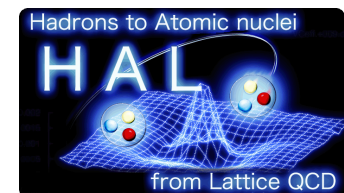


LS force in NN system from Lattice QCD

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for HALQCD collaboration



Future Prospects of Hadron Physics at J-PARC and Large Scale Computational Physics

About This Work:

Recently Hadron-hadron potential from Lattice QCD was proposed.

In this talk, we will report our first attempt at determining potentials in **parity odd sector** including the **spin-orbit force** in **NN system**.

Table of Contents :

What is Lattice QCD ?

How to construct potentials in Lattice QCD

Extension to parity-odd and LS force

Numerical Results in NN system

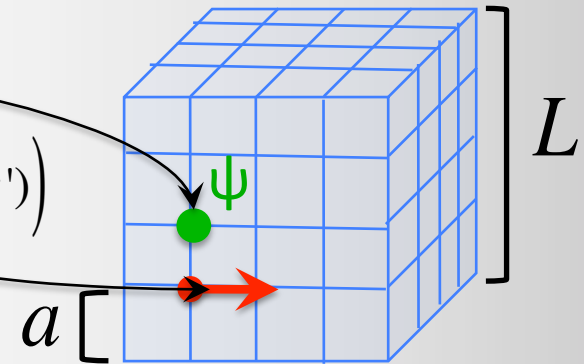
This work

Lattice QCD is strong tool of non-PT calculation of QCD

define the field on sites.

quark field ψ

Gauge field (Gauge link) $U_\mu(\vec{x}) \equiv \exp\left(ig \int_x^{x+a} dx'^\mu A_\mu(x')\right)$



Path-integral is performed with numerically

$$\langle O \rangle = \frac{1}{Z} \int DU D\psi D\bar{\psi} O[\psi, \bar{\psi}, U] \exp(-S_{\text{QCD}}[\psi, \bar{\psi}, U])$$

Take the limit: $a \longrightarrow 0$

Lattice QCD does not need scattering data.



We can calculate potentials based on first principle of QCD

NN potential from Lattice QCD: HAL's method

NBS wave function

$$\text{p} \xrightarrow{\vec{x}} \text{n} \equiv \phi(\vec{x})$$

$$\phi(\vec{r}; E) \equiv \langle 0 | N(\vec{x}) N(\vec{y}) | B = 2; E \rangle$$

derivation:

S.Aoki, T.Hatsuda, N.Ishii,
PTP123(2010)89

Schrodinger type Equation:

$$\begin{aligned} \left(\frac{\Delta}{m_N} + E \right) \phi(\vec{r}; E) &= \int d^3y \underbrace{U(\vec{r}; \vec{r}')}_{\text{"potential"}} \phi(\vec{r}'; E) \\ &= \left[V_C^{(+)}(r) + V_T^{(+)}(r) S_{12} + V_{LS}^{(+)}(r) \vec{L} \cdot \vec{S} \right] P^+ \phi(\vec{r}; E) \\ &\quad + \left[V_C^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] P^- \phi(\vec{r}; E) + \mathcal{O}(\nabla^2) \end{aligned}$$

$$V_C \equiv V_0(\vec{r}) + V_\sigma(\vec{r})(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

After expansion of the U around $\vec{r}$, with taking in to symmetries, we can obtain Okubo-Mulushck formula.

For the moment, parts has been calculated from Lattice.

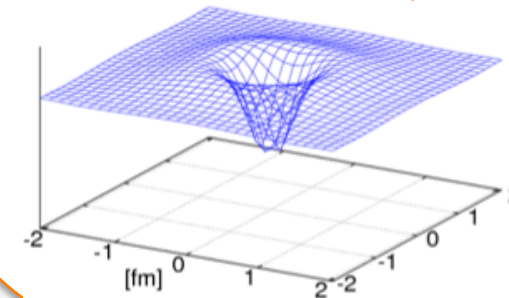
ex) 1S_0 (L=0 S=0) case

$$S_{12} = \begin{cases} \neq 0 & : S=1 \\ =0 & : S=0 \end{cases} \left(\frac{\nabla^2}{m_N} + E \right) \phi(\vec{r}; E) = \left[V_C^{(+)}(r) + \cancel{V_T^{(+)}(r)S_{12}} + \cancel{V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}} \right] \phi(\vec{r}; E)$$

Effective Scrodinger Equation:

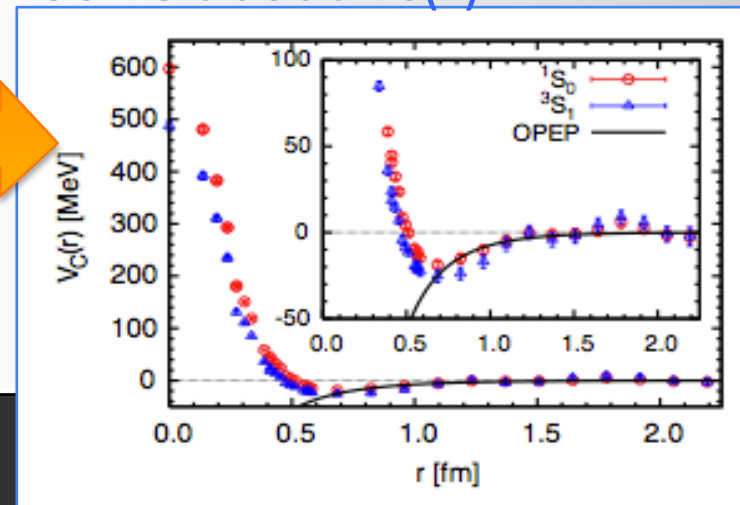
$$\left(\frac{\Delta}{m_N} + E \right) \phi(\vec{r}; E) = V_C(\vec{r}) \phi(\vec{r}; E)$$

from Lattice QCD

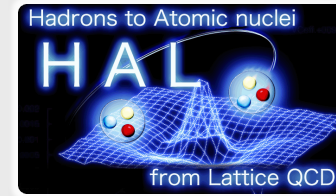


$$\Rightarrow V_C(r) = \left(\frac{1}{m_N} \frac{\Delta \phi(r; E)}{\phi(r; E)} + E \right)$$

Solve about $V_C(r)$

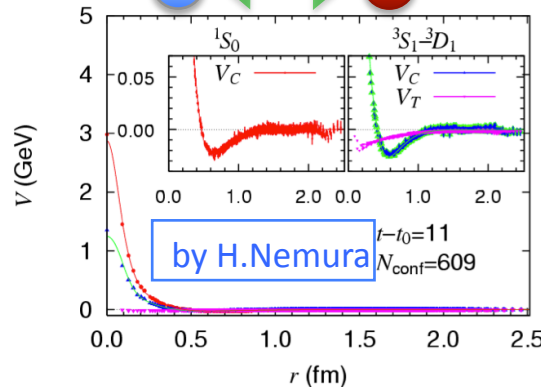


S.Aoki T.Doi T.Hatsuda Y.Ikeda T.Inoue N.Ishii
H.Nemura K.Sasaki for HAL QCD Coll.



This method was extended to YY, YN, meson-baryon system
and three body force.
and various potentials has been calculated.

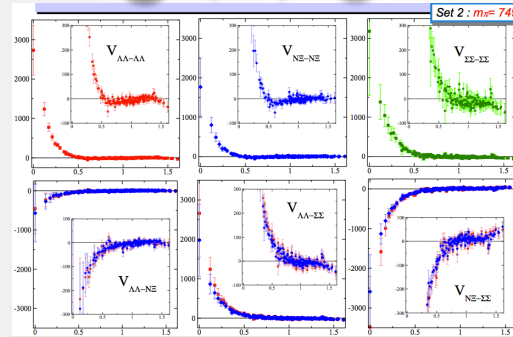
Y-N potential



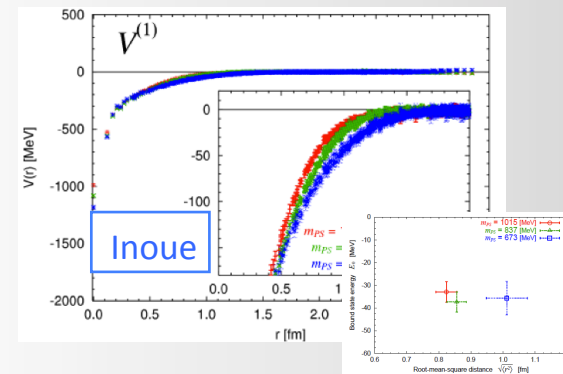
Y-N, Y-Y potential



K. Sasaki

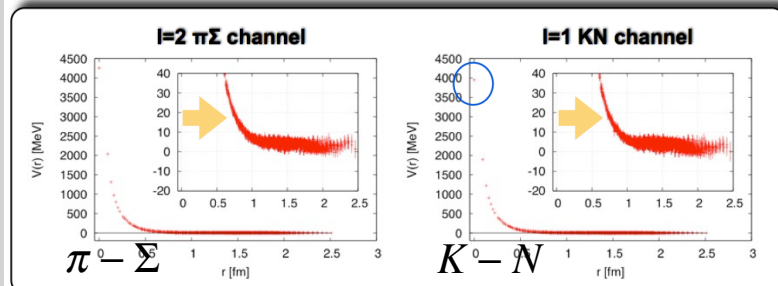


H dibaryon in SU(3)



