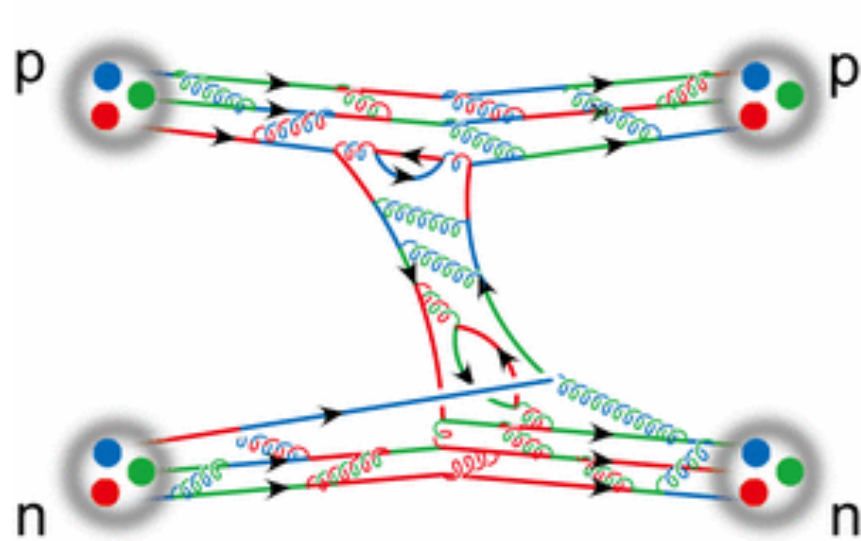


Future prospect of hadron physics from lattice QCD

Sinya Aoki
University of Tsukuba



Workshop on 'Future Prospects of Hadron Physics
at J-PARC and Large Scale Computer Physics'
Feb. 9-11, 2012@IQBRC, Tokai, Japan

SPIRE(Strategic Program for Innovative Research)

Field 5 “The origin of matter and the universe”

SPIRE: MEXT’s program to promote scientific and engineering applications of K computer.
(FY2011-15)

Field 1	Predictive life sciences, medical care and drug design	RIKEN
Field 2	New material and energy creation	Institute of Solid State Physics, Univ. of Tokyo
Field 3	Global change prediction for disaster prevention and reduction	JAMSTEC
Field 4	Industrial Innovation	Institute of Industrial Science, Univ. of Tokyo
Field 5	Origin of matter and the universe	RCCS, Univ. of Tsukuba

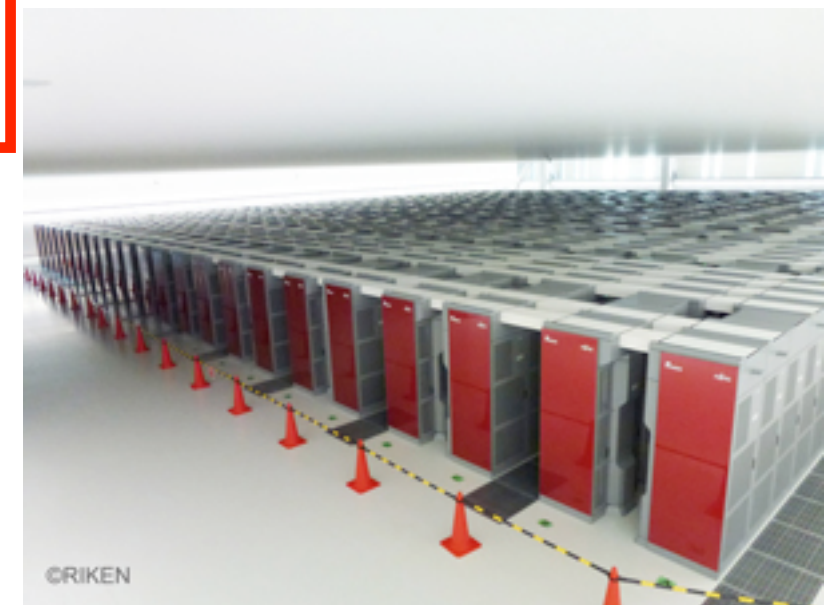
1. Lattice QCD
2. Nucleus
3. Supernova Explosion
4. Early Star Formation





O(10)Peta Flops supercomputer
starts to operate fall in 2012

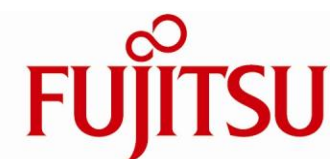
$10^{16} = 10$ Peta



Supercomputer “K computer” Takes Consecutive No. 1 in World Ranking

Achieving world's fastest processing speed of 10.51 petaflops and 93.2% operating efficiency

Rank	Computer	Site	Vendor	Country	Maximal LINPACK performance achieved
1	K computer	RIKEN Advanced Institute for Computational Science (AICS)	Fujitsu	Japan	10,510
2	Tianhe-1A	National Supercomputing Center in Tianjin	NUDT	China	2,566
3	Jaguar	DOE/SC/Oak Ridge National Laboratory	Cray Inc.	US	1,759
4	Nebulae	National Supercomputing Centre in Shenzhen (NSCS)	Dawning	China	1,271
5	TSUBAME 2.0	GSIC Center, Tokyo Institute of Technology	NEC/HP	Japan	1,192
6	Cielo	DOE/NNSA/LANL/SNL	Cray Inc.	US	1,110
7	Pleiades	NASA/Ames Research Center/NAS	SGI	US	1,088
8	Hopper	DOE/SC/LBNL/NERSC	Cray Inc.	US	1,054
9	Tera-100	Commissariat a l'Energie Atomique (CEA)	Bull SA	France	1,050
10	Roadrunner	DOE/NNSA/LANL	IBM	US	1,042

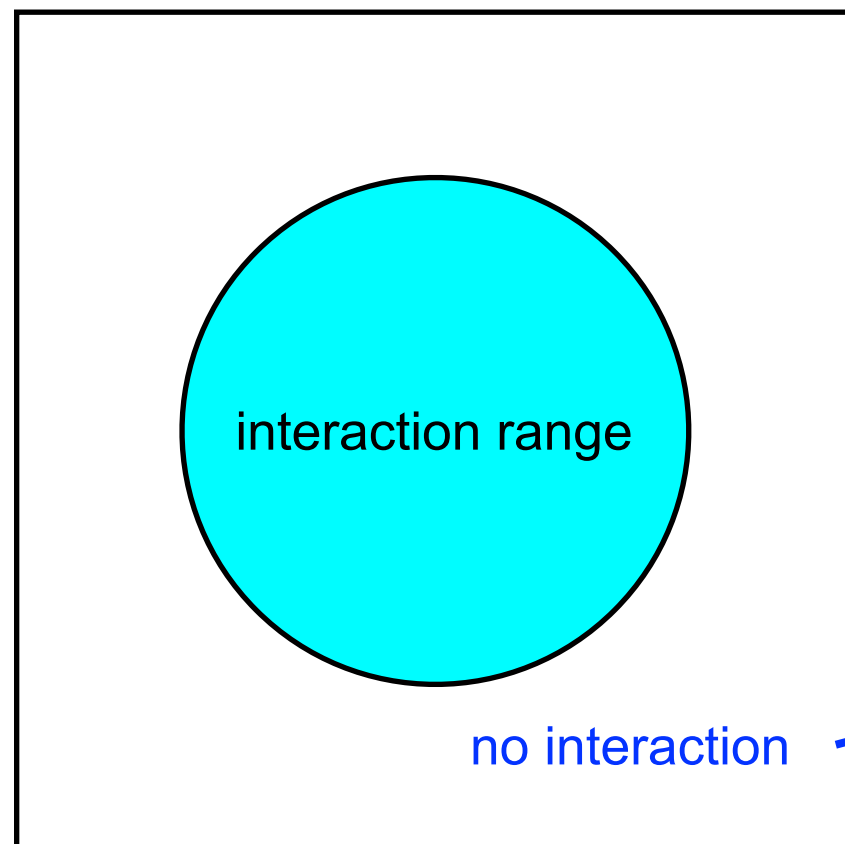


RIKEN
Fujitsu Limited
November 14, 2011

1. Introduction

Extraction of hadronic interaction in lattice QCD

two particles in the finite box



energy $E = \frac{k_n^2}{m_N}$ for free theory $\mathbf{k}_n = \frac{2\pi}{L}\mathbf{n}$

Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x} + \mathbf{r}, 0) N(\mathbf{x}, 0) | 6q, E \rangle$$

QCD eigenstate with energy E

L $N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

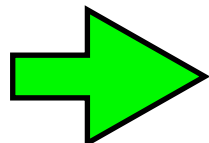
Asymptotic behavior

$$r = |\mathbf{r}| \rightarrow \infty$$

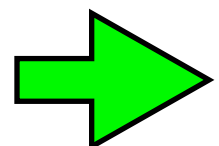
partial wave

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

Finite volume

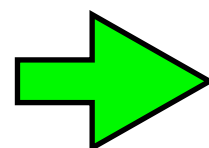


allowed value: k_n^2 different from free theory



two particle energy

$$E = \frac{k_n^2}{m_N}$$



Lueshcer's formula for phase shift

$$\delta_l(k_n)$$

Extract information inside the interaction range as

$$[\epsilon_k - H_0]\varphi_E(\mathbf{x}) = \int d^3y U(\mathbf{x}, \mathbf{y})\varphi_E(\mathbf{y}) \quad \epsilon_k = \frac{\mathbf{k}^2}{2\mu} \quad H_0 = \frac{-\nabla^2}{2\mu}$$

Calculate observables such as phase shift and binding energy, solving the Schroedinger Eq. in the **infinite volume** with this “potential”.

Properties

- Potential gives correct phase shifts below inelastic threshold **by construction**.
- **(HAL) Scheme**: Potential depends on the choice of $N(\mathbf{x})$. (cf. running coupling)
- Potential is non-local but can be **energy independent**.
- Velocity expansion: $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta^3(\mathbf{x} - \mathbf{y})$ Okubo-Marshak (1958)

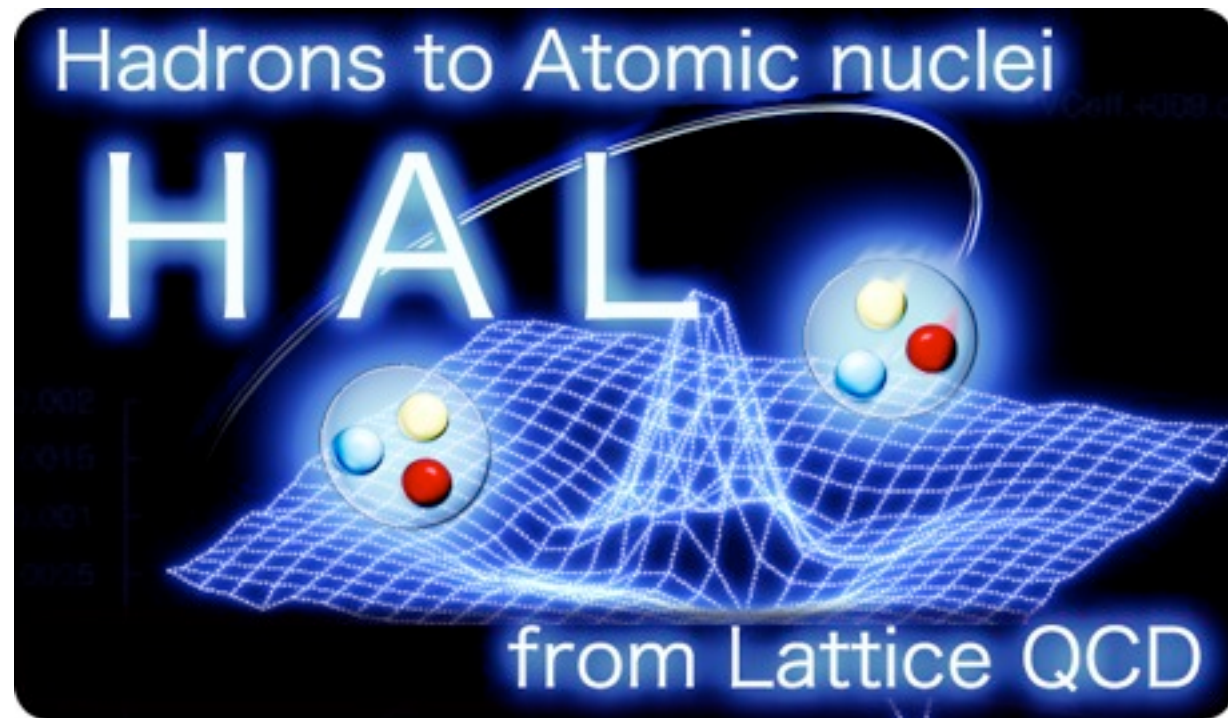
$$V(\mathbf{x}, \nabla) = \underbrace{V_0(r)}_{\text{LO}} + \underbrace{V_\sigma(r)(\sigma_1 \cdot \sigma_2)}_{\text{LO}} + \underbrace{V_T(r)S_{12}}_{\text{LO}} + \underbrace{V_{\text{LS}}(r)\mathbf{L} \cdot \mathbf{S}}_{\text{NLO}} + \underbrace{O(\nabla^2)}_{\text{NNLO}}$$

tensor operator
 $S_{12} = \frac{3}{r^2}(\sigma_1 \cdot \mathbf{x})(\sigma_2 \cdot \mathbf{x}) - (\sigma_1 \cdot \sigma_2)$
spins

$V_A(\mathbf{x})$ **local and energy independent** coefficient function
(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

