結合チャネル複素スケーリング法による Λ(1405) 及び K^-pp の研究

KEK Theory Center / IPNS/ J-PARC branch
Akinobu Doté

1. Introduction

2. coupled-channel Complex Scaling Method applied to a two-body system of $K^{bar}N$-πY

A. D., T. Inoue, T. Myo, NPA 912, 66 (2013)

3. Three-bodys system of K^-pp studied with "coupled-channel Complex Scaling Method + Feshbach method"

4. Summary and future plan

J-PARC branch Harada-san's meeting,
03.Mar.'14 @ KEK Tokai campus

1. Introduction

Kaonic nuclei = Nuclear system with Anti-kaon "K^{bar}"

Attractive $K^{bar}N$ interaction!

Excited hyperon Λ(1405) = a quasi-bound state of K^- and proton

- ✓ *Difficult to explain by naive 3-quark model*
- ✓ *Consistent with "repulsive nature" indicated by $K^{bar}N$ scattering length and 1s level shift of kaonic hydrogen atom*

$\boxed{\Lambda(1405)\ 1/2^-}$ $I(J^P) = 0(\frac{1}{2}^-)$

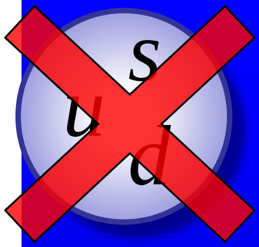
Mass $m = 1405.1^{+1.3}_{-1.0}$ MeV

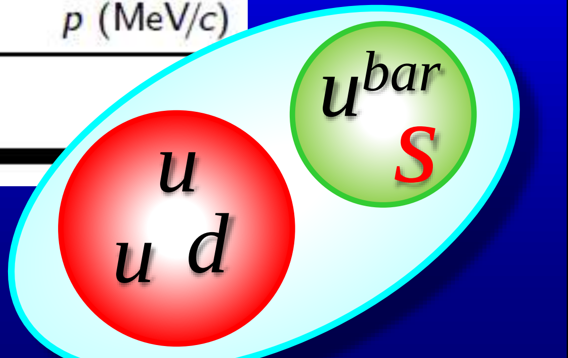
Full width $\Gamma = 50 \pm 2$ MeV

Below $\overline{K}N$ threshold

$K^{bar}N$ threshold = 1435MeV

Λ(1405) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\Sigma\pi$	100 %	



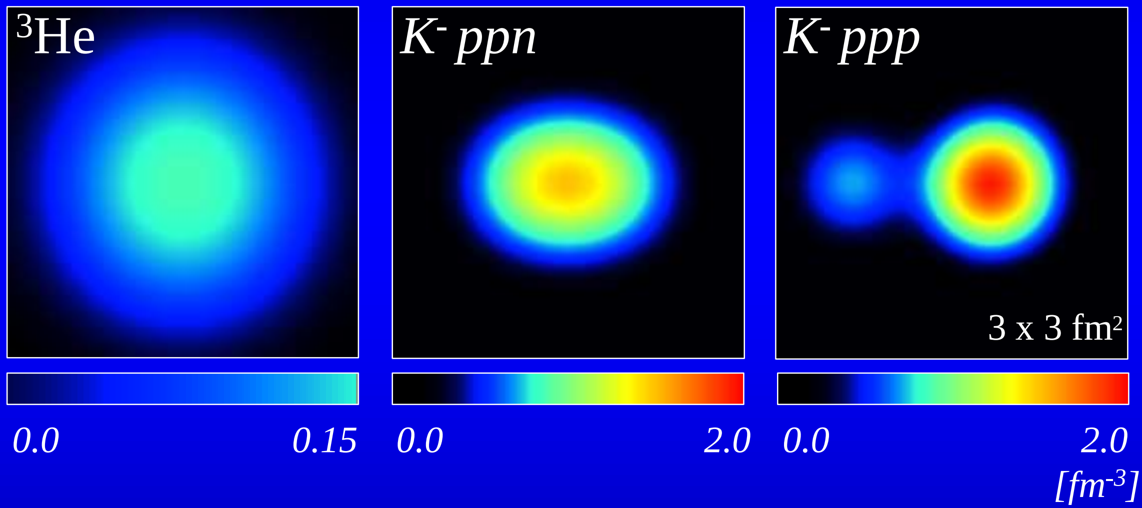


T. Hyodo and D. Jido,
Prog. Part. Nucl. Phys. 67, 55 (2012)

Kaonic nuclei = Exotic system !?

Phenomenologically derived I=0 $K^{bar}N$ potential … Very attractive

Deeply bound (Total B.E. ～100MeV)
Highly dense state formed in a nucleus
Interesting structures that we have never seen in normal nuclei…

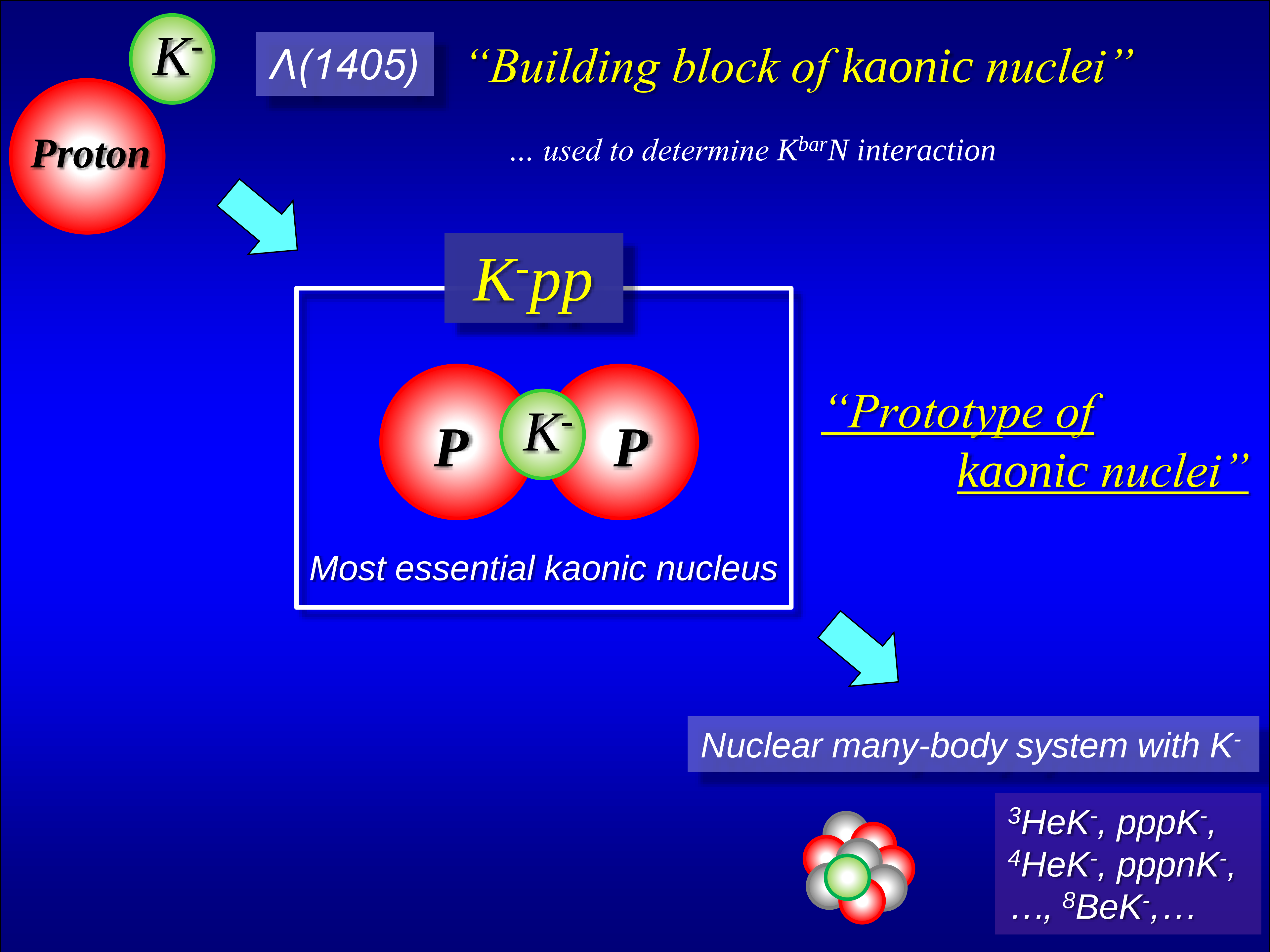


Antisymmetrized Molecular Dynamics method with a phenomenological $K^{bar}N$ potential

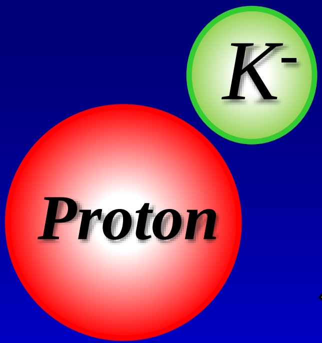
A. Dote, H. Horiuchi, Y. Akaishi and T. Yamazaki, PRC70, 044313 (2004)

Relate to various interesting physics such as …

- *Restoration of chiral symmetry in dense matter*
- *Interesting structure*
- *Neutron star*



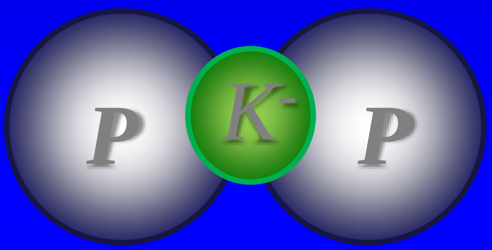
K-
Proton
Λ(1405)
"Building block of kaonic nuclei"
… used to determine K^bar N interaction
K-pp
P
K-
P
Most essential kaonic nucleus
"Prototype of kaonic nuclei"
Nuclear many-body system with K-
3HeK-, pppK-,
4HeK-, pppnK-,
…, 8BeK-,…



$\Lambda(1405)$ *"Building block of kaonic nuclei"*

… used to determine $K^{bar}N$ interaction

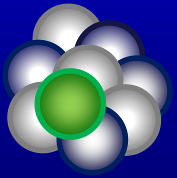
K^-pp



Most essential kaonic nucleus

"Prototype of kaonic nuclei"

Nuclear many-body system with K^-



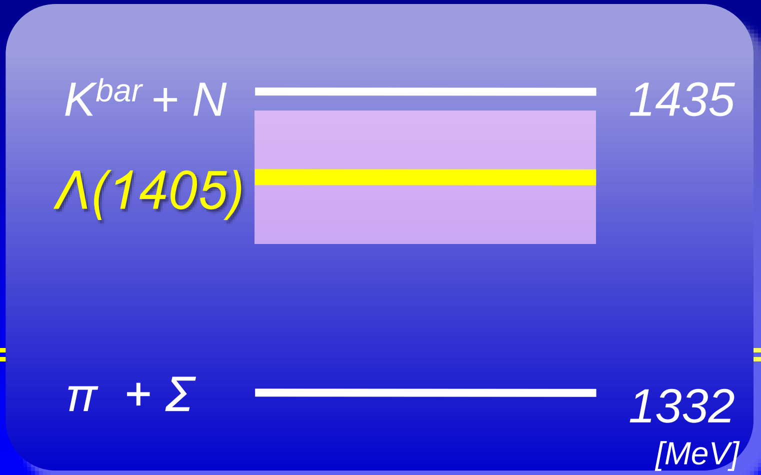
$^3HeK^-$, $pppK^-$, $^4HeK^-$, $pppnK^-$, …, $^8BeK^-$, …

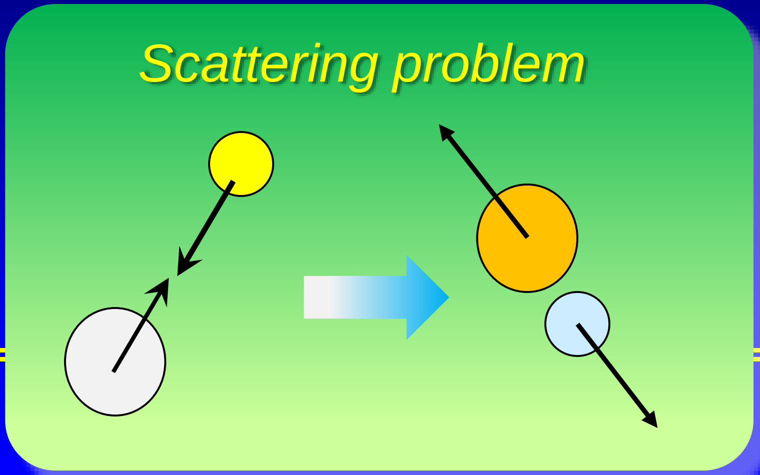
2. coupled-channel Complex Scaling Method

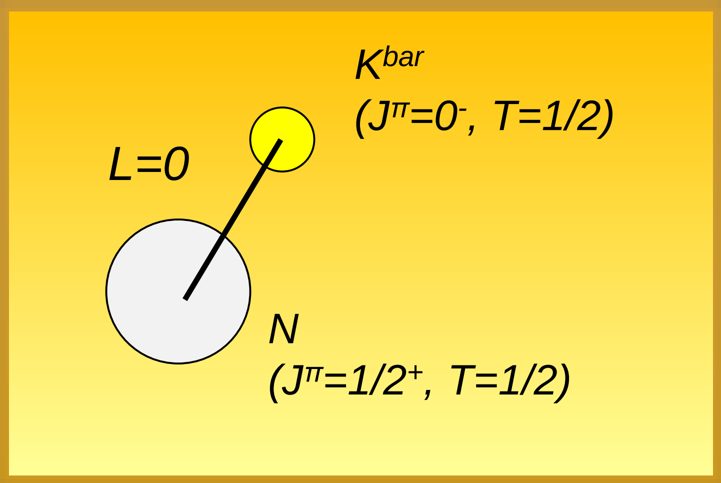
applied to

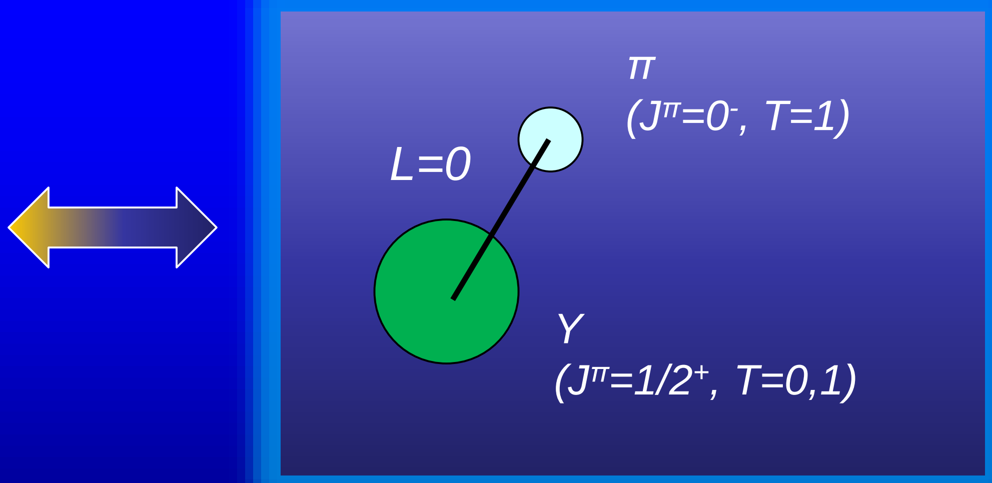
a two-body system of $K^{bar}N-\pi Y$

$K^{bar}N$ system with c.c. Complex Scaling Method





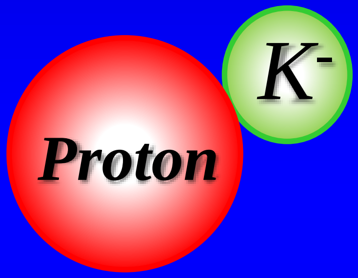




$K^{bar}N$-πY coupled system with s-wave and isospin-0,1 state

2-1. Formalism of ccCSM

Λ(1405) = a building block of kaonic nuclei



- *For Resonance states*
- *For Scattering problem*

Complex Scaling Method for Resonance

Complex rotation of coordinate (Complex scaling)

$$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$$

$$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$$

$$\Rightarrow \quad E = \langle \Phi | H | \Phi \rangle = \langle \Phi_\theta | H_\theta | \Phi_\theta \rangle$$

By Complex scaling, …

- *Resonant state*

$$\Phi_R \sim \exp(ik_R r) = \exp\{i(\kappa - i\gamma) r\}$$

κ, γ : real, >0

Divergent

$$\Rightarrow \quad U(\theta)\Phi_R \sim \exp\{i(\kappa - i\gamma) r e^{i\theta}\}$$

$$= \underline{\exp\{-r(\kappa \sin\theta - \gamma\cos\theta)\}} \cdot \exp\{ir(\kappa\cos\theta + \gamma\sin\theta)\}$$

$$\to \quad 0 \qquad (\tan\theta > \gamma/\kappa)$$

Damping under some condition

Complex Scaling Method for Resonance

Complex rotation of coordinate (Complex scaling)

$U(\theta): \quad \mathbf{r} \to \mathbf{r}\, e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}\, e^{-i\theta}$

$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$

$\Rightarrow$

$E = \langle \Phi | H | \Phi \rangle = \langle \Phi_\theta | H_\theta | \Phi_\theta \rangle$

By Complex scaling,

- Resonance wave function: divergent function ⇒ damping function

⇒ *Boundary condition is the same as that for a bound state.*

- *Resonance energy (pole position) doesn't change.*

ABC theorem

"The energy of bound and resonant states is independent of scaling angle θ."

