Workshop on `Future Prospects of Hadron Physics at J-PARC and Large Scale Computational Physics'

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# Possibility of multi-antikaonic nuclei with hyperon-mixing

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# 1. Introduction  Strangeness nuclear physics at J-PARC

S= −1, −2

- Kaon-Nucleon (KN) interaction
- Kaon dynamics in nuclear medium

Kaonic nuclei ( bound state of single $K^-$ meson) [ Y.Akaishi and T.Yamazaki, Phys.Rev. C65 (2002) 044005. ]

→ Multi kaonic nuclear cluster

High energy

$(p\ p \rightarrow K^+ + K^- p\ p)$

$(p\ p \rightarrow K^+ K^+ + K^- K^- p\ p)$

[M. Hassanvand, Y.Akaishi, T.Yamazaki, Phys.Rev. C84, 015207 (2011). ]

J-PARC [ http://j-parc.jp/ ]


- Hyperon (Y)-N , YY interactions
- Hypernuclei

  ΛΛ hypernuclei, Ξ hypernuclei …

  [B.F.Gibson et al. Eds. Nucl. Phys. A835(2010) 1.]

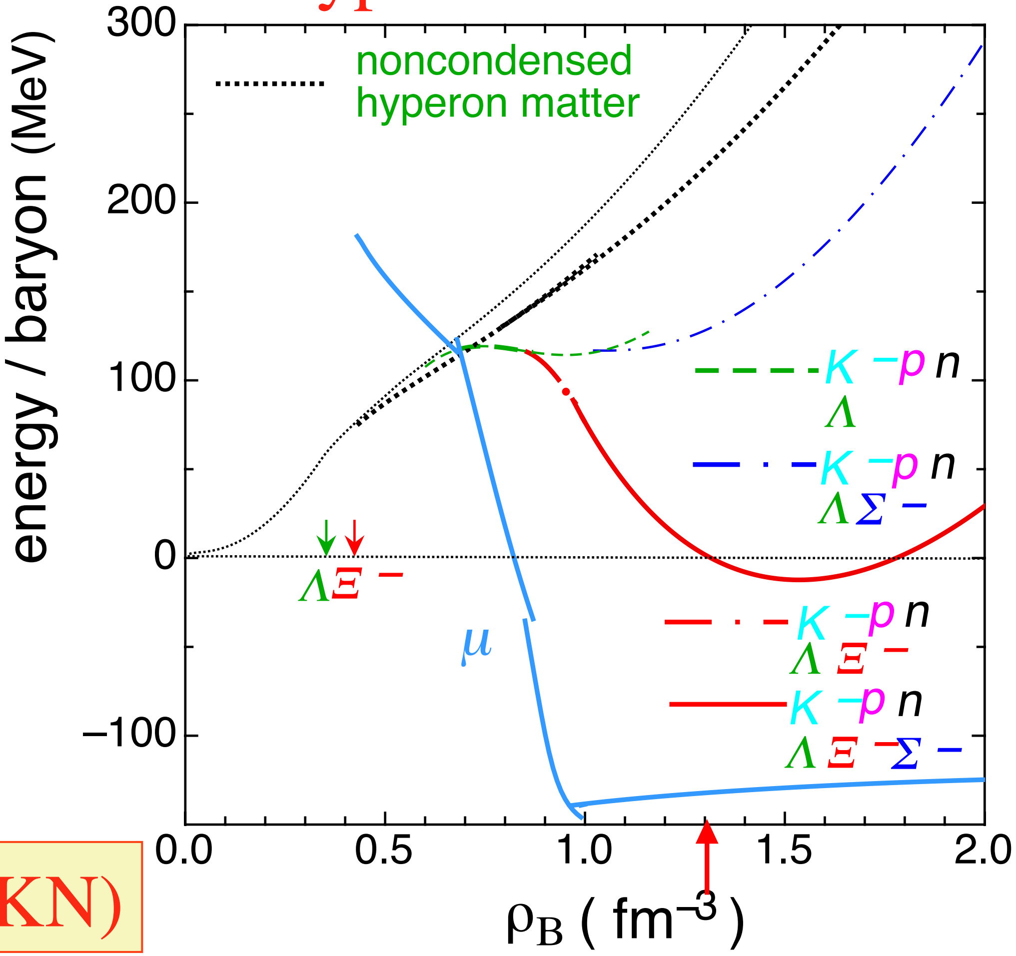
→ |S| >>1 (in neutron stars)

- Kaon condensation , Hyperon-mixing

# Coexistence of kaon condensation and hyperons in neutron stars

--- possibility of Self-bound states ---

[ T. Muto, Phys. Rev. C 77 (2008) 015810.]




A local energy minimum =>

highly dense self-bound object

For finite nuclei

|S| = 0, 1, 2, 3,・・

**Multi-Antikaonic Nuclei (MKN)**

relativistic mean-field theory (RMF)

[T. Muto, T. Maruyama and T. Tatsumi, Phys. Rev. C79, 035207 (2009). ]

central region: high density $\rho_B \sim 3.5\ \rho_0$

Meson-exchange models (MEM)

[ c.f. D. Gazda, E. Friedman, A. Gal, J. Mares, Phys. Rev. C76, 055204 (2007); Phys. Rev. C77, 045206 (2008). ]

A possible existence of antikaonic nuclear bound states with hyperon-mixing for finite nuclei within the RMF framework

Search for the ground state of MKN consisting of antikaons and hyperon-mixing

c.f. [D. Gazda, E. Friedman, A. Gal, J. Mares, Phys. Rev. C 80, 035205 (2009).]