2. Activities using the 2nd method

HAL QCD Collaboration

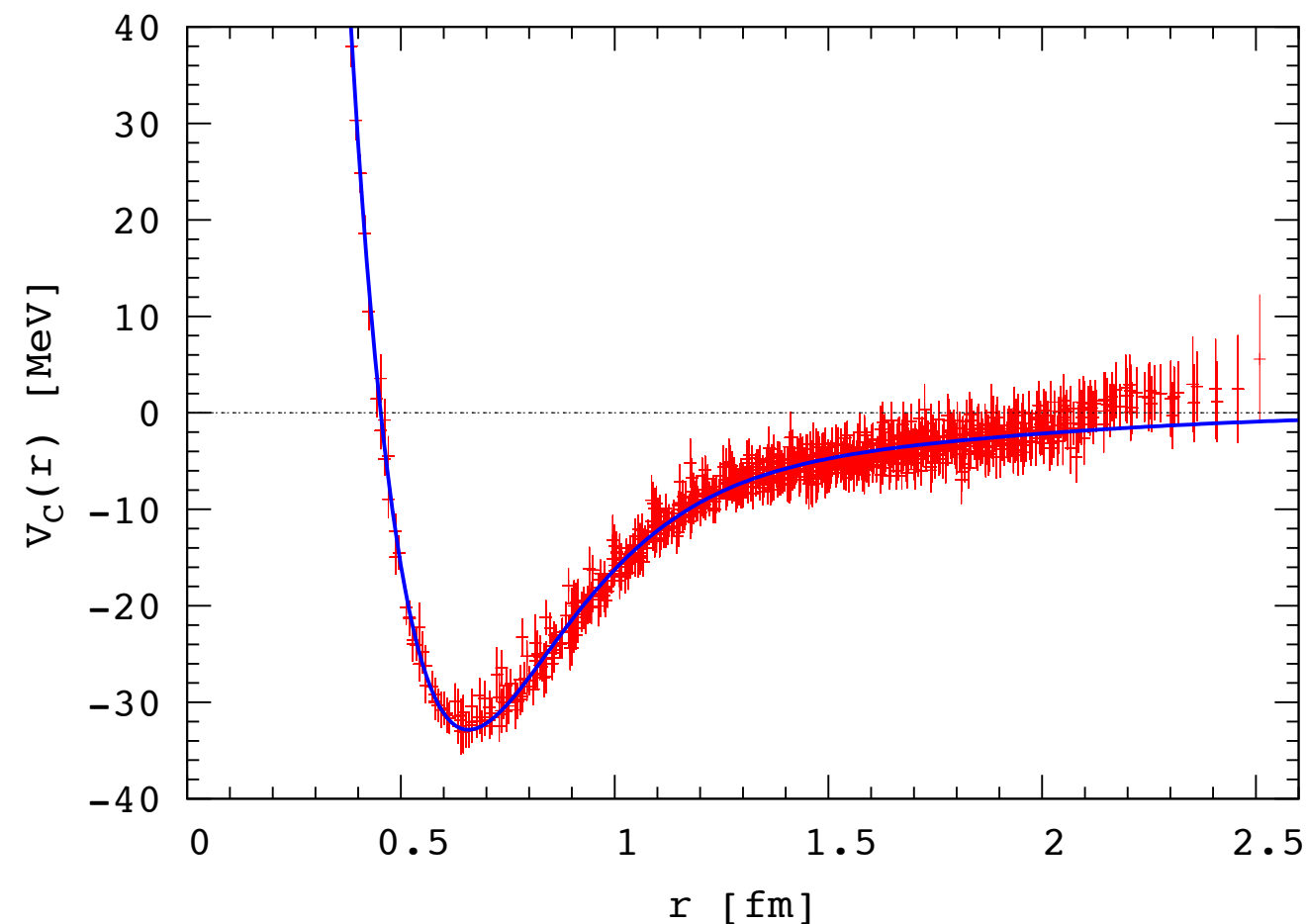


Sinya Aoki (U. Tsukuba)
Bruno Charron (U. Tokyo)
Takumi Doi (Riken)
Tetsuo Hatsuda (Riken/U. Tokyo)
Yoichi Ikeda (TIT)
Takashi Inoue (Nihon U.)
Noriyoshi Ishii (U. Tsukuba)
Keiko Murano (Riken)
Hidekatsu Nemura (U. Tsukuba)
Kenji Sasaki (U. Tsukuba)
Masanori Yamada (U. Tsukuba)

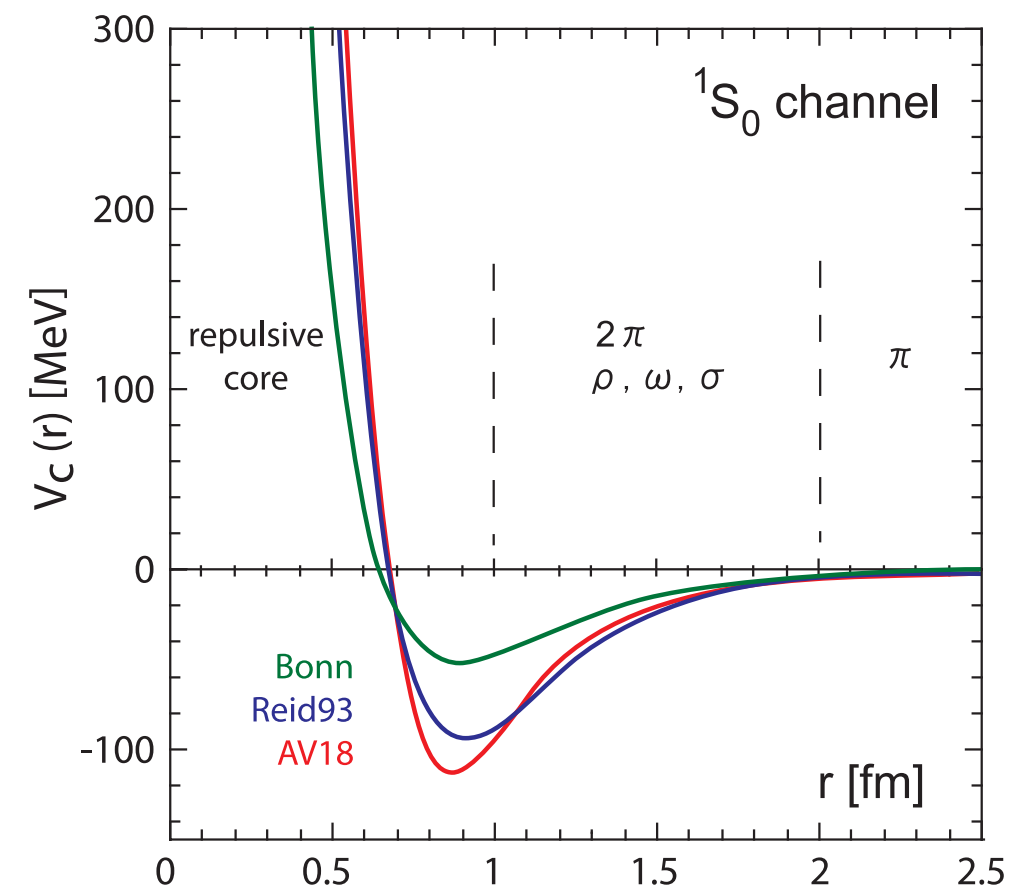
Example: NN potential

2+1 flavor QCD, $I=1$ potential (in preparation)

$a=0.09\text{fm}$, $L=2.9\text{fm}$ $m_\pi \simeq 700\text{ MeV}$



phenomenological potential



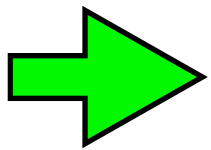
Qualitative features of NN potential are reproduced.

- (1) attractions at medium and long distances
- (2) repulsion at short distance(repulsive core)

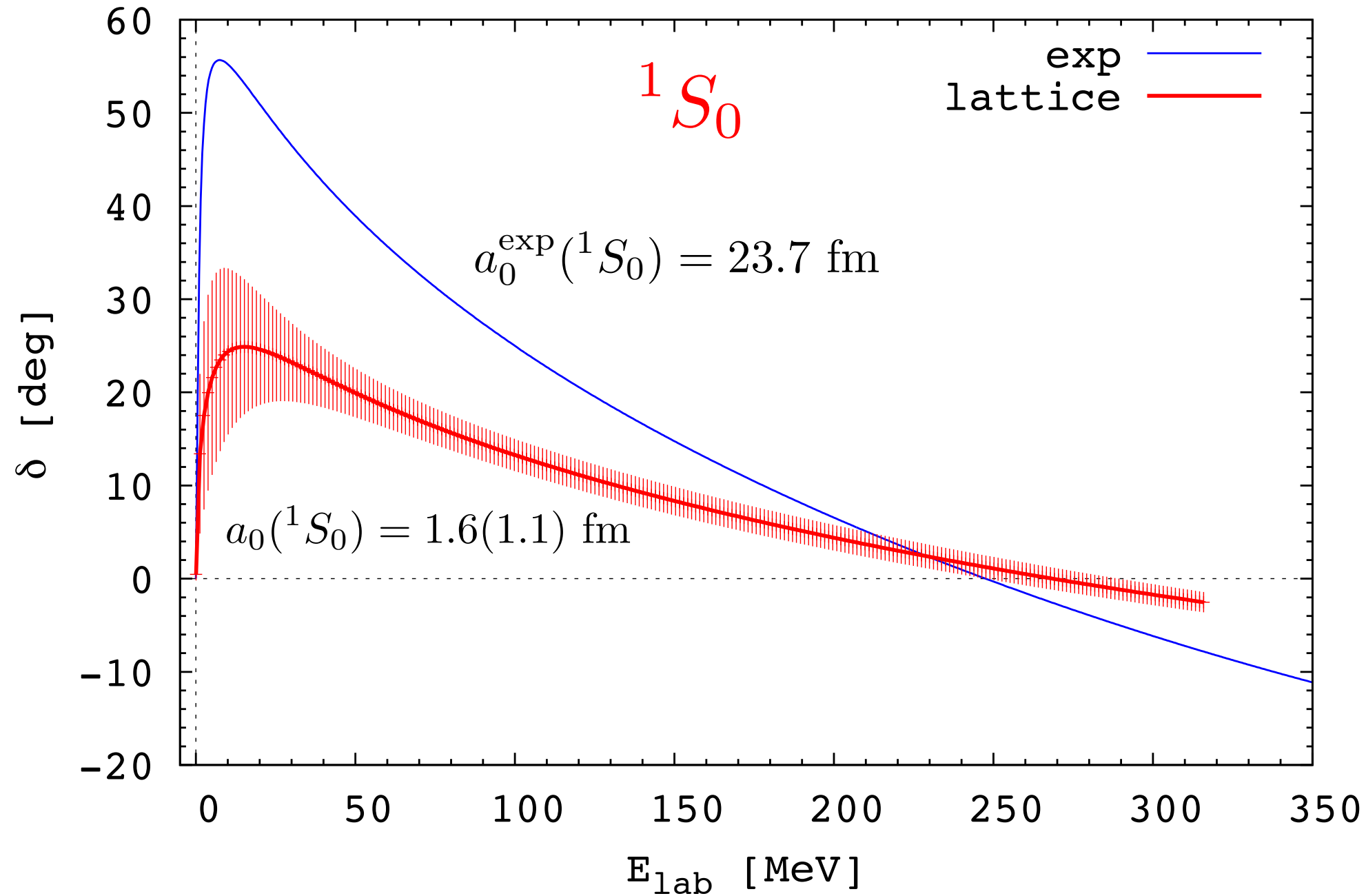
1st paper(quenched QCD): Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in
Nature Research Highlights 2007

NN potential



phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass on “K” computer.

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at **different energy** become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

$$m_\pi \simeq 0.53 \text{ GeV}$$

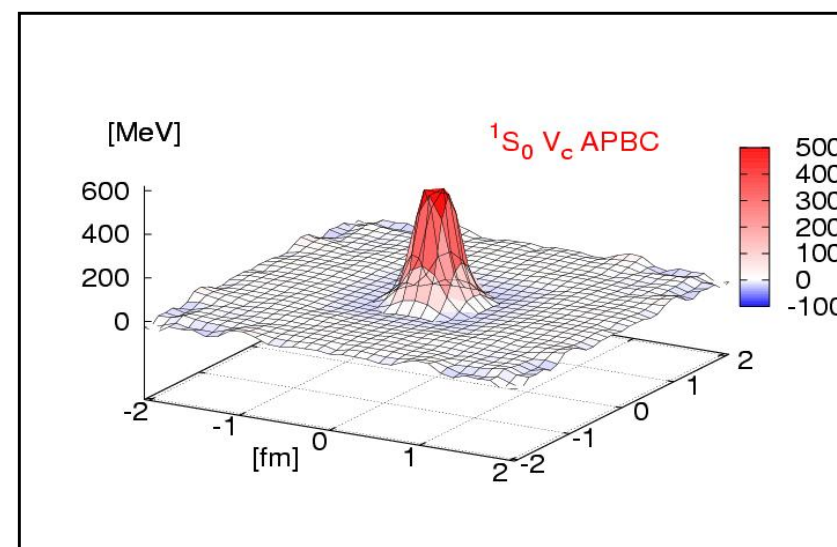
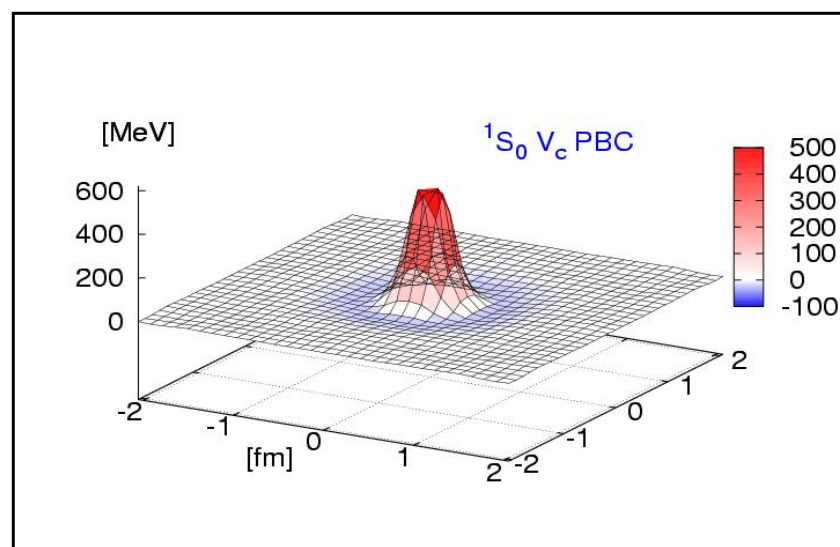
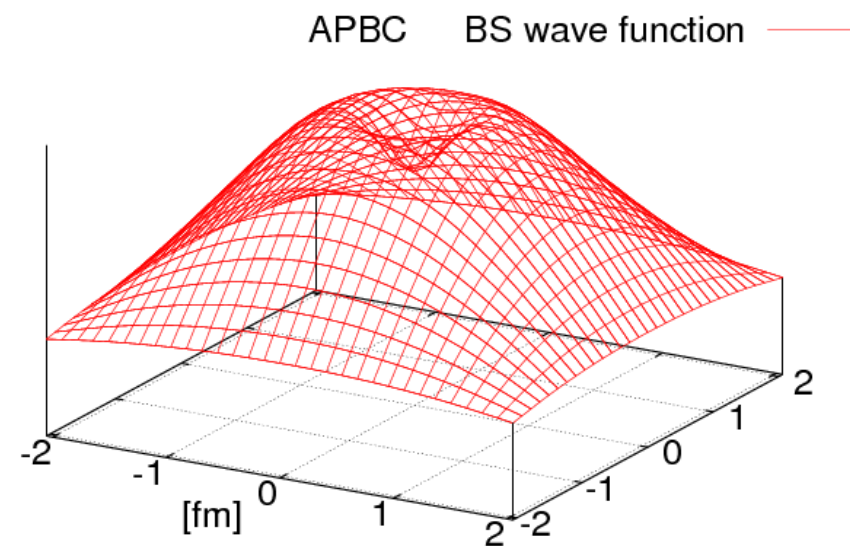
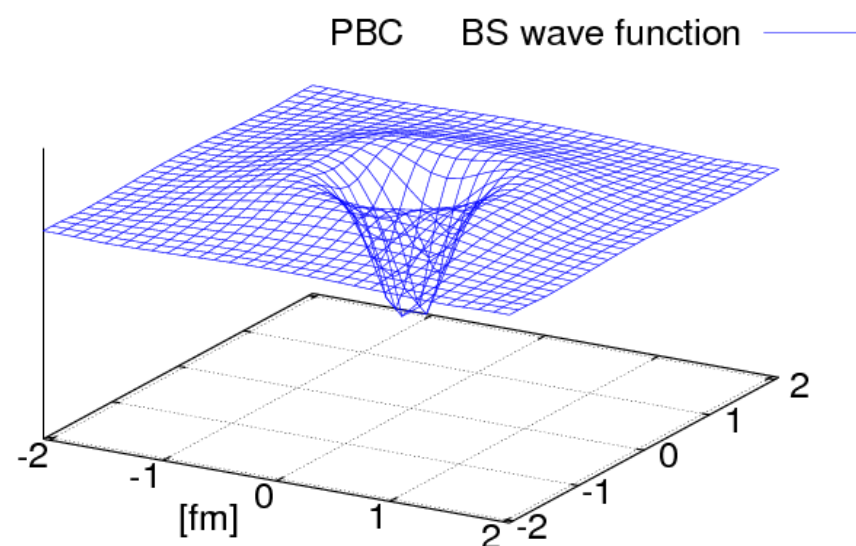
$$a=0.137\text{fm}$$