† J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269.
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

Diagonalize H_θ with Gaussian base,

we can obtain resonant states, in the same way as bound states!

Complex Scaling Method for Resonance

Test calculation with a phenomenological potential

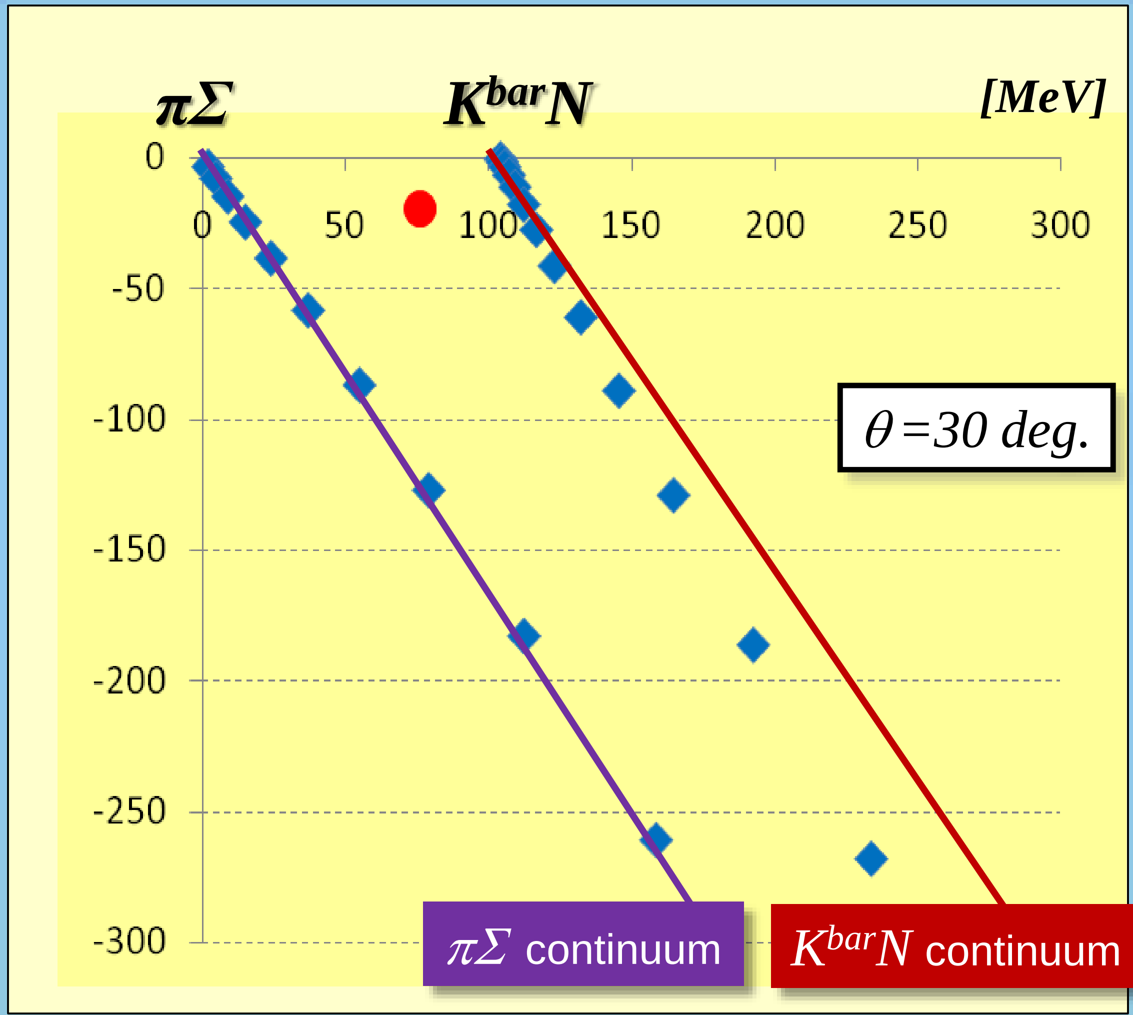
Akaishi-Yamazaki potential[†]

- *Local Gaussian form*
- *Energy independent*

$$V_{AY,I=0} = V_{\alpha\beta}^{(I=0)} \exp\left[-\left(r / 0.66\ \mathrm{fm}\right)^2\right]$$

$$V_{\alpha\beta}^{(I=0)} = \begin{array}{cc} K^{bar}N & \pi\Sigma \\ \end{array}$$

$$V_{\alpha\beta}^{(I=0)} = \begin{array}{c} \begin{array}{cc} \bar{K}N & \pi\Sigma \end{array} \\ \begin{pmatrix} -436 & -412 \\ -412 & 0 \end{pmatrix} \end{array} [\mathrm{MeV}]$$



B. E. ($K^{bar}N$) = 28.2 MeV
Γ = 40.0 MeV

… Nominal position of Λ(1405)

[†]Y. Akaishi and T. Yamazaki, PRC65, 044005 (2002)

2-1. Formalism of ccCSM

<u>Calculation of $K^{bar}N$ scattering amplitude</u>

- *For Resonance states*
- *<u>For Scattering problem</u>*

Calc. of scattering amplitude with CSM

PHYSICAL REVIEW C **75**, 044602 (2007)

Scattering amplitude without an explicit enforcement of boundary conditions

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(Received 15 December 2006; published 11 April 2007)

It has been known for some time that for short range potentials scattering observables can be calculated using complex coordinates. We will show that the standard uniform complex scaling can be applied to calculate the scattering amplitude even in the presence of a long range interaction. The main advantage of the application of the complex scaling to the scattering problem is that the direct imposition of the complicated scattering boundary condition can be avoided. As a result, the scattering problem can be solved using only square integrable functions. The method will be applied not only for potential scattering but for the coupled-channel reaction model. As an application we calculate the phase shifts of the charge exchange reaction $^3H(p,n)^3He$.

With help of the CSM, all problems for bound, resonant and scattering states can be treated with Gaussian base!

Calc. of scattering amplitude with CSM

A. T. Kruppa, R. Suzuki and K. Katō, PRC 75, 044602 (2007)

1. Separate incoming wave $j_l(kr)$

$$H_l\psi_{l,k}^{(+)}(r) = E\psi_{l,k}^{(+)}(r)$$

$$\psi_{l,k}^{(+)}(r) = \hat{j}_l(kr) + \psi_{l,k}^{sc}(r).$$

$$\psi_{l,k}^{sc}(r) \underset{r\to\infty}{\to} k f_l(k)\hat{h}_l^{(+)}(kr),$$

- Unknown
- Non square-integrable

$$h_l^{(+)}(x) \to \exp\left[i\left(kr - \frac{l\pi}{2}\right)\right]$$

2. Complex scaling $r \to re^{i\theta}$

$$(E - H_l)\psi_{l,k}^{sc}(r) = V(r)\hat{j}_l(kr).$$

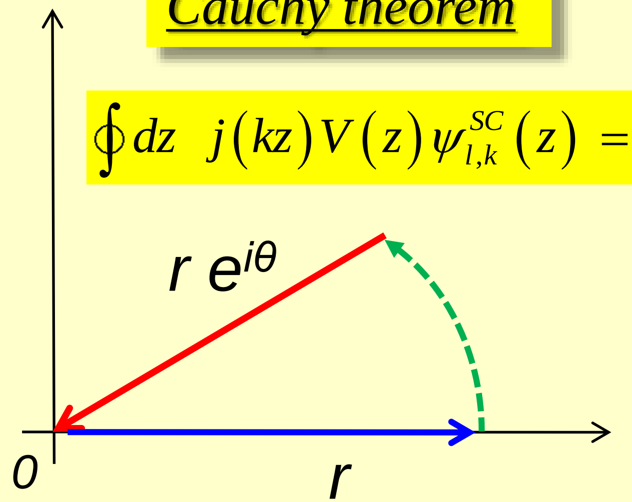
$$\psi_{l,k}^{sc,\theta}(r) \underset{r\to\infty}{\to} e^{i\theta/2} k f_l(k) i^{-l} e^{ikr\cos\theta - kr\sin\theta}$$

square-integrable for $0 < \vartheta < \pi$

Expanding with square-integrable basis function (ex: Gaussian basis)

Cauchy theorem

$$\oint dz \;\; j(kz)V(z)\psi_{l,k}^{SC}(z) = 0$$



…with help of Cauchy theorem

$$\frac{}{\hbar^2 k^2}\int_0 \cdots$$ Born term is OK!

$$f_l^{sc}(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\,\hat{j}_l(kr)V(r)\psi_{l,k}^{sc}(r)$$

Scattered part is unknown.

Scattered wave function along $re^{i\vartheta}$ is known!

By Cauchy theorem, we can obtain as

$$\boxed{f_l^{sc}(k) = -\frac{2m}{\hbar^2 k^2}e^{i\theta/2}\int_0^\infty dr\,\hat{j}_l(kre^{i\theta})V(re^{i\theta})\psi_{l,k}^{sc,\theta}(r)}$$

Calc. of scattering amplitude with CSM

A. T. Kruppa, R. Suzuki and K. Katō, PRC 75, 044602 (2007)

Scattering problem can be solved as bound-state problem by matrix calculation!

Equation to be solved: $(E - H_l(\theta))\psi_{l,k}^{sc,\theta}(r) = e^{i\theta/2}V(re^{i\theta})\hat{j}_l(kre^{i\theta})$,

$\psi_{l,k}^{sc,\theta}(r) \underset{r\to\infty}{\to} e^{i\theta/2}kf_l(k)i^{-l}e^{ikr\cos\theta - kr\sin\theta}$ *can be expanded with Gaussian basis*

square-integrable for 0 < ϑ < π

$$\psi_{l,c}^{(c_0),\theta}(r) = \sum_{j=1}^{N} t_{c,j}^{(c_0)}(\theta)G_j(r)$$

Linear equation to be solved with matrix calculation!

$$\sum_j \left[(EO_{ij} - H_{c,ij}^{l,\theta})t_{c,j}^{(c_0)}(\theta) - \sum_{c'=1}^{n} V_{cc',ij}^{\theta} t_{c',j}^{(c_0)}(\theta) \right] = b_{cc_0,i}^{\theta}$$

where each matrix element indicates $O_{ij} = \langle G_i|G_j\rangle$, $H_{c,ij}^{l,\theta} = \langle G_i|H_c^{l,\theta}|G_j\rangle$, $V_{cc',ij}^{\theta} = \langle G_i|V_{cc'}(re^{i\theta})|G_j\rangle$ and

$$b_{cc_0,i}^{\theta} = e^{i\theta/2}\int_0^{\infty} dr\, G_i(r)V_{cc_0}(re^{i\theta})\hat{j}_l(kre^{i\theta}).$$

2-2. Set up

- $K^{bar}N$ potential
- Kinematics

"KSW-type potential" … chiral-SU(3) based

Effective Chiral Lagrangian

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$

$$\mathcal{L}^{(1)} = \mathrm{tr}\left(\overline{\Psi}_B(i\gamma_\mu D^\mu - M_0)\Psi_B\right) + F\,\mathrm{tr}\left(\overline{\Psi}_B\gamma_\mu\gamma_5[A^\mu,\Psi_B]\right) + D\,\mathrm{tr}\left(\overline{\Psi}_B\gamma_\mu\gamma_5\{A^\mu,\Psi_B\}\right),$$

$$D^\mu\Psi_B = \partial^\mu\Psi_B + [\Gamma^\mu,\Psi_B]$$

$$\Gamma^\mu = \frac{1}{8f^2}[\phi,\partial^\mu\phi] + O(\phi^4)$$

Pseudopotential $V_{ij}(r) = \frac{C_{ij}}{2f^2}\sqrt{\frac{M_iM_j}{s\omega_i\omega_j}}\,\delta^3(r)$

- Delta-function type (→ Yukawa / separable type)
- Up to order q^2

N. **K**aiser, P. B. **S**iegel and W. **W**eise, NPA 594, 325 (1995)

"KSW-type potential"

$$V_{ij}^{(I)}(r) = -\frac{C_{ij}^{(I)}}{8f^2}(\omega_i+\omega_j)\sqrt{\frac{M_iM_j}{s\,\omega_i\,\omega_j}}\,g_{ij}^{(I)}(r)$$

$$g_{ij}^{(I)}(r) = \frac{1}{\pi^{3/2}a_{ij}^{(I)3}}\exp\left[-\left(r/a_{ij}^{(I)}\right)^2\right]$$

$$\begin{array}{ccc} & \bar{K}N & \pi\Sigma \\ C_{ij}^{(I=0)} = & \multicolumn{2}{c}{\begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ & 4 \end{pmatrix}} \end{array}$$

$a^{(I)}_{ij}$: range parameter [fm]

- Local Gaussian form in r-space
  → Easy to handle in many-body calculation with Gaussian base
- Weinberg-Tomozawa term
  → Relative strength between channels determined by the SU(3) algebra
- Energy dependence
  ← Chiral SU(3) theory

Constrained by $K^{bar}N$ scattering length (Martin's value)

$a_{KN(I=0)} = -1.70+i0.67$fm, $a_{KN(I=1)} = 0.37+i0.60$fm A.D.Martin, NPB179, 33(1979)

Kinematics

• Semi-relativistic

$$H_{SR} = \sum_{c=K^{bar}N,\pi Y} \left[\sqrt{m_c^2 + \mathbf{p}^2} + \sqrt{M_c^2 + \mathbf{p}^2} \right] |c\rangle\langle c| + V_{KSW}$$

KSW-type potential

$$V_{ij}^{(I)}(r) = -\frac{C_{ij}^{(I)}}{8 f_\pi^2} (\omega_i + \omega_j) \sqrt{\frac{M_i M_j}{s\, \omega_i \omega_j}}\, g_{ij}^{(I)}(r)$$

"Relativistic" flux factor

• Non-relativistic

$$H_{NR} = \sum_{c=K^{bar}N,\pi Y} \left[m_c + M_c + \frac{\mathbf{p}^2}{2\mu_c} \right] |c\rangle\langle c| + V_{KSW(NR)}$$

→ "NRv1" Non-rela. approx. version 1

$$V_{ij}^{(I)}(r) = -\frac{C_{ij}^{(I)}}{8 f_\pi^2} (\omega_i + \omega_j) \frac{1}{\sqrt{s}} \sqrt{\frac{M_i M_j}{\mu_i\, \mu_j}}\, g_{ij}^{(I)}(r)$$

Comparison of formula of differential cross section between non-rela. and rela.