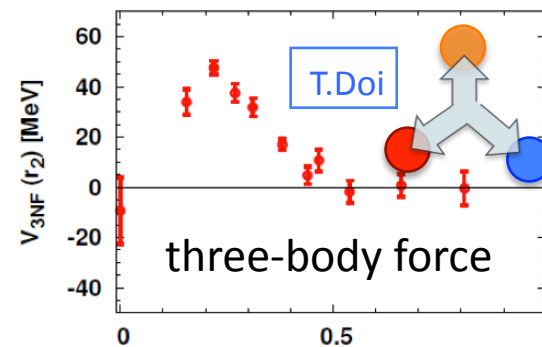
$I=2 \pi \Sigma = \pi^+ \Sigma^+ \rightarrow (u\bar{d})(uus)$

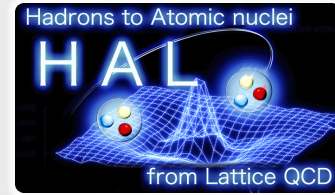
$I=1 KN = K^+ p \rightarrow (u\bar{s})(uud)$



Y. Ikeda

meson-baryon potential





$V_C^{(+)} \quad V_T^{(+)}$ has been calculated from Lattice QCD.

However, $V_{LS}^{(+)}$, $V_C^{(-)}$, $V_T^{(-)}$, $V_{LS}^{(-)}$ is not yet.

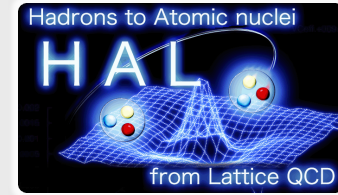
$$\left(\frac{\nabla^2}{m_N} + E \right) \phi(\vec{r}; E) = \left[V_C^{(+)}(r) + V_T^{(+)}(r) S_{12} + V_{LS}^{(+)}(r) \vec{L} \cdot \vec{S} \right] P^+ \phi(\vec{r}; E) \\ + \left[V_C^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] P^- \phi(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

These remaining potentials are ..

➤ needed for complete determination of potentials

Especially, spin-orbit force is important for ..

- magic numbers in nuclei
- Is-splitting of the (hyper) nuclear spectra
- super fluid in neutron star



$V_C^{(+)} \quad V_T^{(+)}$ has been calculated from Lattice QCD.

However, $V_{LS}^{(+)}$, $V_C^{(-)}$, $V_T^{(-)}$, $V_{LS}^{(-)}$ is not yet.

$$\left(\frac{\nabla^2}{m_N} + E \right) \phi(\vec{r}; E) = \left[V_C^{(+)}(r) + V_T^{(+)}(r) S_{12} + V_{LS}^{(+)}(r) \vec{L} \cdot \vec{S} \right] P^+ \phi(\vec{r}; E) \\ + \left[V_C^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] P^- \phi(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

This work

In this work we have extended HAL method to LS force.
LS force with parity minus sector in **NN system** is calculated.

Extension for LS force
with S=1 system.

$$\left(\frac{\nabla^2}{m_N} + E \right) \phi^{(-)}(\vec{r}; E) = \left[V_C^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] \phi^{(-)}(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

calculation of V_c , V_T and V_{LS}

parity minus , $S=1$ system:

$$\left(\frac{\nabla^2}{m_N} + E \right) \phi^{(-)}(\vec{r}; E) = \left[V_c^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] \phi^{(-)}(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

$\phi^{(-)}(\vec{r}) = \psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$: linear independent each other

$$\left(\frac{\nabla^2}{2m} + E \right) \psi^{(1)}(\vec{r}) = V_c^{(-)} \psi^{(1)}(\vec{r}) + V_T^{(-)} S_{12} \psi^{(1)}(\vec{r}) + V_{LS}^{(-)} \vec{L} \cdot \vec{S} \psi^{(1)}(\vec{r})$$

$$\left(\frac{\nabla^2}{2m} + E \right) \psi^{(2)}(\vec{r}) = V_c^{(-)} \psi^{(2)}(\vec{r}) + V_T^{(-)} S_{12} \psi^{(2)}(\vec{r}) + V_{LS}^{(-)} \vec{L} \cdot \vec{S} \psi^{(2)}(\vec{r})$$

$$\left(\frac{\nabla^2}{2m} + E \right) \psi^{(3)}(\vec{r}) = V_c^{(-)} \psi^{(3)}(\vec{r}) + V_T^{(-)} S_{12} \psi^{(3)}(\vec{r}) + V_{LS}^{(-)} \vec{L} \cdot \vec{S} \psi^{(3)}(\vec{r})$$

NBS wave function $\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$ are calculated from Lattice QCD.

calculation of V_c , V_T and V_{LS}

$$\left(\frac{\nabla^2}{m_N} + E \right) \phi^{(-)}(\vec{r}; E) = \left[V_c^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] \phi^{(-)}(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

Solve about V_c , V_T , V_{LS}

$$\begin{pmatrix} \left(\nabla^2 / 2m \right) \psi^{(1)}(\vec{r}) \\ \left(\nabla^2 / 2m \right) \psi^{(2)}(r) \\ \left(\nabla^2 / 2m \right) \psi^{(3)}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \psi^{(1)}(\vec{r}) & S_{12} \psi^{(1)}(\vec{r}) & \vec{L} \cdot \vec{S} \psi^{(1)}(\vec{r}) \\ \psi^{(2)}(\vec{r}), & S_{12} \psi^{(2)}(\vec{r}), & \vec{L} \cdot \vec{S} \psi^{(2)}(\vec{r}), \\ \psi^{(3)}(\vec{r}), & S_{12} \psi^{(3)}(\vec{r}), & \vec{L} \cdot \vec{S} \psi^{(3)}(\vec{r}), \end{pmatrix} \begin{pmatrix} V_c^{(-)}(r) - E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

V_c , V_T , V_{LS} can be obtained from three wave function

$$\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$$

calculation of V_c , V_T and V_{LS}

$$\left(\frac{\nabla^2}{m_N} + E \right) \phi^{(-)}(\vec{r}; E) = \left[V_c^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] \phi^{(-)}(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

Solve about V_c , V_T , V_{LS}

$$\begin{pmatrix} \psi^{(1)}(\vec{r}) & S_{12} \psi^{(1)}(\vec{r}) & \vec{L} \cdot \vec{S} \psi^{(1)}(\vec{r}) \\ \psi^{(2)}(\vec{r}), & S_{12} \psi^{(2)}(\vec{r}), & \vec{L} \cdot \vec{S} \psi^{(2)}(\vec{r}), \\ \psi^{(3)}(\vec{r}), & S_{12} \psi^{(3)}(\vec{r}), & \vec{L} \cdot \vec{S} \psi^{(3)}(\vec{r}), \end{pmatrix}^{-1} \begin{pmatrix} (\nabla^2 / 2m) \psi^{(1)}(\vec{r}) \\ (\nabla^2 / 2m) \psi^{(2)}(r) \\ (\nabla^2 / 2m) \psi^{(3)}(\vec{r}) \end{pmatrix} = \begin{pmatrix} V_c^{(-)}(r) - E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

V_c , V_T , V_{LS} can be obtained from three wave function

$$\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$$

Here, $\psi^{(1)}(\vec{r})$, $\psi^{(2)}(\vec{r})$, $\psi^{(3)}(\vec{r})$ must be

- Parity Minus
- linear independent each other
- Orbit alangular momentum $L \neq 0$

$$\begin{pmatrix} \psi^{(1)}(\vec{r}) & S_{12} \psi^{(1)}(\vec{r}) & \vec{L} \cdot \vec{S} \psi^{(1)}(\vec{r}) \\ \psi^{(2)}(\vec{r}), & S_{12} \psi^{(2)}(\vec{r}), & \vec{L} \cdot \vec{S} \psi^{(2)}(\vec{r}), \\ \psi^{(3)}(\vec{r}), & S_{12} \psi^{(3)}(\vec{r}), & \vec{L} \cdot \vec{S} \psi^{(3)}(\vec{r}), \end{pmatrix}^{-1} \begin{pmatrix} (\nabla^2 / 2m) \psi^{(1)}(\vec{r}) \\ (\nabla^2 / 2m) \psi^{(2)}(r) \\ (\nabla^2 / 2m) \psi^{(3)}(\vec{r}) \end{pmatrix} = \begin{pmatrix} V_C^{(-)}(r) - E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