K. Murano, N. Ishii, S. Aoki, T. Hatsuda

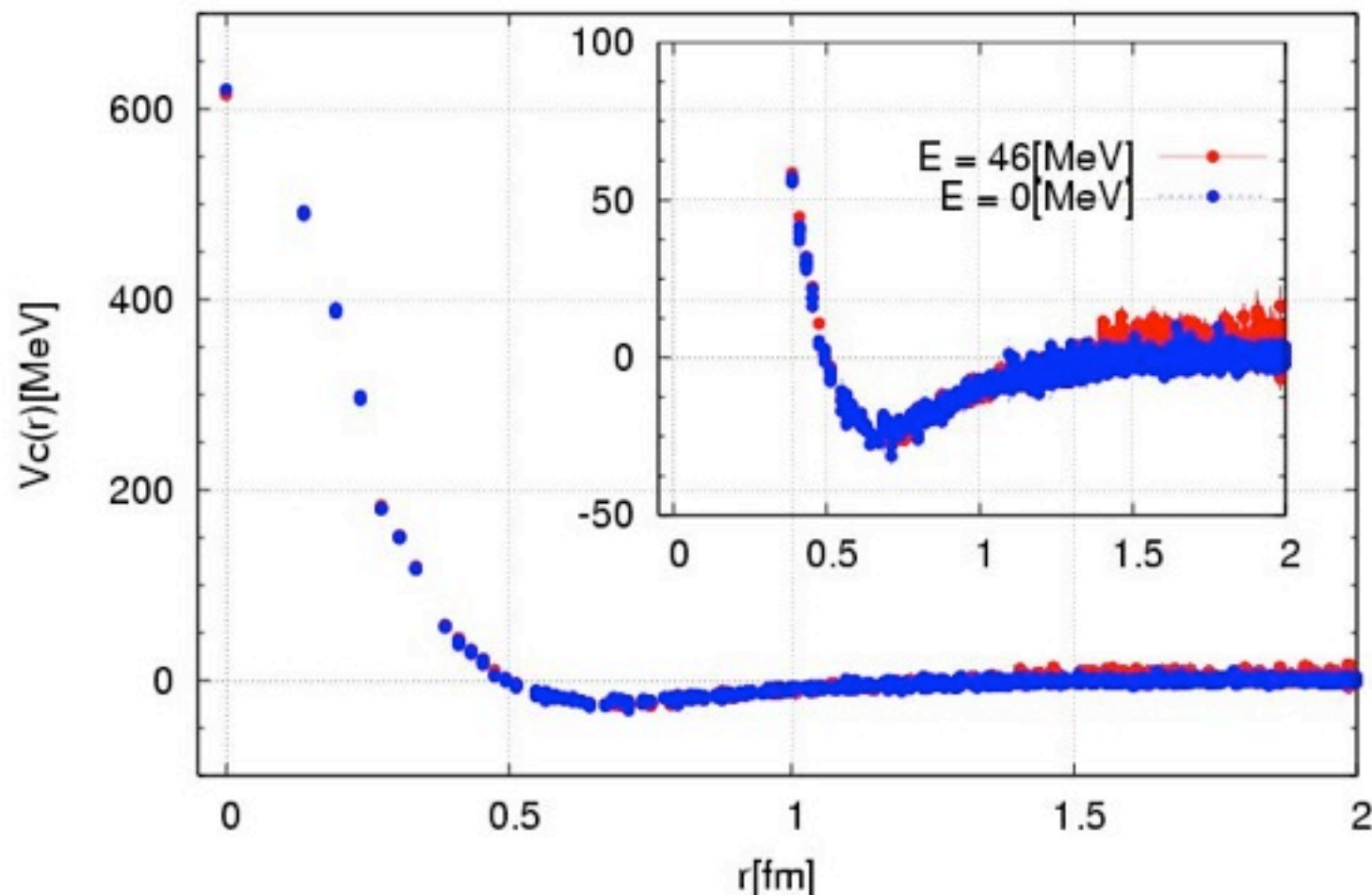
PTP 125 (2011)1225.

● PBC ($E \sim 0$ MeV)

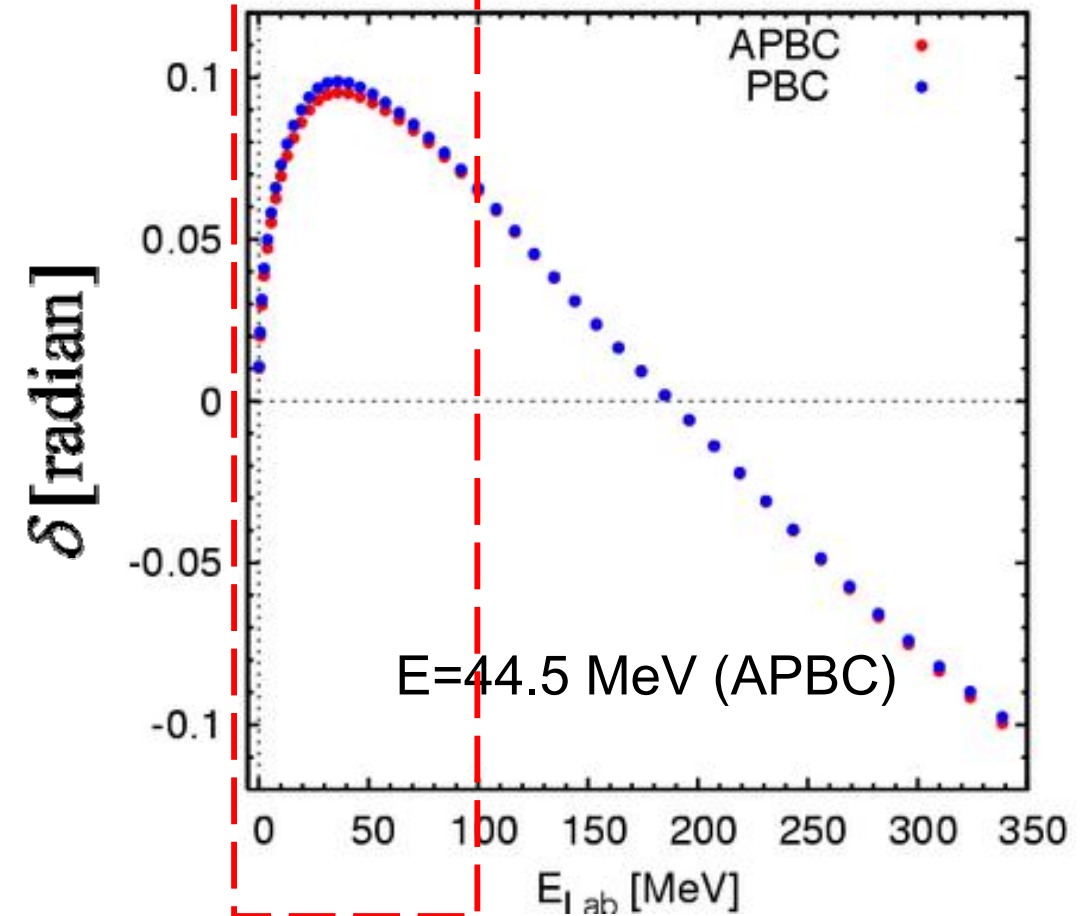
● APBC ($E \sim 46$ MeV)



$V_c(r; {}^1S_0)$: PBC v.s. APBC $t=9$ ($x=\pm 5$ or $y=\pm 5$ or $z=\pm 5$)



phase shifts from potentials



Higher order terms turn out to be very small at low energy in HAL scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(cf. convergence of ChPT, convergence of perturbative QCD)

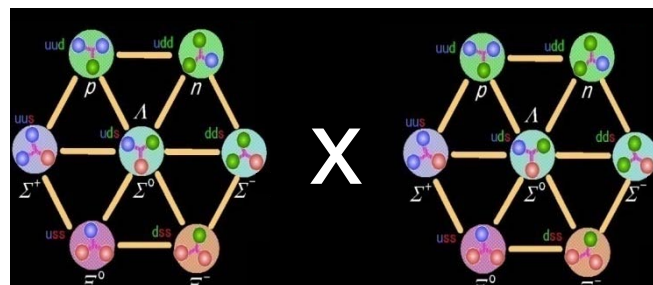
3. Future prospect

- predictions
 - hyperon potential
 - H-dibaryon and other exotics
- Nuclei structure
 - potentials from LQCD -> many body calculations for nucleons
 - hyperon potential(LQCD) -> a few body calculations <-> experiments
 - many body forces from lattice QCD <-> ChPT
 - light nuclei from LQCD <-> a few body calculations
- Nuclear matter at finite density
 - still no reliable method in LQCD
 - potentials from LQCD -> many body calculation at finite density
 - LQCD -> EFT, models -> nuclear matter at finite density

Baryon Potentials in the flavor SU(3) symmetric limit

$$m_u = m_d = m_s$$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

6 independent potentials in flavor-basis

$$\begin{array}{ll} V^{(27)}(r), & V^{(8s)}(r), & V^{(1)}(r) & \longleftarrow & {}^1S_0 \\ V^{(10^*)}(r), & V^{(10)}(r), & V^{(8a)}(r) & \longleftarrow & {}^3S_1 \end{array}$$

3-flavor QCD $a=0.12$ fm

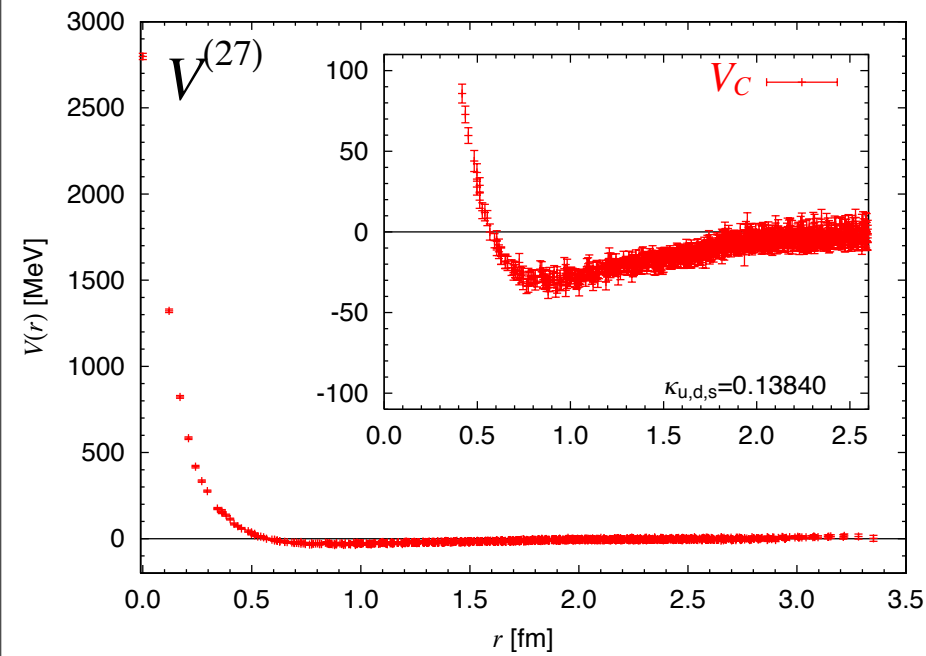
Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591

$L=2$ fm

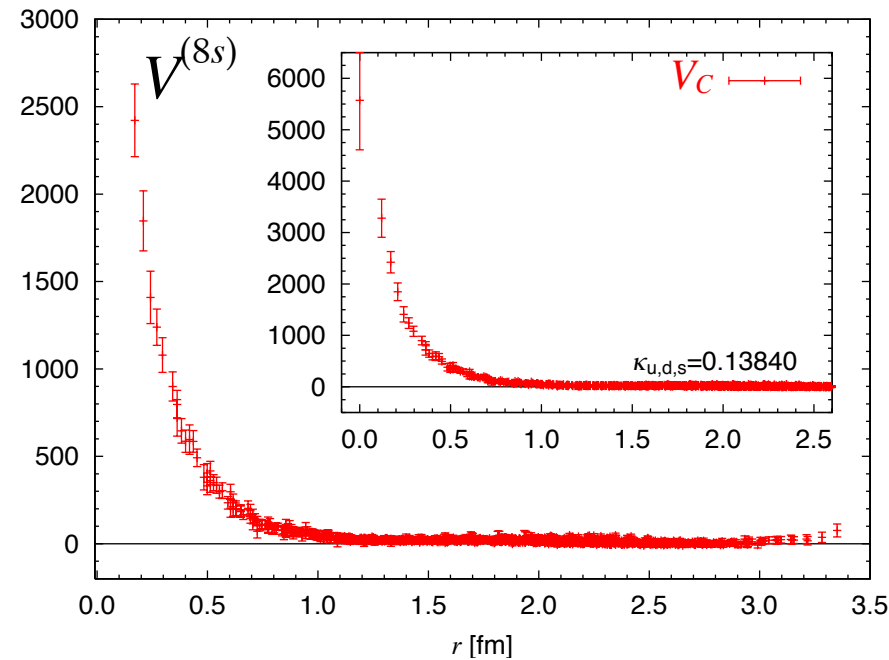
Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