$\omega_i \sim \mu_i$

→ "NRv2" Non-rela. approx. version 2

$$V_{ij}^{(I)}(r) = -\frac{C_{ij}^{(I)}}{8 f_\pi^2} (\omega_i + \omega_j) \sqrt{\frac{1}{m_i\, m_j}}\, g_{ij}^{(I)}(r)$$

@ non-rela. limit (small p^2)

2–3. Result

A. D., T. Inoue, T. Myo,
NPA 912, 66 (2013)

- *I=0 channel*
  - *I=0 $K^{bar}N$ scattering length*
    Range parameters of KSW-type potential
  - *Scattering amplitude*
  - *Resonance pole*
  - *Lower pole*

$I=0\ K^{bar}N$ scattering length / Range parameters

$$V_{ij}^{(I=0)}(r) = V_{ij}^{(I=0)}\left(\sqrt{s}\right)\ \frac{1}{\pi^{3/2} d_{ij}{}^{3}} \exp\left[-\left(r/d_{ij}\right)^2\right]$$

Range parameters:

$$d_{ij} = \begin{pmatrix} d_{K^{bar}N,K^{bar}N} & d_{K^{bar}N,\pi\Sigma} \\ & d_{\pi\Sigma,\pi\Sigma} \end{pmatrix}$$

$$d_{K^{bar}N,\pi\Sigma} \equiv \left(d_{K^{bar}N,K^{bar}N} + d_{\pi\Sigma,\pi\Sigma}\right)/2$$

Pion decay constant "f_π" as a parameter

f_π = 90 ~ 120 MeV

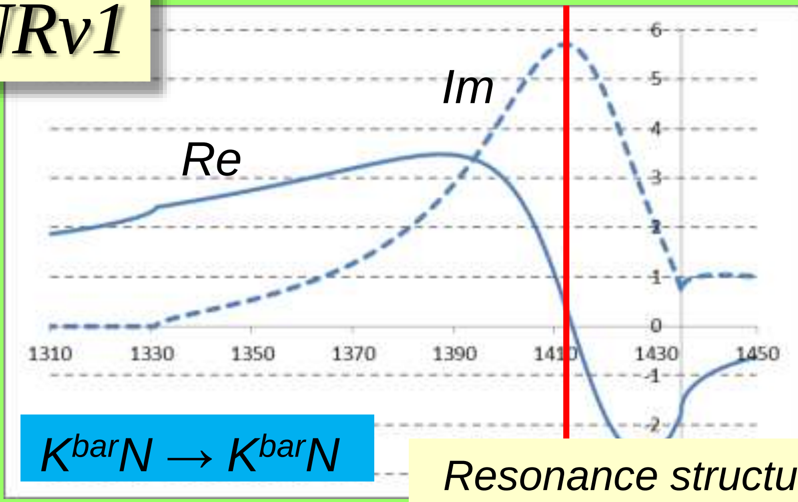
f_π=110 MeV	Kinematics KSW-type pot.	Non-rela. NR-v1	 NR-v2	Semi-rela. SR-A	 SR-B	Martin
Range parameters [fm]	$d_{KN,KN}$ $d_{\pi\Sigma,\pi\Sigma}$	0.44 0.61	0.44 0.64	0.50 0.71	0.37 0.35	
KbarN scatt. length [fm]	Re $a^{I=0}_{KN}$ Im $a^{I=0}_{KN}$	−1.701 0.681	−1.700 0.681	−1.700 0.681	−1.696 0.681	−1.70 0.68

- ✓ *Found sets of range parameters ($d_{KN,KN}$, $d_{\pi\Sigma,\pi\Sigma}$) to reproduce the Martin's value.*
- ✓ *Two sets found in semi-rela. case. (SR-A, SR-B)*

Scattering amplitude - Non-rela. case -

f_π=110 MeV

NRv1



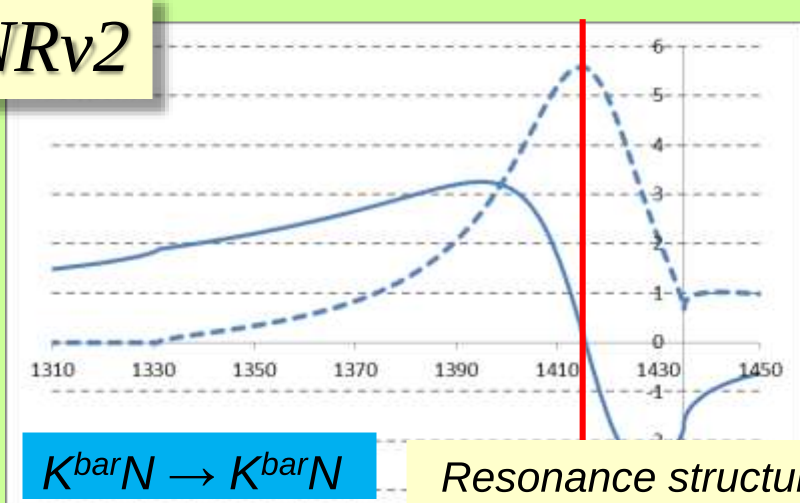
$K^{bar}N \rightarrow K^{bar}N$

Resonance structure
at 1413 MeV

$\pi\Sigma \rightarrow \pi\Sigma$

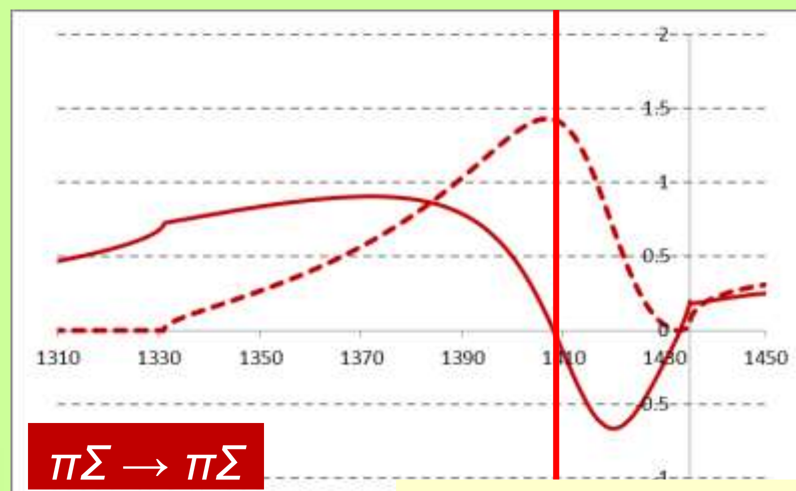
at 1405 MeV

NRv2



$K^{bar}N \rightarrow K^{bar}N$

Resonance structure
at 1416 MeV



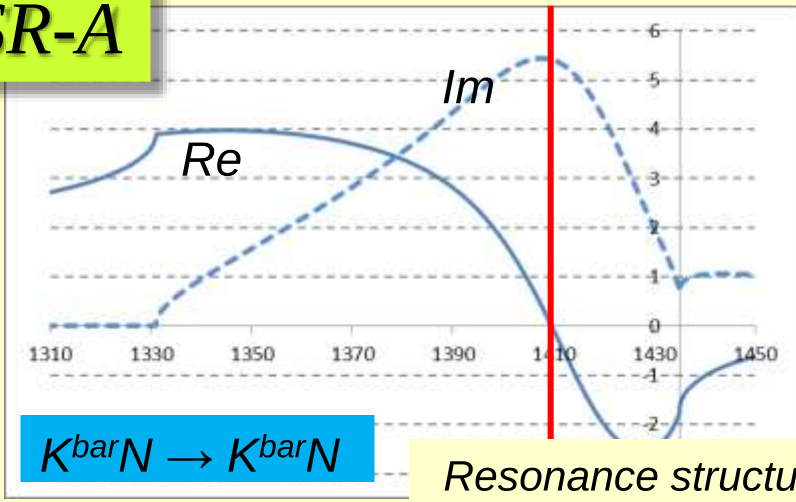
$\pi\Sigma \rightarrow \pi\Sigma$

at 1408 MeV

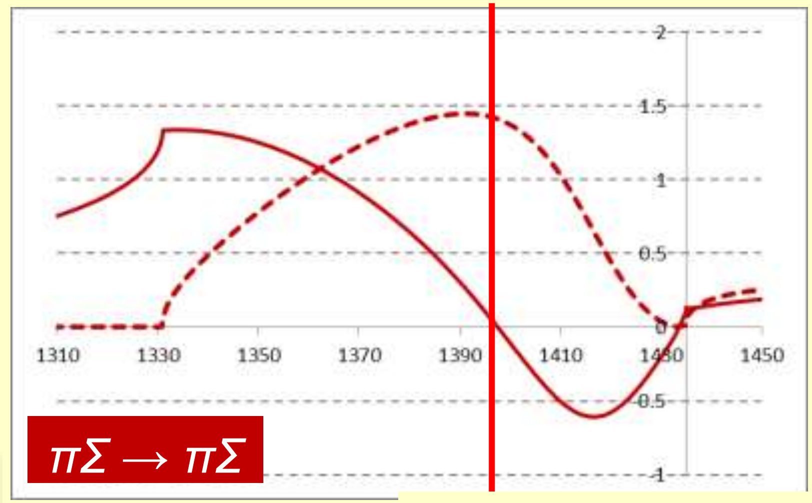
Scattering amplitude - Semi-rela. case -

f_π=110 MeV

SR-A

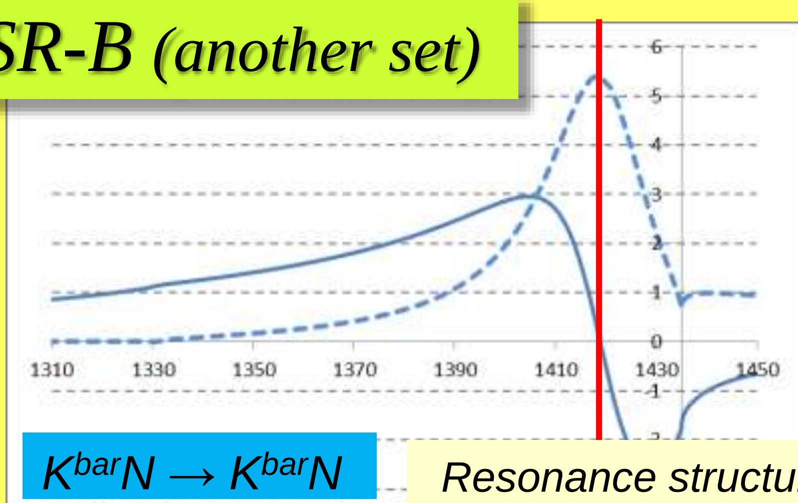


Resonance structure
at 1410 MeV

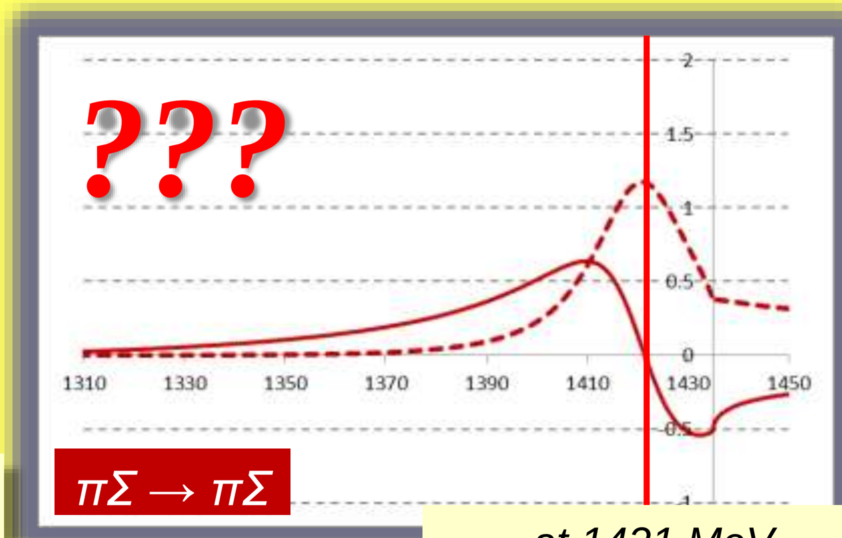


at 1398 MeV

SR-B (another set)



Resonance structure
at 1419 MeV



at 1421 MeV

Resonance pole

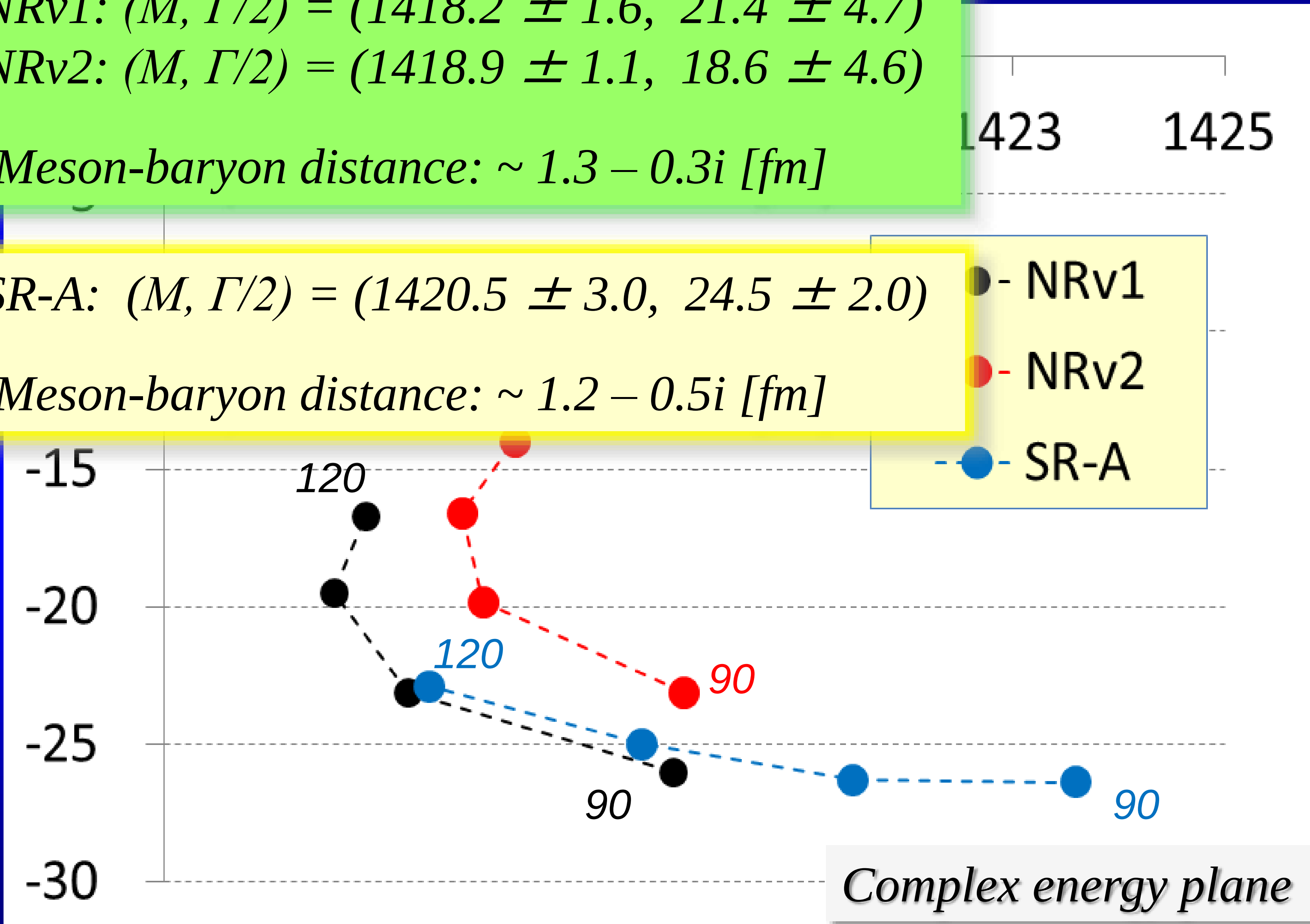
f_π=90 - 120MeV

NRv1: $(M, \Gamma/2) = (1418.2 \pm 1.6,\ 21.4 \pm 4.7)$
NRv2: $(M, \Gamma/2) = (1418.9 \pm 1.1,\ 18.6 \pm 4.6)$

