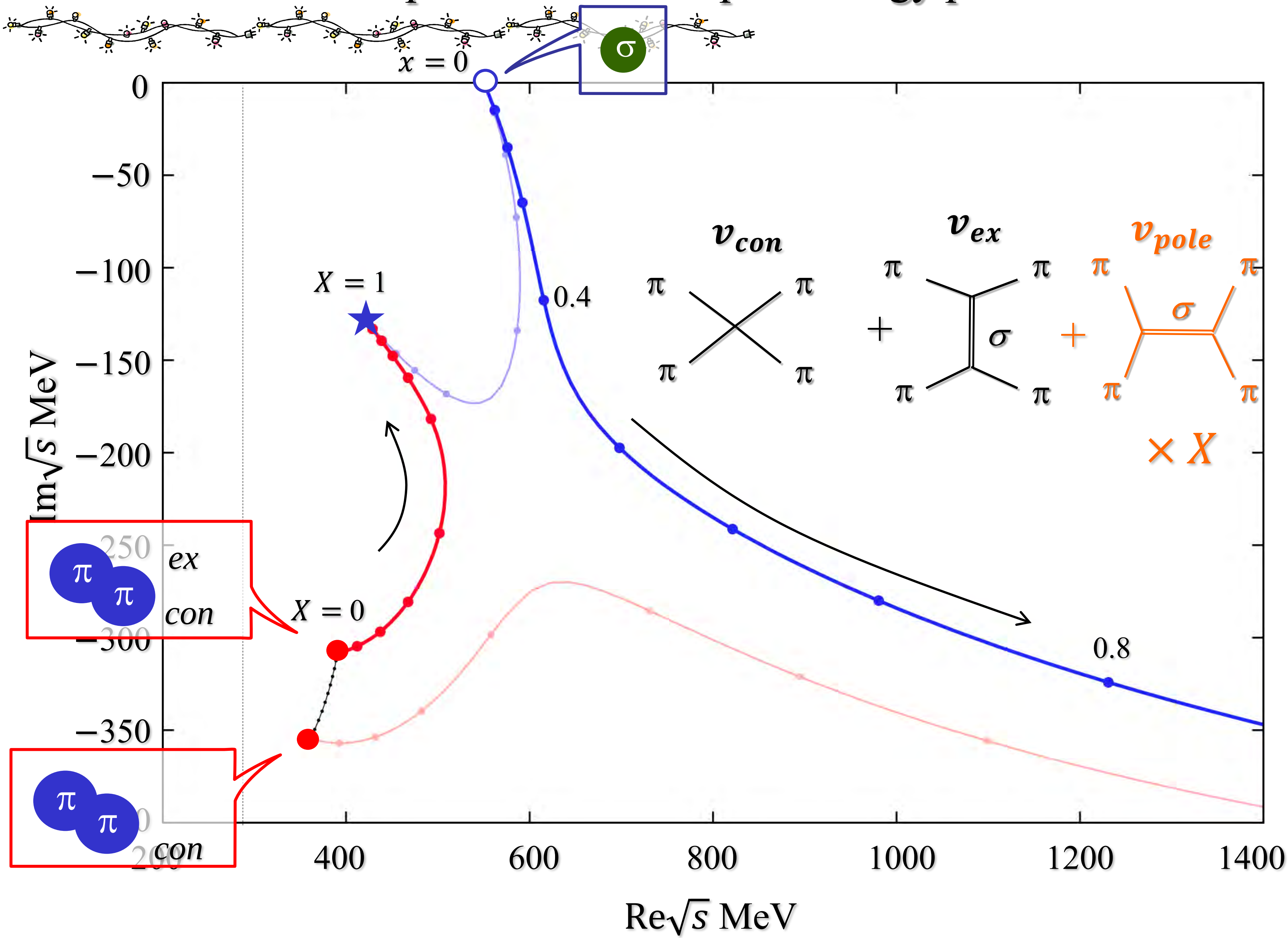
$$t = v + vGt = (v^{-1} - G)^{-1}$$

Numerical results : pole-flow in complex energy plane

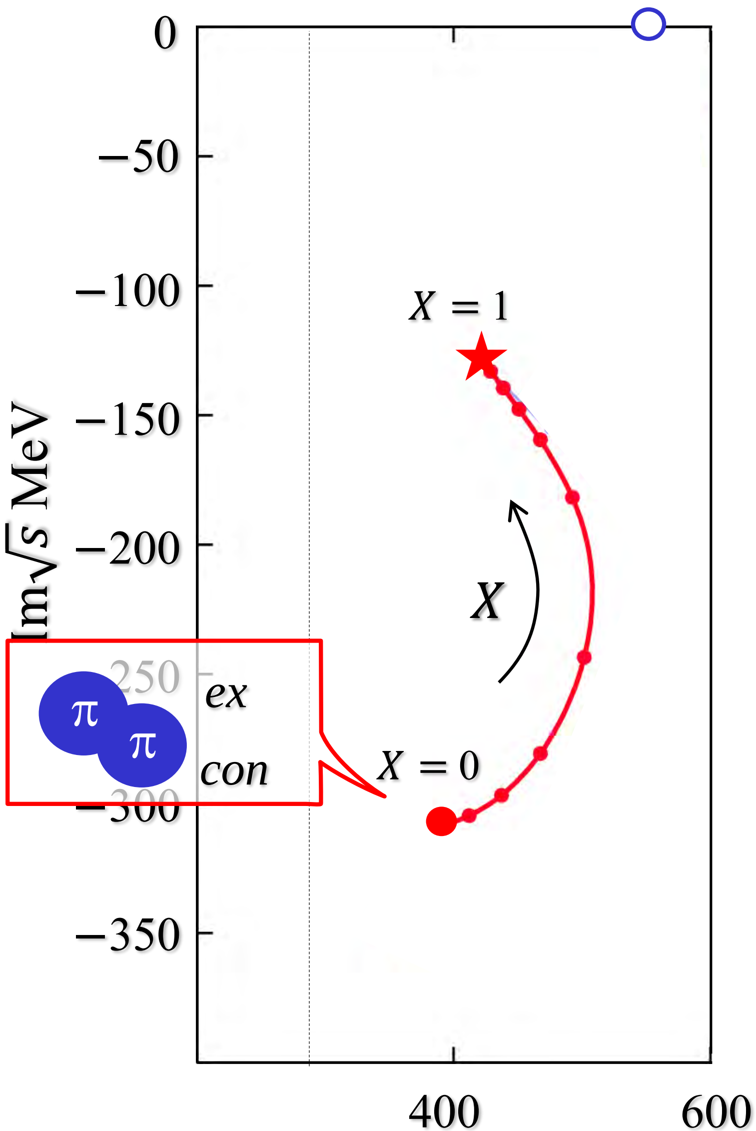
T. Hyodo, D. Jido, T. Kunihiro, NPA848(10)341