Here, $\psi^{(1)}(\vec{r})$, $\psi^{(2)}(\vec{r})$, $\psi^{(3)}(\vec{r})$ must be

- Parity Minus
- linear independent each other
- Orbit alangular momentum $L \neq 0$

➡ In this work, we calculated : 3P_0 , 3P_1 and 3P_2

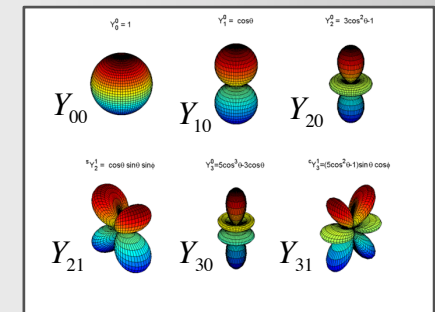
How to construct them ?

How to construct Parity odd, with $L \neq 0$

In previous works...

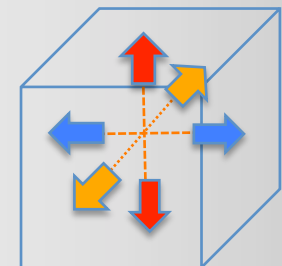
two nucleon in **rest frame** (wall sorce)

- **Parity Plus**
- orbital angular momentum **$L=0$**



In this work :

- Impose the momentum to the nucleon (quark)
- construct the **$L=1$** (on the cubic group : T1) state with combining 6-different direction of momentum.



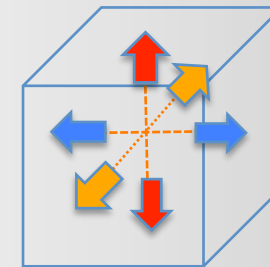
In this work, we calculated : 3P_0 , 3P_1 and 3P_2

How to construct Parity odd state

Parity :

parity plus $\phi(\vec{r})^{P=+} = \phi(\vec{r}; +\vec{k}) + \phi(\vec{r}; -\vec{k})$

parity minus $\phi(\vec{r})^{P=-} = \phi(\vec{r}; +\vec{k}) - \phi(\vec{r}; -\vec{k})$

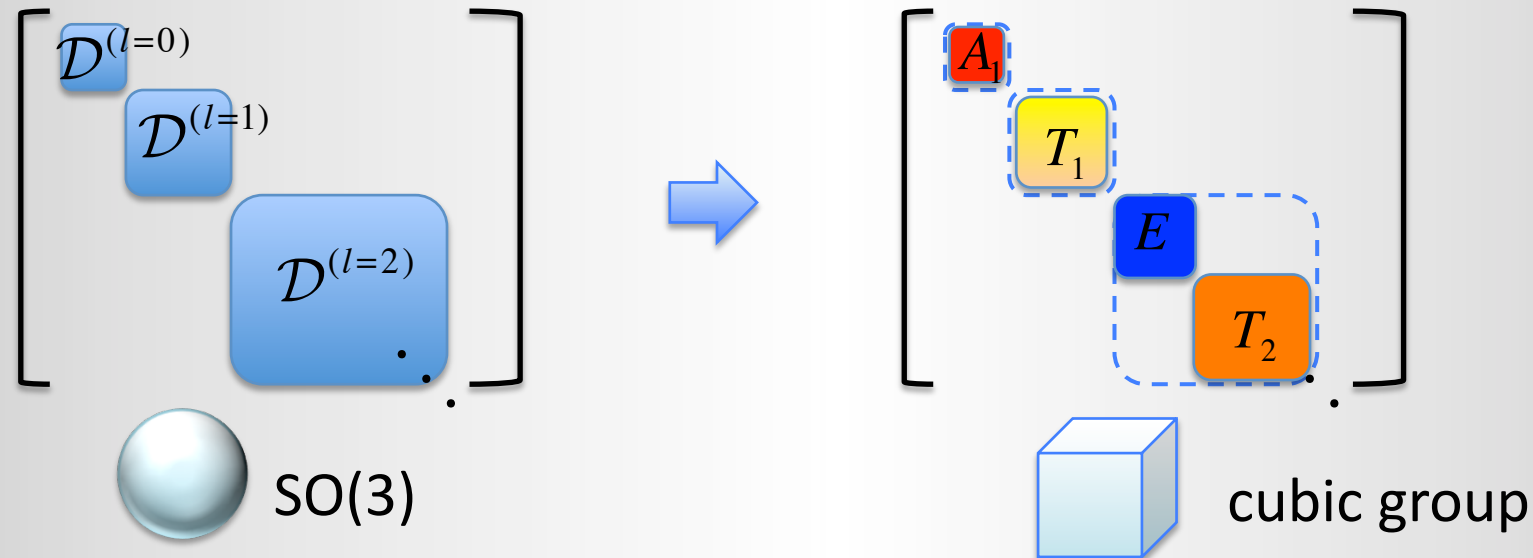


$$\bar{N}_\beta(f) \equiv \sum_{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3} \epsilon_{abc} (\bar{u}_a(\mathbf{x}_1) C \gamma_5 \bar{d}_b(\mathbf{x}_2)) \bar{d}_{c,\beta}(\mathbf{x}_3) \cdot \exp(-i\vec{k} \cdot \vec{r})$$

- Impose the momentum to the nucleon (quark)
- subtracting NBS waves with opposite direction of momentum

How to construct $L \neq 0$ states

representation of $SO(3)$ reduce to
“irreducible representation on the cubic group”



angular momentum, Spin $\rightarrow$ irreducible representation of cubic group

$$\mathcal{L}, \mathcal{S}, \mathcal{J} \longrightarrow A_1, A_2, E, T_1, T_2$$

projection operator : in SO(3) case

An irreducible representation has an orthogonality
(Group Orthogonality Theorem):

$$\int d\theta d\phi D_{\mu\nu}^{(l)}(\theta, \phi)^* D_{\mu'\nu'}^{(l)}(\theta, \phi) = \frac{4\pi}{2l+1} \delta_{l,l'} \delta_{\mu,\mu'} \delta_{\nu,\nu'}$$

We can extract definite angular momentum (representation on SO(3))

$$\psi(\vec{x}) = \psi^{(l=0)}(\vec{x}) + \psi^{(l=1)}(\vec{x}) + \dots$$

$$\psi(R(\theta, \phi) \vec{x}) = \mathcal{D}^{(l=0)}(\theta, \phi) \psi^{(l=0)}(\vec{x}) + \mathcal{D}^{(l=1)}(\theta, \phi) \psi^{(l=1)}(\vec{x}) + \dots$$


$$\frac{2l+1}{4\pi} \int d\theta d\phi D^{(l=L)}(\theta, \phi)^* \psi(R(\theta, \phi) \vec{x}) = \psi^{(l=L)}(\vec{x})$$

projection operator : in cubic group case

An irreducible representation has an orthogonality
(Group Orthogonality Theorem):

$$\sum_{i=0} D_{\mu\nu}^{(\Gamma)}(g_i)^* D_{\mu'\nu'}^{(\Gamma')}(g_i) = \frac{24}{d_\Gamma} \delta_{\Gamma,\Gamma'} \delta_{\mu,\mu'} \delta_{\nu,\nu'}, \quad \Gamma = A_1, A_2, E, T_1, T_2$$

In same way, we can extract definite representation **on cubic group**
(~ angular momentum)

$$\psi(\vec{x}) = \psi^{(A_1)}(\vec{x}) + \psi^{(A_2)}(\vec{x}) + \dots$$

$$\psi(\mathbf{R}(g_i) \vec{x}) = \mathbf{D}^{(A_1)}(g_i) \psi^{(A_1)}(\vec{x}) + \mathbf{D}^{(A_2)}(g_i) \psi^{(A_2)}(\vec{x}) + \dots$$


$$\frac{d_\Gamma}{24} \sum_{i=0}^{24} D_{\mu,\nu}^{(\Gamma)}(g_i)^* \psi_\nu(\mathbf{R}(g_i) \vec{x}) = \psi_\mu^{(\Gamma)}(\vec{x})$$

projection operator : in cubic group case

An irreducible representation has an orthogonality
(Group Orthogonality Theorem):