**Meson-baryon distance: ~ 1.3 – 0.3i [fm]*

SR-A: $(M, \Gamma/2) = (1420.5 \pm 3.0,\ 24.5 \pm 2.0)$

**Meson-baryon distance: ~ 1.2 – 0.5i [fm]*



T. Sekihara, T. Hyodo, Phys. Rev. C 87 (2013) 045202.

“Size” of resonance state

<r^2> is a complex value on the resonance pole.

approximation. Combining two approaches, we evaluate t[h]
complex “mean distance” between $\bar{K}N$ on the pole position
1426 − 17i MeV as

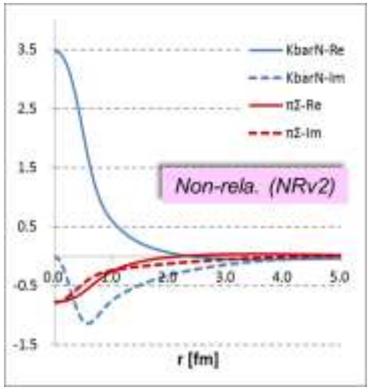
$$\sqrt{\langle r^2\rangle_{\bar{K}N}} = 1.22 - 0.63i \text{ fm}, \quad \left|\sqrt{\langle r^2\rangle_{\bar{K}N}}\right| = 1.37 \text{ fm}. \tag{36}$$

(b) $T^{\mu}_{\gamma ij}$

via electro-magnetic form factor

The mean-squared distance of $\bar{K}N$ inside $\Lambda(1405)$ on the po[le]
was also calculated in Ref. [37] by using the complex scali[ng]
method with an effective coupled-channel potential. The resu[lt]
on the pole at 1419 − 14i MeV is

$$\sqrt{\langle r^2\rangle_{\bar{K}N}} = 1.21 - 0.49i \text{ fm}, \quad \left|\sqrt{\langle r^2\rangle_{\bar{K}N}}\right| = 1.31 \text{ fm}. \quad (3$$



Directly calculated from C.S. wfnc.

Where is the lower pole?

Double-pole nature of chiral potential

D. Jido, J.A. Oller, E. Oset, A. Ramos, U.-G. Meißner, Nucl. Phys. A 725 (2003) 181.
T. Hyodo, W. Weise, Phys. Rev. C 77 (2008) 035204.

	B and Γ	Coupling to	
Higher pole	Shallow and narrow	$K^{bar}N$	Close to $K^{bar}N$ threshold
Lower pole	Deep and broad	πΣ	*Generated by energy dependence*

Search for the deeper pole by

CSM with complex-range Gaussian base

S. Ohtsubo, Y. Fukushima, M. Kamimura, E. Hiyama, arXiv: 1302.4256 [nucl-th]

$$G_n^{(\pm)}(r) = r^l \exp\left[-(1 \pm i\omega)\left(\frac{r^2}{2b_n^2}\right)\right]$$

$$\Rightarrow$$

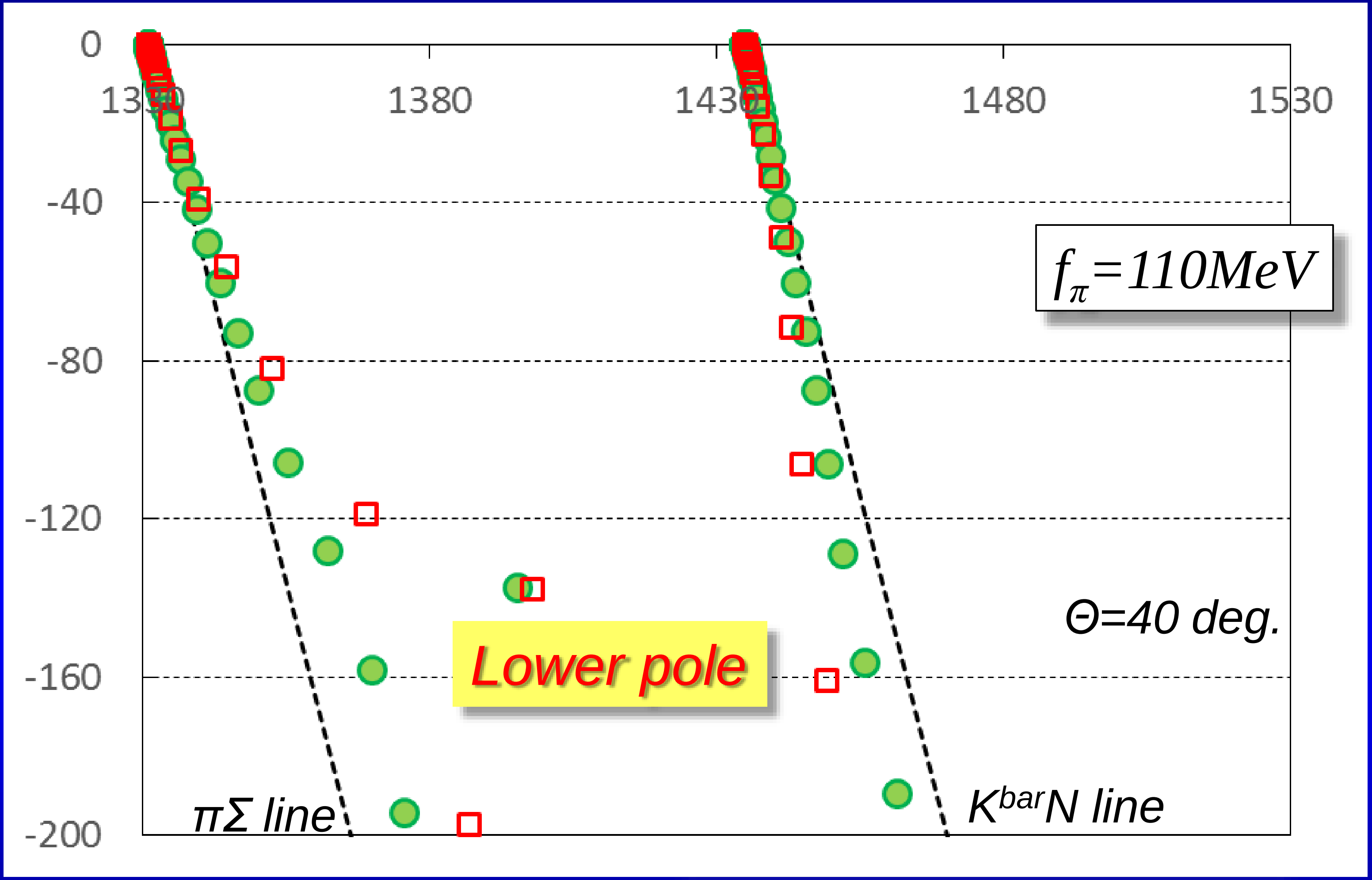
$$G_n^{(COS)}(r) = \frac{1}{2}\left\{G_n^{(+)}(r) + G_n^{(-)}(r)\right\} = r^l \exp\left[-\left(\frac{r^2}{2b_n^2}\right)\right] \cos\left[\omega\left(\frac{r^2}{2b_n^2}\right)\right]$$

$$G_n^{(SIN)}(r) = \frac{-1}{2i}\left\{G_n^{(+)}(r) - G_n^{(-)}(r)\right\} = r^l \exp\left[-\left(\frac{r^2}{2b_n^2}\right)\right] \sin\left[\omega\left(\frac{r^2}{2b_n^2}\right)\right]$$

Improve wave function of continuum states due to {cos, sin} oscillation

Clear separation of resonance pole from continuum states, even if it has broad width.

Where is the lower pole?



Real range Gaussian

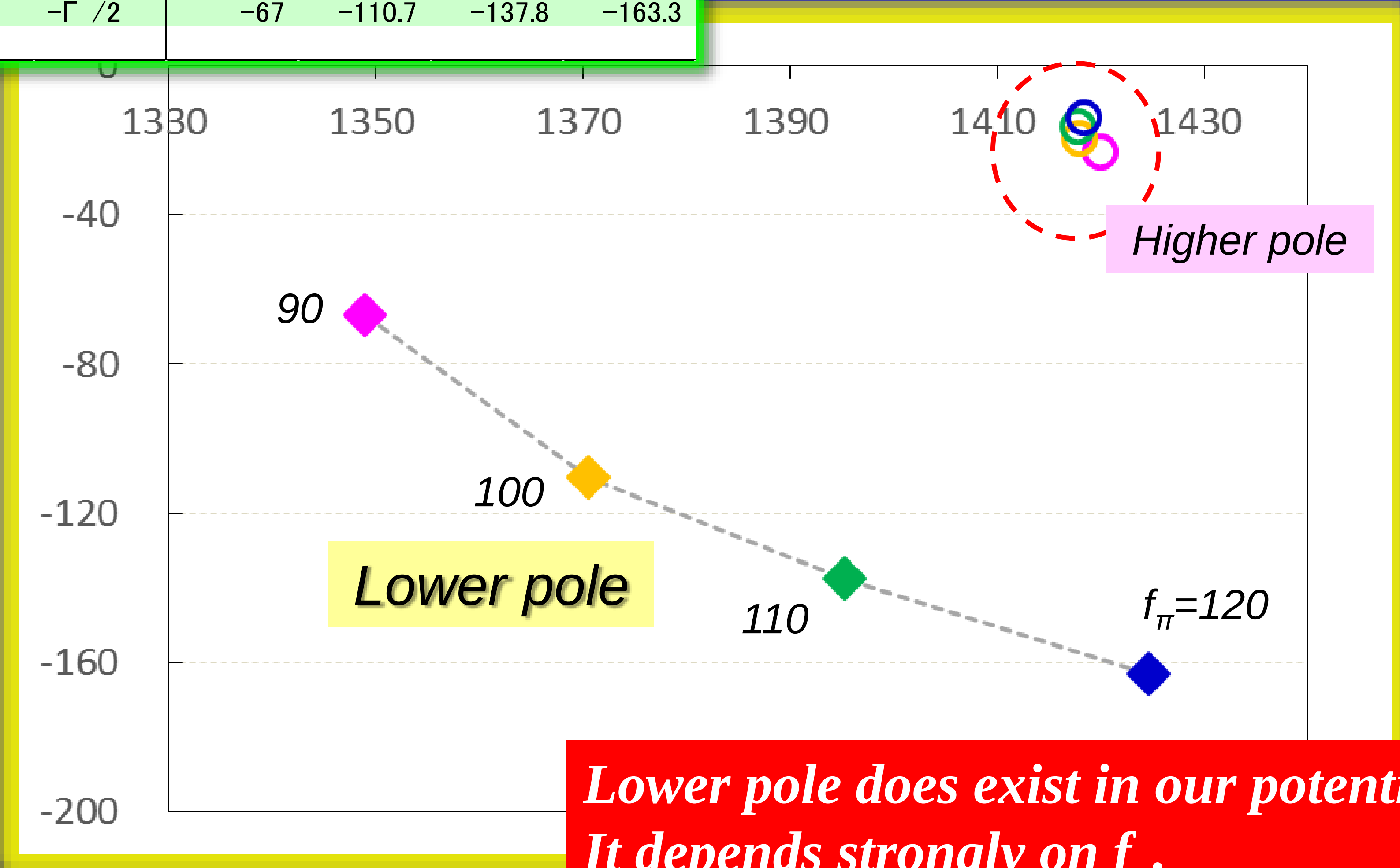
□ ω=0.0

Complex range Gaussian

● ω=2.0

Where is the lower pole?

	fpi	90	100	110	120
Higher pole	M	1419.9	1418.0	1417.8	1418.3
	−Γ /2	−23.1	−19.8	−16.6	−14.0
Lower pole	M	1349	1370.5	1395.4	1424.7
	−Γ /2	−67	−110.7	−137.8	−163.3



Lower pole does exist in our potential.
It depends strongly on f_π.

2−3. Result

- *I=1 channel*

A. D., T. Inoue, T. Myo,
NPA 912, 66 (2013)

  - *I=1 $K^{bar}N$ scattering length*
    Range parameters of KSW-type potential
  - *Scattering amplitude*

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

6 range parameters, but the scattering length provides 2 real values…

- ***"Iso-symmetric choice"***
  $\{d_{KN,KN}, d_{KN,\pi\Sigma}, d_{\pi\Sigma,\pi\Sigma}\}$ *fixed to I=0 channel ones* *Ref) chiral unitary model*
- *Ignore* $\{d_{\pi\Sigma,\pi\Lambda}, d_{\pi\Lambda,\pi\Lambda}\}$ *in case of only WT term, due to SU(3) algebra*

⇒ *Search* $d_{KN,\pi\Lambda}$ *to reproduce Re or Im* $a^{I=1}{}_{KbarN}$ *of Matin's value.*

$$d_{ij}^{(I=1)} = \begin{pmatrix} d^{(I=0)}_{K^{bar}N,K^{bar}N} & d^{(I=0)}_{K^{bar}N,\pi\Sigma} & d_{K^{bar}N,\pi\Lambda} \\ & d^{(I=0)}_{\pi\Sigma,\pi\Sigma} & \cancel{d_{\pi\Sigma,\pi\Lambda}} \\ & & \cancel{d_{\pi\Lambda,\pi\Lambda}} \end{pmatrix}$$

SU(3) C.G. for I=1

$$C^{(I=1)}{}_{ij} = \begin{pmatrix} 1 & -1 & -\sqrt{3/2} \\ & 2 & 0 \\ & & 0 \end{pmatrix}$$

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

6 range parameters, but the scattering length provides 2 real values…

- ***"Iso-symmetric choice"***
  $\{d_{KN,KN}, d_{KN,\pi\Sigma}, d_{\pi\Sigma,\pi\Sigma}\}$ *fixed to I=0 channel ones* *Ref) chiral unitary model*
- *Ignore* $\{d_{\pi\Sigma,\pi\Lambda}, d_{\pi\Lambda,\pi\Lambda}\}$ *in case of only WT term, due to SU(3) algebra*

⇒ *Search* $d_{KN,\pi\Lambda}$ *to reproduce Re or Im* $a^{I=1}_{KbarN}$ *of Matin's value.*

	Kinematics KSW−type pot. Condition	Non−rela. NRv2 Re fitted	 Im fitted	Semi−rela. SR−A Re fitted	 Im fitted	Martin
Range parameters [fm]	d(KN, KN)	0.438	0.438	0.499	0.499	
	d(π Σ , π Σ)	0.636	0.636	0.712	0.712	
	d(KN, π Λ)	0.301	0.445	0.354	0.467	
KbarN scatt. length [fm]	Re	**0.372**	**0.657**	**0.371**	**0.659**	0.37
	Im	**1.504**	**0.599**	**1.493**	**0.6**	0.6

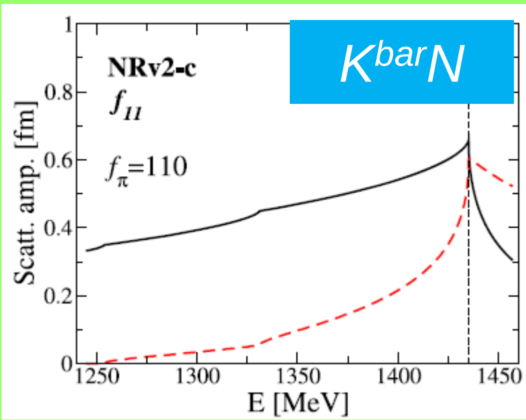
When Re $a^{I=1}_{KbarN}$ is reproduced, Im $a^{I=1}_{KbarN}$ deviates largely from Martin's value!