Modification of kaon dynamics due to increase in strangeness number S

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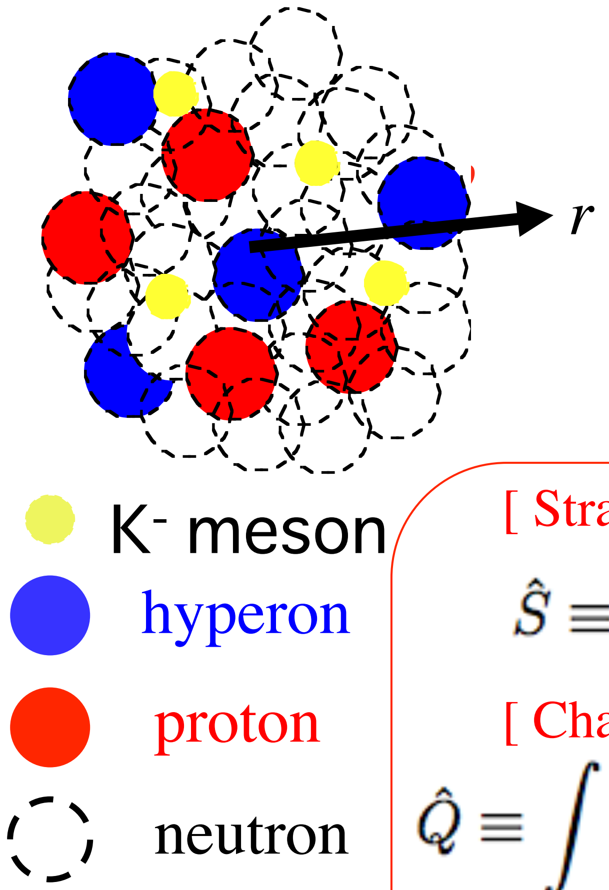
Relation to kaon condensation in neutron stars

## 2. Formulation

### 2-1 Outline of Kaon-condensed hypernuclei

Multi-$\bar{K}$ Nuclei




[ Initial target nucleus ]

$A = N + Z$ : mass number

$Z$: the number of proton

|S|: the number of the embedded $K^-$

Assume : Spherical symmetry

Local density approximation for baryons

$(p, n, \Lambda, \Sigma^-, \Xi^-)$

[ Strangeness conservation ]

$$\hat{S} \equiv \int d^3r \left(\rho_{K^-}(r) + \rho_\Lambda(r) + \rho_{\Sigma^-}(r) + 2\rho_{\Xi^-}(r)\right) = |S|$$

[ Charge conservation ]

$$\hat{Q} \equiv \int d^3r \left(\rho_p(r) - \rho_{K^-}(r) - \rho_{\Sigma^-}(r) - \rho_{\Xi^-}(r)\right) = Z - |S|$$

[ Baryon number conservation ]

$$\hat{N}_B \equiv \int d^3r \left(\rho_p(r) + \rho_n(r) + \rho_\Lambda(r) + \rho_{\Sigma^-}(r) + \rho_{\Xi^-}(r)\right) = A$$

## 2-2 Relativistic Mean-Field (RMF) theory

### Baryon-Baryon interaction $(p, n, \Lambda, \Sigma^-, \Xi^-)$

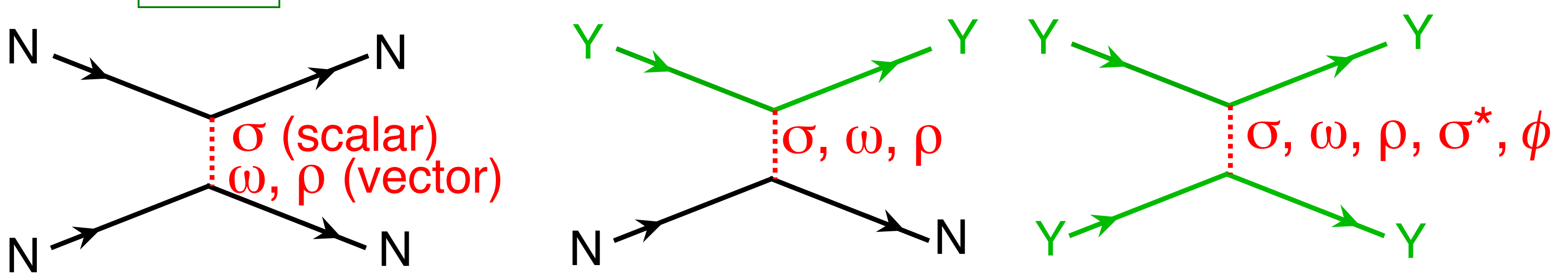
$$\mathcal{L}_{B,M} = \sum_B \overline{B}(i\gamma^\mu D_\mu - m_B^*)B + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) + \frac{1}{2}\left(\partial^\mu\sigma^*\partial_\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right)$$
$$- \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}R^{\mu\nu}R_{\mu\nu} + \frac{1}{2}m_\rho^2 R^\mu R_\mu - \frac{1}{4}\phi^{\mu\nu}\phi_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu$$
$$- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} ,$$

$$D^\mu \equiv \partial^\mu + ig_{\omega B}\omega^\mu + ig_{\rho B}\vec{\tau}\cdot\vec{R}^\mu + ig_{\phi B}\phi^\mu + iQA^\mu$$

$$m_B^*(r) = m_B - g_{\sigma B}\sigma(r) - g_{\sigma^* B}\sigma^*(r)$$

N-N $(N = p, n)$

Y-N, Y-Y $(Y = \Lambda, \Sigma^-, \Xi^-)$




$\sigma^* \sim \bar{s}s \qquad \phi \sim \bar{s}\gamma s$

## 2-3 $\bar{K}-N, \bar{K}-\bar{K}$ interactions

Nonlinear effective chiral Lagrangian

[ D. B. Kaplan and A. E. Nelson, Phys. Lett. B 175 (1986) 57. ]

Meson fields ($K^{\pm}$) (nonlinear representation)

$$\Sigma \equiv e^{2i\Pi/f} \qquad \Pi = \pi_a T_a = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & K^+ \\ 0 & 0 & 0 \\ K^- & 0 & 0 \end{pmatrix}$$