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In same way, we can extract definite representation **on cubic group**
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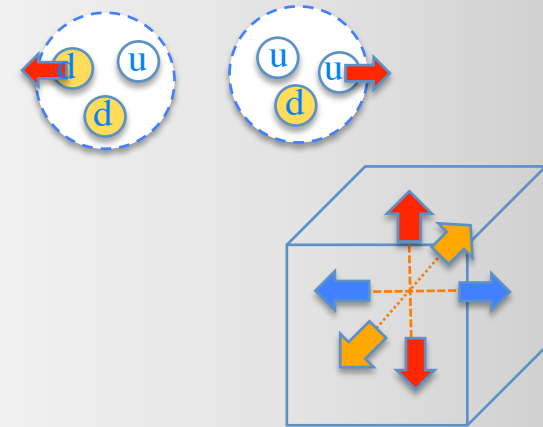
$$\frac{d_\Gamma}{24} \sum_{i=0}^{24} \chi(g_i)^* \psi(\mathbf{R}(g_i) \vec{x}) = \psi^{(\Gamma)}(\vec{x}), \quad \begin{array}{l} \text{charcter} \\ \chi = \text{Tr}\{D\} \end{array}$$

Construction of States

Parity :

parity plus $\phi(\vec{r})^{P=+} = \phi(\vec{r}; +\vec{k}) + \phi(\vec{r}; -\vec{k})$

parity minus $\phi(\vec{r})^{P=-} = \phi(\vec{r}; +\vec{k}) - \phi(\vec{r}; -\vec{k})$



Angular momentum:

we impose momentum to nucleus

Projection operator
based on **cubic group**:
because rotational sym
is broken $O(3) \rightarrow$ cubic group

$$\frac{2l+1}{4\pi} \int d\theta d\phi D^{(l=L)}(\theta, \phi)^* \psi(R(\theta, \phi) \vec{x})$$



$$\frac{d_\Gamma}{24} \sum_{i=0}^{24} D_{\mu, \nu}^{(\Gamma)}(g_i)^* \psi_\nu(R(g_i) \vec{x})$$

Great Orthogonally :

$$\sum_{i=0} D_{\mu \nu}^{(\Gamma)}(g_i)^* D_{\mu' \nu'}^{(\Gamma')}(g_i) = \frac{24}{d_\Gamma} \delta_{\Gamma, \Gamma'} \delta_{\mu, \mu'} \delta_{\nu, \nu'}, \quad \Gamma = A_1, A_2, E, T_1, T_2$$

Numerical Results

Set Up



$N_f=2$

Iwasaki gauge + clover fermion

$\beta=0.195$

$\kappa=0.1375$

$a=0.1555$

$1/a = 1271 \text{ MeV}$

$L^3 \times T = 16^3 \times 32$

$m_N=2165 \text{ MeV}$

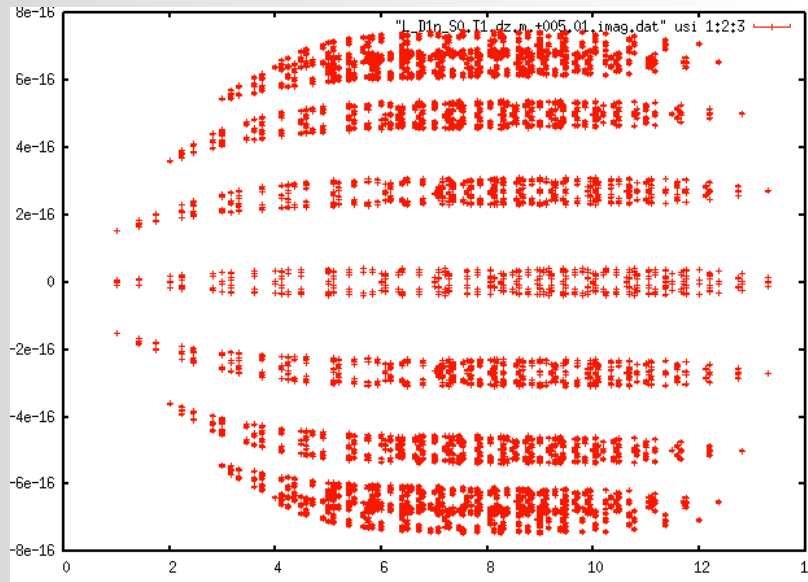
$m_{\pi}=1136 \text{ MeV}$



heavy quark mass
(calculation cost $\propto 1/m_q$)

spin-singlet ($S=0$)
parity minus

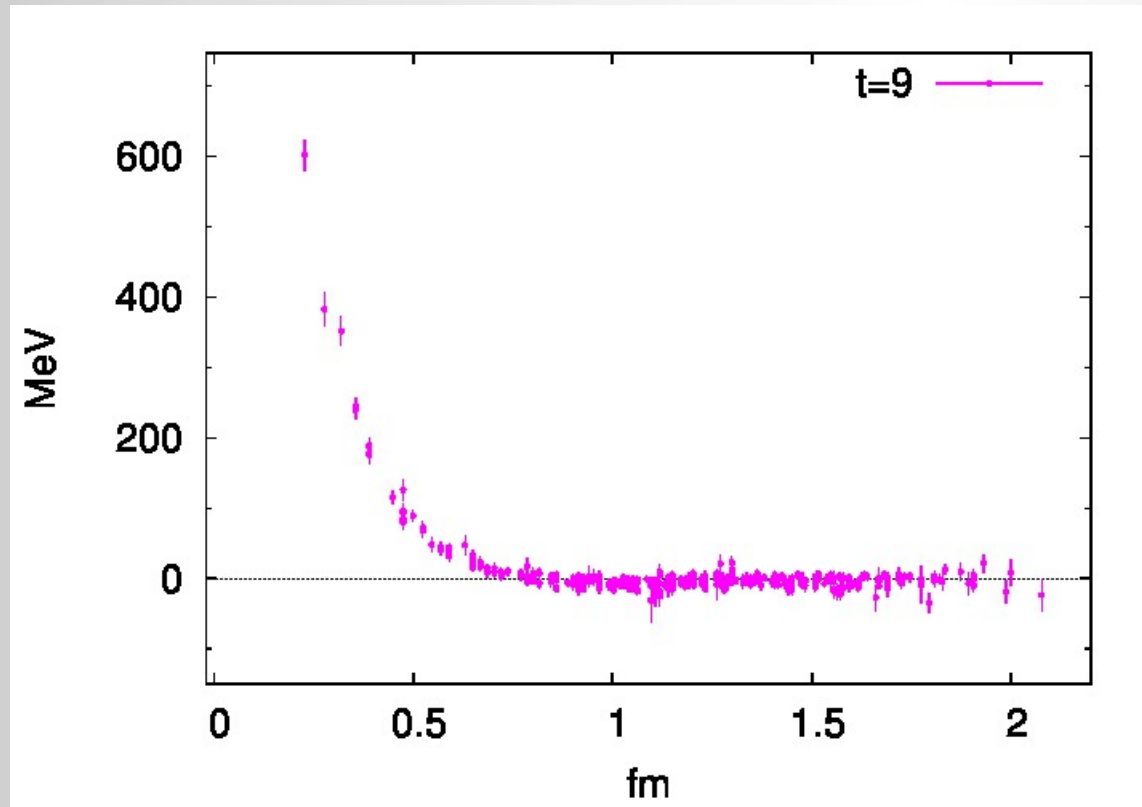
1P1 NBS wave function for S=0



$$\left(\frac{\nabla^2}{m_N} + E \right) \phi^{(-)}(\vec{r}; E) = \left[V_C^{(-)}(r) + \cancel{V_T^{(+)}(r) S_{12}} + \cancel{V_{LS}^{(+)}(r) \vec{L} \cdot \vec{S}} \right] \phi^{(-)}(\vec{r}; E)$$