$L=2-4$ fm

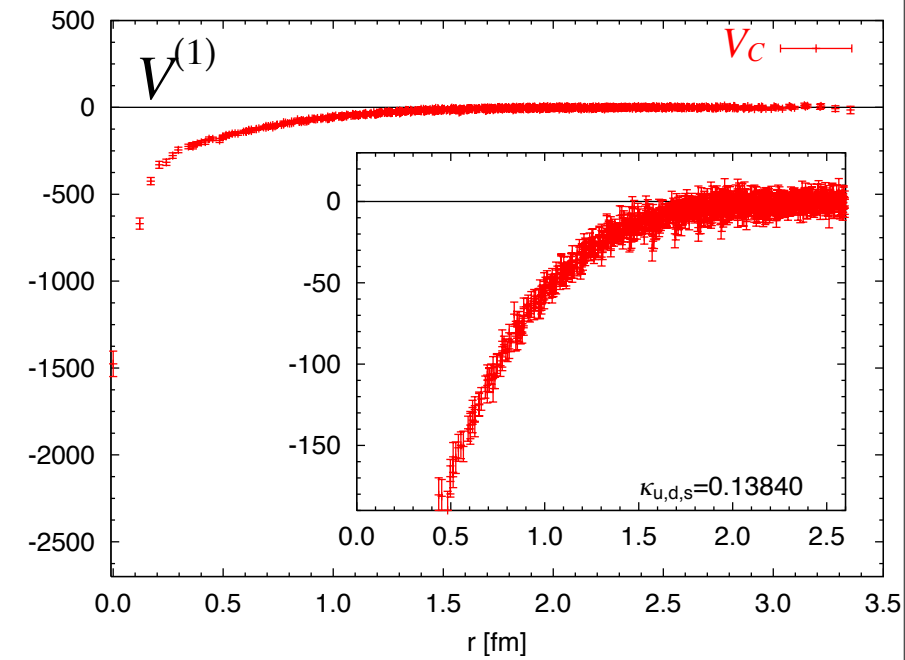
$L \simeq 4$ fm, $m_\pi \simeq 470$ MeV



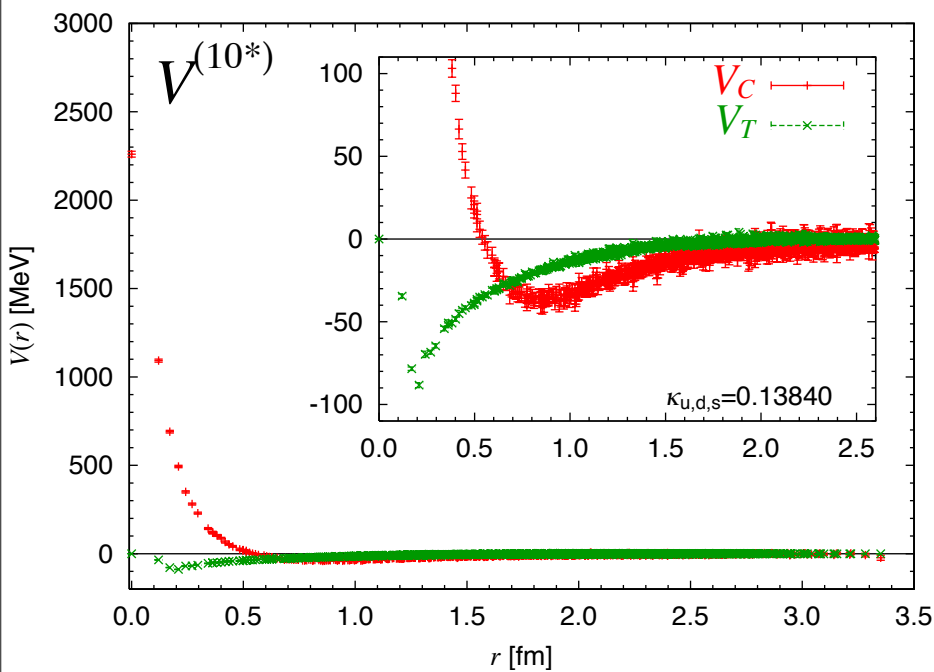
same as NN



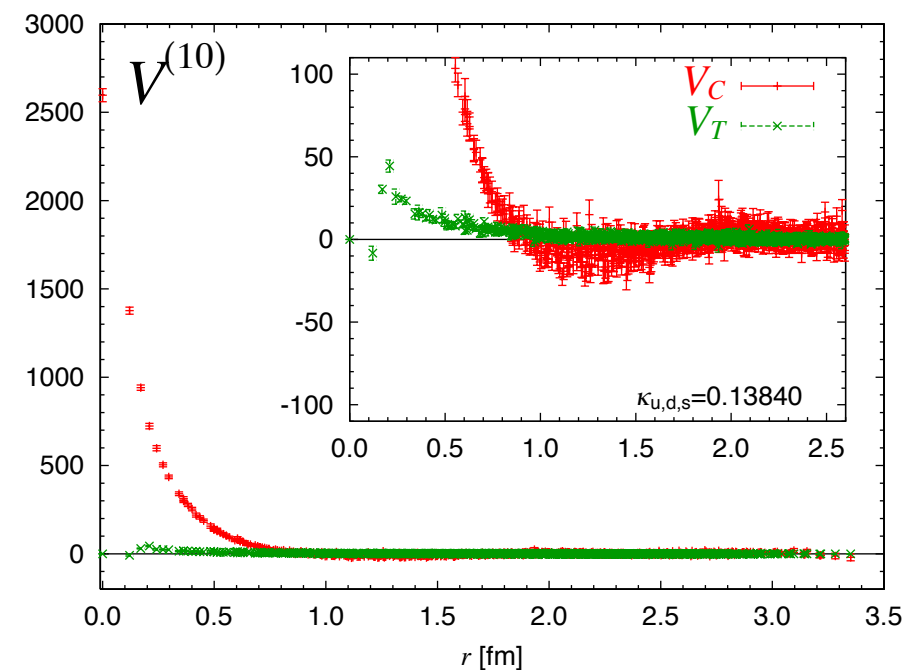
8s: strong repulsive core. repulsion only.



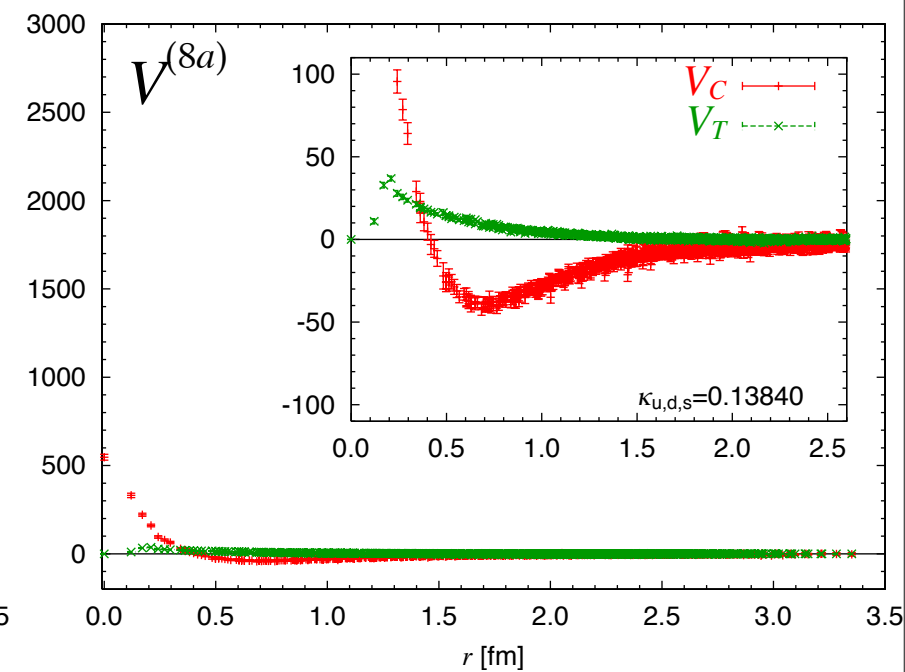
1: attractive instead of repulsive core ! attraction only .



same as NN



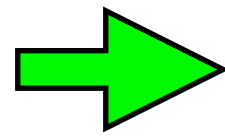
10: strong repulsive core. weak attraction.



8a: weak repulsive core. strong attraction.

Flavor dependences of BB interactions become manifest in SU(3) limit !

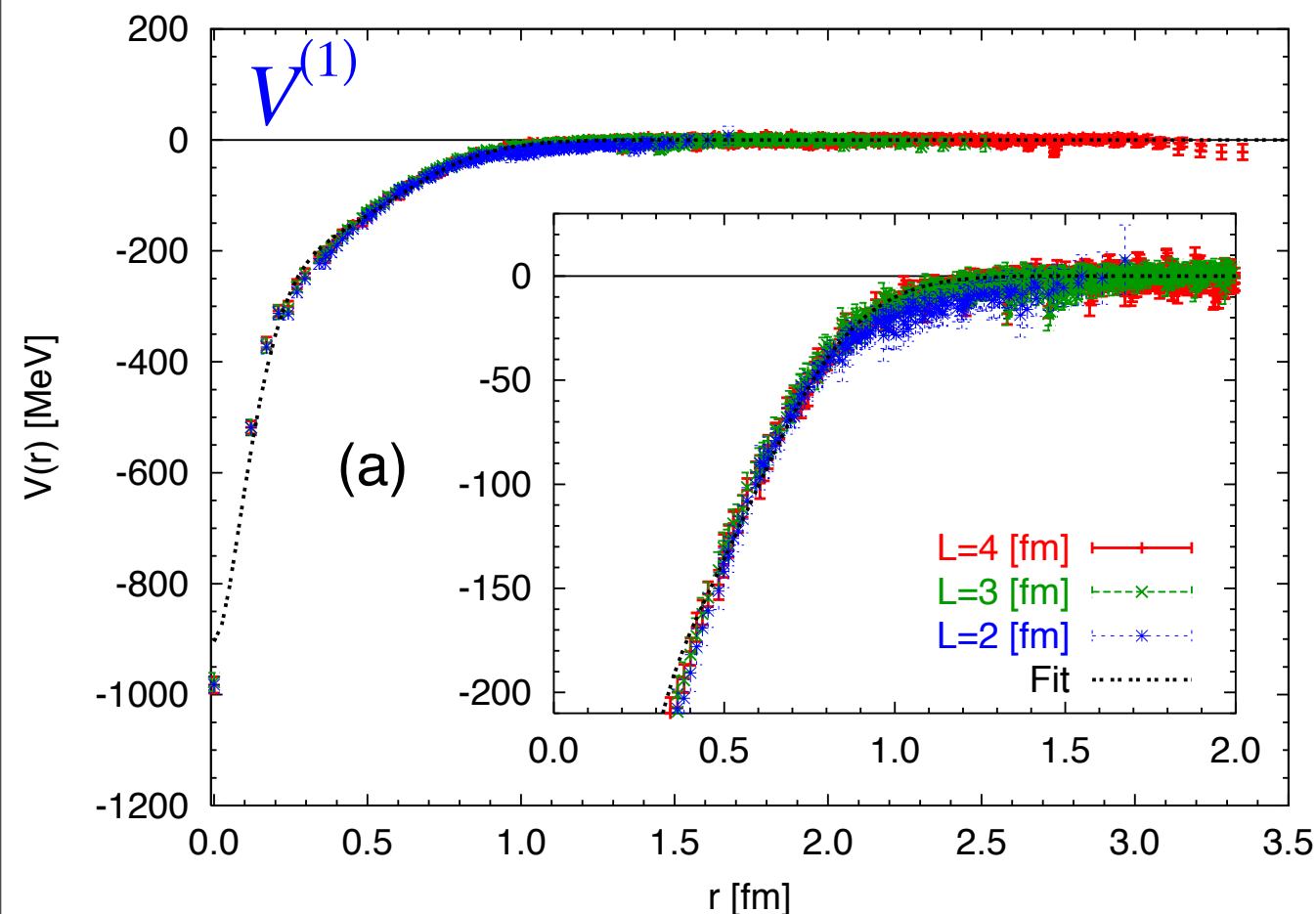
Attractive potential
in the flavor singlet channel



possibility of a bound state (H-dibaryon)

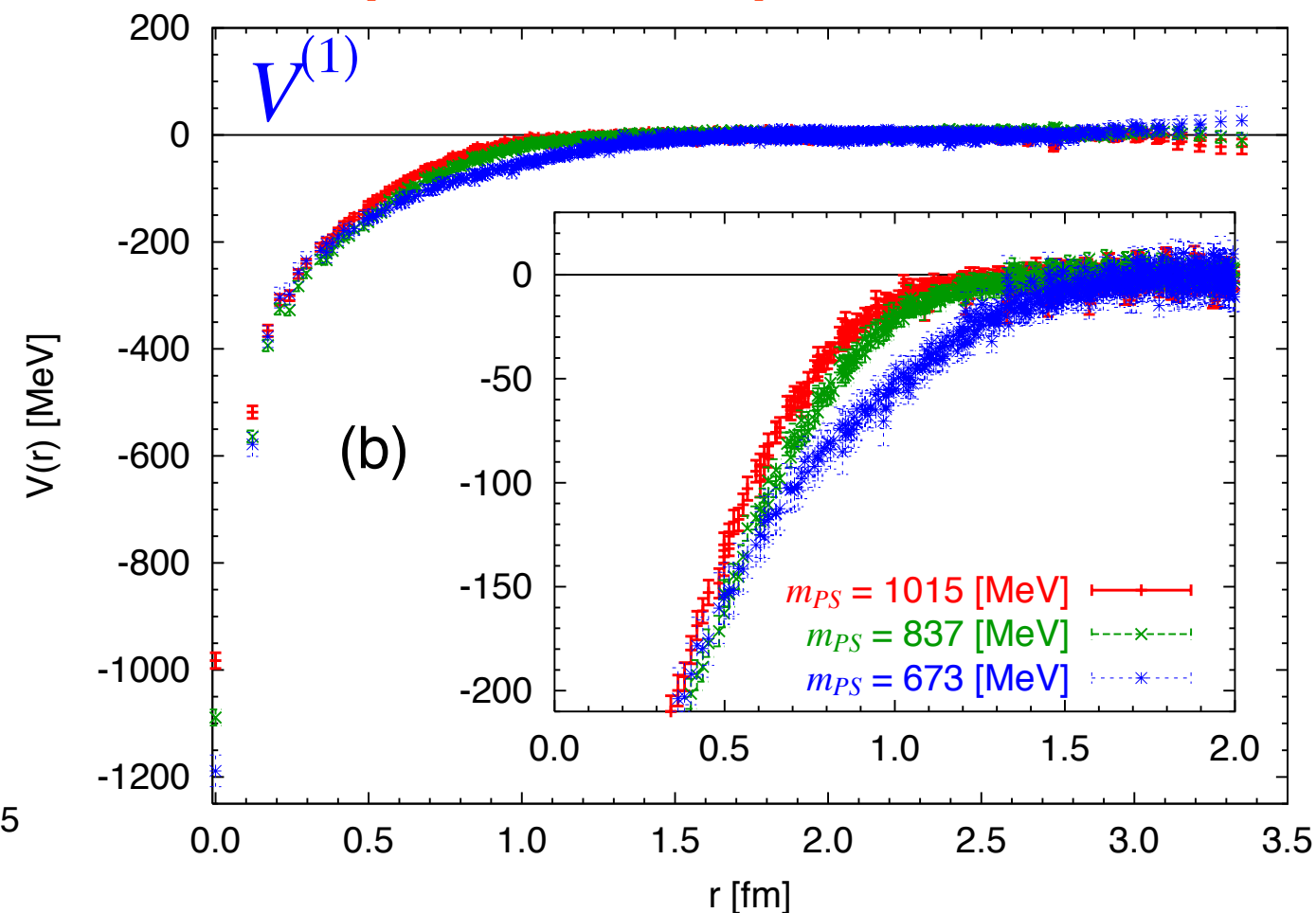
$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

volume dependence



L=3 fm is enough for the potential.