$f$ =93 MeV

Condensate assumption

($K^-$ mesons are condensed in the lowest energy state) → S-wave

$$\langle K^- \rangle = \frac{f}{\sqrt{2}} \theta(\mathbf{r})$$

$$\mathcal{L}_{KB} = \frac{1}{2}\left\{1 + \left(\frac{\sin\theta}{\theta}\right)^2\right\}\partial^\mu K^+ \partial_\mu K^- + \frac{1-\left(\frac{\sin\theta}{\theta}\right)^2}{2f^2\theta^2}\left\{(K^+\partial_\mu K^-)^2 + (K^-\partial_\mu K^+)^2\right\}$$
$$- \left[m_K^2 - \frac{1}{f^2}\sum_{B=p,n,\Lambda,\Sigma^-,\Xi^-} \Sigma_{KB}\bar{B}B\right]\left(\frac{\sin(\theta/2)}{\theta/2}\right)^2 K^+K^-$$
$$+ i\frac{1}{2f^2}\left(\bar{p}\gamma^\mu p + \frac{1}{2}\bar{n}\gamma^\mu n - \frac{1}{2}\bar{\Sigma}^-\gamma^\mu\Sigma^- - \bar{\Xi}^-\gamma^\mu\Xi^-\right)\left(\frac{\sin(\theta/2)}{\theta/2}\right)^2 (K^+\partial_\mu K^- - \partial_\mu K^+ K^-)$$

S-wave scalar int.

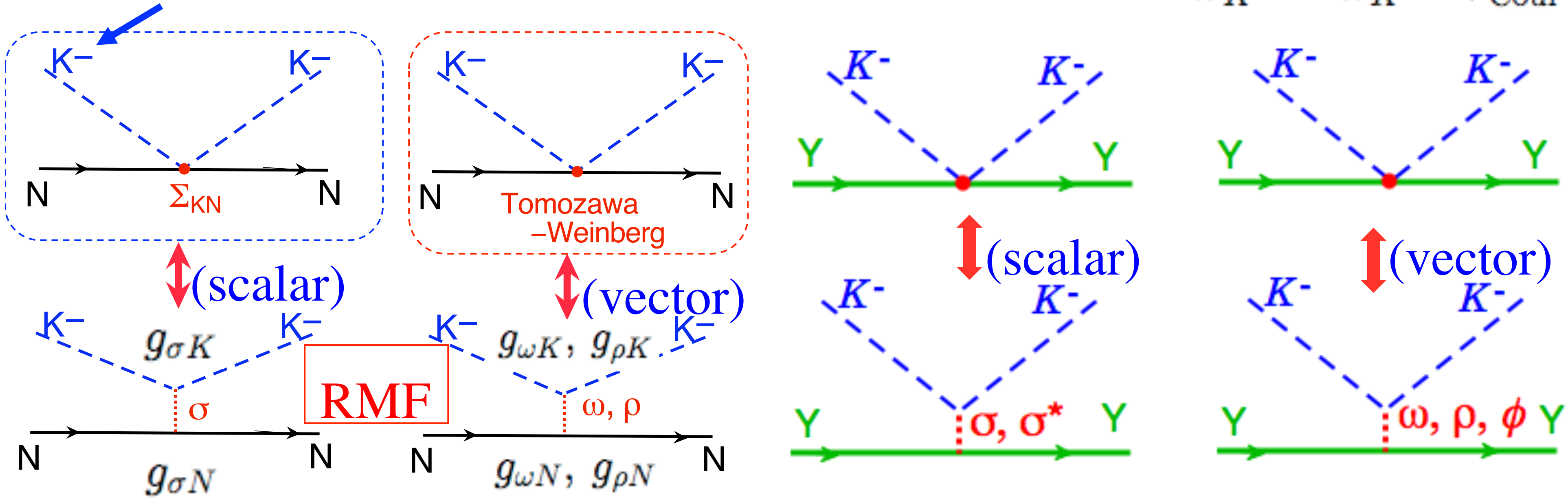
$$m_K^{*2} \equiv m_K^2 - 2g_{\sigma K} m_K \sigma - 2g_{\sigma^* K} m_K \sigma^*$$

S-wave vector int.

$$X_0 \equiv g_{\omega K}\omega_0 + g_{\rho K} R_0 + g_{\phi K}\phi_0$$

kaon fields ($K^{\pm}$) (nonlinear representation)

$$\tilde{\omega}_{K^-} = \omega_K - V_{\rm Coul}$$




## 2-4 Thermodynamic potential

$$\Omega = \int d^3r \mathcal{H}(r) + \mu_s \hat{S} + \mu_Q \hat{Q} + \nu \hat{N}_B$$

$\delta\Omega = 0$ as $\rho_a \rightarrow \rho_a + \delta\rho_a$

$(a = K^-, p, n, \Lambda, \Sigma^-, \Xi^-)$

$$\omega_{K^-} = \mu_Q - \mu_s$$
$$\mu_p = -(\mu_Q + \nu)$$
$$\mu_n = -\nu$$
$$\mu_\Lambda = -(\mu_s + \nu)$$
$$\mu_{\Sigma^-} = \mu_Q - \mu_s - \nu$$
$$\mu_{\Xi^-} = \mu_Q - 2\mu_s - \nu$$

Chemical equilibrium for strong processes

$$\omega_{K^-} + \mu_p = \mu_\Lambda$$
$$\omega_{K^-} + \mu_n = \mu_{\Sigma^-}$$
$$\omega_{K^-} + \mu_\Lambda = \mu_{\Xi^-}$$

## 2-5 Equations of motion for meson fields

(coherent state)

$\langle K^- \rangle = \frac{f}{\sqrt{2}}\theta(\mathbf{r})$

$\delta\Omega/\delta\theta(r) = 0$ K$^-$ field equation ・nonlinear $\bar{K}$-$\bar{K}$ int.