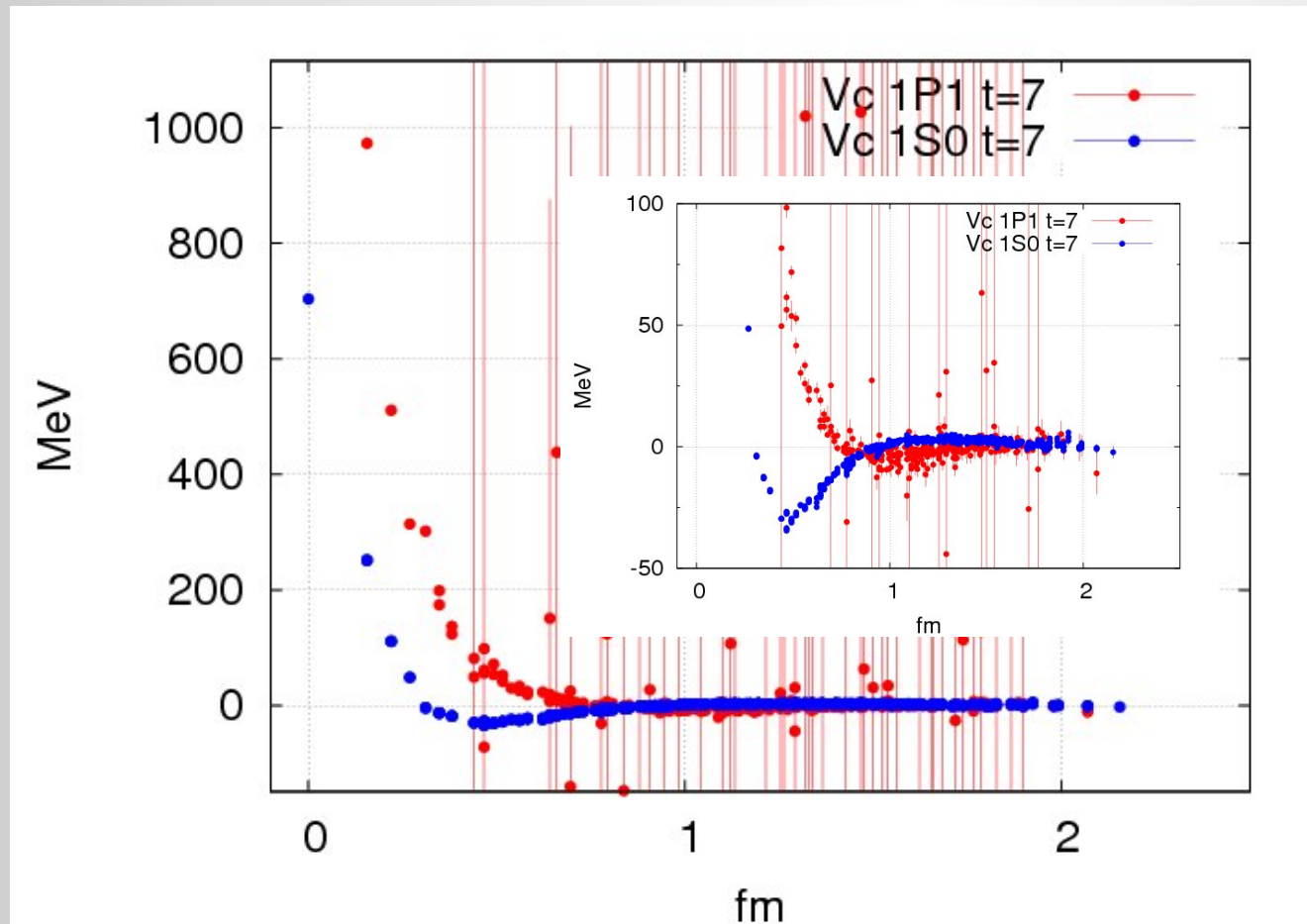
$$\Rightarrow V_{C_{S=0}}^{(-)}(r) = \left(\frac{1}{m_N} \frac{\Delta \phi(r; E)}{\phi(r; E)} + E \right)$$

1P1 ($S=0$) central potential for parity odd



(so3 improved ver, not time-dependent)

- comparison of central potential in **parity even** and one in **parity odd**.

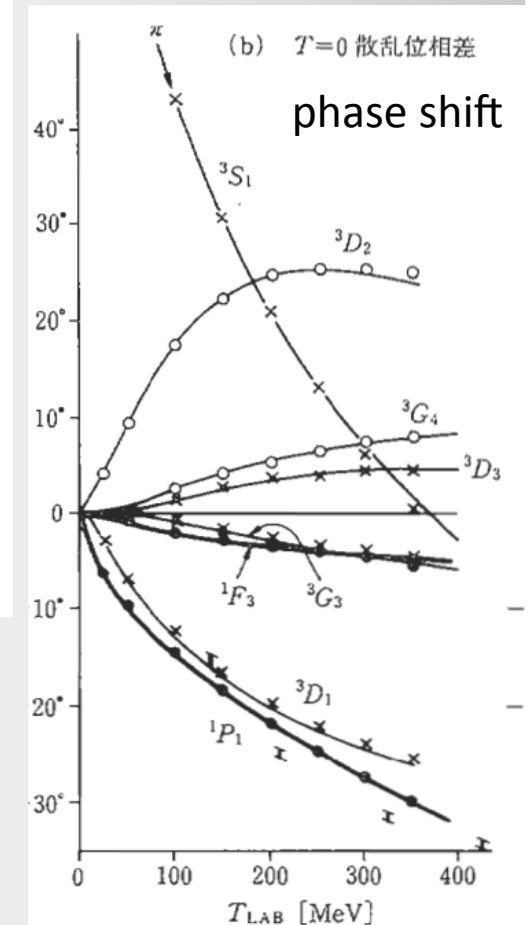


Parity Plus
Parity Minus

- parity odd central force (from 1P1) has strong repulsive core

1P1 cos type source $\ddot{\cdot} E = (2\pi / L)^2 / m_N$

1S0 wall source $\ddot{\cdot}$ shift

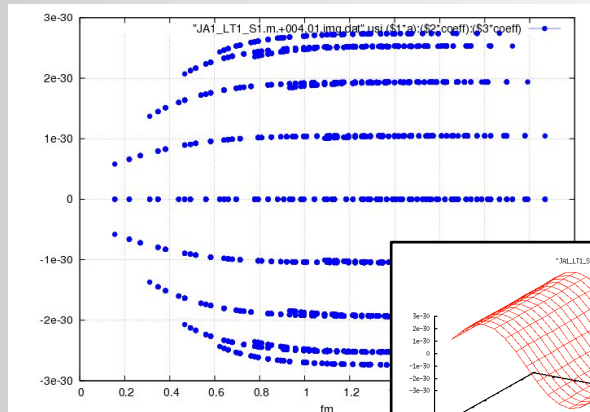


spin-triplet ($S=1$)
parity minus

results : NBS wave function for S=1

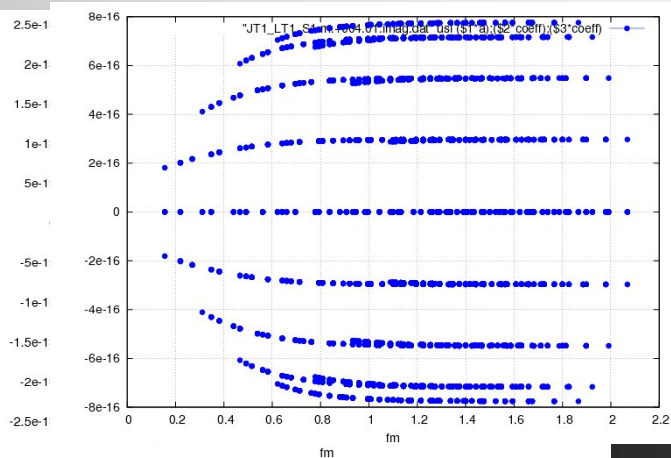
3P0

J=1 (A1) L=1 (T1) (imaginary part)



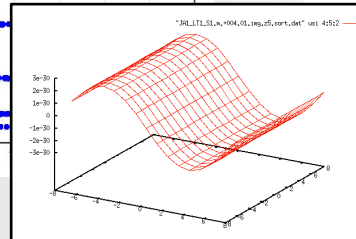
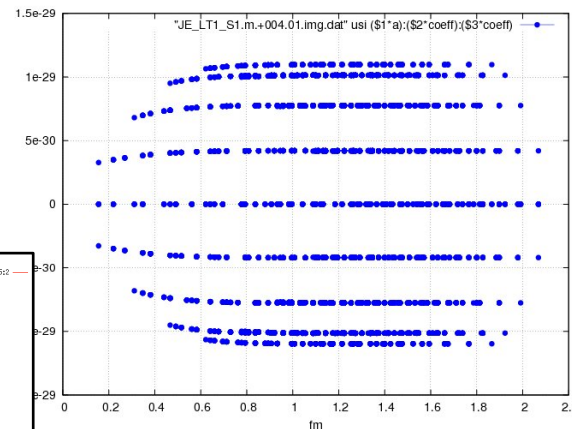
3P1

J=1 (T1) L=1 (T1)



3P2

J=2 (E) L=1 (T1) (imaginary part)