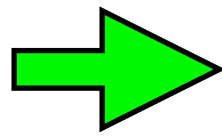
pion mass dependence



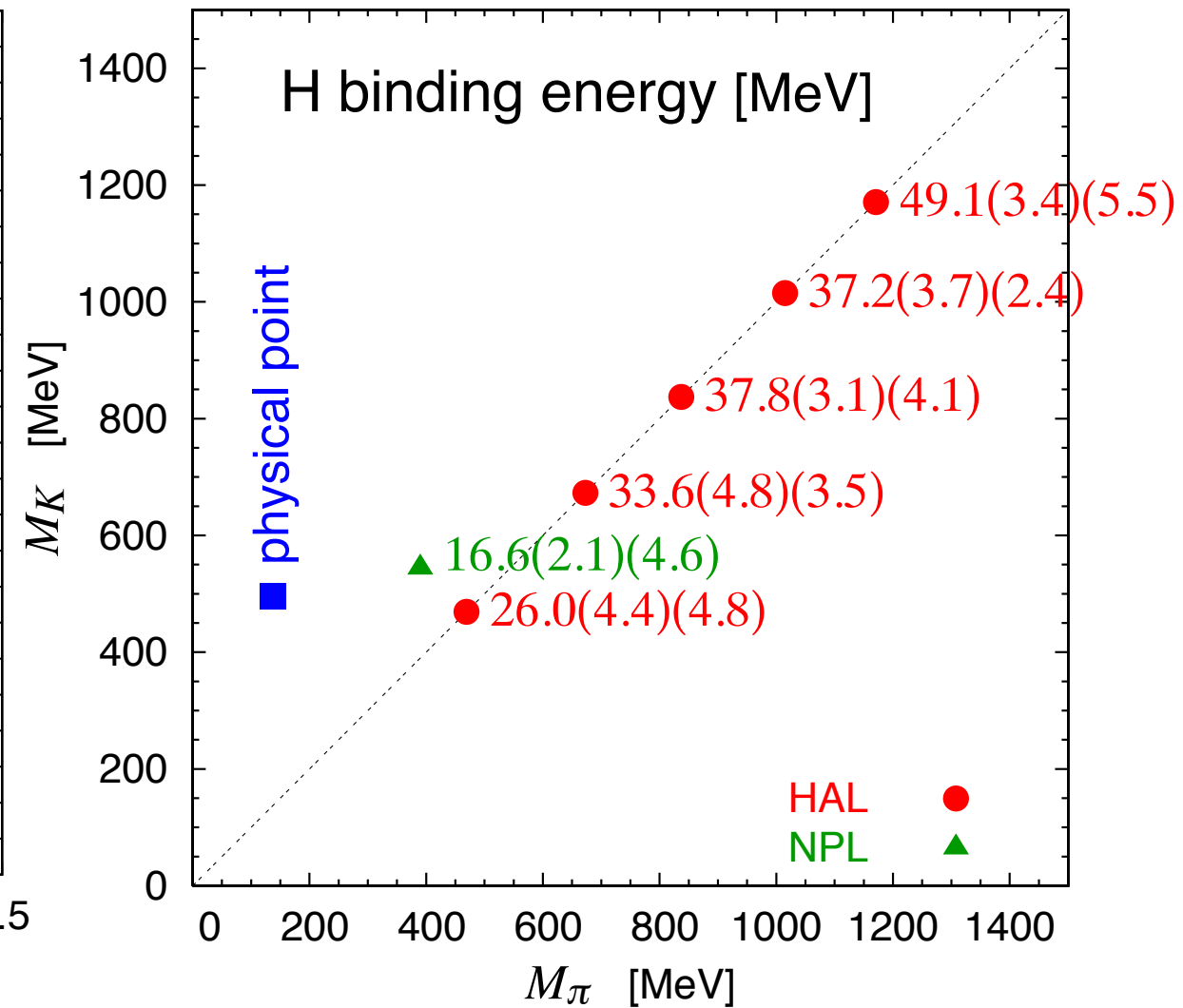
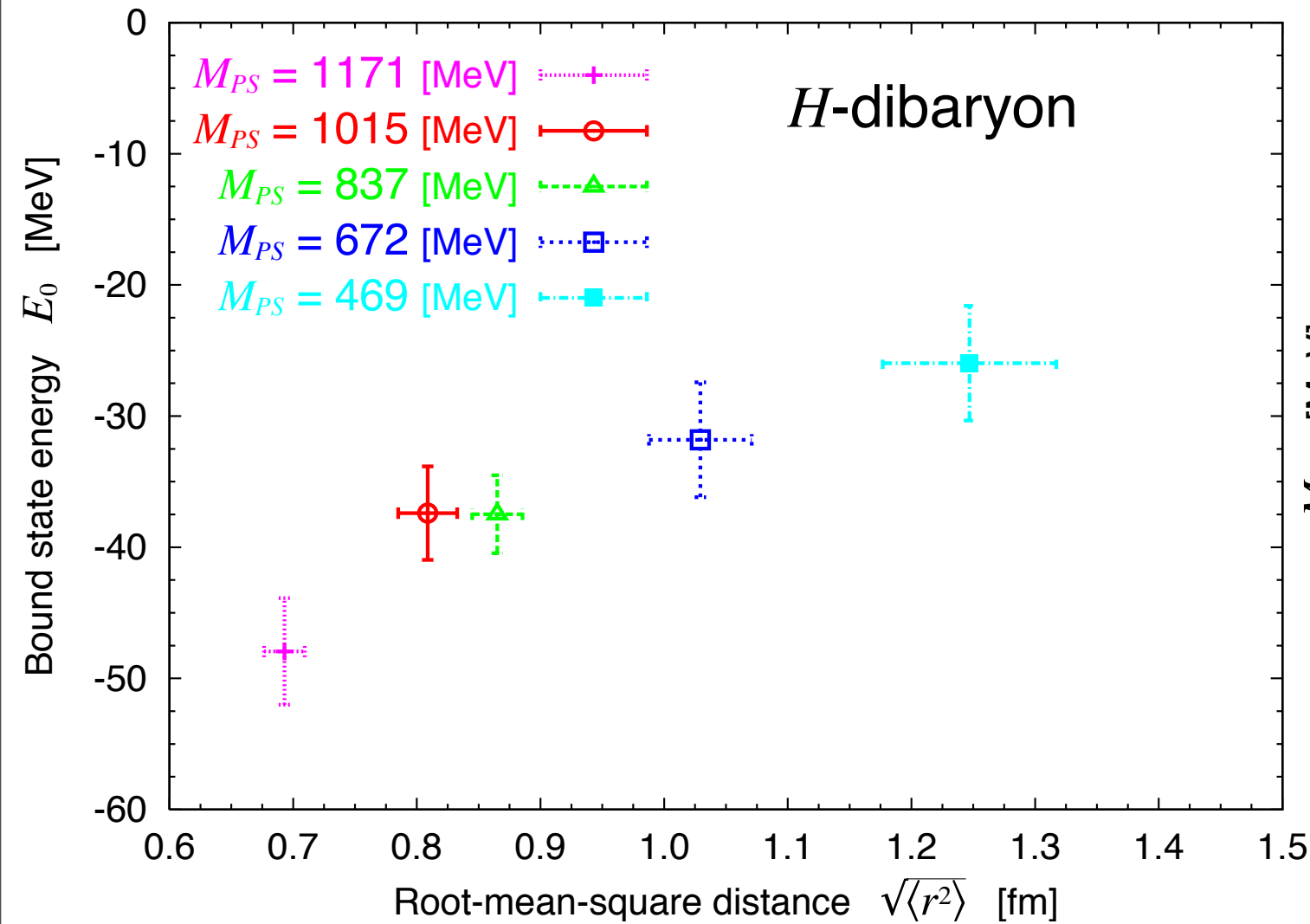
lighter the pion mass, stronger the attraction

fit potentials at L=4 fm by
$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

Solve Schroedinger equation
in **the infinite volume**



One bound state (H-dibaryon) exists.

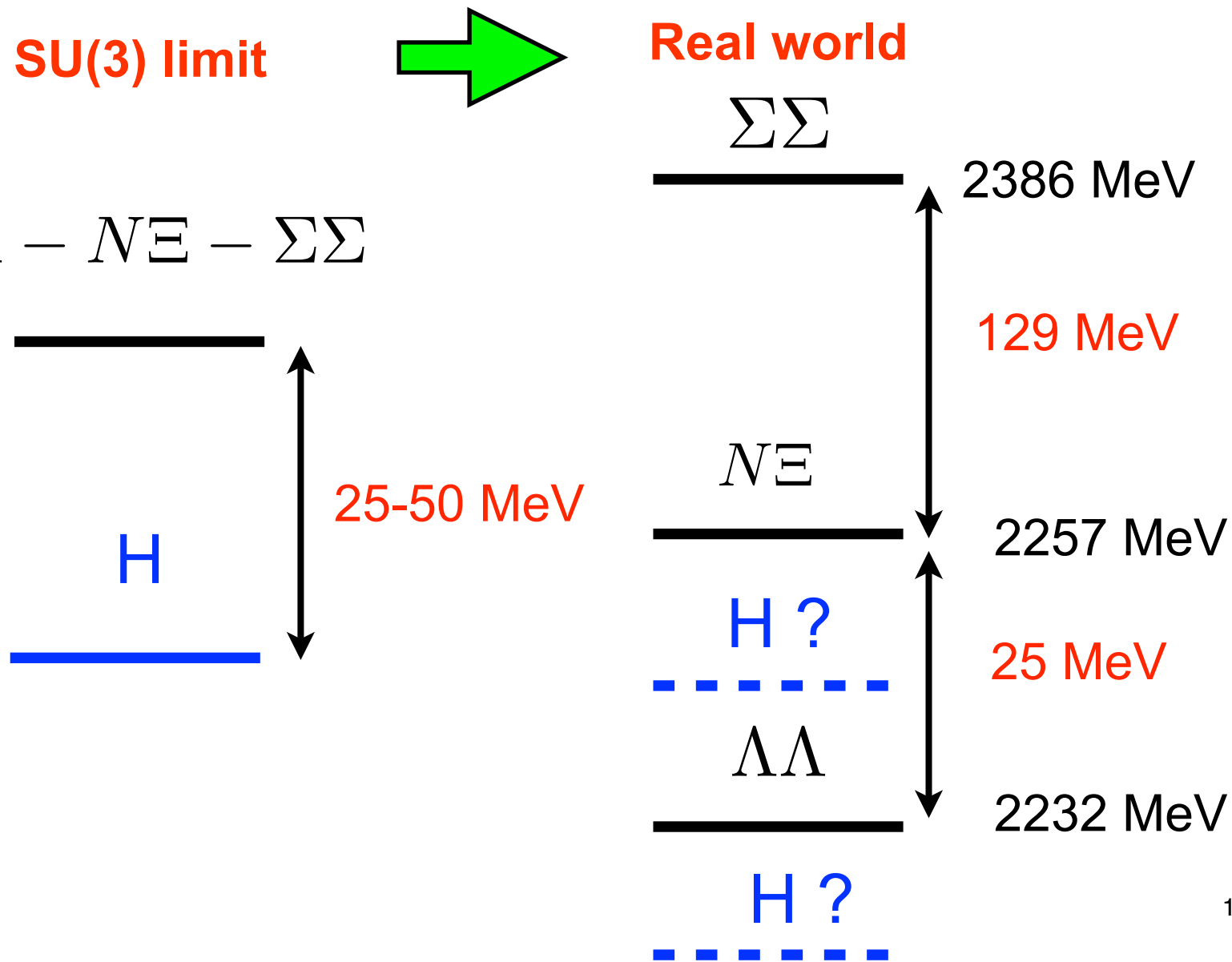


An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

Real world ?