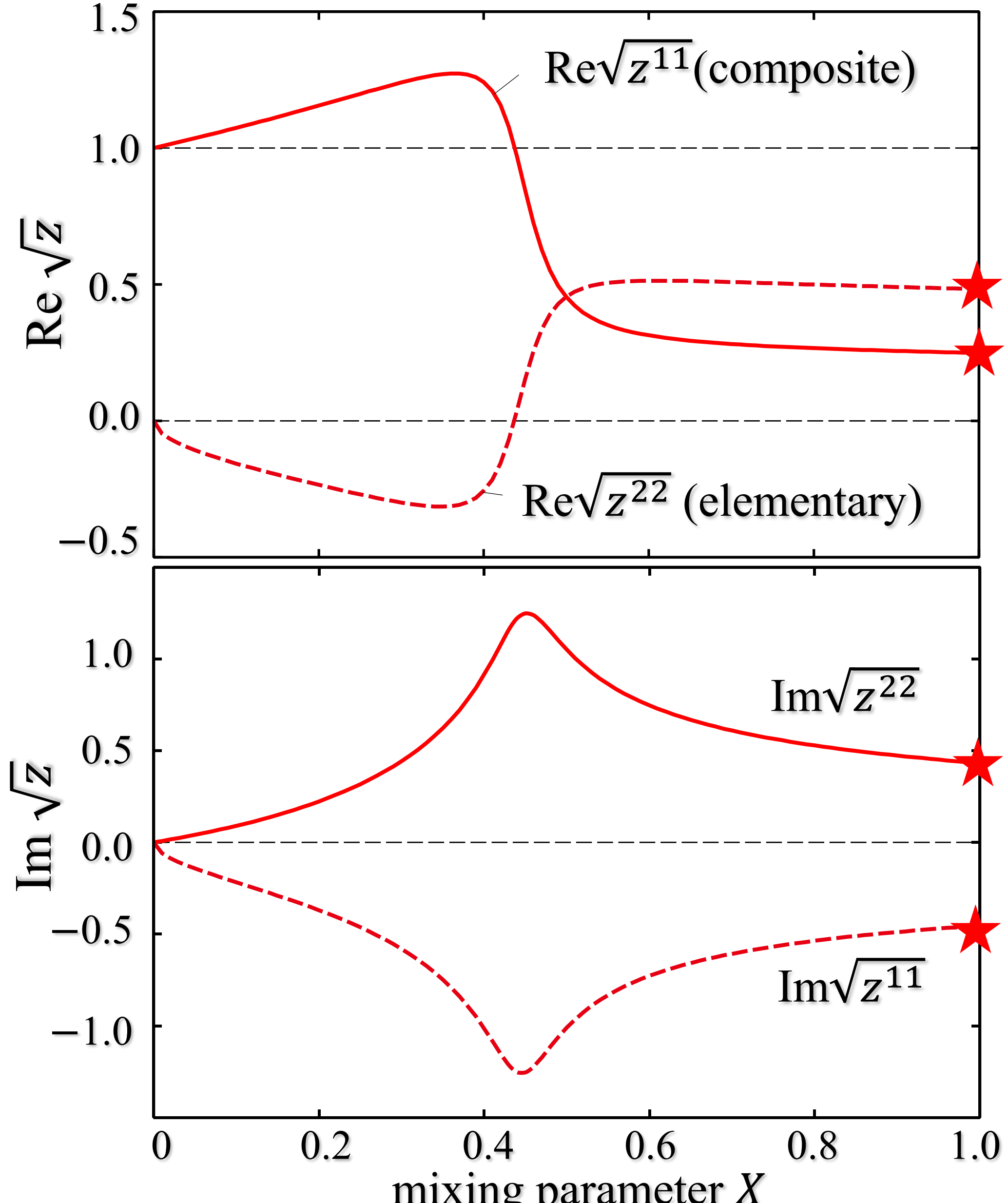


Numerical results : pole-flow in complex energy plane

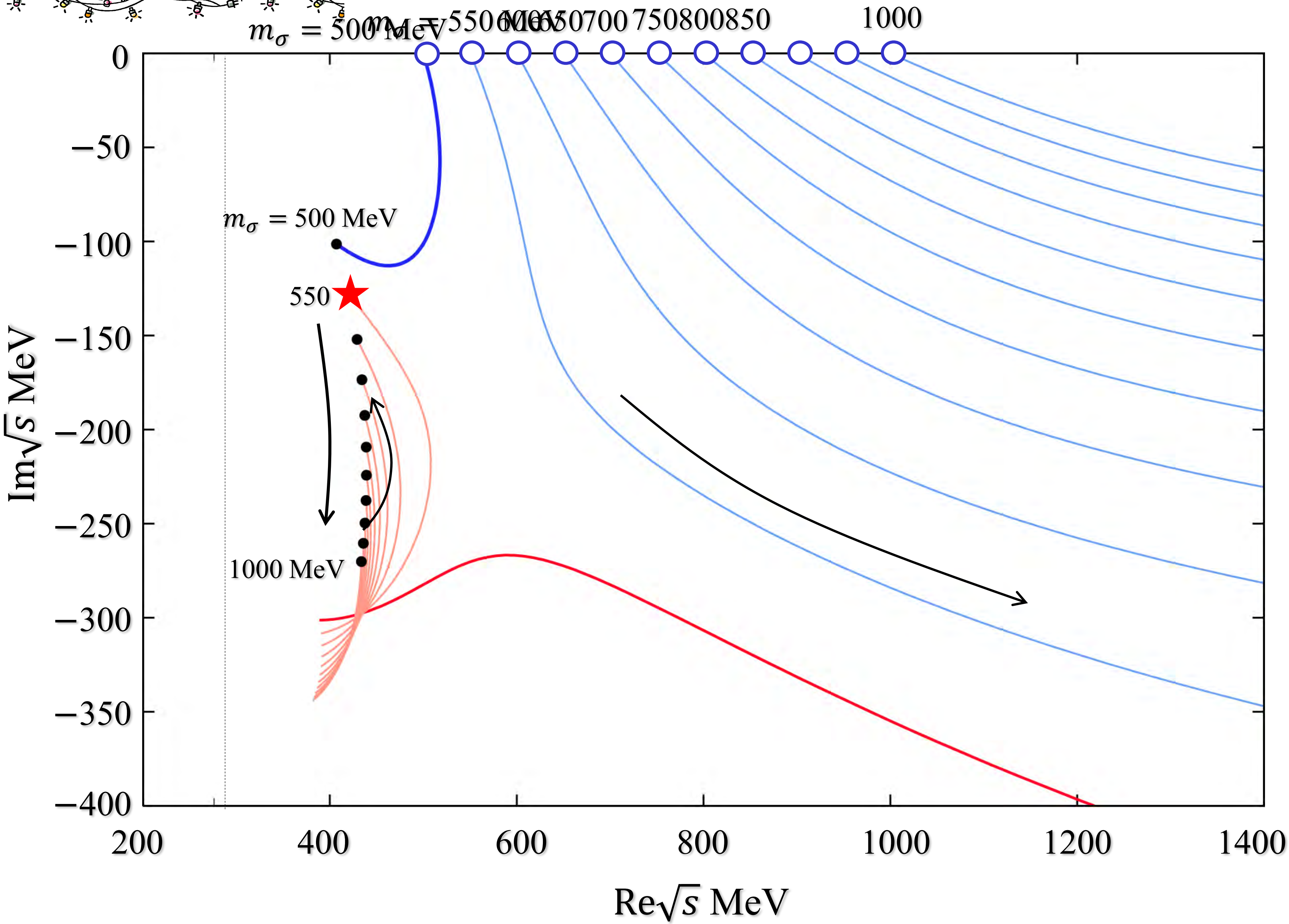


Numerical results : pole-flow in complex energy plane

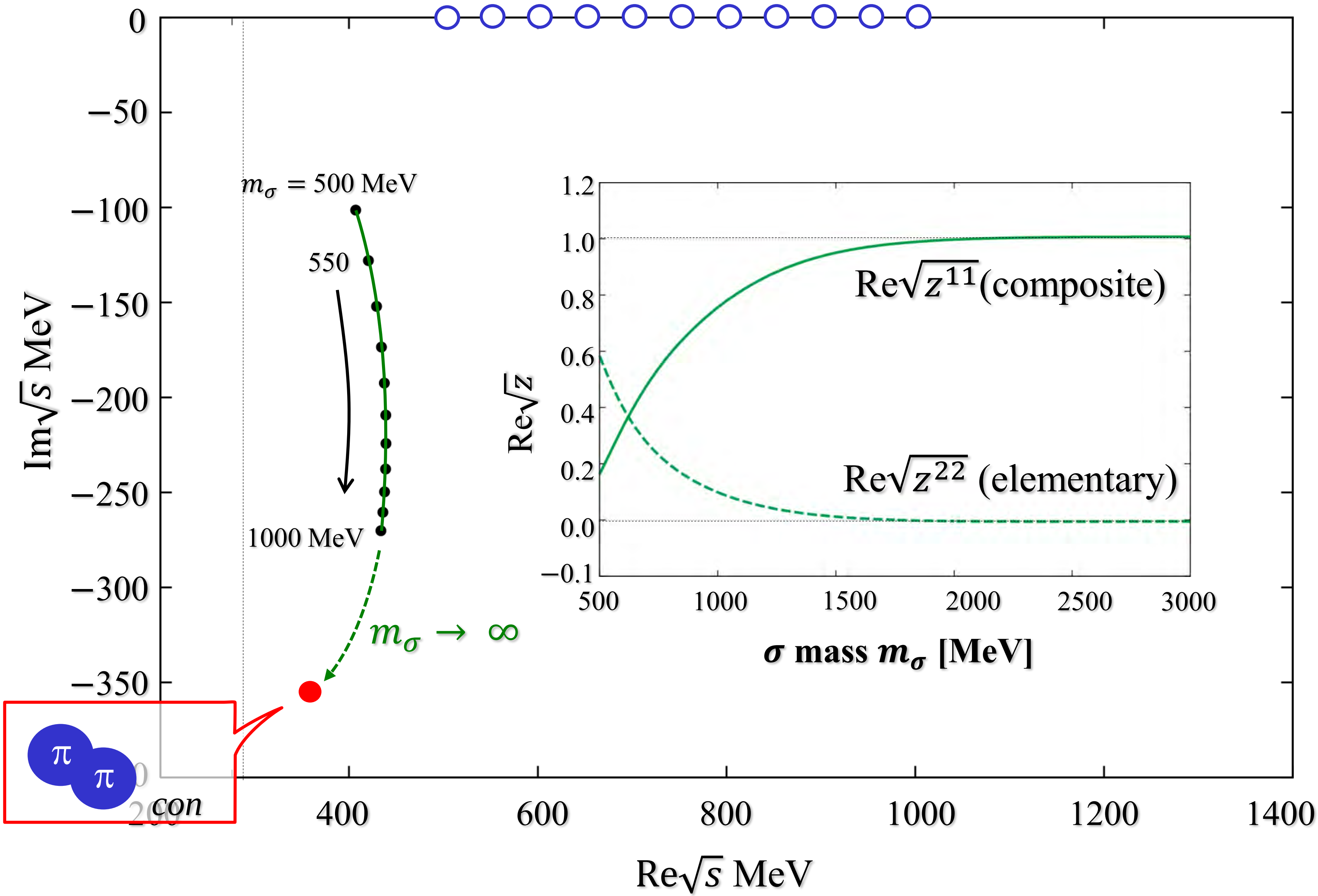