$a=0.1555$
 $1/a = 1271 \text{ MeV}$
 $L^3 \times T = 16^3 \times 32$
 $mN=2165 \text{ MeV}$

$$\psi(\vec{r}) = R(r)Y_{1m}(\theta, \phi)$$

We have obtained

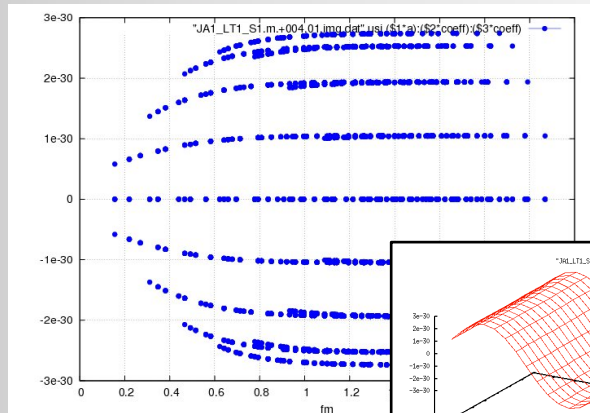
3P0, 3P1, 3P2, 3F2, 3F3

NBS wave functions

results : NBS wave function for S=1

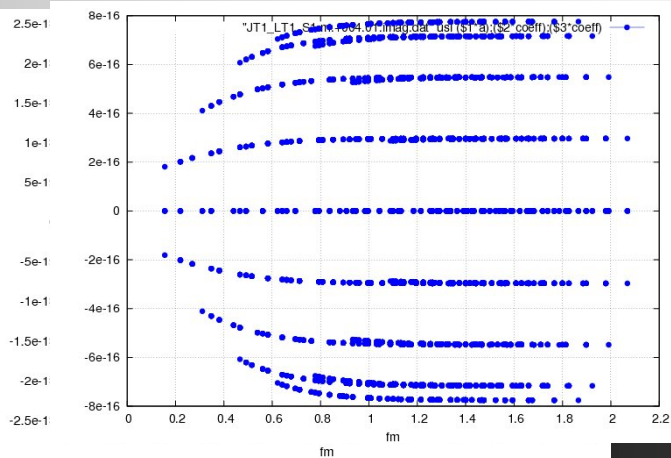
3P0

J=1 (A1) L=1 (T1) (imaginary part)



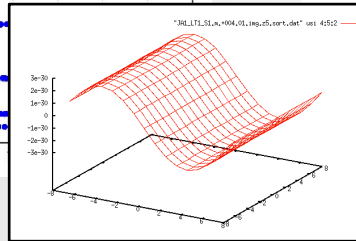
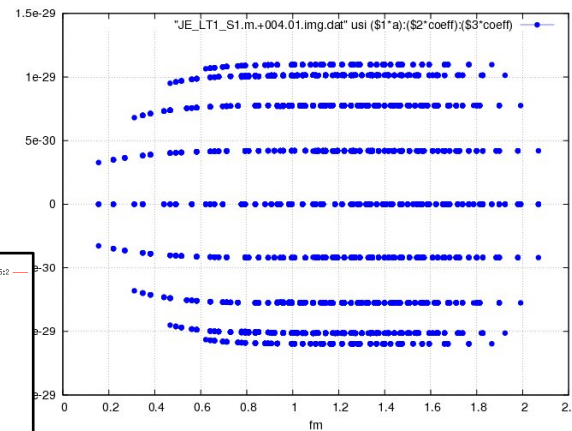
3P1

J=1 (T1) L=1 (T1)



3P2

J=2 (E) L=1 (T1) (imaginary part)



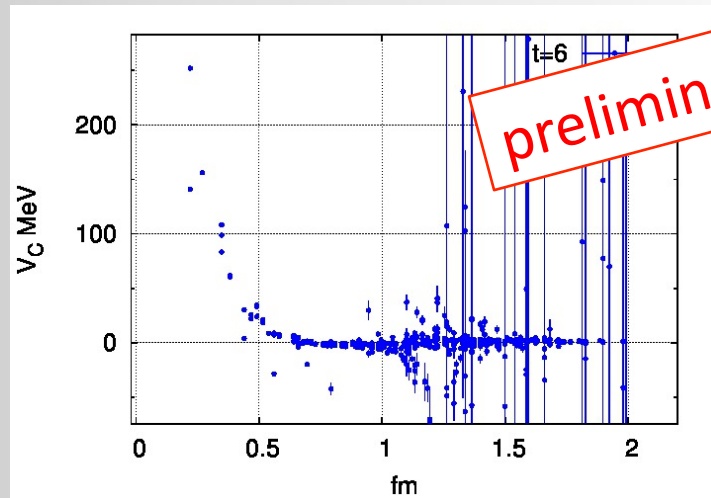
a=0.1555
1/a = 1271 MeV
 $L^3 \times T = 16^3 \times 32$
mN=2165 MeV

$$\psi(\vec{r}) = R(r)Y_{1m}(\theta, \phi)$$

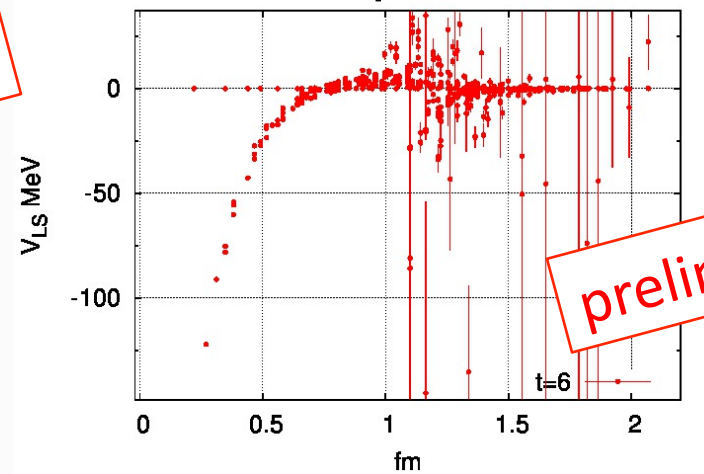
$$\begin{pmatrix} (\nabla^2 / 2m) {}^3P_0(\vec{r}) \\ (\nabla^2 / 2m) {}^3P_1(\vec{r}) \\ (\nabla^2 / 2m) {}^3P_2(\vec{r}) \end{pmatrix} = \begin{pmatrix} {}^3P_0(\vec{r}) & S_{12} {}^3P_0(\vec{r}) & \vec{L} \cdot \vec{S} {}^3P_0(\vec{r}) \\ {}^3P_1(\vec{r}) & S_{12} {}^3P_1(\vec{r}) & \vec{L} \cdot \vec{S} {}^3P_1(\vec{r}) \\ {}^3P_2(\vec{r}) & S_{12} {}^3P_2(\vec{r}) & \vec{L} \cdot \vec{S} {}^3P_2(\vec{r}) \end{pmatrix} \begin{pmatrix} V_C^{(-)}(r) - E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