H-dibaryon with the flavor SU(3) breaking

$$m_u = m_d \neq m_s$$



$$m_\pi \simeq 470 \text{ MeV}$$

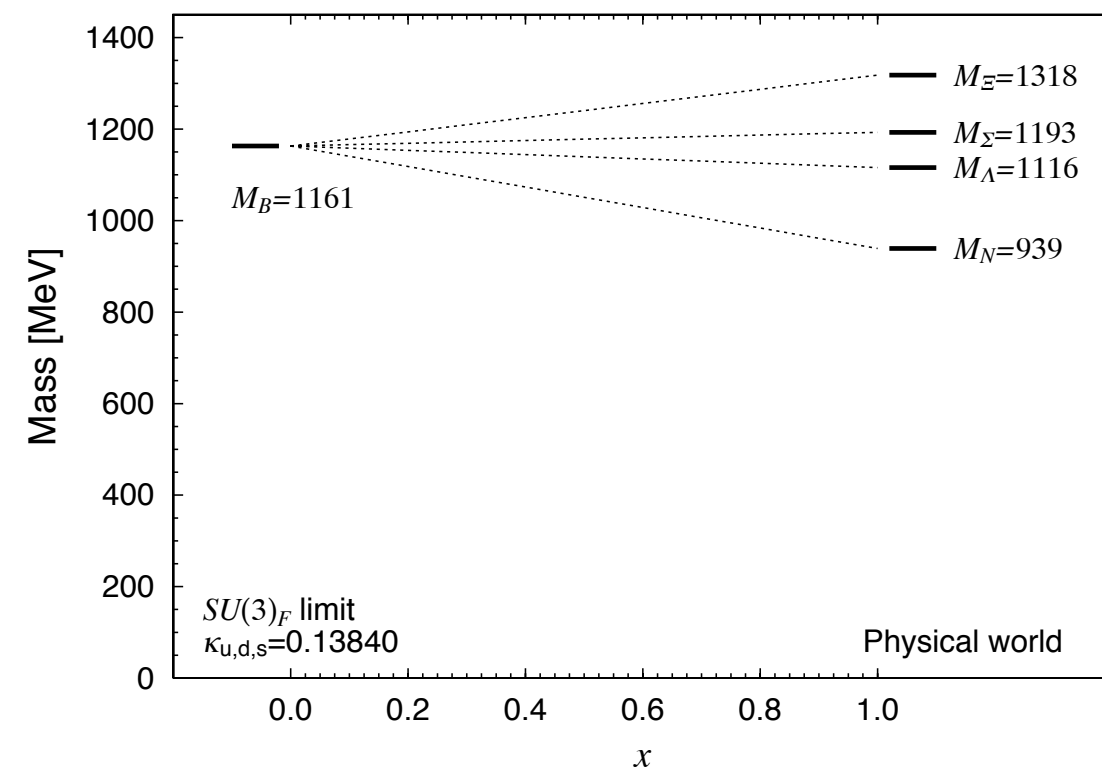
$$m_\pi \simeq 135 \text{ MeV}$$

Our approximation for SU(3) breaking

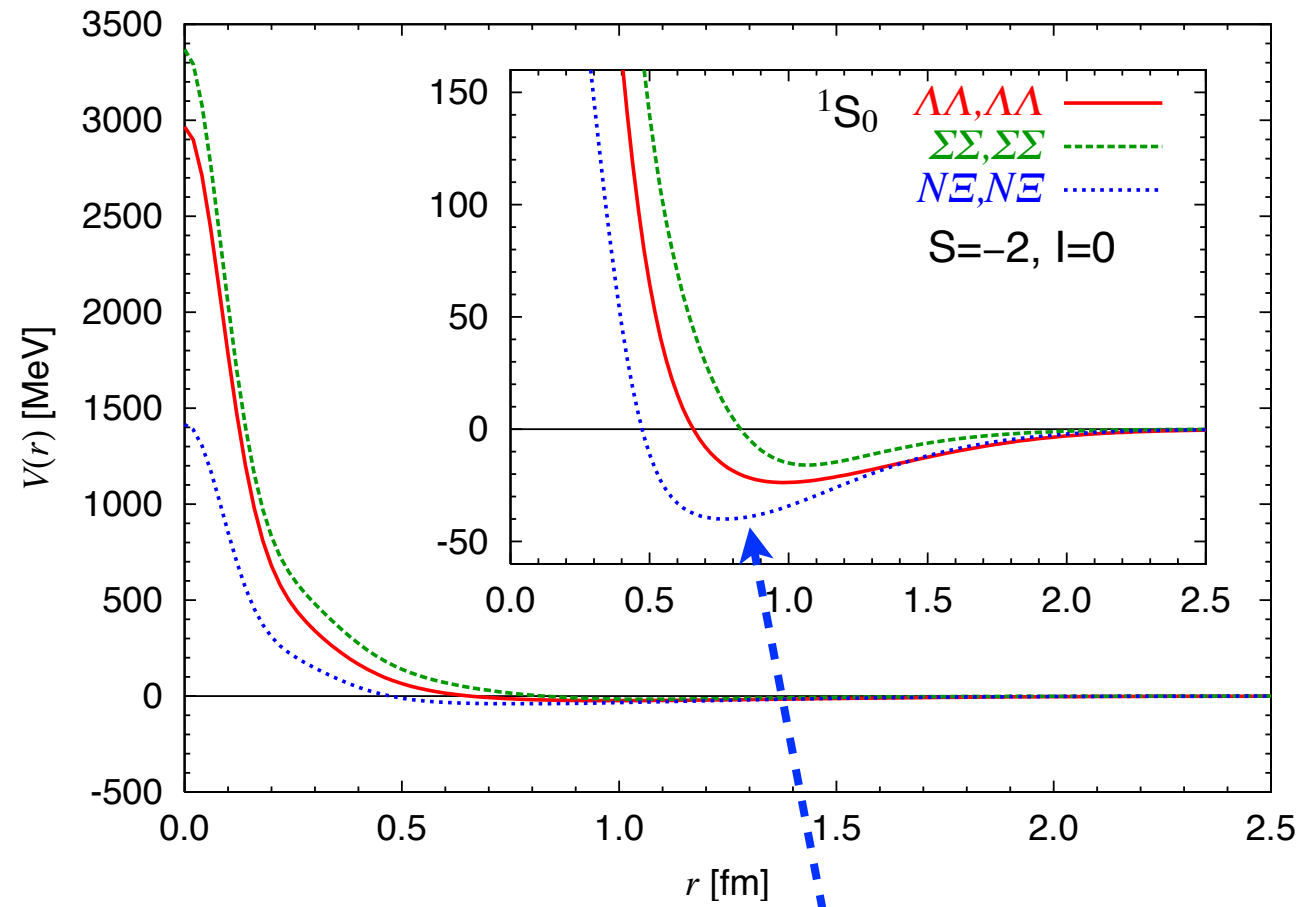
1. Linear interpolation of octet baryon masses

$$M_Y(x) = (1 - x)M_Y^{\text{SU}(3)} + xM_Y^{\text{Phys}}$$

2. Potentials in particle basis in SU(3) limit



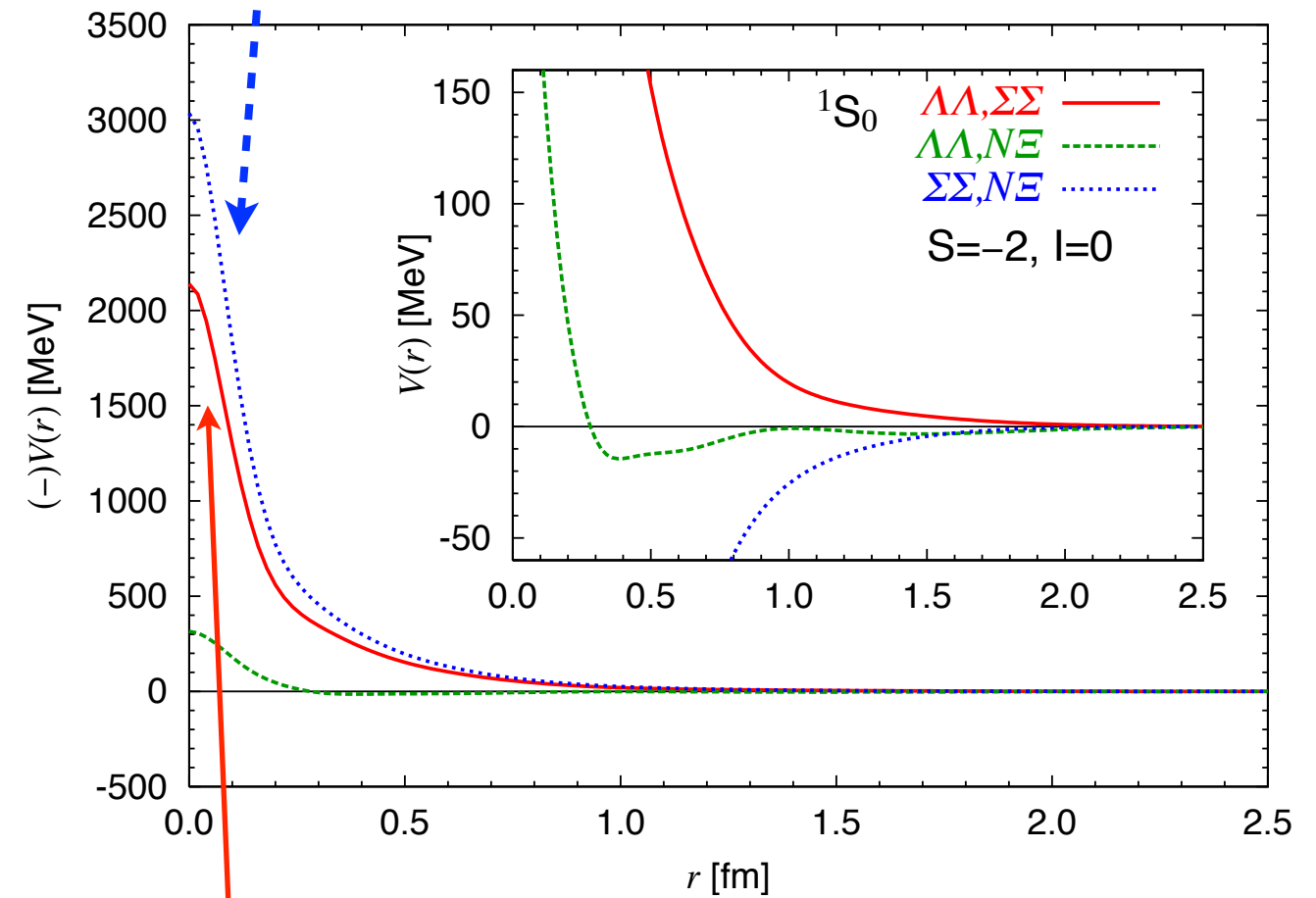
diagonal potential



most attractive

sizable

transition potential

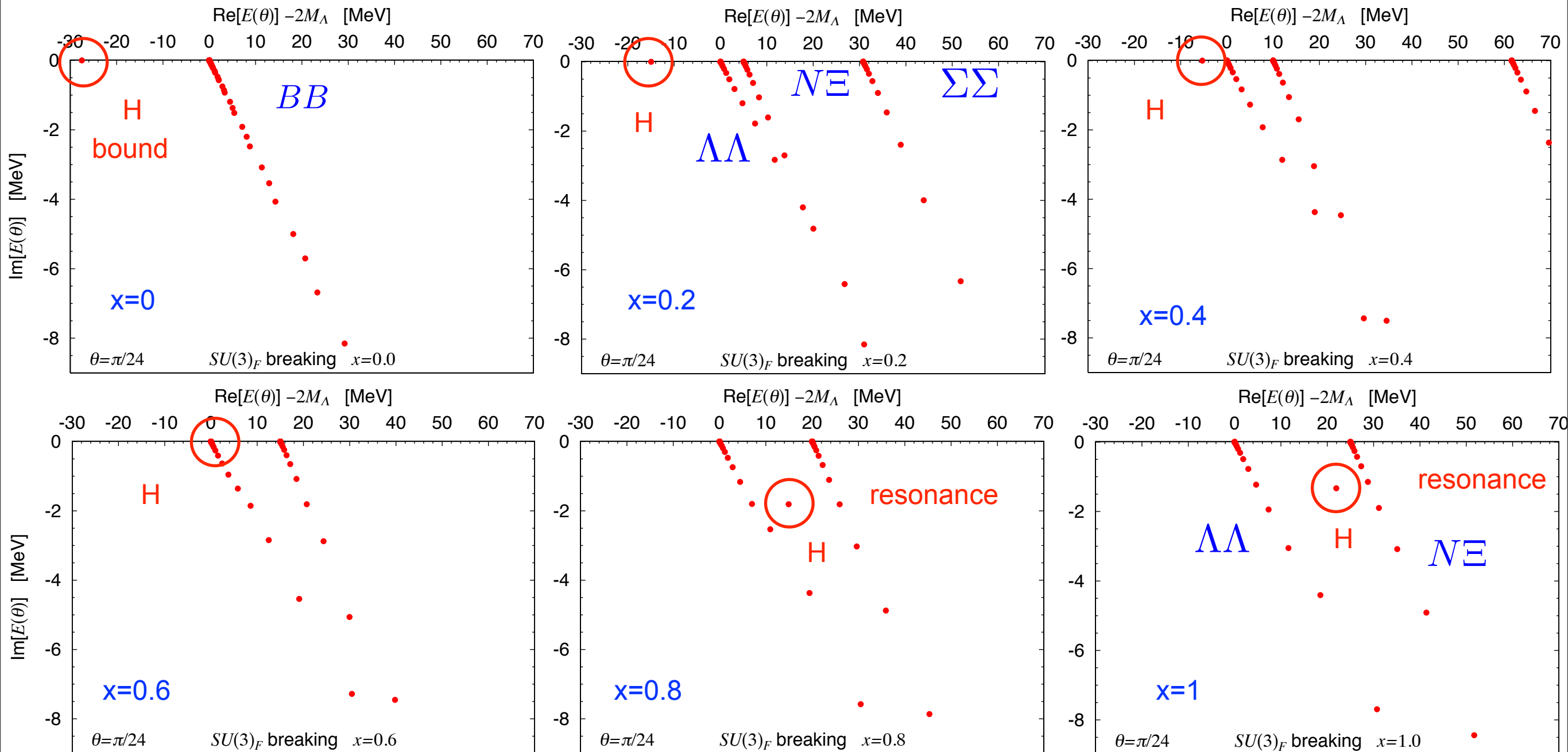


sizable

This part needs to be improved.

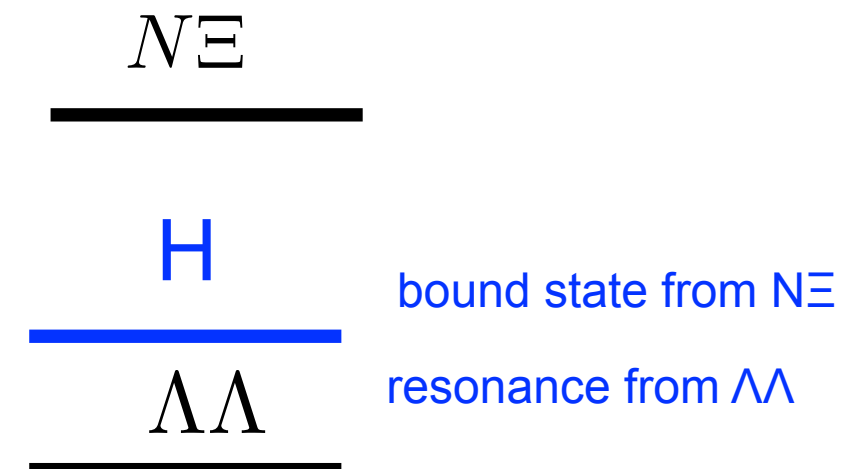
The direct calculation of potentials in 2+1 flavor QCD is in progress.

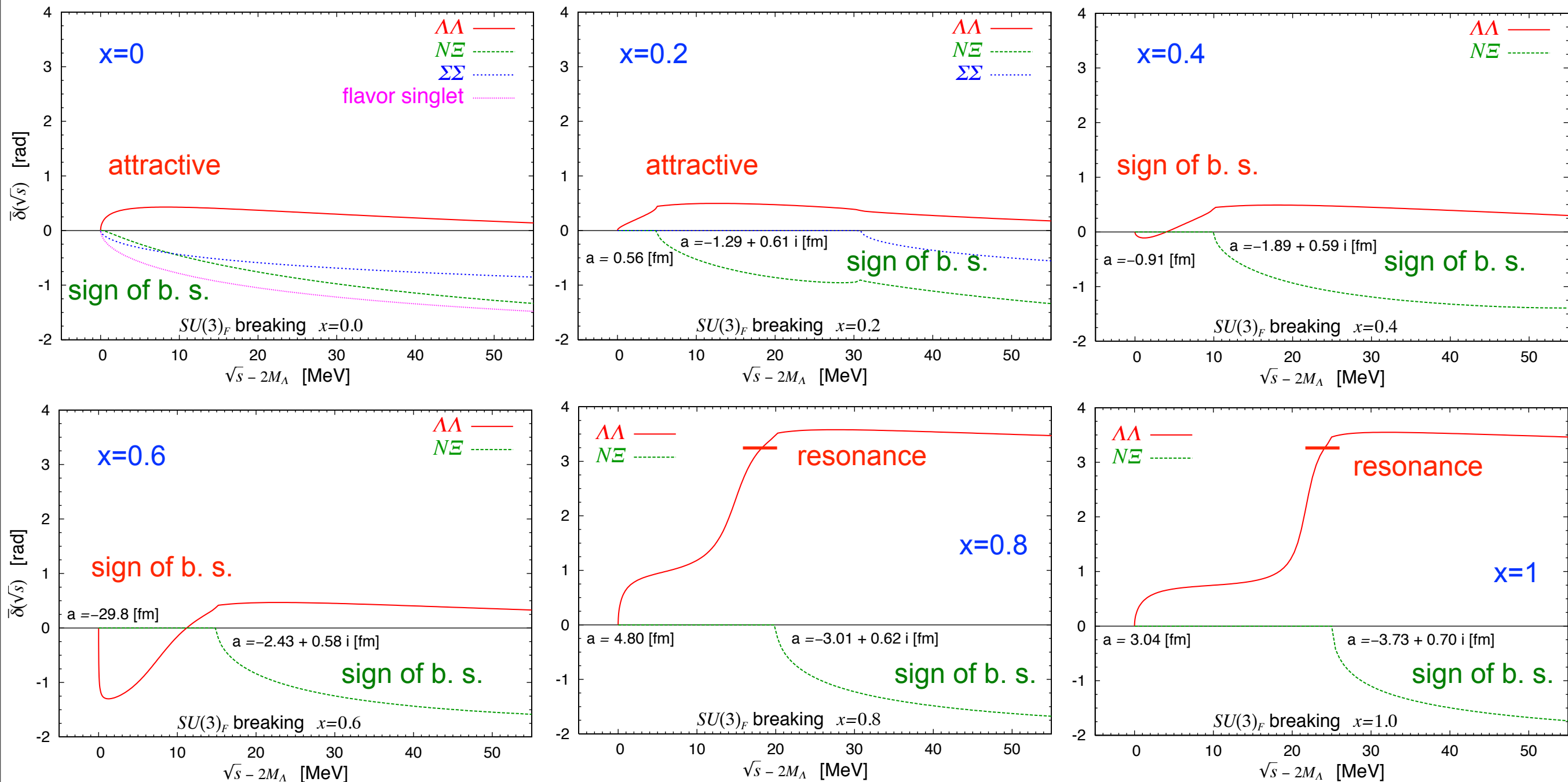
K. Sasaki *et al.* (HAL QCD Coll.), Lat 2011



H-dibaryon seems to become resonance at physical point.