$\widetilde{\omega}_{K^-} = \omega_{K^-} - V_{\rm Coul}$

$$\nabla^2\theta = \sin\theta\Big[\big(m_K^2 - 2g_{\sigma K}m_K\sigma - 2g_{\sigma^* K}m_K\sigma^*\big) - 2\widetilde{\omega}_{K^-}(g_{\omega K}\omega_0 + g_{\rho K}R_0 + g_{\phi K}\phi_0) - \widetilde{\omega}_{K^-}^2\cos\theta\Big]$$

$$-\nabla^2\sigma + m_\sigma^2\sigma = -\frac{dU}{d\sigma} + g_{\sigma N}(\rho_p^s + \rho_n^s) + g_{\sigma\Lambda}\rho_\Lambda^s + g_{\sigma\Sigma^-}\rho_{\Sigma^-}^s + g_{\sigma\Xi^-}\rho_{\Xi^-}^s + 2f^2 g_{\sigma K}m_K(1-\cos\theta)$$

$$-\nabla^2\omega_0 + m_\omega^2\omega_0 = g_{\omega N}(\rho_p + \rho_n) + g_{\omega\Lambda}\rho_\Lambda + g_{\omega\Sigma^-}\rho_{\Sigma^-} + g_{\omega\Xi^-}\rho_{\Xi^-} - 2f^2 g_{\omega K}\widetilde{\omega}_K(1-\cos\theta)$$

$$-\nabla^2 R_0 + m_\rho^2 R_0 = g_{\rho N}(\rho_p - \rho_n) + g_{\rho\Lambda}\rho_\Lambda - g_{\rho\Sigma^-}\rho_{\Sigma^-} - g_{\rho\Xi^-}\rho_{\Xi^-} - 2f^2 g_{\rho K}\widetilde{\omega}_K(1-\cos\theta)$$

$$-\nabla^2\sigma^* + m_{\sigma^*}^2\sigma^* = g_{\sigma^*\Lambda}\rho_\Lambda^s + g_{\sigma^*\Sigma^-}\rho_{\Sigma^-}^s + g_{\sigma^*\Xi^-}\rho_{\Xi^-}^s + 2f^2 g_{\sigma^* K}m_K(1-\cos\theta)$$

$$-\nabla^2\phi_0 + m_\phi^2\phi_0 = g_{\phi\Lambda}\rho_\Lambda + g_{\phi\Sigma^-}\rho_{\Sigma^-} + g_{\phi\Xi^-}\rho_{\Xi^-} - 2f^2 g_{\phi K}\widetilde{\omega}_K(1-\cos\theta)$$

$$\nabla^2 V_{\rm Coul} = 4\pi e^2(\rho_p - \rho_{\Sigma^-} - \rho_{\Xi^-} - \rho_{K^-})$$

## 2-6 Choice of parameters

--- NN interaction --- gross features of normal nuclei and nuclear matter

- saturation properties of nuclear matter ($\rho_0$ =0.153 fm$^{-3}$)
- binding energy of nuclei and proton-mixing ratio
- density distributions of p and n

} $g_{\sigma N}$, $g_{\omega N}$, $g_{\rho N}$

--- vector meson couplings for Y --- SU(6) symmetry

$$g_{\omega\Lambda} = g_{\omega\Sigma^-} = 2g_{\omega\Xi^-} = \frac{2}{3}g_{\omega N} \qquad g_{\rho\Lambda} = 0 \qquad g_{\rho\Sigma^-} = 4g_{\rho\Xi^-} = 4g_{\rho N}$$

$$g_{\phi\Lambda} = g_{\phi\Sigma^-} = \frac{1}{2}g_{\phi\Xi^-} = -\frac{\sqrt{2}}{3}g_{\omega N}$$

--- scalar meson couplings for Y ---

$$U_\Lambda^N(\rho_0) = -g_{\sigma\Lambda}\sigma + g_{\omega\Lambda}\omega_0 = -27 \text{ MeV} \rightarrow g_{\sigma\Lambda} = 3.84$$

$$U_{\Sigma^-}^N(\rho_0) = -g_{\sigma\Sigma^-}\sigma + g_{\omega\Sigma^-}\omega_0 = 23.5 \text{ MeV}$$ repulsive case → $g_{\sigma\Sigma^-} = 2.28$

$$U_{\Xi^-}^N(\rho_0) = -g_{\sigma\Xi^-}\sigma + g_{\omega\Xi^-}\omega_0 = -16 \text{ MeV} \rightarrow g_{\sigma\Xi^-} = 2.0$$