Numerical results of potentials S=1 parity minus

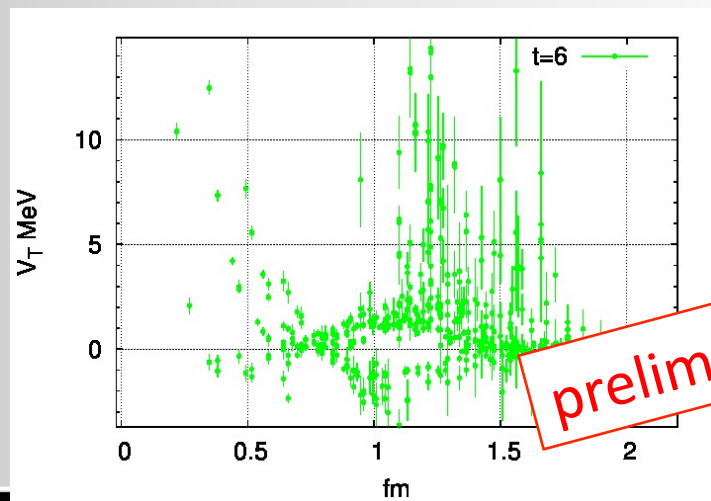
V_c : center force



V_{LS} : spin-orbit force



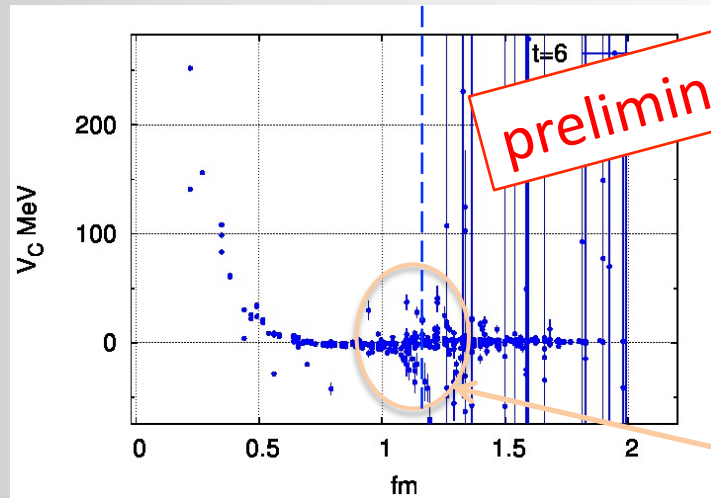
V_T : tensor force



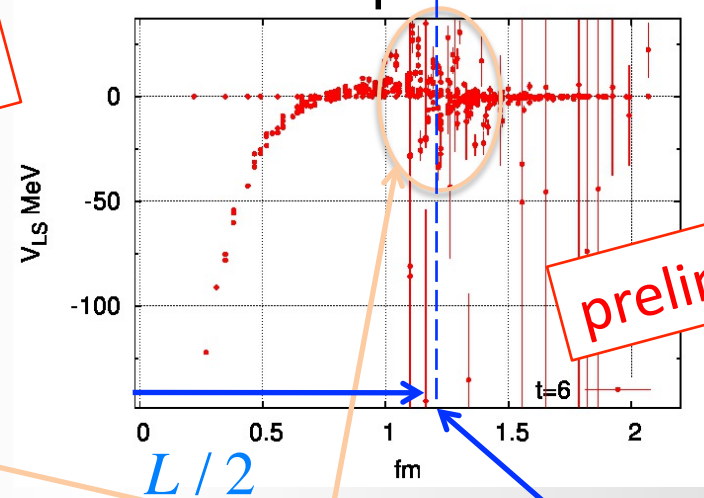
$$\left(\frac{\nabla^2}{m_N} + E \right) \phi(\vec{r}; E) = \left[V_c^{(+)}(r) + V_T^{(+)}(r) S_{12} + V_{LS}^{(+)}(r) \vec{L} \cdot \vec{S} \right] P^+ \phi(\vec{r}; E) \\ + \left[V_c^{(-)}(r) + V_T^{(-)}(r) S_{12} + V_{LS}^{(-)}(r) \vec{L} \cdot \vec{S} \right] P^- \phi(\vec{r}; E) + \mathcal{O}(\nabla^2)$$

Numerical results of potentials S=1 parity minus

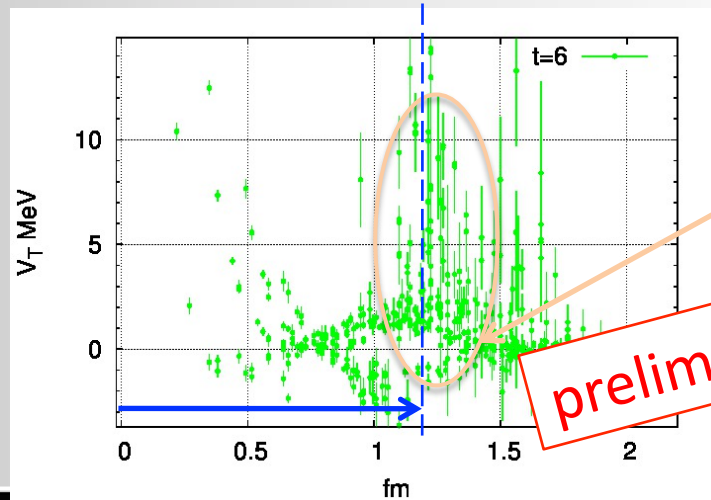
V_C : center force



V_{LS} : spin-orbit force

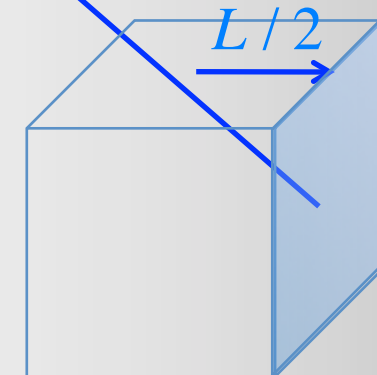


V_T : tensor force



there are strong fluctuation.

← caused by boundary.
NBS wave has strong finite effect

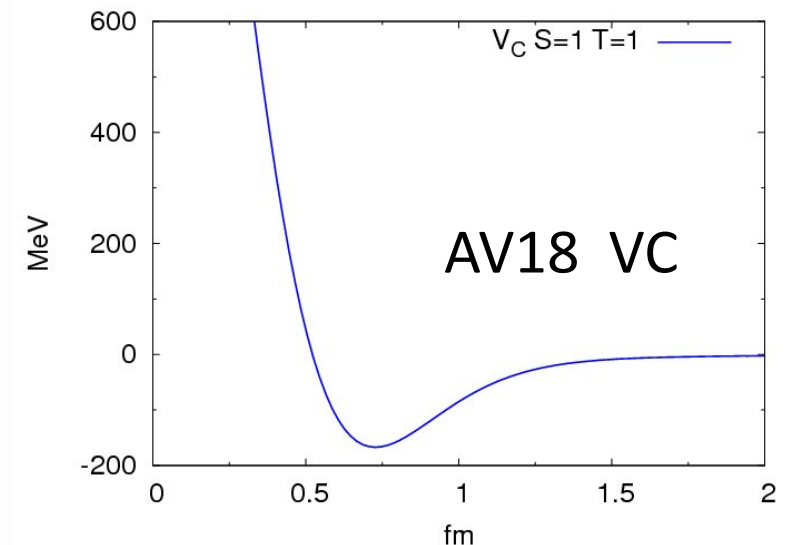
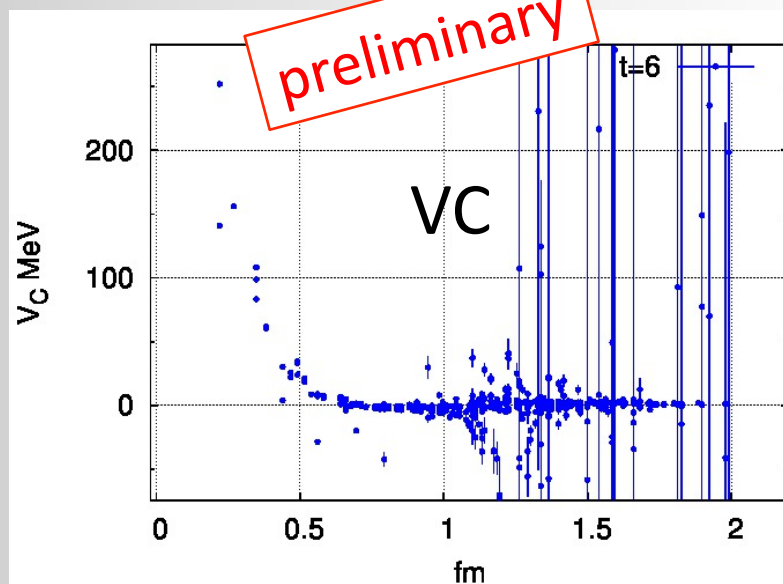


comparison with phenomenological potential (AV18)

central force has repulsive core
(qualitatively same with one with AV18)

Phys. Rev. C **51**, 38 (1995)