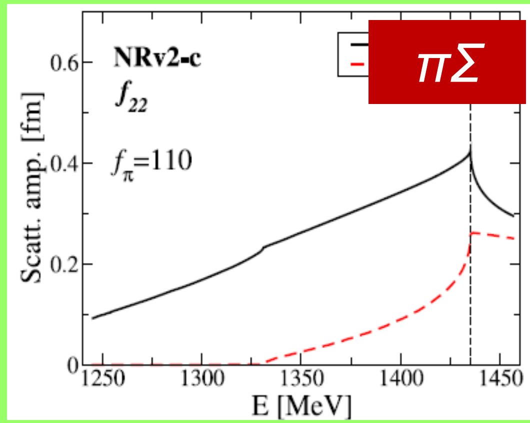
<u>I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$</u>

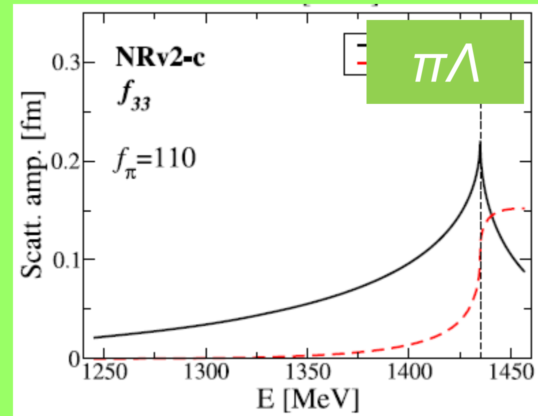
f_π=110 MeV

- ✓ Search $d_{KN,\pi\Sigma}$ to reproduce Im $a^{I=1}_{KbarN}$ of Matin's value.

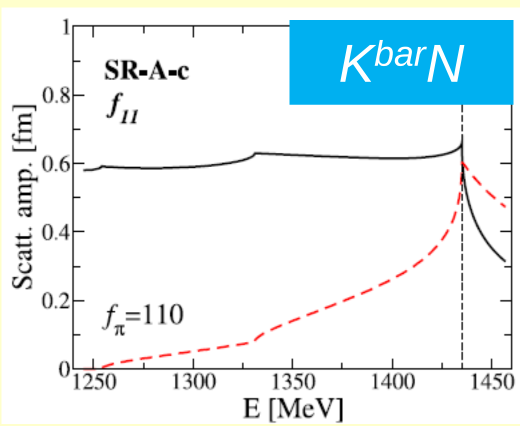
<u>NRv2 (c): $a^{I=1}_{KbarN}$ = 0.657 + i 0.599 fm</u>

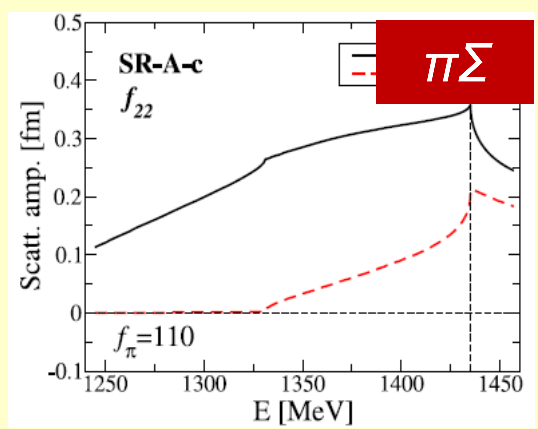


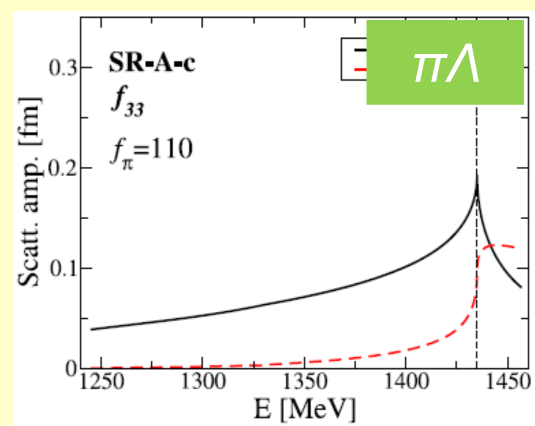




<u>SR-A (c): $a^{I=1}_{KbarN}$ = 0.659 + i 0.600 fm</u>





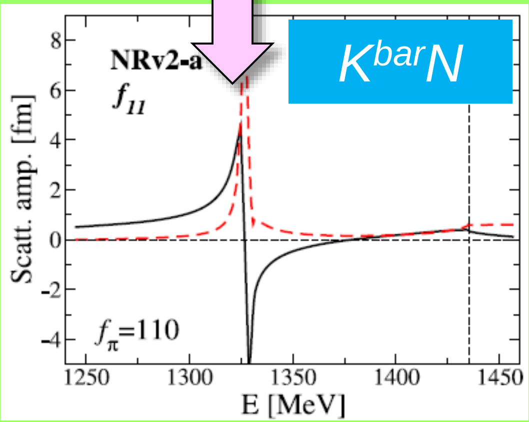


I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

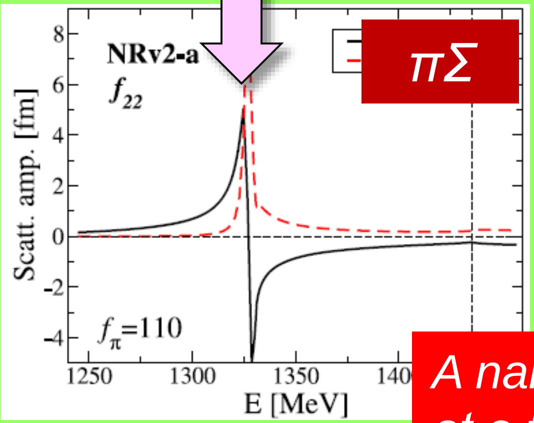
f_π=110 MeV

✓ Search $\{d_{\pi\Sigma,\pi\Sigma}, d_{KN,\pi\Lambda}\}$ to reproduce Re and Im $a^{I=1}_{KbarN}$ of Matin's value.

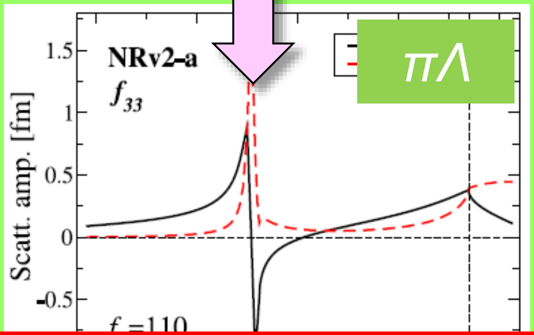
NRv2 (a): $a^{I=1}_{KbarN}$ = 0.376 + i 0.606 fm



$K^{bar}N$



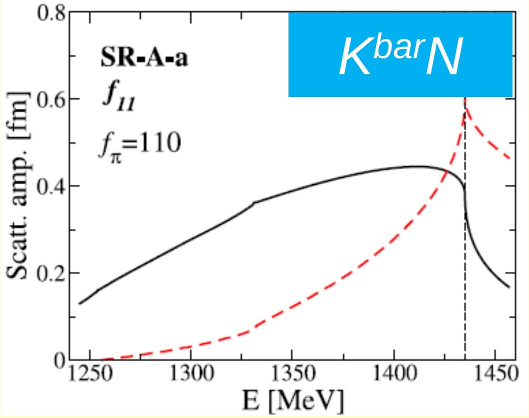
$\pi\Sigma$



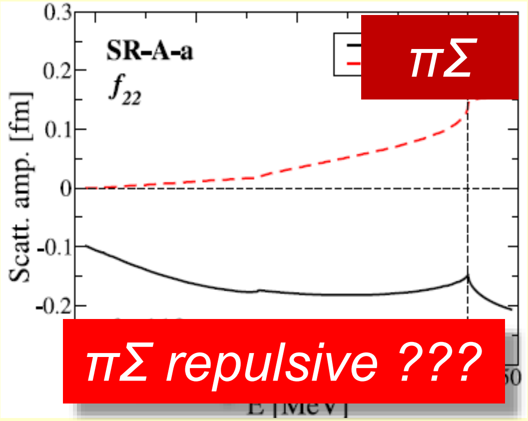
$\pi\Lambda$

A narrow resonance exists at a few MeV below $\pi\Sigma$ threshold

SR-A (a): $a^{I=1}_{KbarN}$ = 0.375 + i 0.605 fm

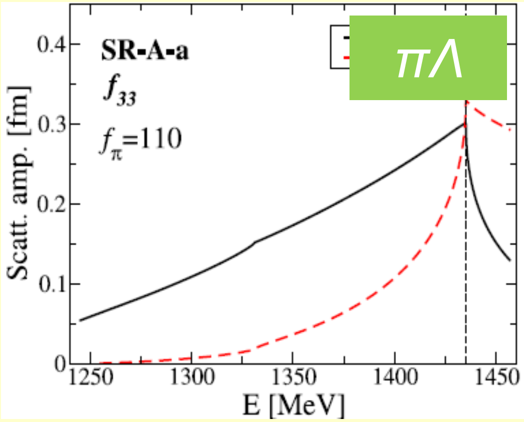


$K^{bar}N$



$\pi\Sigma$

$\pi\Sigma$ repulsive ???



$\pi\Lambda$

2-4. Short summary of $K^{bar}N$-πY study with ccCSM

We have studied comprehensively scattering and resonant states of $K^{bar}N$-πY system with a coupled-channel Complex Scaling Method .
We construct a chiral SU(3) potential with a Gaussian form in r-space

- *In I=0 sector, the resonance structure appears in $K^{bar}N/\pi\Sigma$ scattering amplitude at ~1420MeV.*

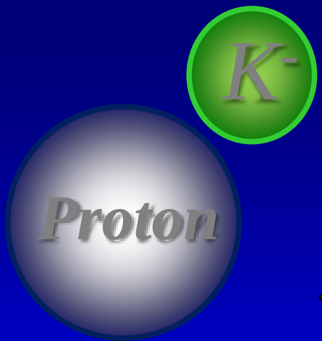
  Double-pole structure is confirmed.
  The lower pole is found by using the complex-range Gaussian base.

  NRv2) Higher pole: $(M, -\Gamma/2) = (1418.9 \pm 1.1,\ 18.6 \pm 4.6)$
  Lower pole: $(M, -\Gamma/2) = (1349, -67)$ ~ $(1425, -163)$
  @ f_π=90 @ f_π=120

- *In I=1 sector, it is difficult to reproduce Re $a^{I=1}_{KbarN}$ of Martin's value within our model, in case of "iso-symmetric choice" of range parameter.*

Future plan

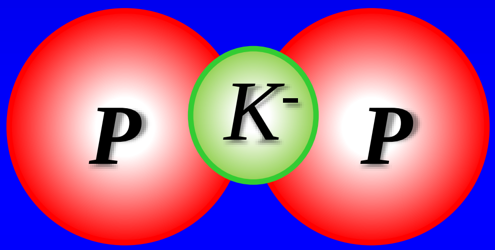
- Detailed study of lower pole
- Updated data of $K^- p$ scattering length by SHIDDARTA



$\Lambda(1405)$ *"Building block of kaonic nuclei"*

... used to determine $K^{bar}N$ interaction

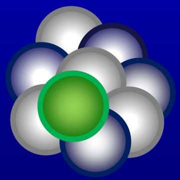
$K^{-}pp$



Most essential kaonic nucleus

"Prototype of kaonic nuclei"

Nuclear many-body system with K^{-}



$^{3}HeK^{-}$, $pppK^{-}$, $^{4}HeK^{-}$, $pppnK^{-}$, …, $^{8}BeK^{-}$, …

3. Three-body system of $K^{-}pp$

studied with

"coupled-channel

Complex Scaling Method

+ Feshbach method"

Theoretical studies of K^-pp

"K^-pp" = Prototype of K^{bar} nuclei

$K^{bar}NN - \pi\Sigma N - \pi\Lambda N$, $J^p=1/2^-$, $T=1/2$

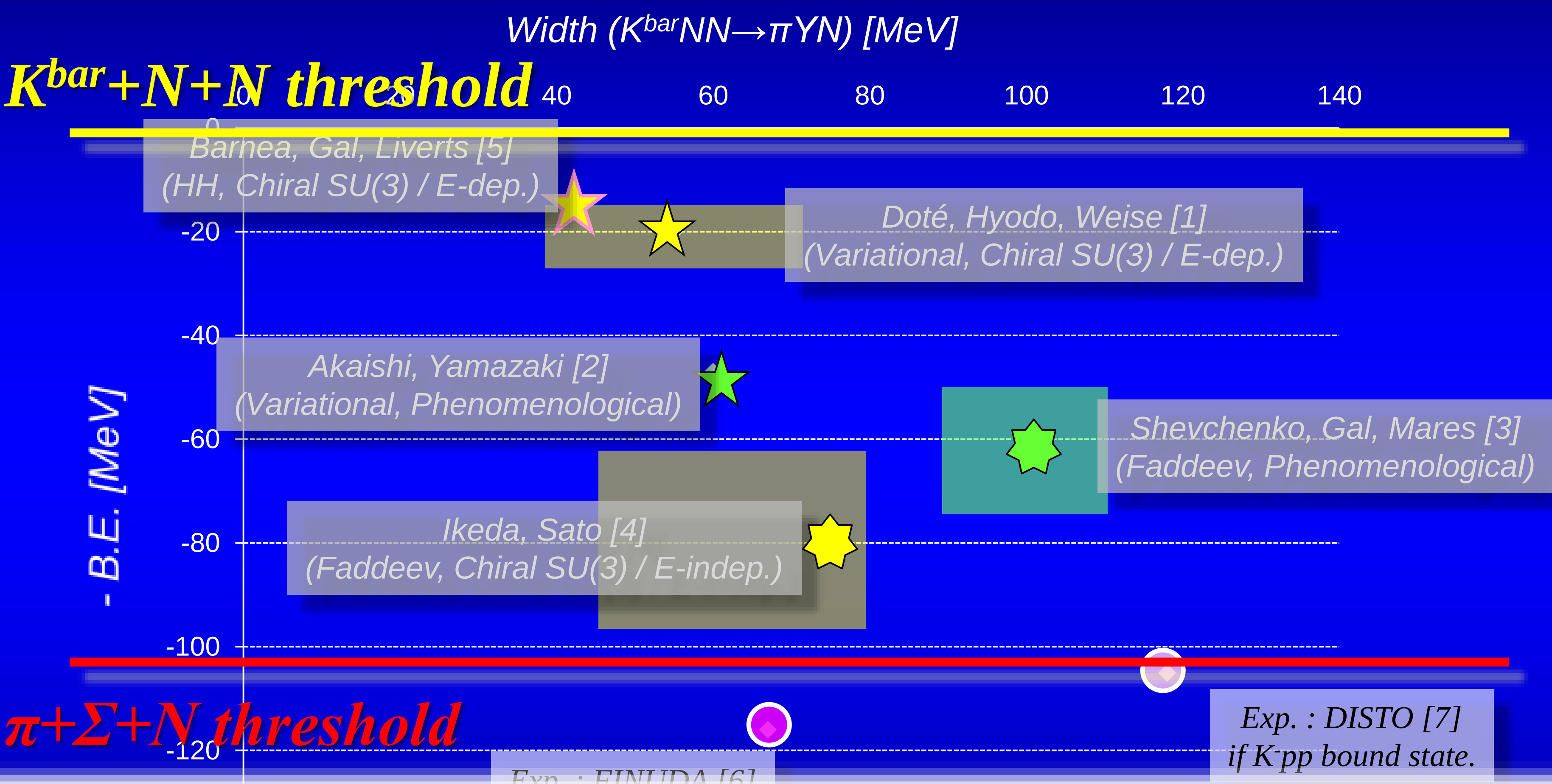
- Doté, Hyodo, Weise — PRC79, 014003(2009)
  Variational with a chiral SU(3)-based $K^{bar}N$ potential
- Akaishi, Yamazaki — PRC76, 045201(2007)
  ATMS with a phenomenological $K^{bar}N$ potential
- Ikeda, Sato — PRC76, 035203(2007)
  Faddeev with a chiral SU(3)-derived $K^{bar}N$ potential
- Shevchenko, Gal, Mares — PRC76, 044004(2007)
  Faddeev with a phenomenological $K^{bar}N$ potential
- Barnea, Gal, Liverts — PLB712, 132(2012)
  Hyperspherical harmonics with a chiral SU(3)-based $K^{bar}N$ potential

- Wycech, Green — PRC79, 014001(2009)
  Variational with a phenomenological $K^{bar}N$ potential (with p-wave)

- Arai, Yasui, Oka/ Uchino, Hyodo, Oka — PTP119, 103(2008) /PTPS 186, 240(2010)
  Λ^* nuclei model
- Nishikawa, Kondo — PRC77, 055202(2008)
  Skyrme model

All studies predict that K^-pp can be bound!