$$g_{\sigma^* N} = g_{\sigma^*\Lambda} = g_{\sigma^*\Sigma^-} = 0$$

--- vector meson couplings for Kaon --- quark and isospin counting rule

$$g_{\omega K} = g_{\omega N}/3 \quad g_{\rho K} = g_{\rho N}$$

$$g_{\phi K} = 6.04/\sqrt{2}$$

--- scalar meson couplings for Kaon ---

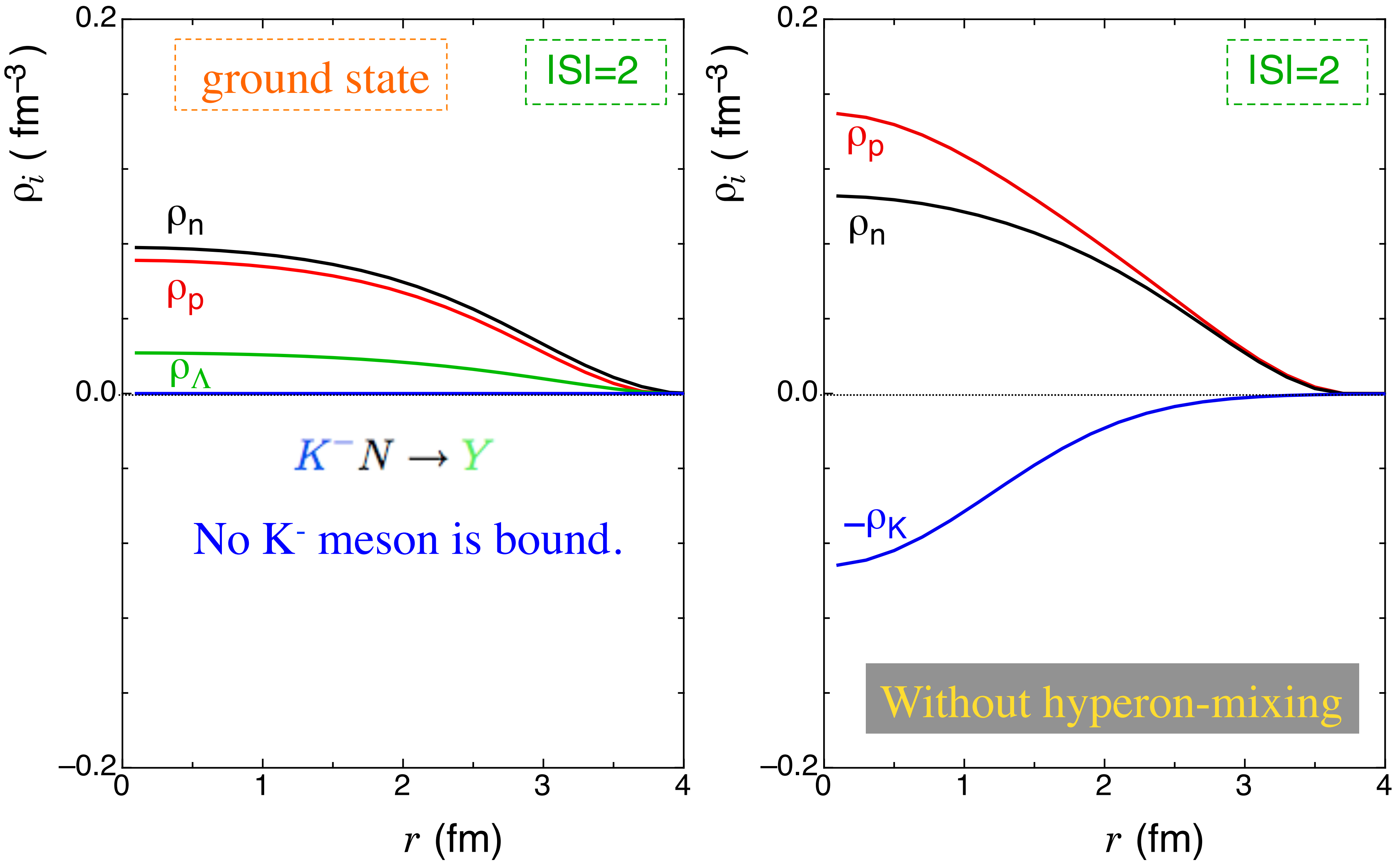
$$U_K = -(g_{\sigma K}\sigma + g_{\omega K}\omega_0) \rightarrow g_{\sigma K}$$

at $\rho_0$ in symmetric nuclear matter

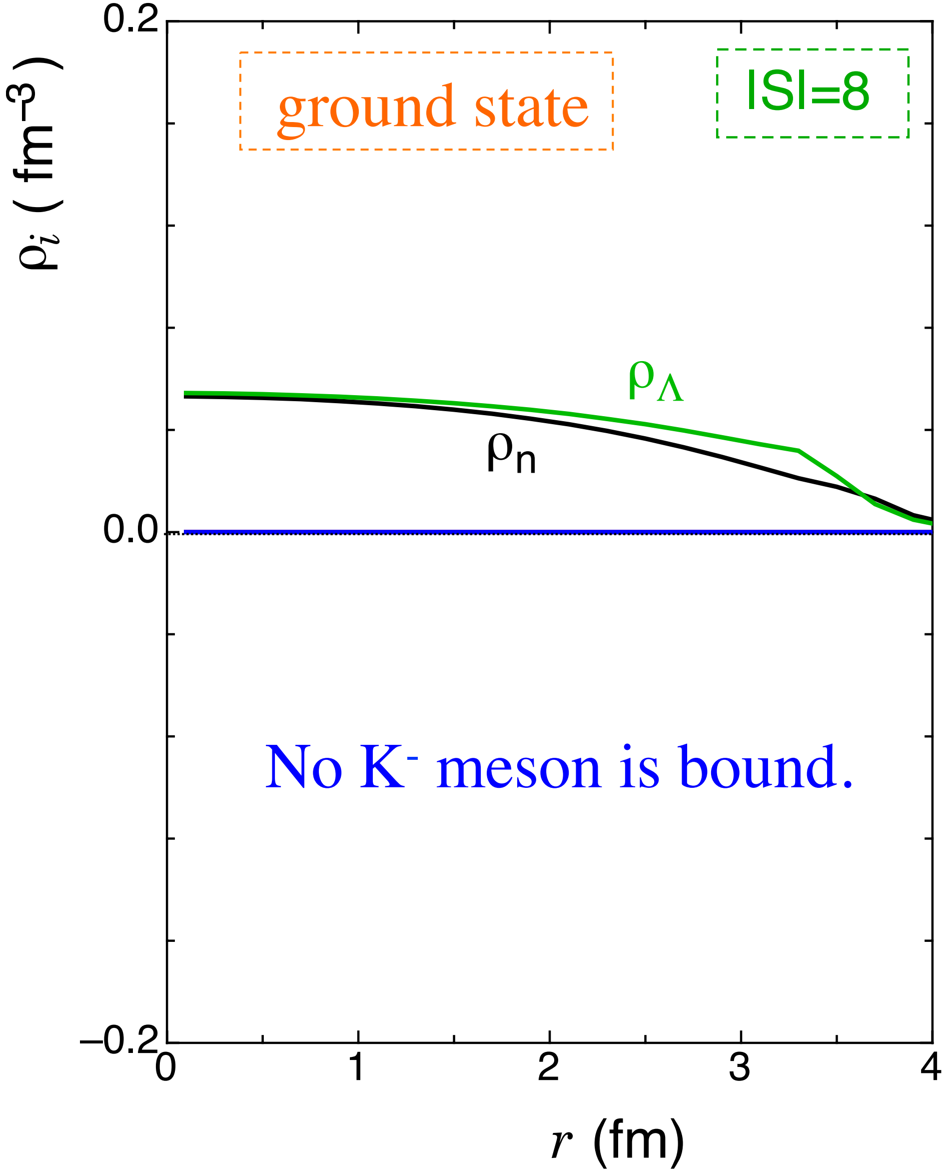
$$g_{\sigma^* K} = 2.65/2 \leftarrow$$ Decay of $f_0(975)$