This needs a direct confirmation by 2+1 flavor QCD.





H couples most strongly $N\Xi$.

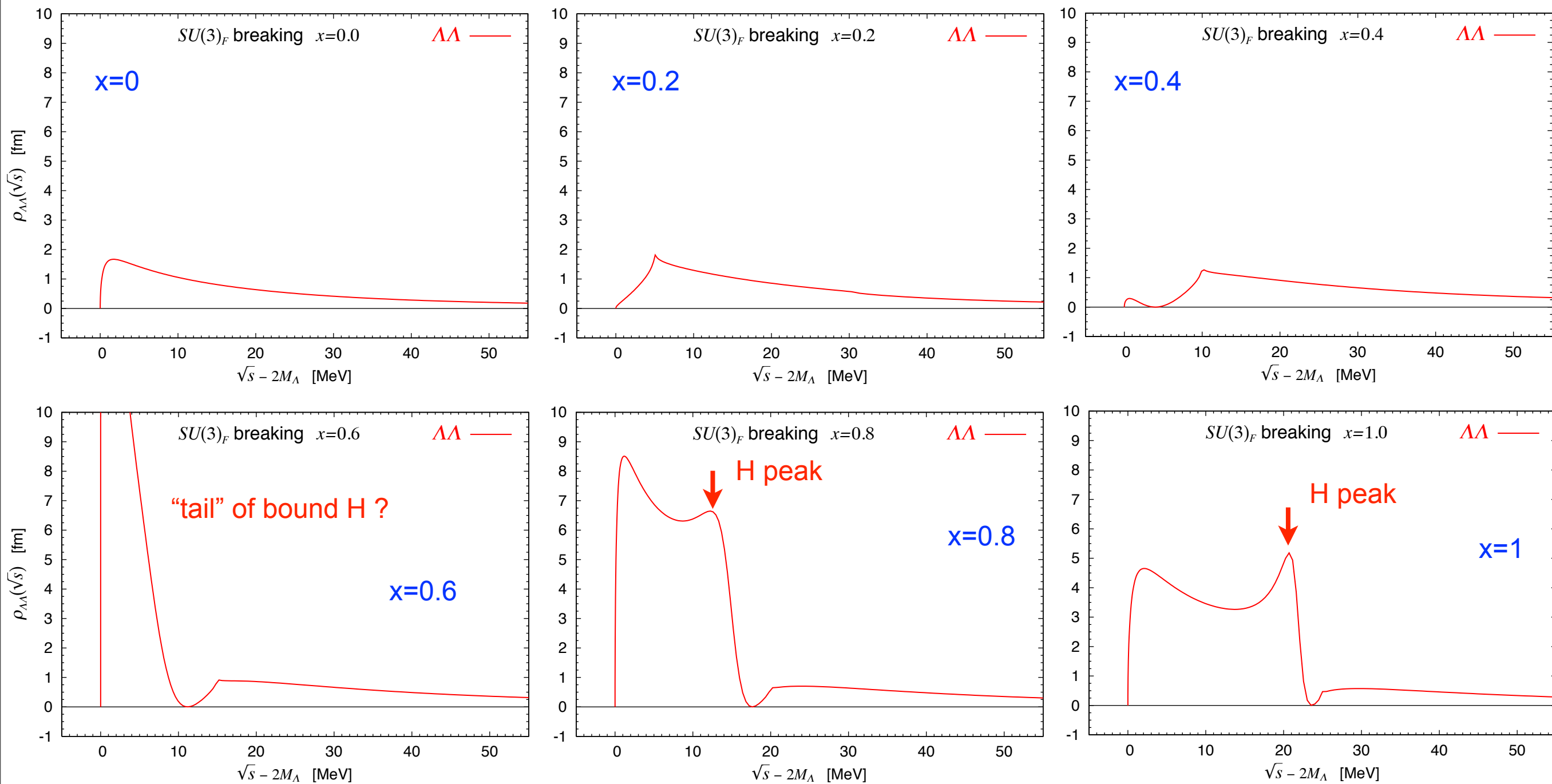
$\Lambda\Lambda$ interaction is attractive.

H has a sizable coupling to $\Lambda\Lambda$ near and above the threshold.

Invariant mass spectrum

$$\Lambda\Lambda \rightarrow \Lambda\Lambda$$

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

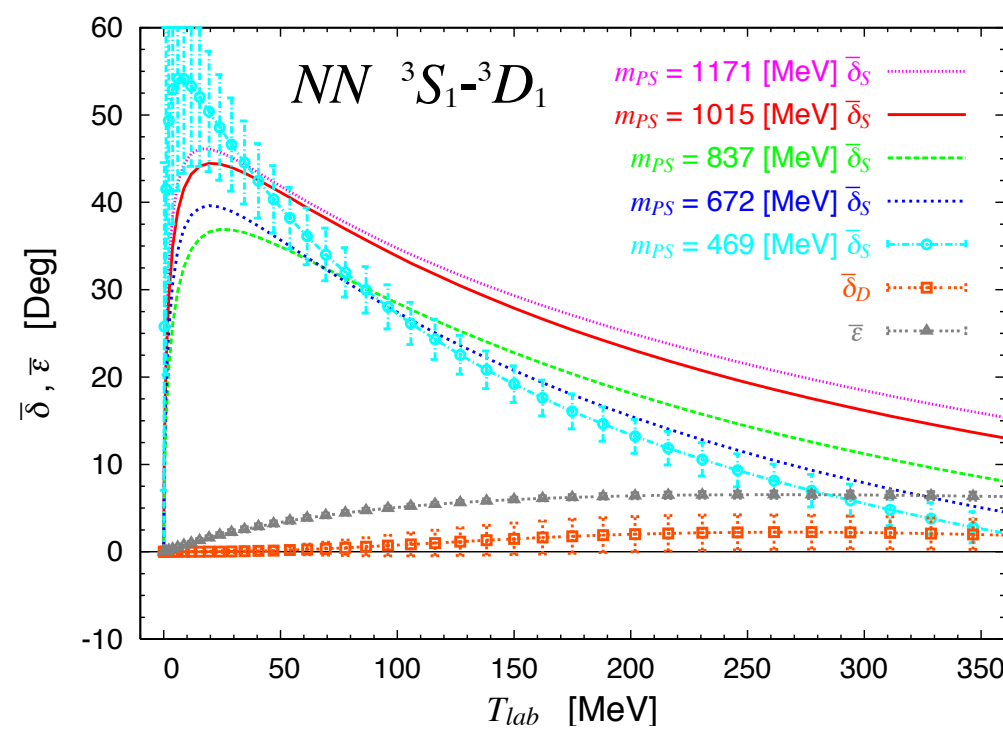
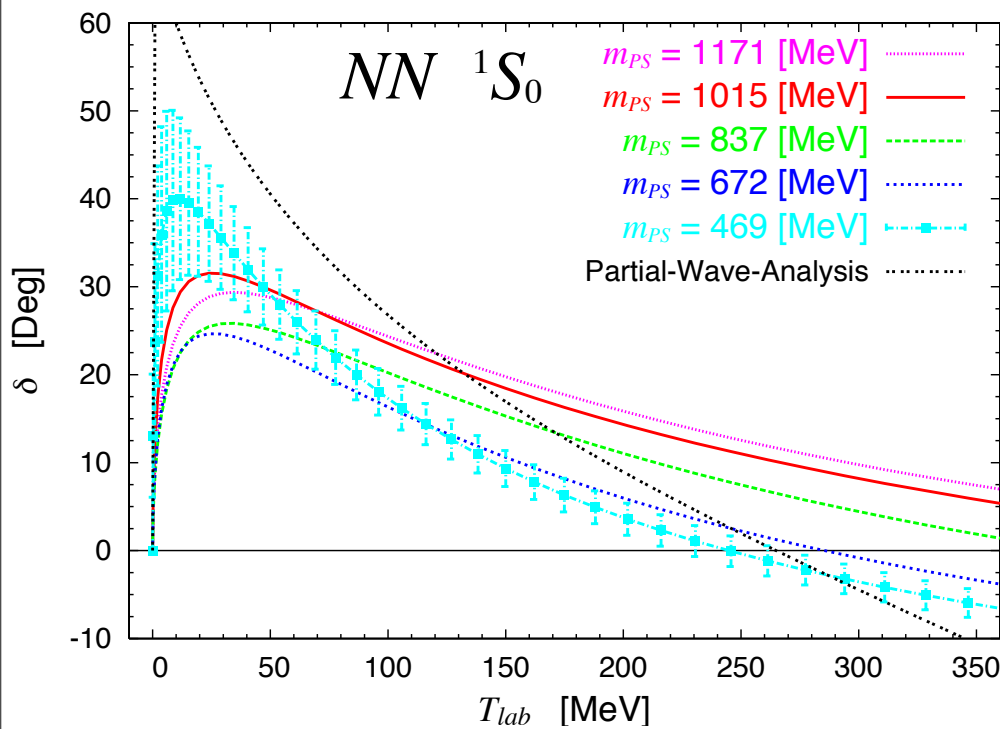


A peak of the resonance H might be observed in experiments !?

Other observables in the flavor SU(3) limit

NN Phase shift, deuteron and 4N state

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]

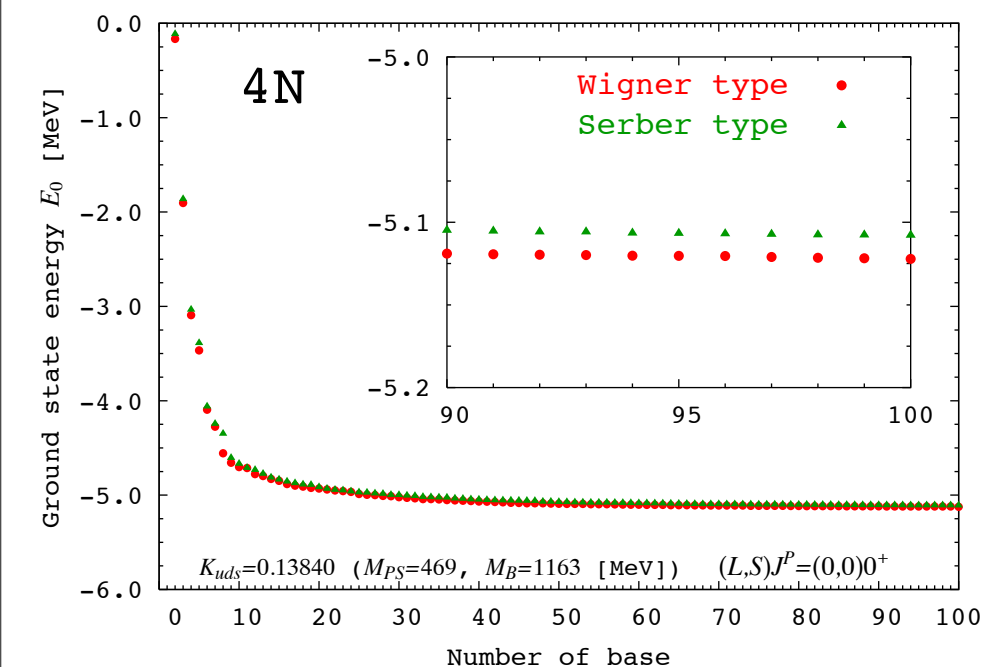


Attraction is stronger in triplet, but no deuteron so far.

Also, no 3N state.