Typical results of theoretical studies of $K^{-}pp$



From theoretical viewpoint,
$K^{-}pp$ exists between K^{bar}-N-N and π-Σ-N thresholds!

[4] PRC76, 035203 (2007)

"Λ(1405)" etc.

<u>3-1. Formalism of</u>

<u>“ccCSM + Feshbach method”</u>

ccCSM + Feshbach method

Elimination of channels by Feshbash method

Schrödinger eq.
in model space "P" and out of model space "Q"

$$\begin{pmatrix} T_P + v_P & V_{PQ} \\ V_{QP} & T_Q + v_Q \end{pmatrix} \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix} = E \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix}$$

Schrödinger eq. in P space :

$$\left(T_P + U_P^{Eff}(E) \right) \Phi_P = E \, \Phi_P$$

Effective potential for P-space

$$\boxed{U_P^{Eff}(E) = v_P + V_{PQ} G_Q(E) V_{QP}}$$

Q-space Green function:

$$\boxed{G_Q(E) = \frac{1}{E - H_{QQ}}}$$

Extended Closure Relation in Complex Scaling

$$H_{QQ}^\theta \left| \chi_n^\theta \right\rangle = \varepsilon_n^\theta \left| \chi_n^\theta \right\rangle$$

$$H_{QQ}^\theta = U(\theta) H_{QQ} U^{-1}(\theta)$$

$$\boxed{\int_C \sum_{R+B} \left| \chi_n^\theta \right\rangle \left\langle \chi_n^\theta \right| = 1}$$

Diagonalize H^θ_{QQ} with Gaussian base,

$$\boxed{\sum_n \left| \chi_n^\theta \right\rangle \left\langle \chi_n^\theta \right| \approx 1}$$

Well approximated

T. Myo, A. Ohnishi and K. Kato, PTP99, 801 (1998)

Express the $G_Q(E)$ with **Gaussian base** using ECR

$$U_P^{Eff}(E) = v_P + V_{PQ} \underbrace{U^{-1}(\theta) G_Q^\theta(E) U(\theta)}_{G_Q(E)} V_{QP}$$

$$\boxed{G_Q^\theta(E) \approx \sum_n \left| \chi_n^\theta \right\rangle \frac{1}{E - \varepsilon_n^\theta} \left\langle \chi_n^\theta \right|}$$

$\left\{ \left| \chi_n^\theta \right\rangle \right\}$ *: expanded with Gaussian base.*

Remark on $U^{Eff}{}_P$

$$U_P^{Eff}(E) = v_P + V_{PQ}\; U^{-1}(\theta) G_Q^{\theta}(E) U(\theta)\; V_{QP}$$

$$\boxed{G_Q^{\theta}(E) \approx \sum_n \left|\chi_n^{\theta}\right\rangle \frac{1}{E-\varepsilon_n^{\theta}} \left\langle \chi_n^{\theta}\right|}$$

Non-local

and E-dep.

Due to the energy dependence …

Schrödinger eq. in P space :

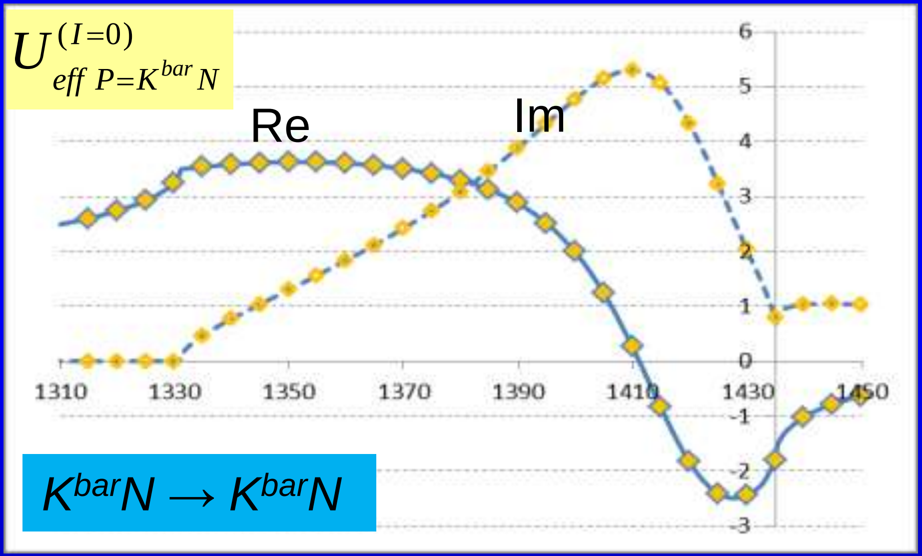
$$\left(T_P + U_P^{Eff}(E)\right)\Phi_P = E\,\Phi_P$$

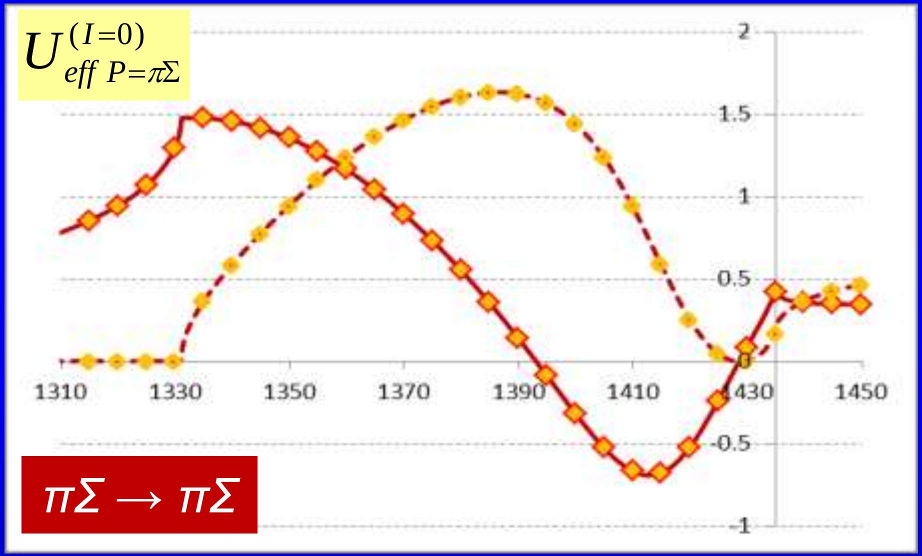
Self-consistency for the energy should be taken into account, when bound and resonant states are considered.

Test of ccCSM+Feshbach on 2-body system

Scattering problem

- Scattering amplitude





KSW - NRv2 (I=0), f_π=110 MeV

Test of ccCSM+Feshbach on 2-body system

Resonance state

Schrödinger eq. in P space : $\left(T_P + U_P^{Eff}(Z)\right)\Phi_P = Z\,\Phi_P$

Resonance → Self-consistency for complex energy "Z"

AY potential (Non-rela. / E-indep.)

†Y. Akaishi and T. Yamazaki, PRC 52 (2002) 044005

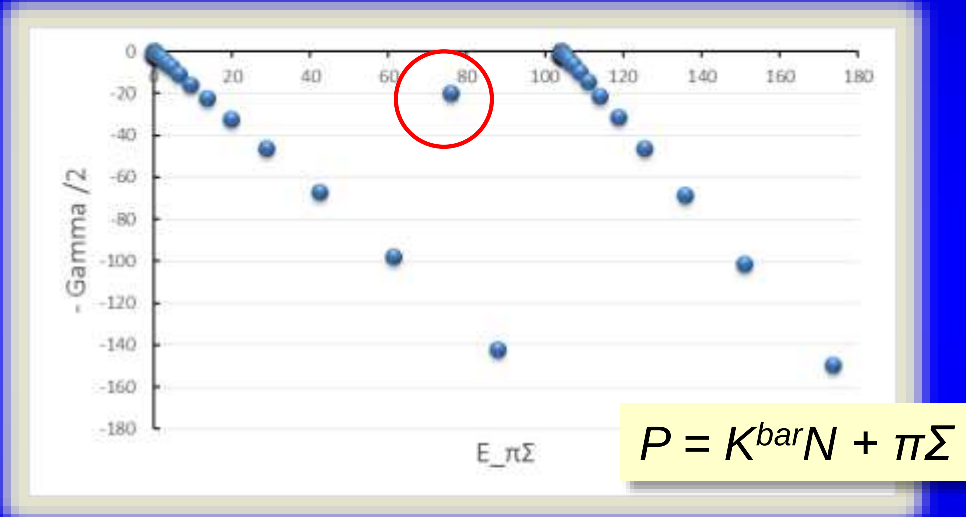
P−space	KN + π Σ	KN	π Σ
B(KN)	28.1698	28.1698	28.1698
Γ /2	20.0288	20.0288	20.0289
[MeV]			
Mean distance			
KN	1.31 − i0.35	1.25 − i0.27	*
π Σ	0.31 − i0.21	*	0.21 + i0.91
Total	1.34 − i0.39	1.25 − i0.27	0.21 + i0.91
[fm]			

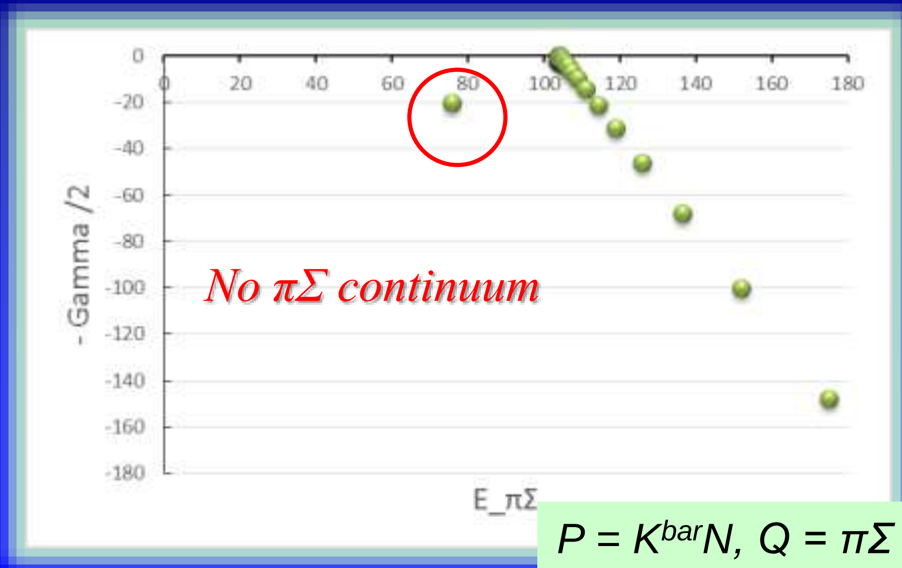
Feshbach+ccCSM

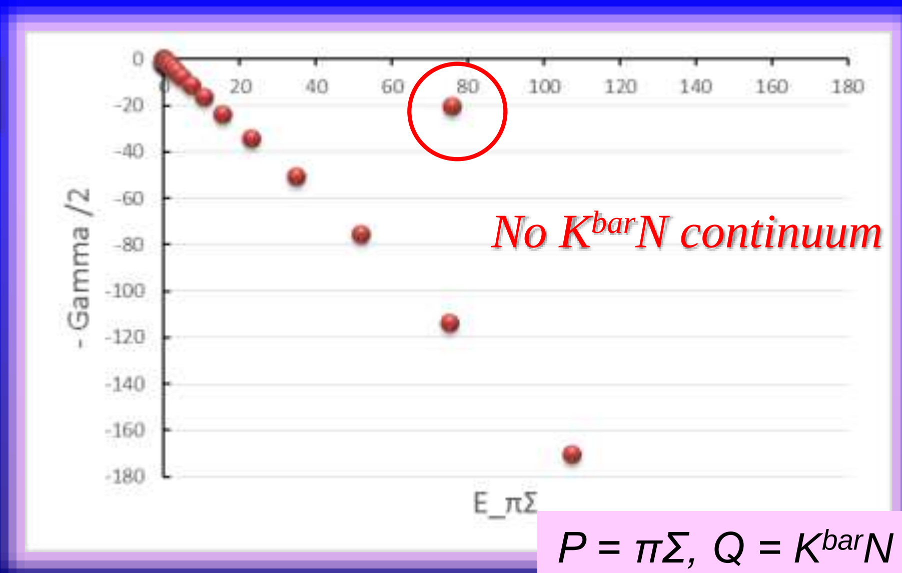
AY potential (Non-rela. / E-indep.)