## 3. Numerical results    3-1 density distributions

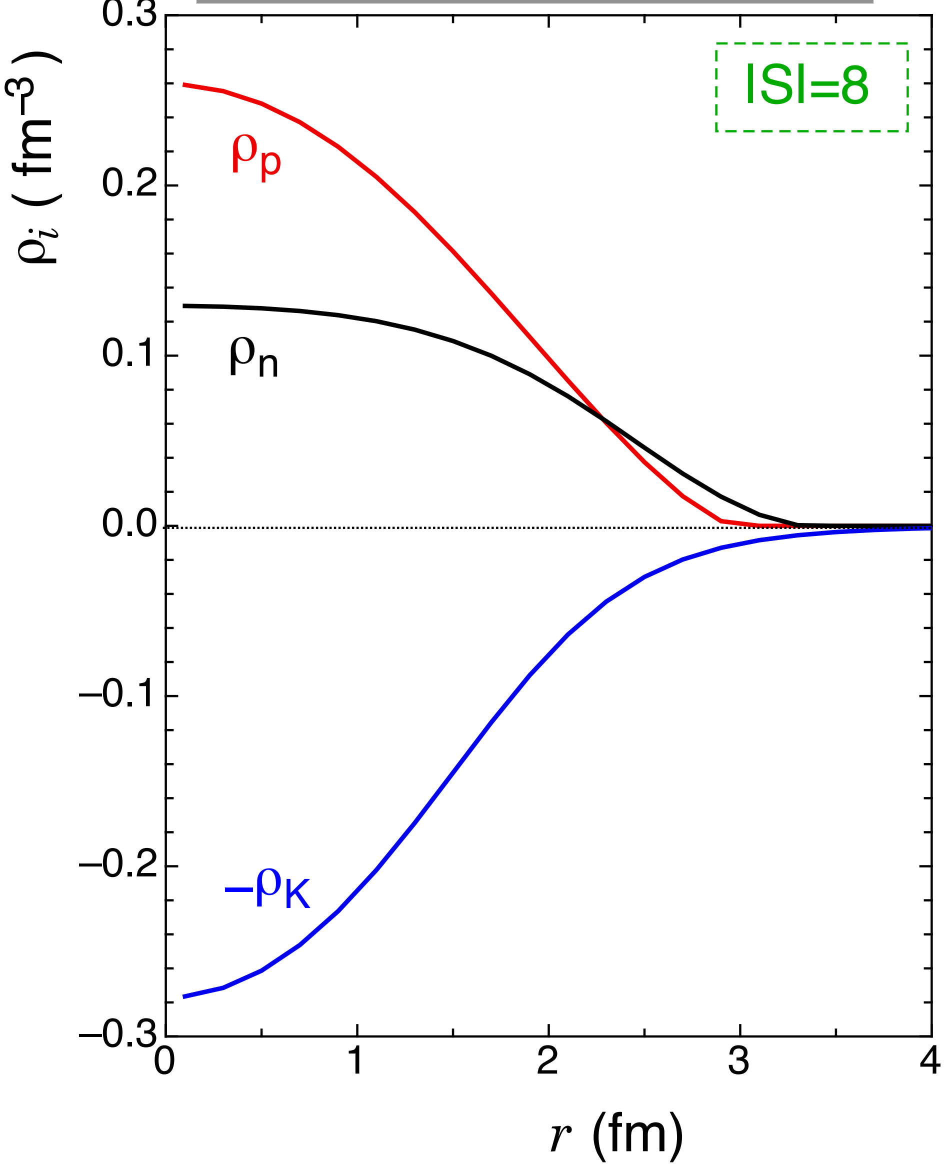
A=15, Z=8 ( $^{15}_{8}O$ )    $U_K = -80$ MeV    ($\Sigma_{KN} \sim 330$ MeV)