binding energy by variational method

$$^4\text{He} \quad (L, S)J^P = (0, 0)0^+$$



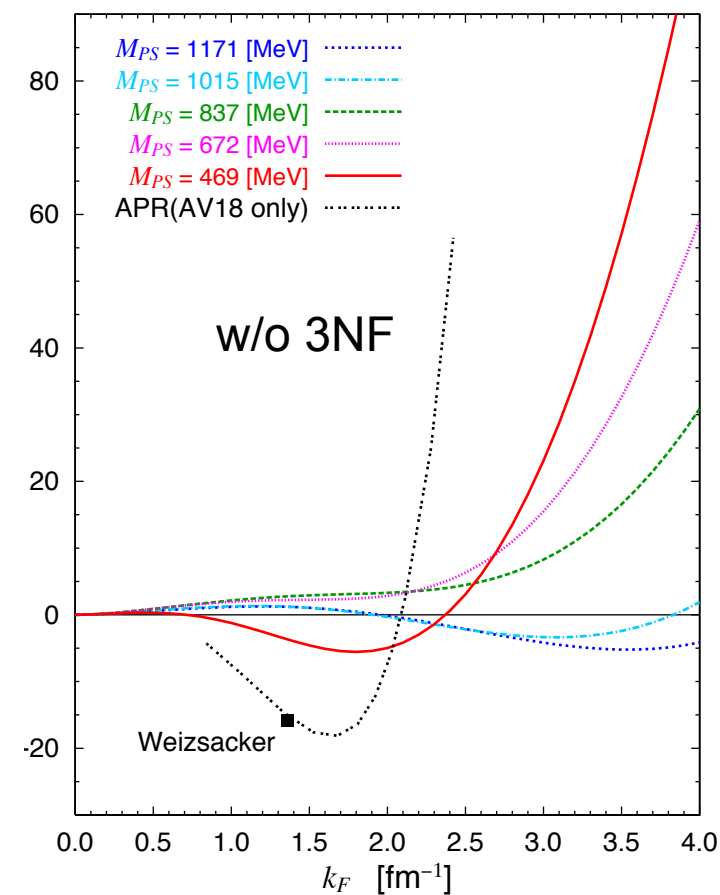
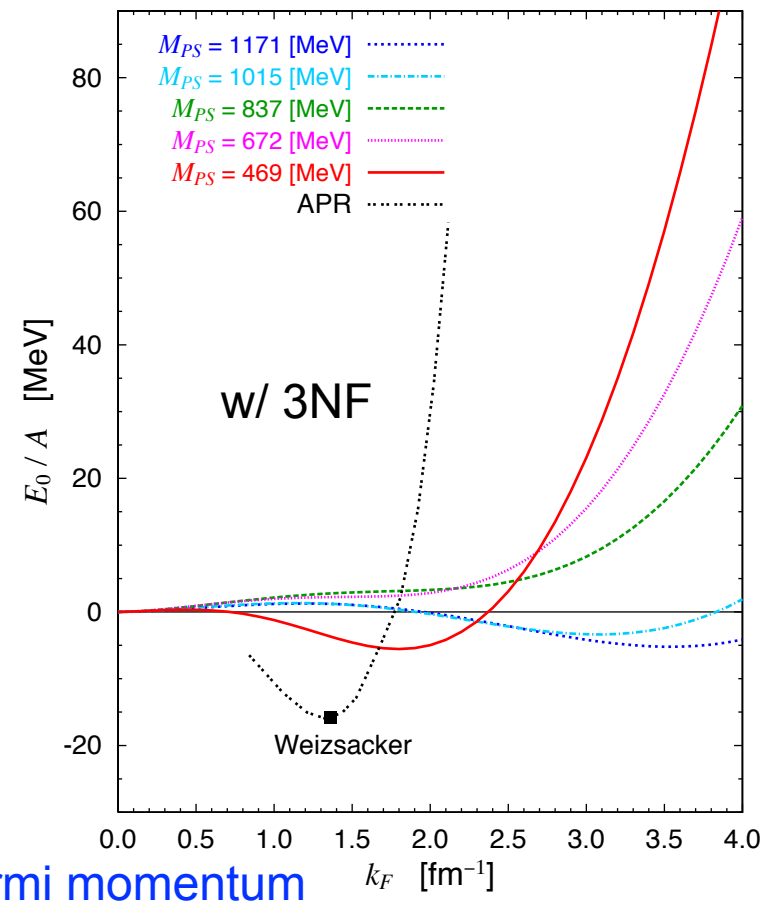
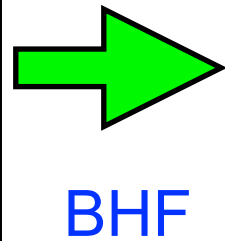
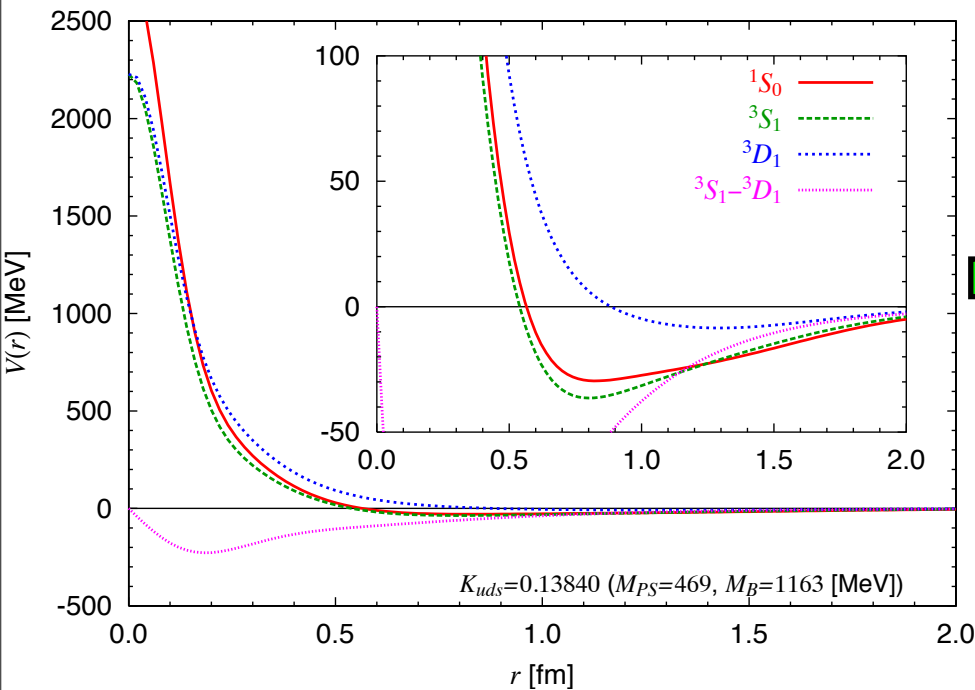
A 4N bound state exists at lightest pion mass.

$$m_{\pi} = 470 \text{ MeV}$$

$$E_{4N} = -5.1 \text{ MeV}$$

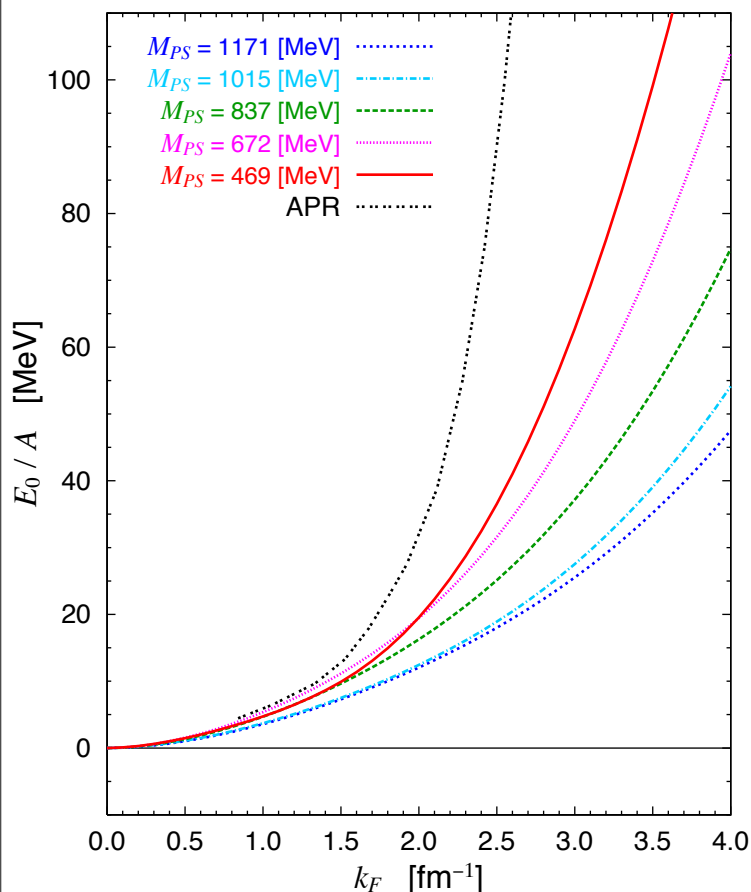
Energy density of Nuclear matter

NN potentials $m_\pi = 470$ MeV



A. Akmal, V.R. Pandharipande, G.G. Ravenhall,
Phys. Rev. C58 1804 (1998)

Neutron matter

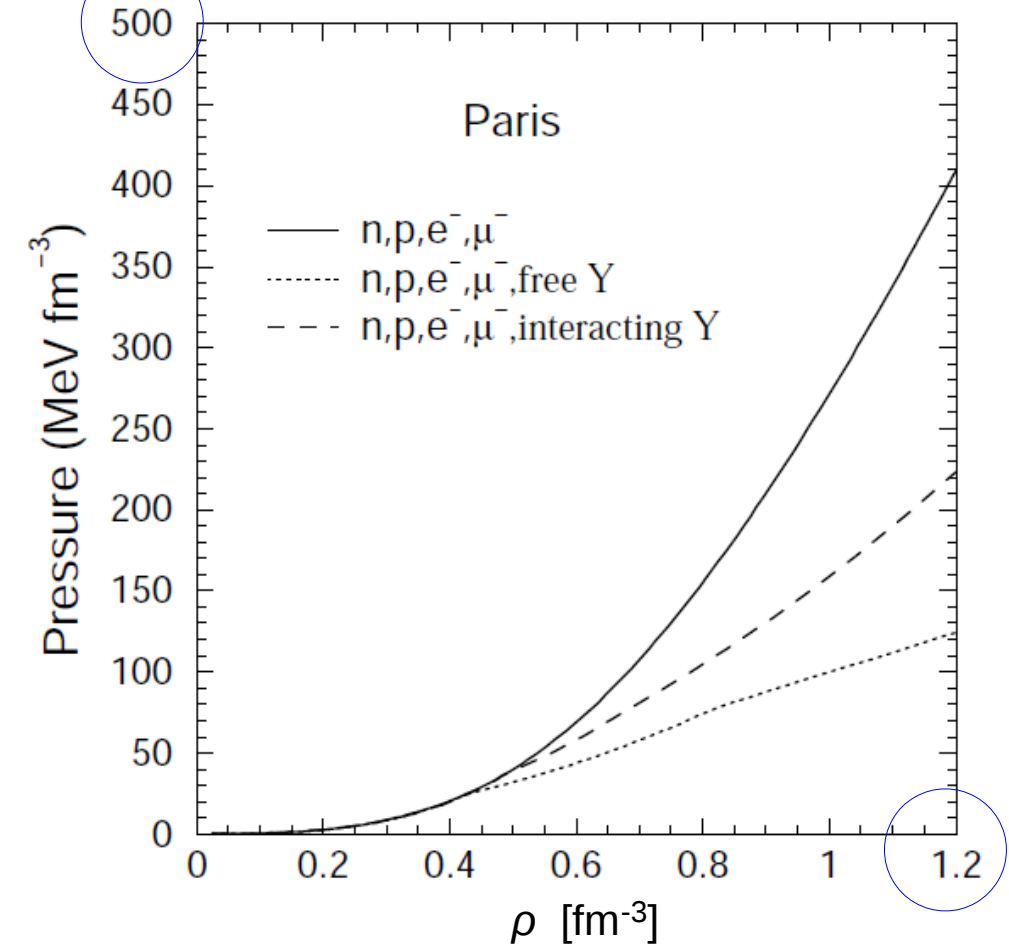
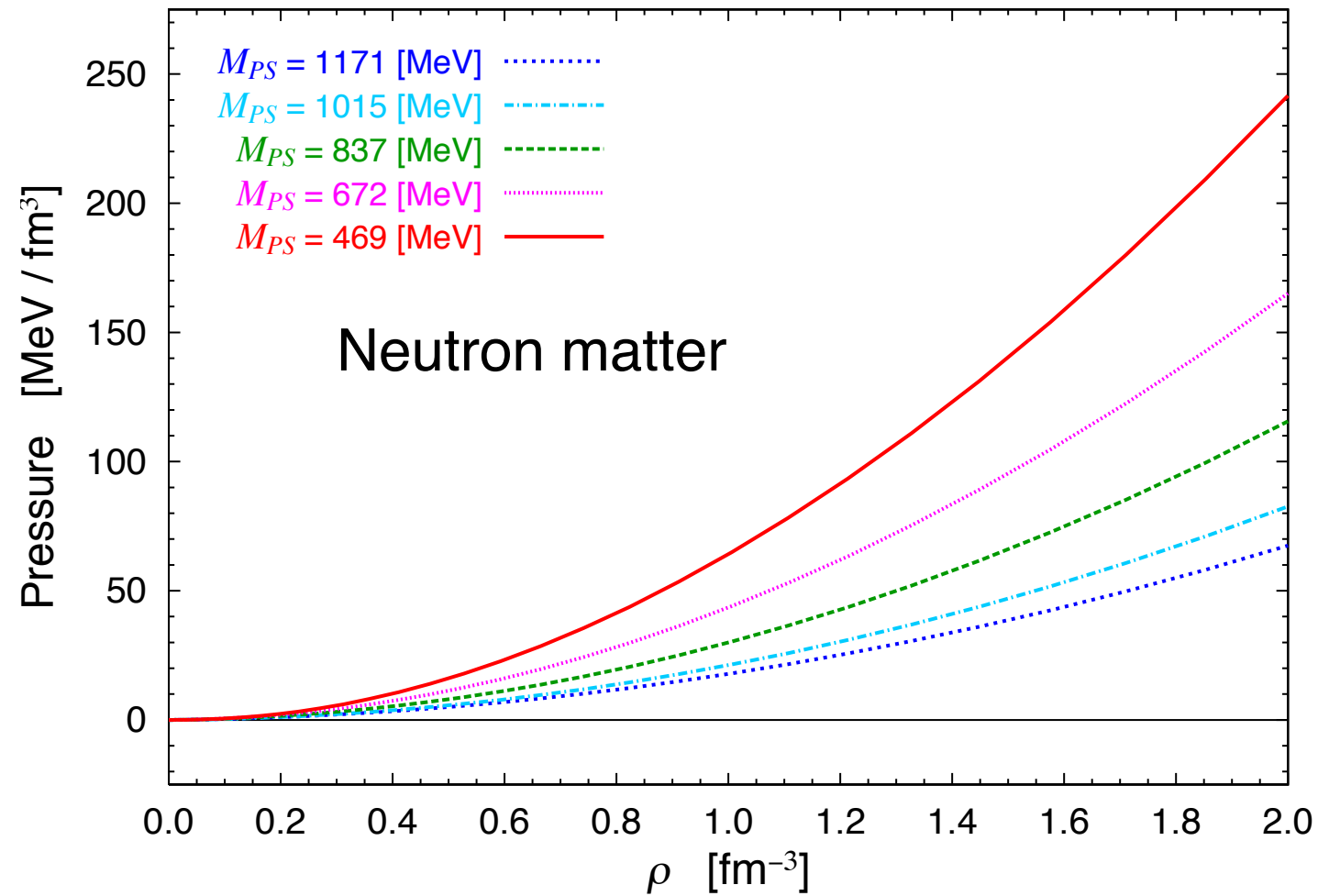


Nuclear matter shows the saturation at the lightest pion mass,
but the saturation point deviates from the empirical one obtained
by Weizsacker mass formula.

No saturation for Neutron matter.

Pressure of Neutron matter

M. Baldo, F. Burgio, H.-J.Schulze,
Phys.Rev. C61, 058801



pressure

$$P = \rho^2 \frac{d(E_0/A)}{d\rho} = \frac{\gamma k_F^4}{18\pi^2} \frac{d(E_0/A)}{dk_F}$$

density

$$\rho = \frac{\gamma k_F^3}{6\pi^2}$$

Our Neutron matter becomes harder as the pion mass decreases,
but it is still softer than phenomenological models.

4. Future prospect

- HAL QCD scheme is shown to be a promising (alternative) method to extract hadronic interactions in lattice QCD.
 - Calculate potential (matrix) in lattice QCD on a **finite box**.
 - Calculate phase shift by solving (coupled channel) Shroedinger equation in **infinite volume**.
 - **bound/resonance/scattering**
- Future directions
 - calculations at the physical pion mass on “**K-computer**”
 - hyperon interactions with the SU(3) breaking
 - Baryon-Meson, Meson-Meson
 - Exotic other than H such as penta-quark, X, Y etc.
 - 3 Nucleon forces
 - Other applications(Alpha-Alpha, Nucleus-Nucleus, Molecule-Molecule)