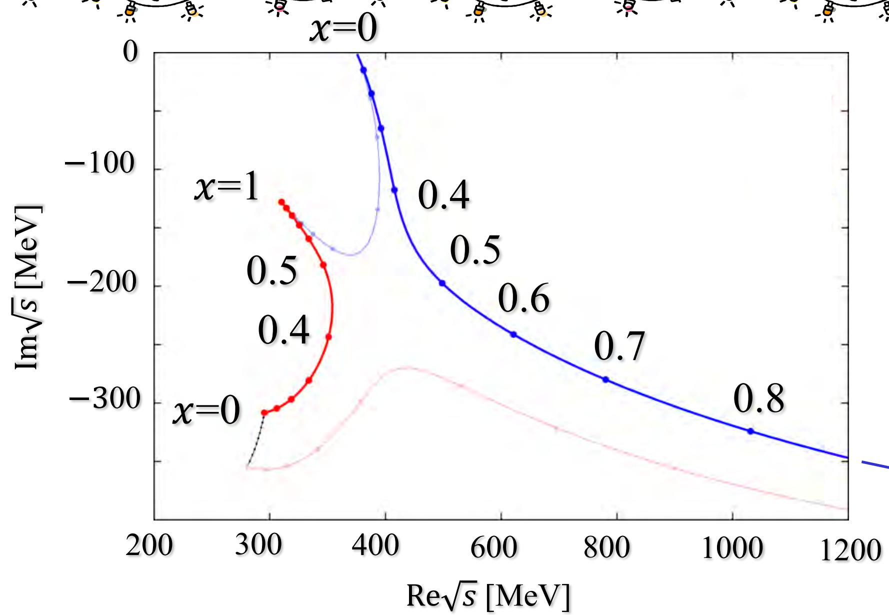
Numerical results : bare σ mass m_σ dependence



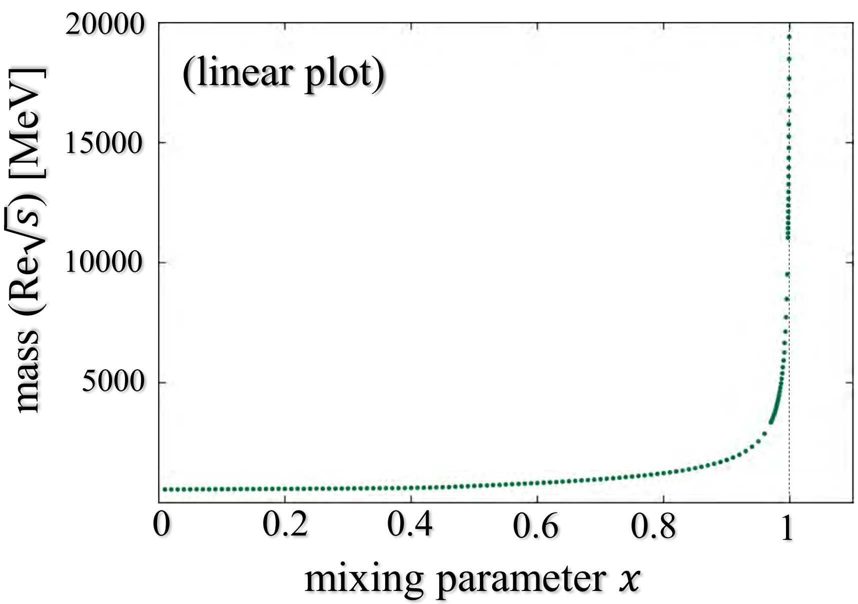
Numerical results : bare σ mass m_σ dependence

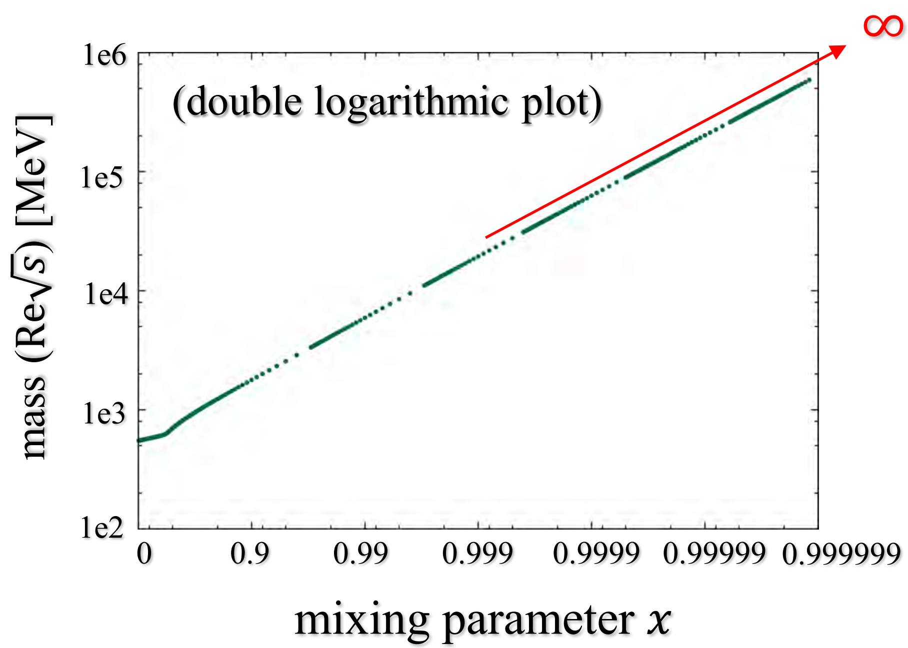


the fate of the other pole