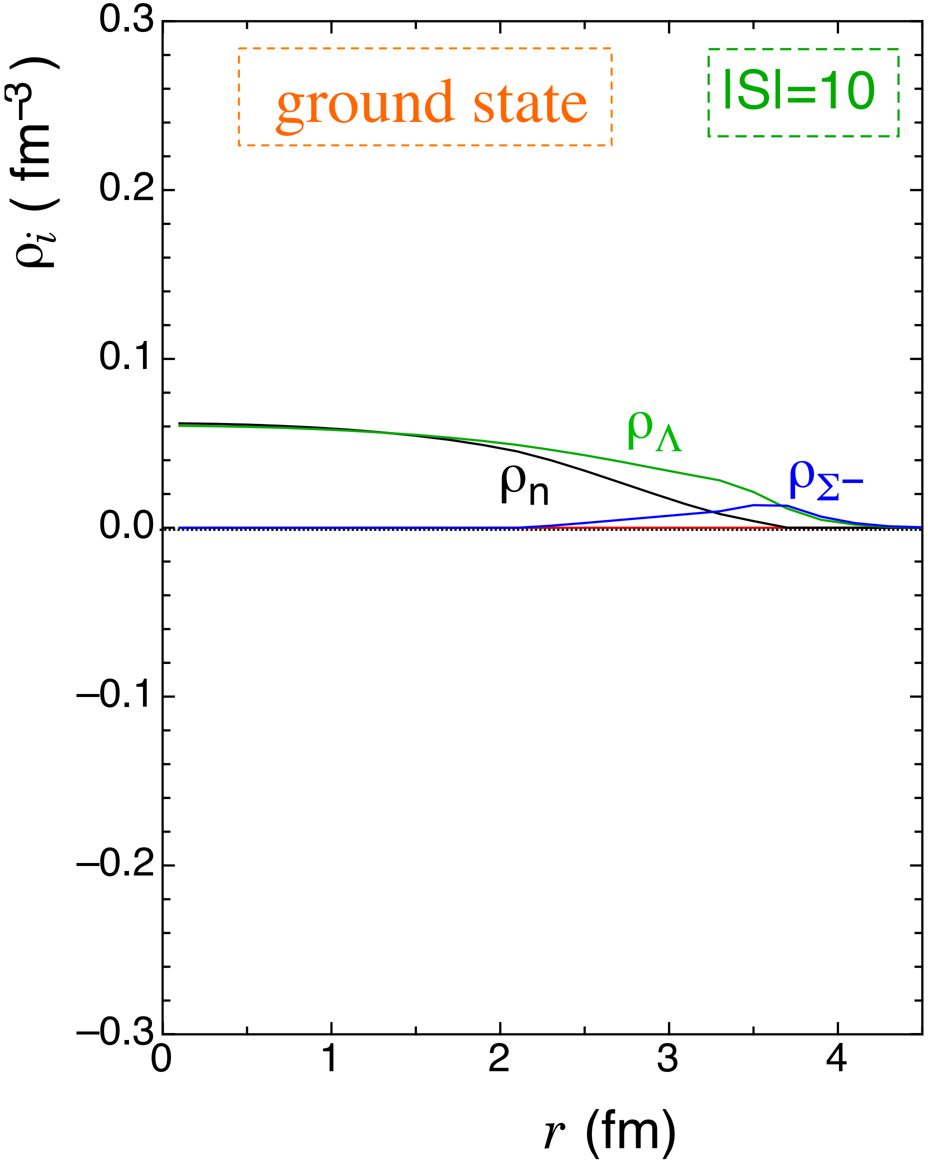
$U_K = -80$ MeV



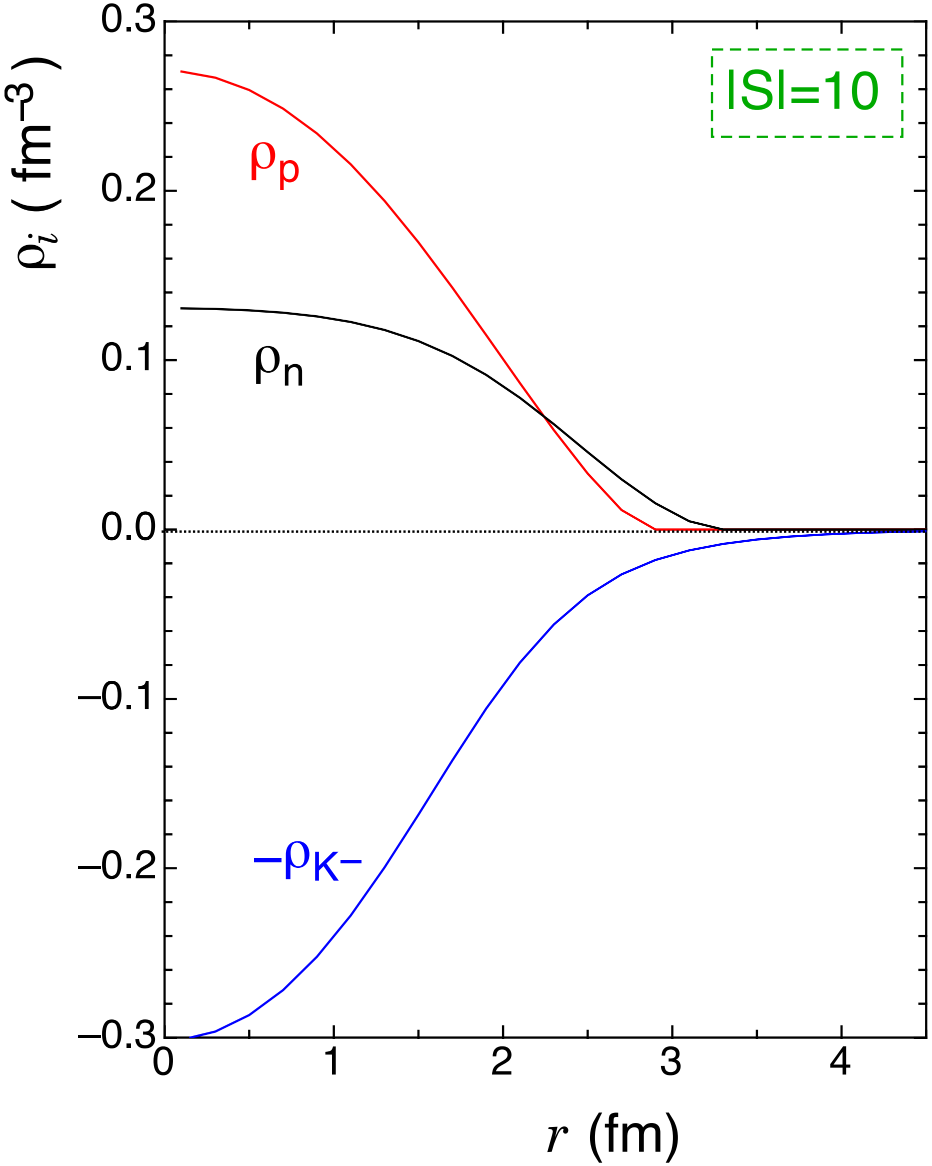

Without hyperon-mixing



$U_K = -80$ MeV

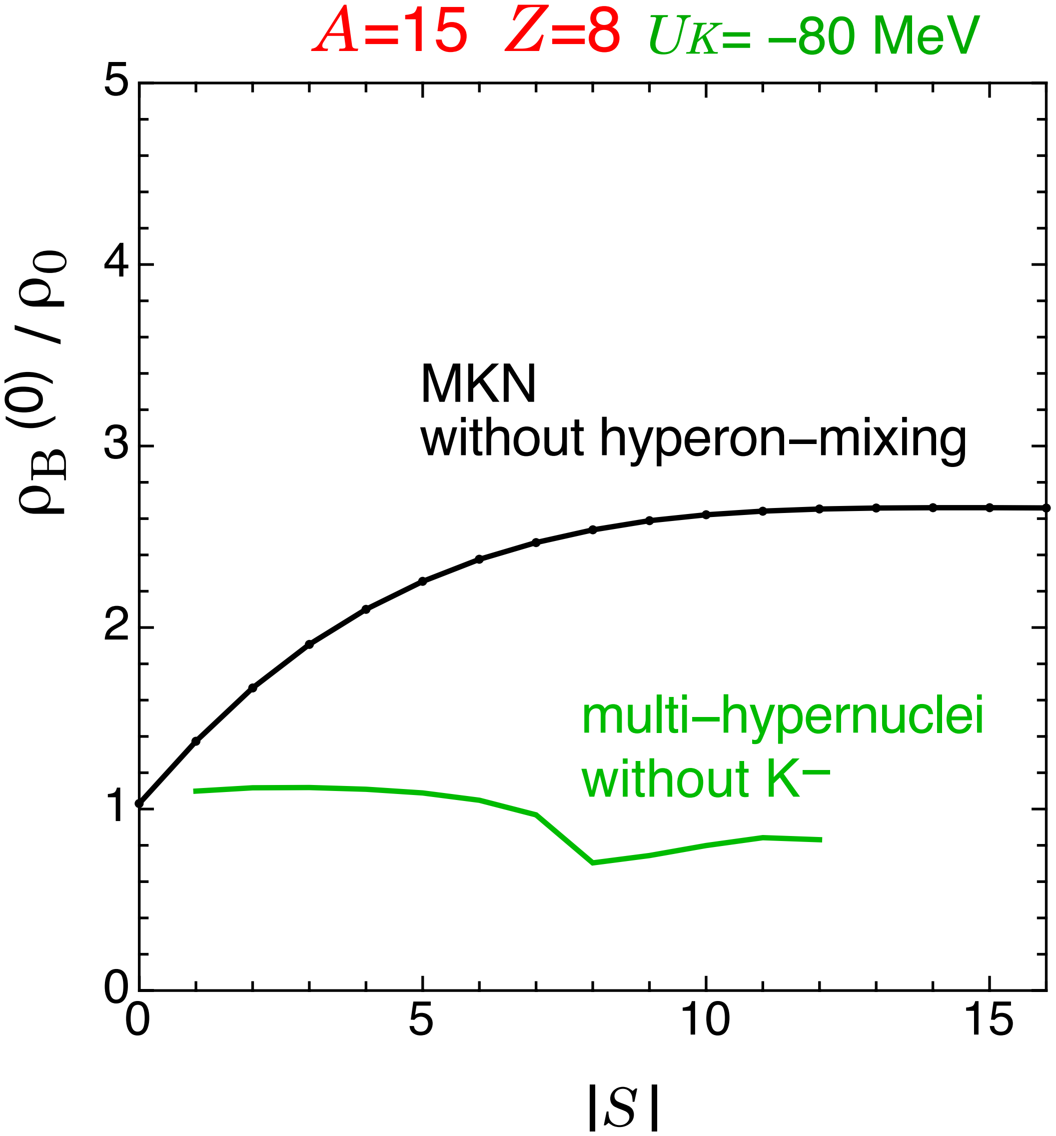



Without hyperon-mixing



# 3-2 |S|-dependence of central density $\rho_B^{(0)}$

$U_K$= - 80 MeV




Central region

For MKN

$K^-$ mesons attract protons due to the strong $I$=0 $\bar{K}N$ int.

$\rho_B^{(0)} \sim 2.6\ \rho_0$ for large |S|

For Multi-hypernuclei

No bound $\bar{K}$ meson

$\rho_B^{(0)} \sim \rho_0$

# 4. Comparison with kaon condensation in neutron stars

Finite system formed in laboratory

$\omega_{K^-} = 400 \sim 450$ MeV $> \mu_\Lambda - \mu_p$ ← Finite effects of nuclei

Chemical equilibrium for strong processes

$K^- p \rightleftharpoons \Lambda, \quad K^- \Lambda \rightleftharpoons \Xi^-, \quad K^- n \rightleftharpoons \Sigma^-$

hard to satisfy

↕

In neutron stars

Infinite matter

chemical equilibrium for weak processes

$n \rightleftharpoons p\, K^- \qquad p\, e^- \rightleftharpoons \Lambda\ (\nu_e) \qquad n\, e^- \rightleftharpoons \Sigma^-\ (\nu_e)$

$n \rightleftharpoons p\, e^-\ (\bar{\nu}_e) \qquad \Lambda\, e^- \rightleftharpoons \Xi^-\ (\nu_e)$

$K^-$ chemical potential : $\omega_{K^-} = \mu = \mu_n - \mu_p < 0$ for high densities