- Complex eigenvalues

$\pi\Sigma$ $\quad$ $K^{bar}N$

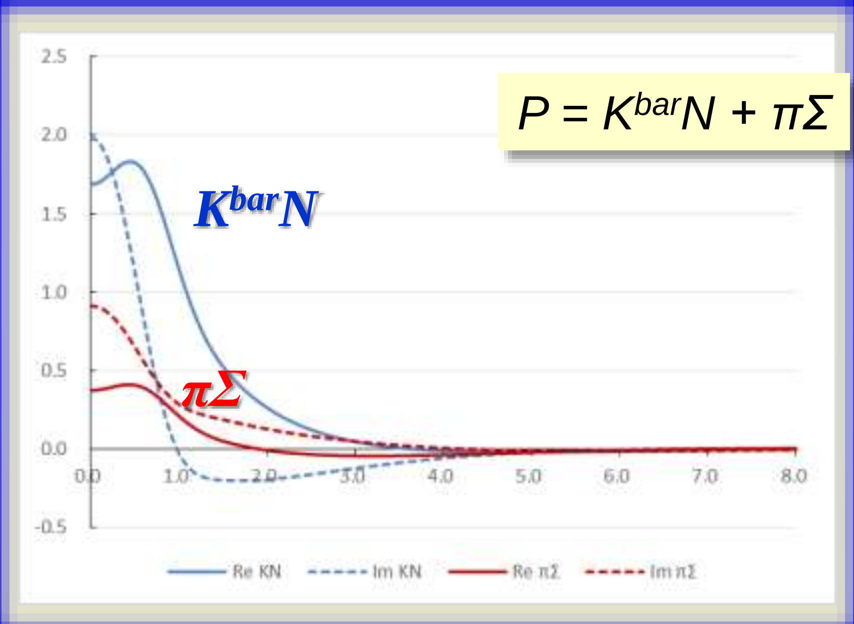


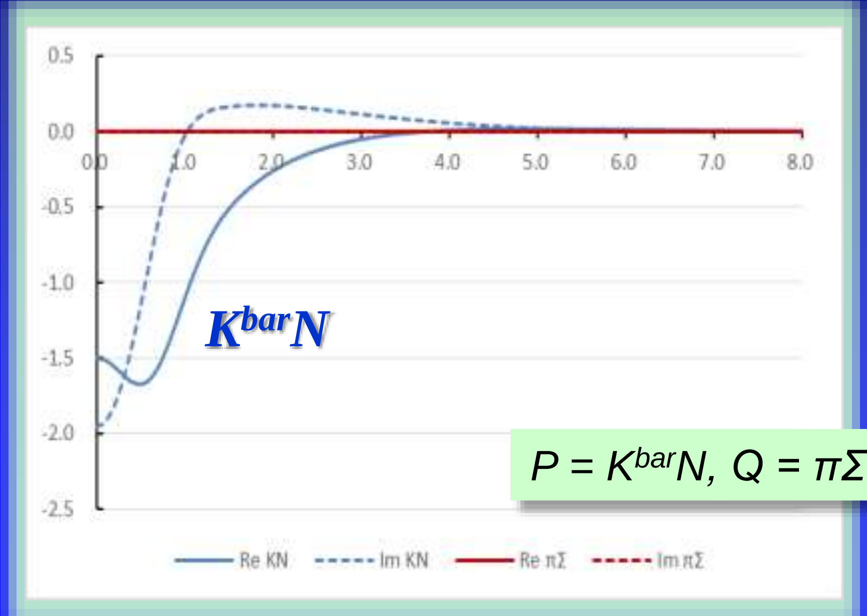


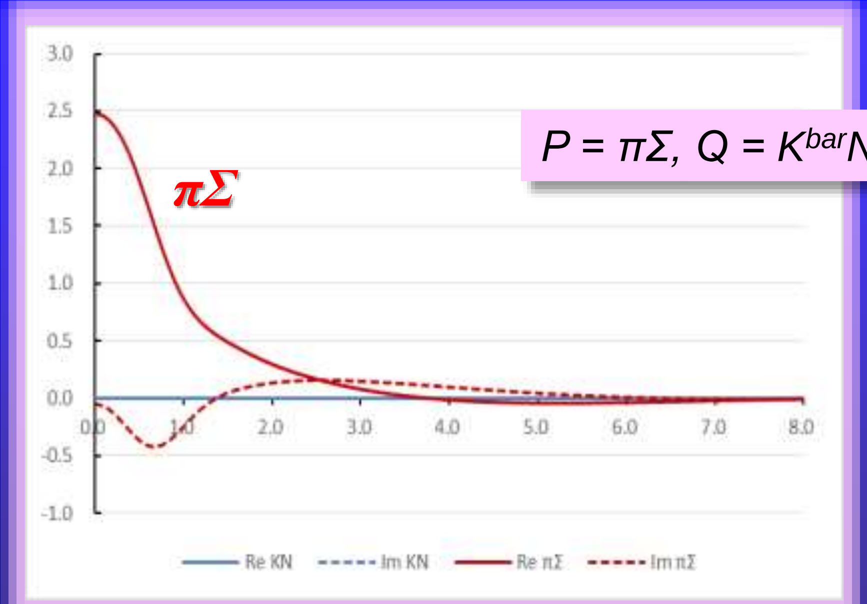


AY potential (Non-rela. / E-indep.)

- Complex wavefunction
  (Phase not adjusted)



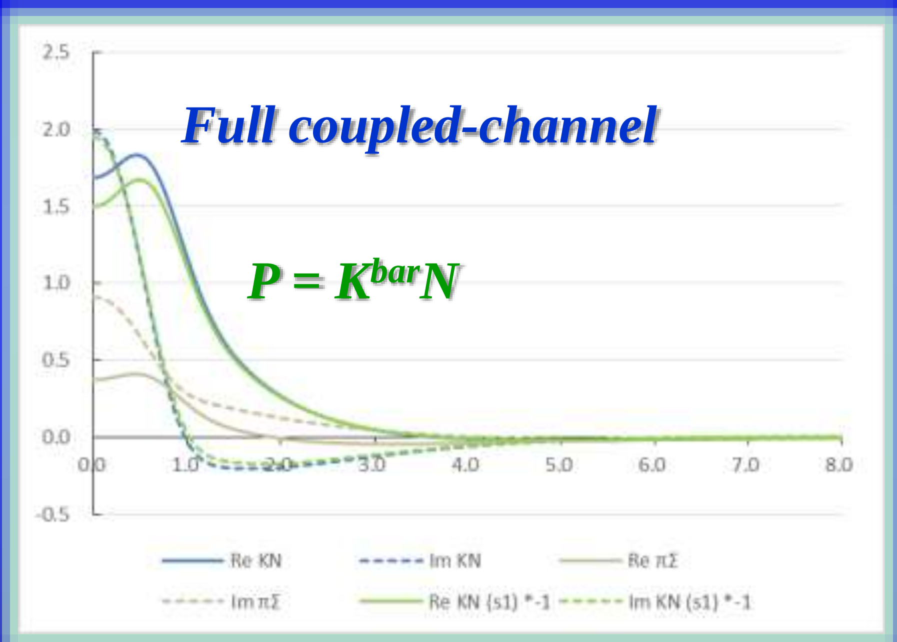




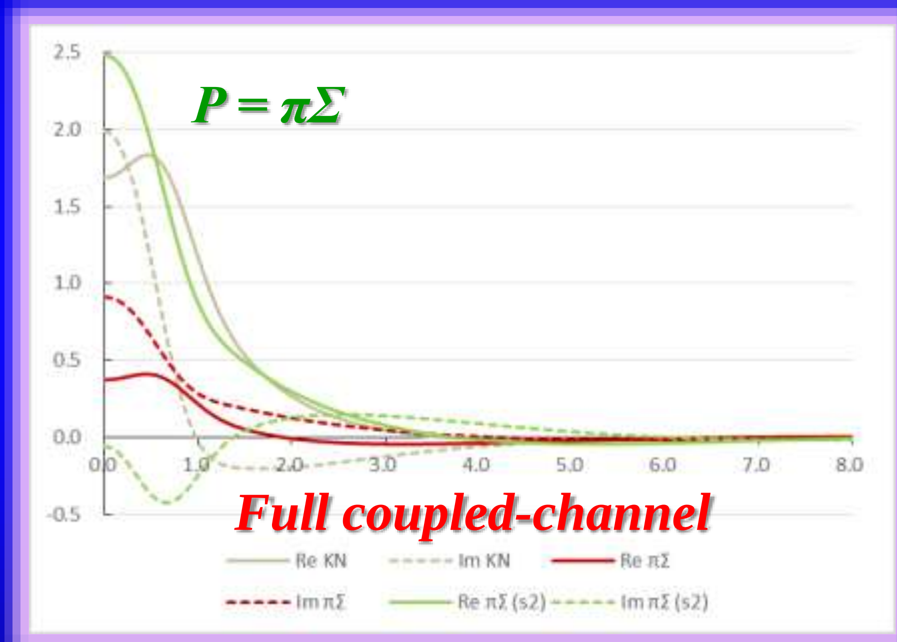
AY potential (Non-rela. / E-indep.)

- Comparison of Complex wavefunction
  (Phase adjusted)

$K^{bar}N$ wave function



πΣ wave function



$K^{bar}N$: dominant component

→ Not so different

πΣ : minor component

→ Very different

KSW potentials (E-dep.)

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8f_\pi^2}\left(\omega_i+\omega_j\right)\times\left[\,flux\ \ factor\,\right]\times g_{ij}(r)$$

- NRv2
  (Non-rela. / E-dep.)

Feshbach+ccCSM

P−space	KN+π Σ	KN	π Σ
B(KN)	17.1605	17.1606	17.1728
Γ /2	16.6176	16.6178	16.613
[MeV]			
Mean distance			
KN	1.37 − i0.37	1.28 − i0.40	*
π Σ	0.37 + i0.04	*	0.23 + i0.93
Total	1.42 − i0.34	1.28 − i0.40	0.23 + i0.93
[fm]			

- SR-A
  (Semi-rela. / E-dep.)

Feshbach+ccCSM

P−space	KN+π Σ	KN	π Σ
B(KN)	15.5336	15.5336	15.524
Γ /2	25.0158	25.015	24.9997
[MeV]			
Mean distance			
KN	1.22 − i0.47	1.07 − i0.46	*
π Σ	0.13 + i0.05	*	0.05 − i0.27
Total	1.22 − i0.47	1.07 − i0.46	0.05 − i0.27
[fm]			

$$\left(T_P + U_P^{Eff}(Z)\right)\Phi_P = Z\,\Phi_P$$

$$v_{ij}(Z)$$

Even when the original interaction has energy dependence, a self-consistent solution with complex energy can be obtained.

3-2. Application of ccCSM + Feshbach method to the three-body system "$K^{-}pp$"

Apply ccCSM + Feshbach method to K^-pp

"K^-pp" … $K^{bar}NN$ - $\pi\Sigma N$ - $\pi\Lambda N$ ($J^\pi=0^-$, $T=1/2$)

In the two-body system, $P = K^{bar}N$, $Q = \pi Y$

$$V\left(K^{bar}N - \pi Y; I = 0,1\right)$$
$$V\left(\pi Y - \pi Y' \quad ; I = 0,1\right)$$

Feshbach + ccCSM $\rightarrow$ $U^{Eff}_{K^{bar}N(I=0,1)}\left(E\right)$

- Schrödinger eq. for $K^{bar}NN$ channel :

$$\left(T_{K^{bar}NN} + V_{NN} + \sum_{i=1,2} U^{Eff}_{K^{bar}N_i(I)}\left(E_{K^{bar}N}\right)\right)\Phi_{K^{bar}NN} = E\,\Phi_{K^{bar}NN}$$

- Trial wave function

$$\left|"K^- pp"\right\rangle = \sum_a C_a^{(KNN,1)}\left\{G_a^{(KNN,1)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) + G_a^{(KNN,1)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_1\right]_{T=1/2}\right\rangle$$

Ch. 1: $K^{bar}NN$, NN: ^1E

$$+ \sum_a C_a^{(KNN,2)}\left\{G_a^{(KNN,2)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) - G_a^{(KNN,2)}\left(-\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)\right\}\left|S_{NN}=0\right\rangle\left|\left[K\left[NN\right]_0\right]_{T=1/2}\right\rangle$$

Ch. 2: $K^{bar}NN$, NN: ^1O

- Basis function = Correlated Gaussian
  …including 3-types Jacobi-coordinates

$$G_a^{(KNN,i)}\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right) = N_a^{(KNN,i)}\exp\left[-\left(\mathbf{x}_1^{(3)},\mathbf{x}_2^{(3)}\right)A_a^{(KNN,i)}\begin{pmatrix}\mathbf{x}_1^{(3)}\\ \mathbf{x}_2^{(3)}\end{pmatrix}\right]$$

Apply ccCSM + Feshbach method to K^-pp

"K^-pp" … $K^{bar}NN$ - $\pi\Sigma N$ - $\pi\Lambda N$ ($J^\pi=0^-$, $T=1/2$)

Chiral SU(3)-based potential

A. D., T. Inoue, T. Myo, Nucl. Phys. A 912, 66 (2013)

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8 f_\pi^2}\left(\omega_i + \omega_j\right)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\, g_{ij}(r) \qquad g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^3}\exp\left[-\left(r/d_{ij}\right)^2\right]$$

Self-consistency for complex $K^{bar}N$ energy

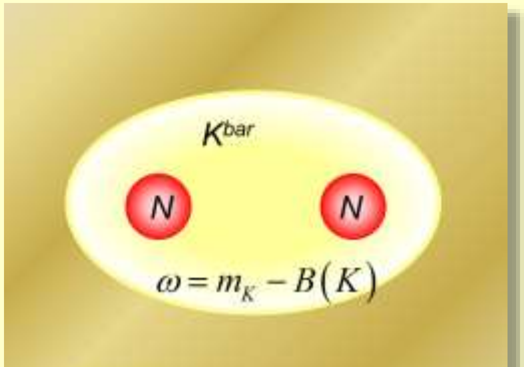
A. D., T. Hyodo, W. Weise, PRC79, 014003(2009)

… Similarly to the DHW study

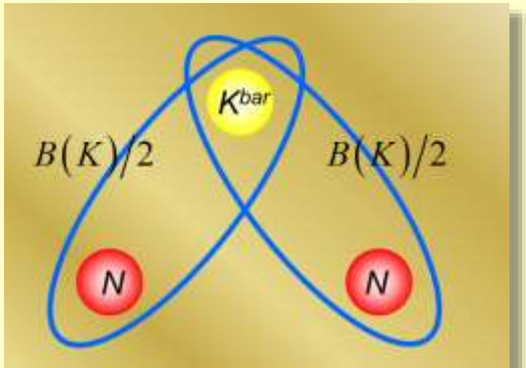
1. Kaon's binding energy: $B(K) \equiv -\left\{ \langle H \rangle - \langle H_{Nucl} \rangle \right\}$ $\qquad H_{Nucl}$: Hamiltonian of nuclear part

2. Define $K^{bar}N$ energy in two ways

$$E_{KN} = M_N + \omega = \begin{cases} M_N + m_K - B(K) & \text{: Field picture} \\ M_N + m_K - B(K)/2 & \text{: Particle picture} \end{cases}$$



K^{bar} field surrounds two nucleons which are almost static.