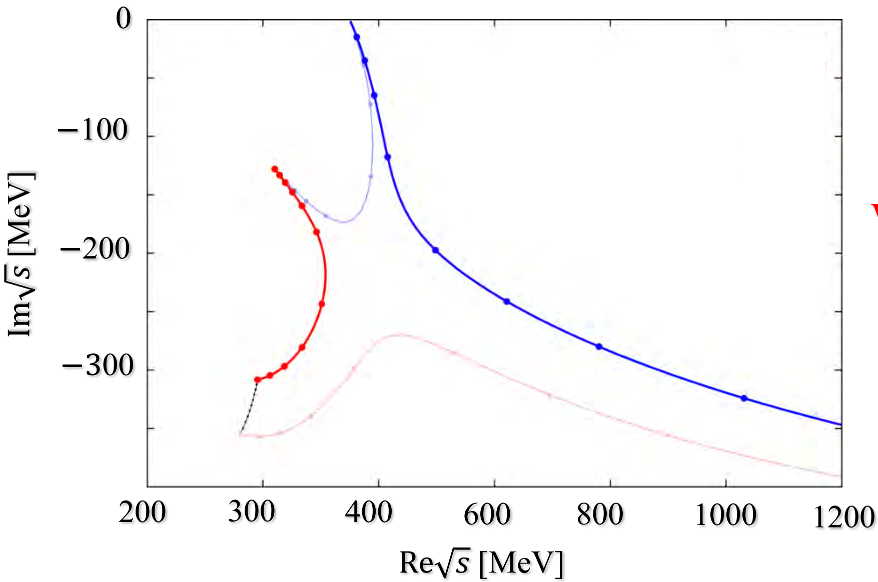


? → **disappear**





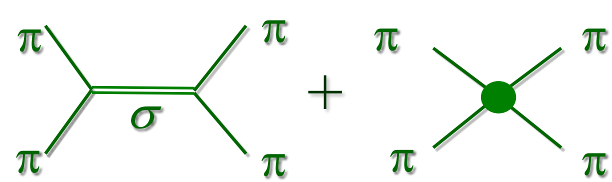
the fate of the other pole



Although we can define **two basis states** as independent degrees of freedom, we do **NOT** necessarily have **two physical states.**