R. B. Wiringa, V. G. J. Stoks and R. Schiavilla,



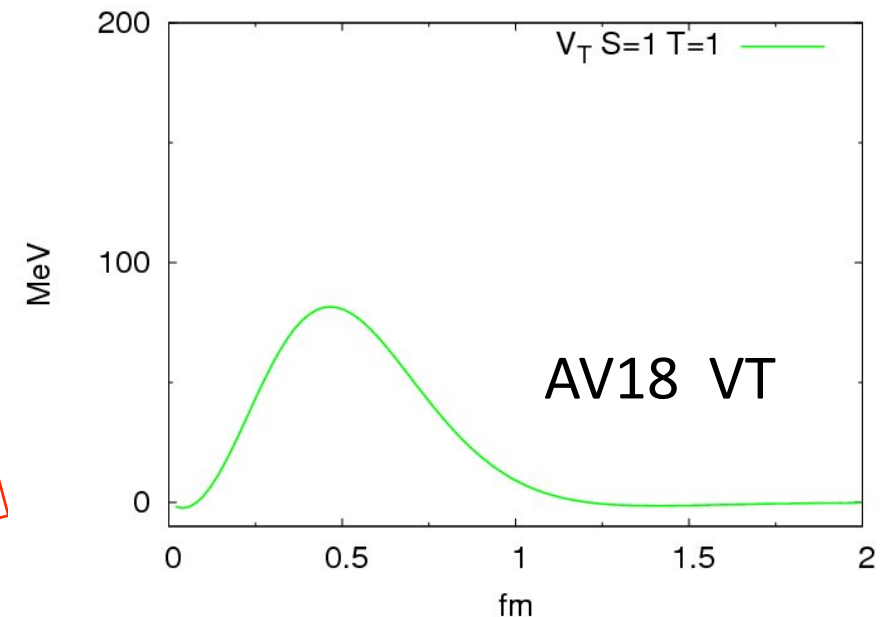
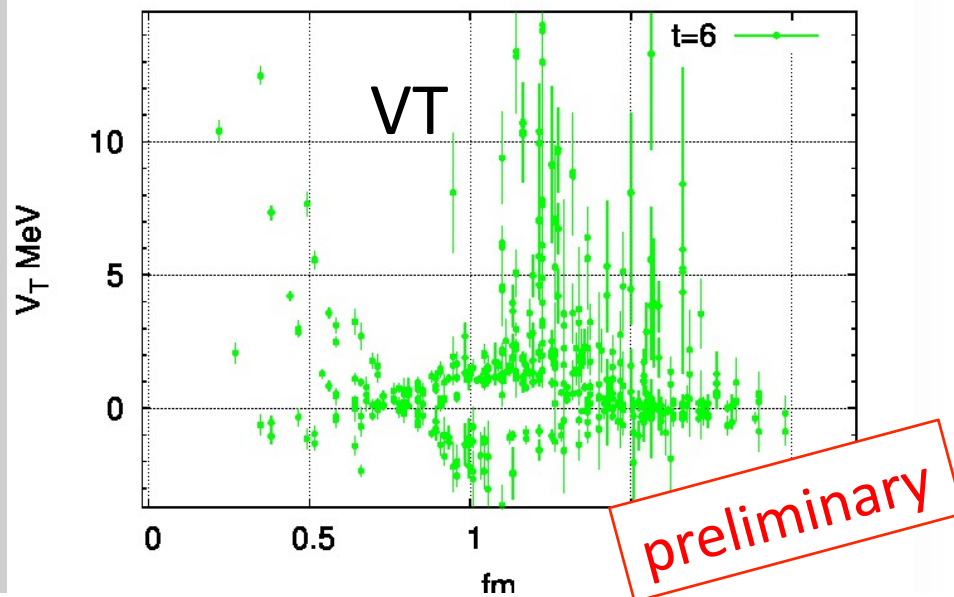
$$v_{ST}^R(NN) = v_{ST,NN}^c(r) + v_{ST,NN}^{l2}(r)L^2 + v_{ST,NN}^t(r)S_{12} + v_{ST,NN}^{ls}(r)\mathbf{L}\cdot\mathbf{S} + v_{ST,NN}^{ls2}(r)(\mathbf{L}\cdot\mathbf{S})^2$$

comparison with phenomenological potential (AV18)

tensor force is weak repulsive force
(qualitatively same with one with AV18)

Phys. Rev. C **51**, 38 (1995)

R. B. Wiringa, V. G. J. Stoks and R. Schiavilla,



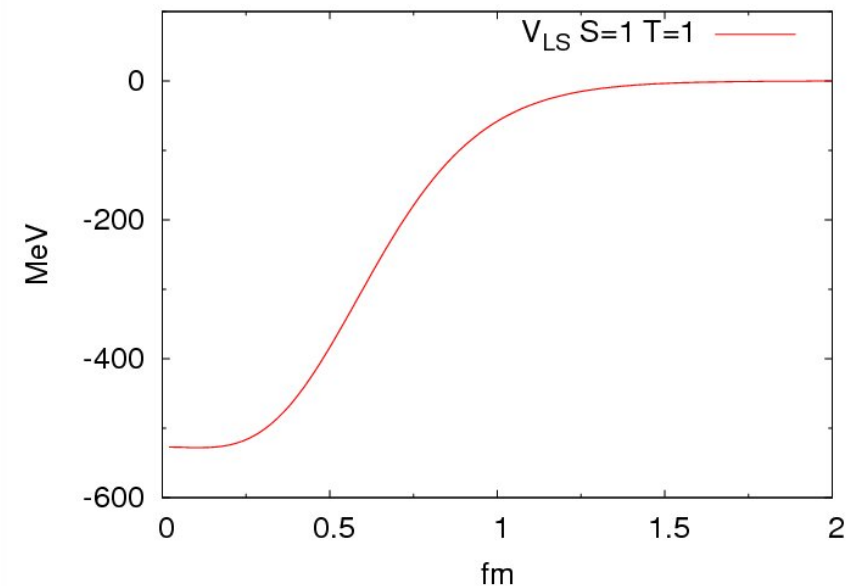
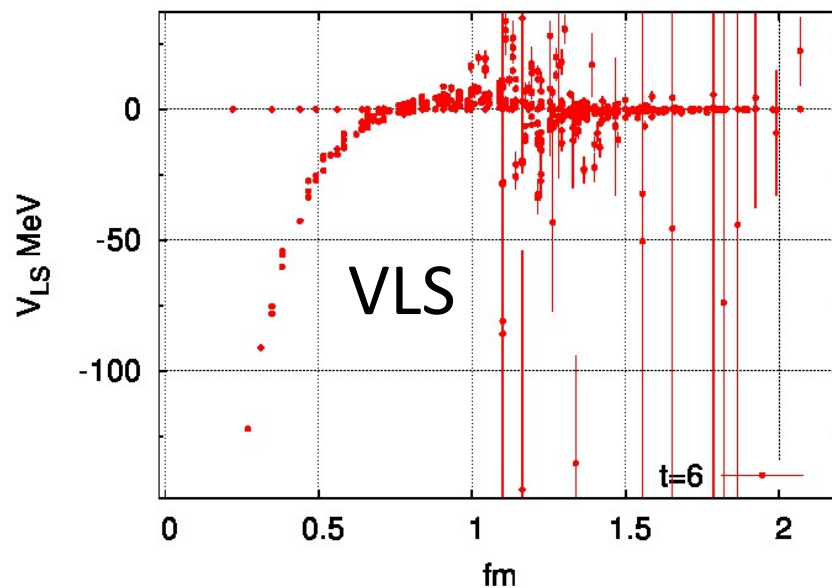
$$v_{ST}^R(NN) = v_{ST,NN}^c(r) + v_{ST,NN}^{l2}(r)L^2 + \boxed{v_{ST,NN}^t(r)S_{12}} + v_{ST,NN}^{ls}(r)\mathbf{L}\cdot\mathbf{S} + v_{ST,NN}^{ls2}(r)(\mathbf{L}\cdot\mathbf{S})^2$$

comparison with phenomenological potential (AV18)

LS force is strong attractive force
(qualitatively same with one with AV18)

Phys. Rev. C **51**, 38 (1995)

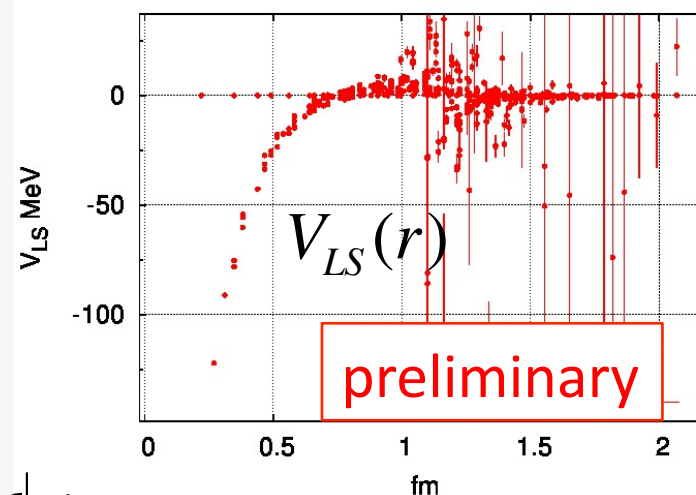
R. B. Wiringa, V. G. J. Stoks and R. Schiavilla,



$$v_{ST}^R(NN) = v_{ST,NN}^c(r) + v_{ST,NN}^{l2}(r)L^2 + v_{ST,NN}^t(r)S_{12} + \boxed{v_{ST,NN}^{ls}(r)\mathbf{L}\cdot\mathbf{S}} + v_{ST,NN}^{ls2}(r)(\mathbf{L}\cdot\mathbf{S})^2$$

Summary

We are currently preparing the calculation of potentials including **LS forces** for parity minus sector.



Future work

➤ parity plus LS force

- **physical point** ($m_{\pi} = 135$ MeV)
- applied for **YN** and **YY** interaction

will be performed in K-Computer (2012).