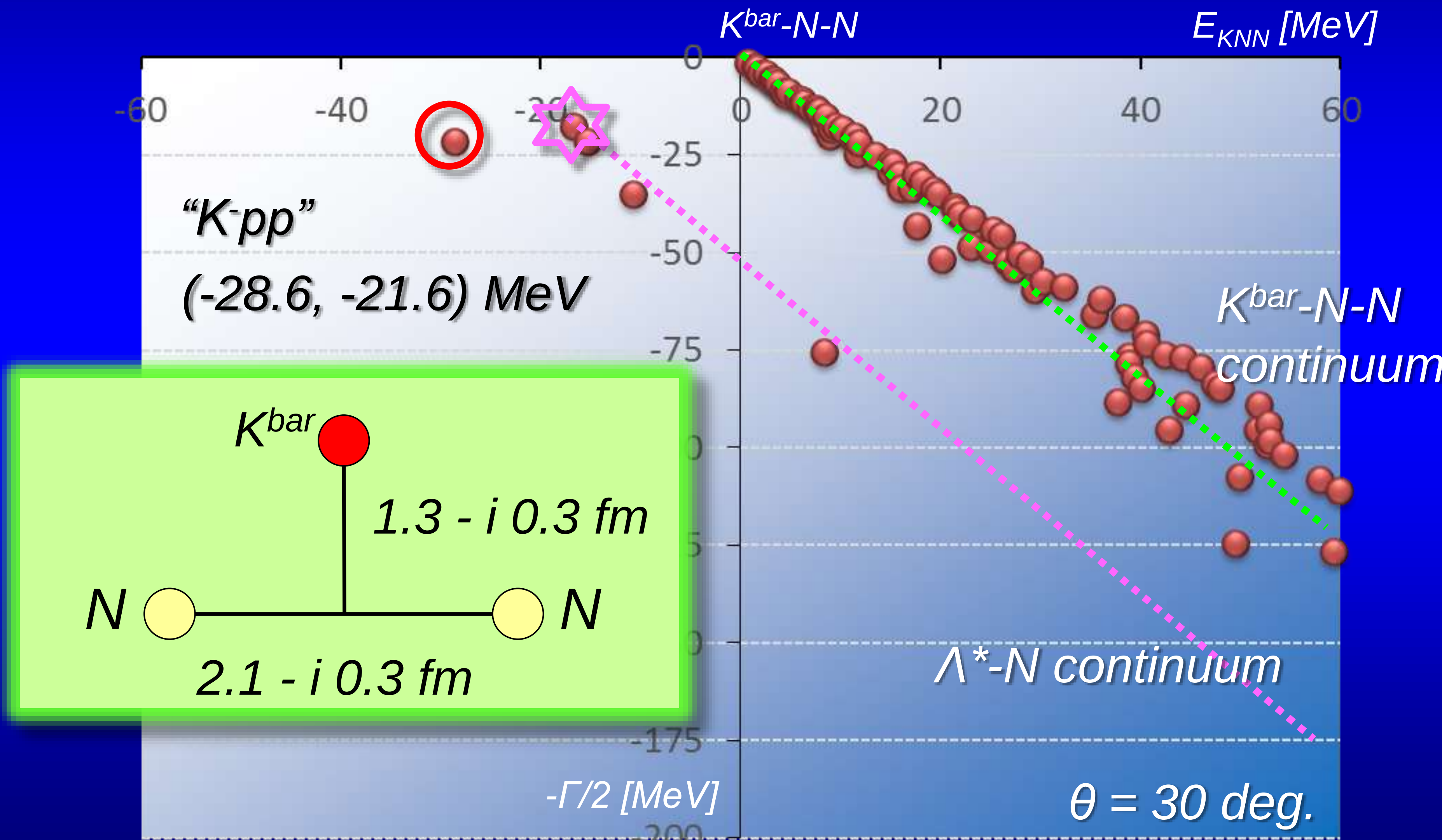


K^{bar} is bound by each nucleon with $B(K)/2$ binding energy.

Result: KSW-NRv2 potential (f_π=110MeV)

NN potential : Av18 (Central + spin-spin)

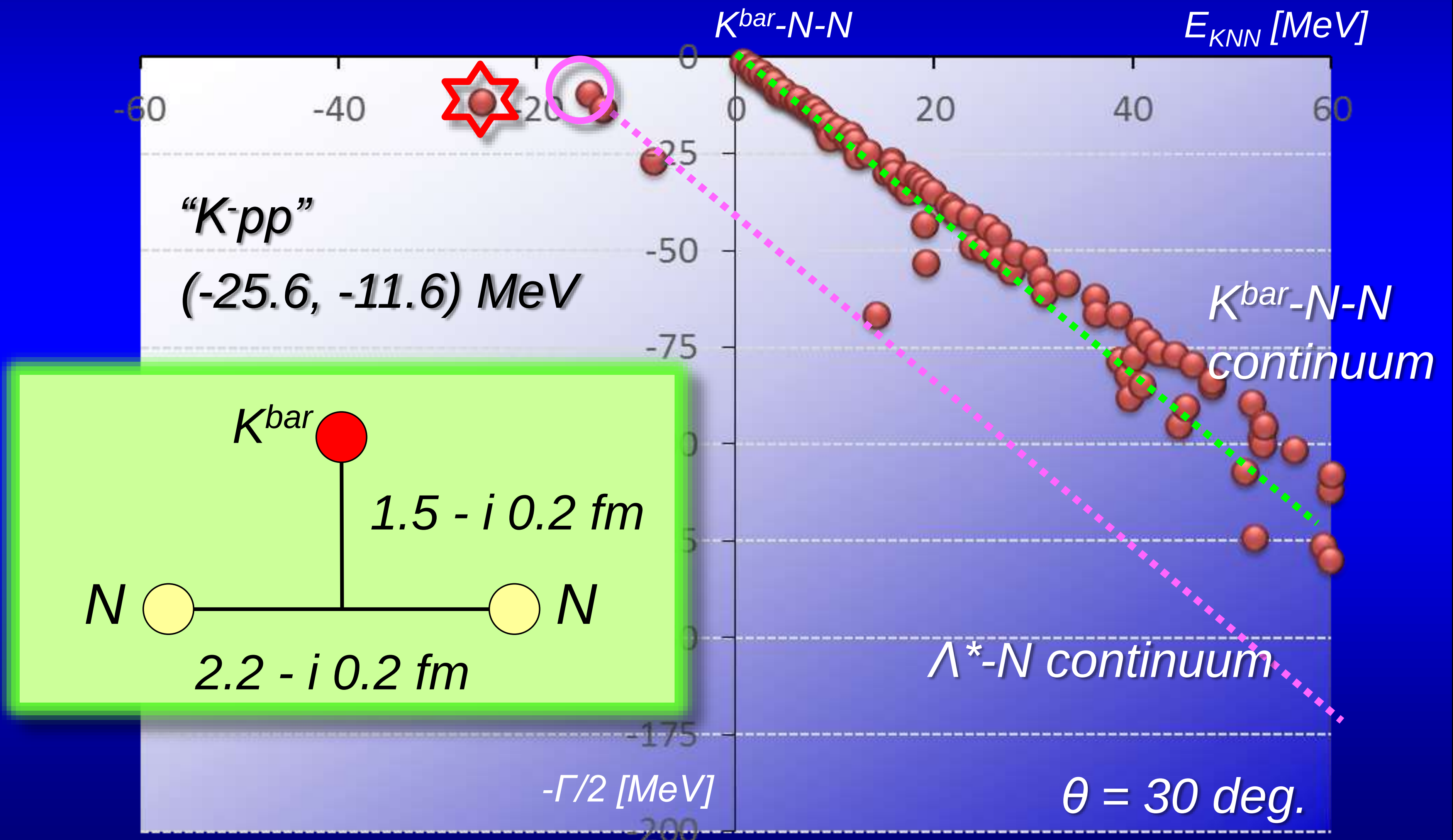
$K^{bar}N$ energy is fixed at Λ*. (self-consistent at Λ*)



Result: KSW-NRv2 potential (f_π=110MeV)

NN potential : Av18 (Central + spin-spin)

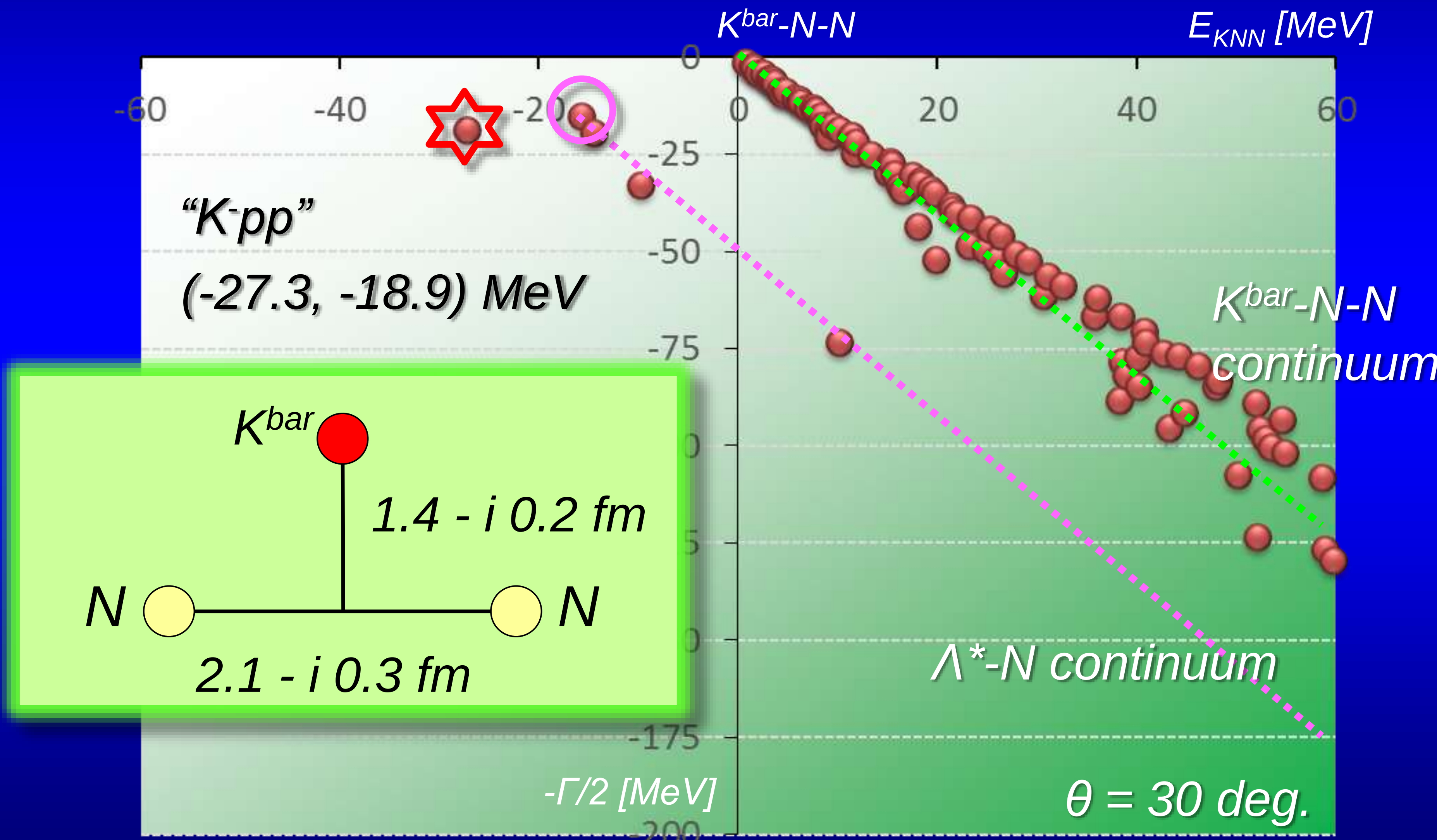
$K^{bar}N$ energy is self-consistent at K^-pp. ("Field picture")



Result: KSW-NRv2 potential (f_π=110MeV)

NN potential : Av18 (Central + spin-spin)

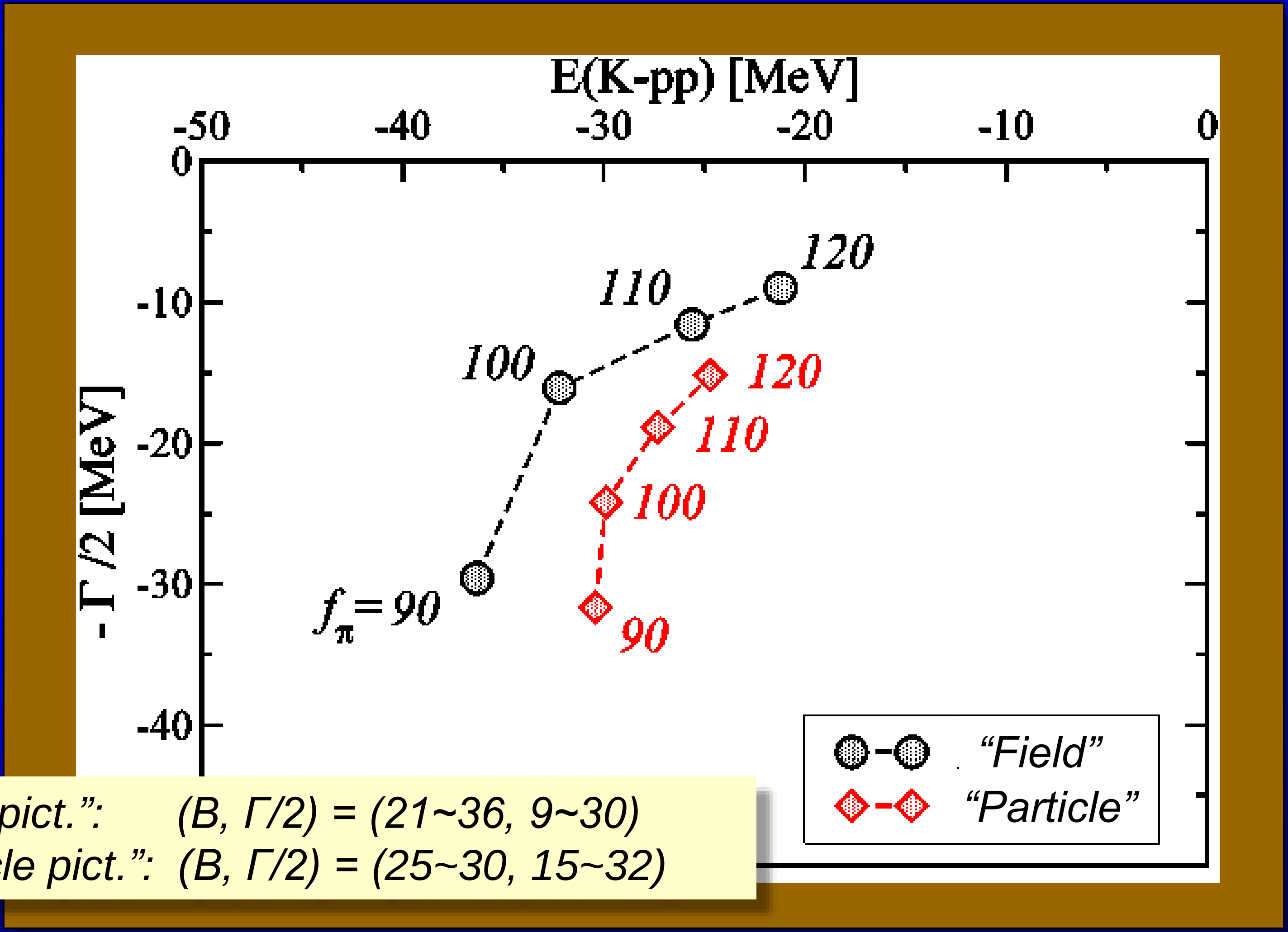
$K^{bar}N$ energy is self-consistent at K^-pp. ("Particle picture")



Result: KSW-NRv2 potential

NN potential : Av18 (Central + spin-spin)

f_π=90~120MeV / "Field picture" or "Particle picture"



"Field pict.": (B, Γ/2) = (21~36, 9~30)
"Particle pict.": (B, Γ/2) = (25~30, 15~32)

4. Summary and future plans

4. Summary and future plans

Λ(1405) and K^-pp are investigated with a coupled-channel Complex Scaling Method

Treatment of resonance and coupled-channel problem is important.

K^{bar}N-πY system

- Scattering problem as well as resonance one is treated in the framework of ccCSM.
  → *Construct a Chiral SU(3)-based potential (Gaussian-type)*
- *Double-pole structure of Λ(1405) is confirmed with help of complex-range Gaussian base.*

K^-pp studied with ccCSM+Feshbch method

- *Effective single-channel potential derived in ccCSM*
- *Self-consistency for kaon's complex energy*

NRv2 potential case

(B, Γ/2) = (21~36, 9~30) MeV : "Field picture"
(25~30, 15~32) MeV : "Particle pict."

Mean NN distance ~ 2.2 fm

Future plans

- Detailed analysis of the lower pole of Λ*
- Further study of K^-pp
  … Other version of K^{bar}N potential, semi-relativistic kinematics
  Detailed analysis of the structure
- Full-coupled channel calculation of K^-pp
- Construct K^{bar}N potential, based on the latest data of SIDDAHRTA