The fate of the other pole depends on the coefficients of these terms

$$t_s^{(tree)} = v_s = \frac{(s-m_\pi^2)^2}{f_\pi^2}\frac{1}{s-m_\sigma^2} - \frac{1}{f_\pi^2}(s-m_\pi^2)$$

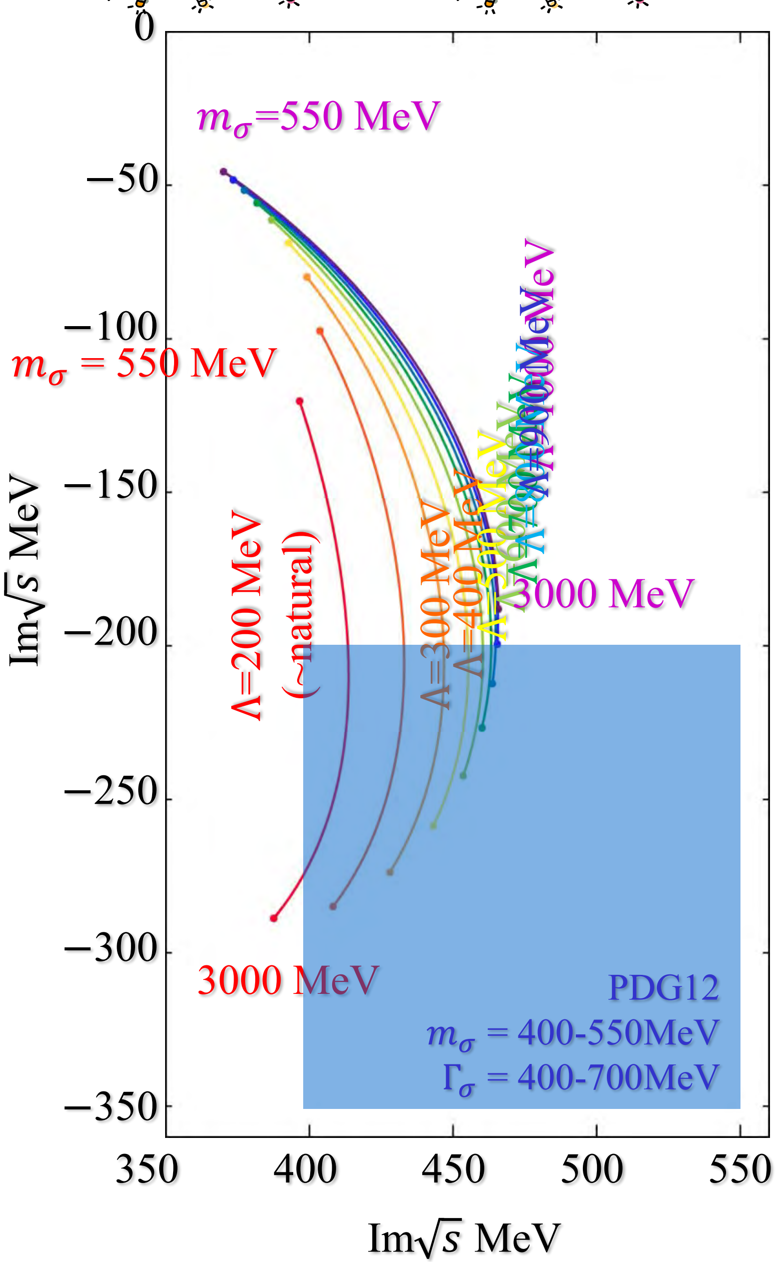


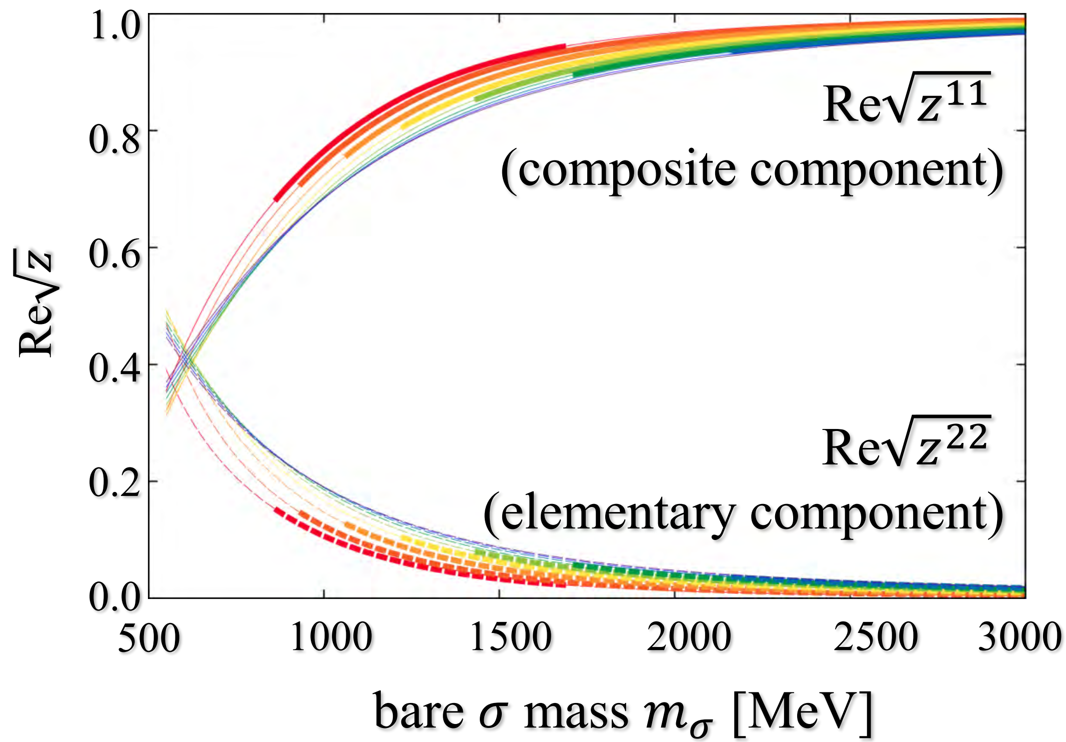
<u>total potential (interaction kernel) at large *s*</u>