## 5. Summary and outlook

We have considered a possible existence of kaonic bound nuclei with hyperon-mixing in a framework of the RMF combined with nonlinear effective chiral Lagrangian for $\bar{K}-B$ and $\bar{K}-\bar{K}$ interactions.

- Due to the finite effects of nuclei, $K^-$ mesons do not receive attractions from surrounding baryons, and the lowest energy for $K^-$ mesons ($\omega_{K^-}$ ~ 400 MeV) is not lowered in comparison with the case of infinite matter ($\mu$ <~ 200 MeV).

→ All the strangeness initially carried by $K^-$ mesons is absorbed into nucleons and is taken over by hyperons through $K^- N \to Y$.

Central density $\rho_B^{(0)} \sim \rho_0$

- No $\Xi^-$ -mixing → Take into account of nonmesonic processes,

$\Lambda\Lambda \rightleftarrows \Xi^- p \quad \Lambda\Sigma^- \rightleftarrows \Xi^- n$

in addition to mesonic process $K^-\Lambda \rightleftarrows \Xi^-$

## Outlook

**Role of hyperons (Y)**

- inelastic channel coupling effects (kaon decay width . . . )

$$\bar{K}N \rightarrow \pi\Lambda, \pi\Sigma$$

- Role of P-wave KNY interactions

Quasi-baryons

$$|\widetilde{\Lambda}_{\pm}\rangle = \cos\phi \cdot |\Lambda_{\pm 1/2}\rangle \pm i \sin\phi \cdot |p_{\pm 1/2}\rangle$$

$$|\widetilde{n}_{\pm}\rangle = \cos\phi' \cdot |n_{\pm 1/2}\rangle \pm i \sin\phi' \cdot |\Sigma^{-}_{\pm 1/2}\rangle$$

- Realistic framework beyond the local density approximation for baryons