$$v_{con} + v_{ex} + X \times v_{pole} \xrightarrow[\text{large } s]{} -\frac{3(1-X)}{f_\pi^2} s + O(s^0)$$

unitarity …?

pole-position and z^{11}, z^{22} @ different cut-off Λ and bare mass m_σ





in unit of MeV

Λ MeV	m_σ MeV	Re$\sqrt{z^{11}}$	Re$\sqrt{z^{22}}$	z^{22}
200	860-1690	0.68-0.94	0.15-0.02	$-0.04-0.08i$
600	1430 <	0.85 <	< 0.08	$-0.008-0.02i$
1000	9240 <	0.997 <	< 0.002	$< 10^{-5}$

[1] S.Weinberg, PR137(65)B672
[2] D. Lurie, A.J.Macfarlane, PR136(64)B816
[3] T.Hyodo, D.Jido, A. Hosaka, PRC85(12)015201

Simple question …

How is z^{22} connected to
the "compositeness condition $Z = 0$"[1,2,3] ?

Nagahiro-Hosaka, in progress

z^{22} ↔ "compositeness condition Z=0" ?

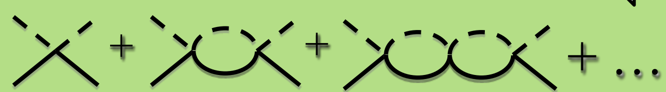
[1] S.Weinberg, PR137(65)B672
[2] D. Lurie, A.J.Macfarlane, PR136(64)B816
D. Lurie, *Particle and Fields*, 1968
[3] T.Hyodo, D.Jido, A. Hosaka, PRC85(12)015201

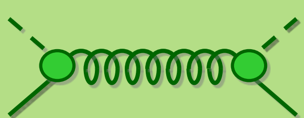
Compositeness condition

four-Fermi theory w/o "elementary"

$$\mathcal{L}_F = -g_0 \bar{\psi}\gamma_5\psi\,\bar{\psi}\gamma_5\psi$$

$$T_F = \frac{v_F}{1 - v_F \Pi(s)}$$

$=$ 

$=$ 

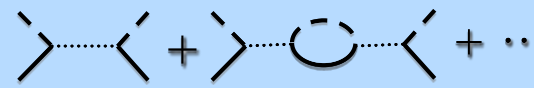
$$Z = 0$$

[2]

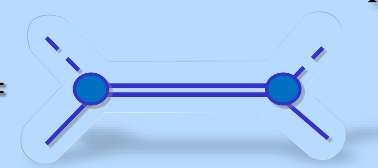
Yukawa theory w/o four-Fermi

$$\mathcal{L}_Y = iG_0 \bar{\psi}\gamma_5\psi\,\phi$$

$$T_Y = G_0 \frac{1}{s - m_0^2 - G_0^2 \Pi(s)} G_0$$

$=$ 

$$= G_R \frac{1}{s - m^{*2} - G_R^2\,\Pi_c'(s)} G_R$$

$=$ 

compositeness condition

$$Z = 1 + G_R^2 \Pi'(m^*) = 0$$

w.f. renormalization of Yukawa theory

Yukawa Theory as a kind of "scale",

which does not have potential to develop a composite state.

Yukawa theory vs. sigma model in nonlinear rep.

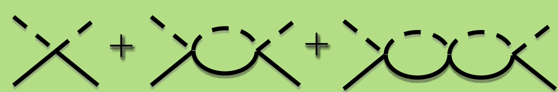
Compositeness condition

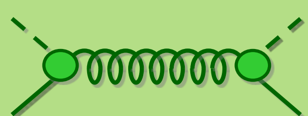
nonlinear σ model w/o "elementary σ"

$(m_\sigma \to \infty)$

$$v_{4\pi} = -(2s - m_\pi^2)/f_\pi^2$$

$$T_F = \frac{v_{4\pi}}{1 - v_{4\pi}\Pi(s)}$$

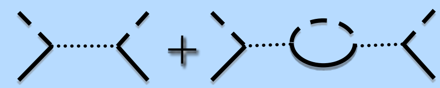
=  + …

= 

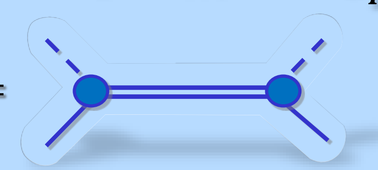
Yukawa theory w/o four π ?

$$\mathcal{L}_Y = G_0\, \phi_\pi \phi_\pi \phi_\sigma$$

$$T_Y = G_0 \frac{1}{s - m_0^2 - G_0^2 \Pi(s)} G_0$$

=  + ⋯

$$= G_R \frac{1}{s - m^{*2} - G_R^2\, \Pi_c'(s)} G_R$$

= 

compositeness condition

$$Z = 1 + G_R^2 \Pi'(m^*) = 0$$

w.f. renormalization of Yukawa theory

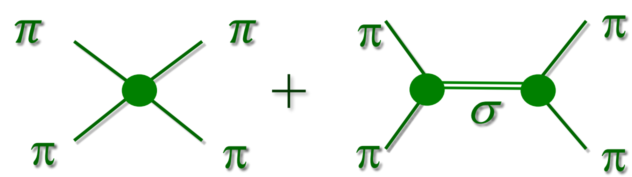
We don't have a Yukawa theory equivalent with the sigma model.

Equivalent representation(s)

<u>Nonlinear reprentation</u> (contains composite and elementary)

$$A^{NL}(s) = -\frac{1}{f_\pi^2}(s-m_\pi^2) + \frac{(s-m_\pi^2)^2}{f_\pi^2}\boxed{\frac{1}{s-m_\sigma^2}}$$

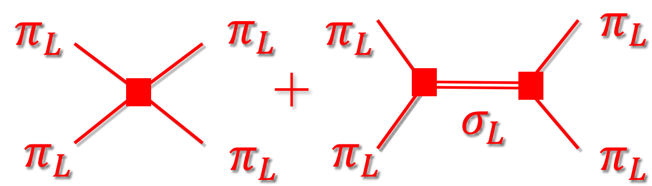
contains composite and elementary



<u>linear representation</u>

$$A^{L}(s) = \frac{1}{f_\pi^2}(m_\sigma^2-m_\pi^2) + \frac{(m_\sigma^2-m_\pi^2)^2}{f_\pi^2}\boxed{\frac{1}{s-m_\sigma^2}}$$

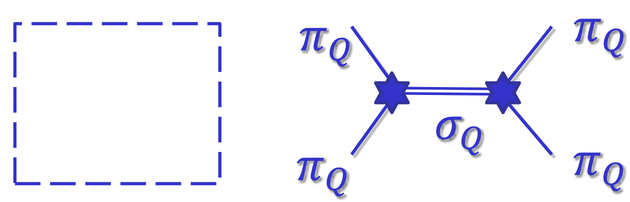
elementary and repulsive 4π contact



<u>“quasi-particle” (“準粒子”) (すべてを σ-poleとみなす)</u>

$$A^{Q}(s) = \frac{(s-m_\pi^2)(m_\sigma^2-m_\pi^2)^2}{f_\pi^2}\boxed{\frac{1}{s-m_\sigma^2}}$$

“elementary” and no contact



<u>s-wave BS equation</u>

$$T = [V^{-1} - G]^{-1}$$

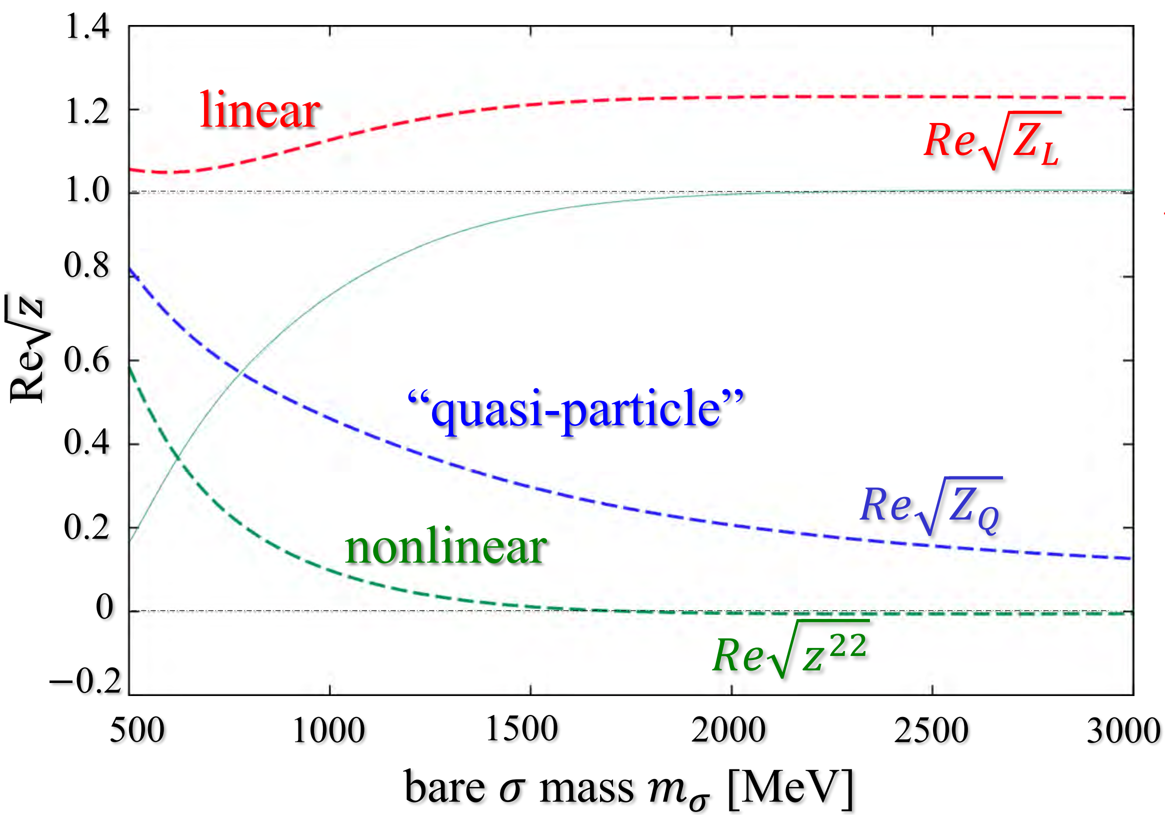
$$= G_R^2 \frac{1}{s - m^{*2}}$$

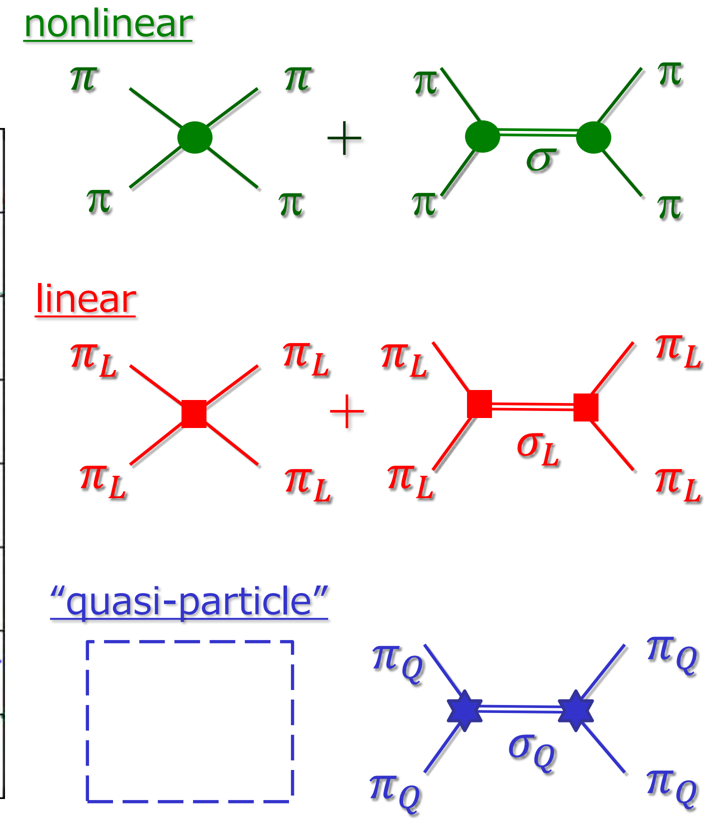
<u>$I = 0$ amplitude</u>

$$t^{tree} = 3A(s) + [A(t) + A(u)]_{s-wave} \equiv V$$

→ calculate the wave function renormalization” for each “bare σ”

Equivalent representation(s)





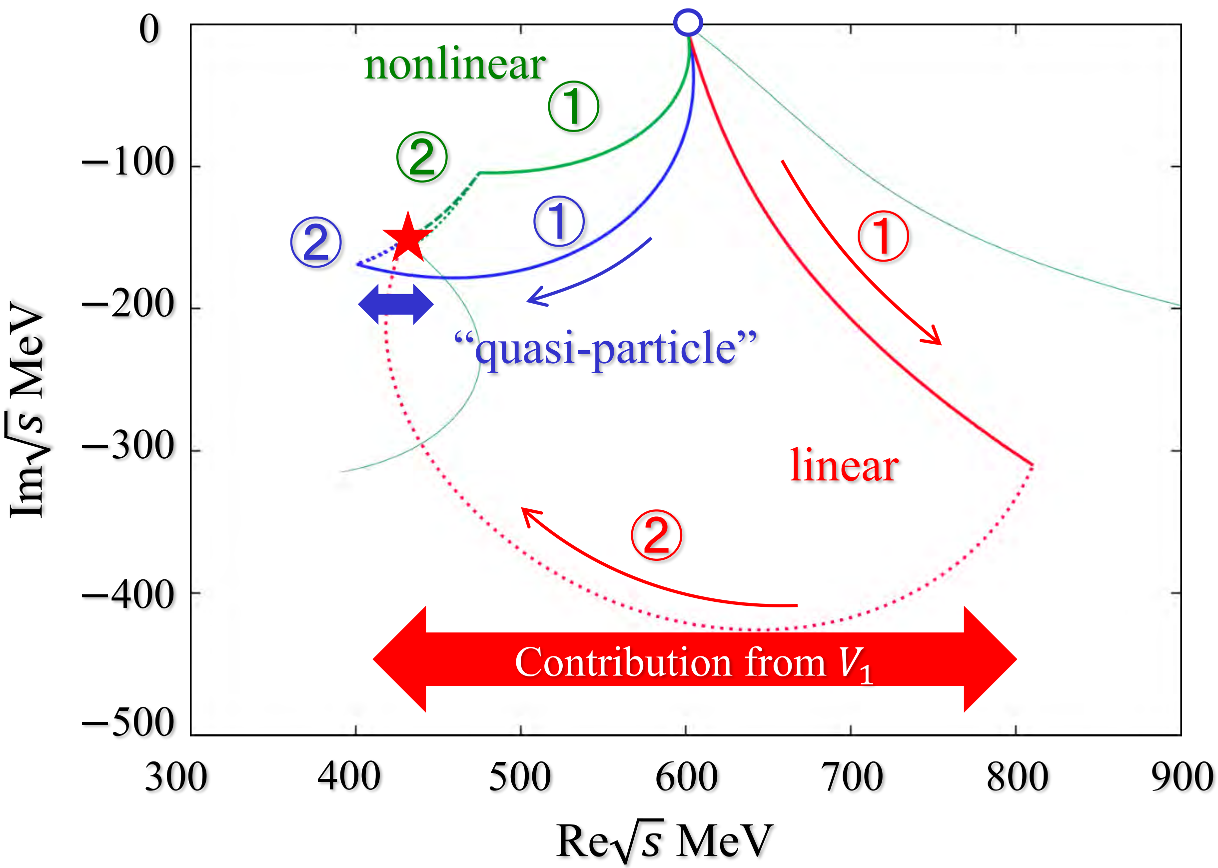
- ✓ Z ↛ 0 as $m_\sigma \to \infty$ in linear rep. / $Z \to 0$ in nonlinear and "quasi-partile"
- ✓ Z are different at different representations
  → it depends on the definition of "elementary" particle
- ✓ We first need to define "what is the elementary particle" .

Contribution from other terms than pole term

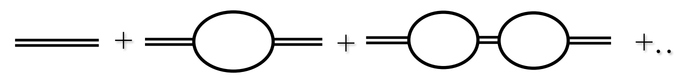
$V^{\mathrm{total}} = V_1 + V_{s\text{-pole}}$

大部分をpole項に押しつけることで、
V_1(残り)を摂動で扱えるようにする。
Weinberg, PR130(63)776

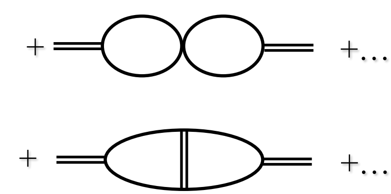
bare σ mass m_σ= 600 MeV



① one-loop correction (V_{s-pole})



② contact (etc.) correction
(contribution from V_1)



- ✓ V_1 in the linear representation is large
- ✓ "quasi-particle" rep. seems to be that of [Weinberg,PR130]. But, … what is this?
- ✓ 'Nonlinear + two level analysis' gives us a reasonable physical interpretation

summary

- **Mixing property of σ meson in nonlinear rep. by means of two level prob.**
  - Mixture of a $\pi\pi$ composite and “elementary” σ
  - We have only one physical pole unlike the a_1 case (with hidden Lagrangian)
  - Physical σ is almost “$\pi\pi$ composite” and the component of “elementary” is small within the present model setting.
  - bare σ mass (or CDD pole) is closely connected to cut-off. We need an extra condition to fix cut-off scale.

- **Representation dependence of wave function renormalization Z**
  - **“Compositeness condition $Z = 0$” $\leftrightarrow$ z^{22}**
  - **Generally**, it depends on the definition of “elementary particle”

- **What is the most “economical” basis ?**
  - or an approach from different axis (such as behavior expected in finite T/ρ) ?