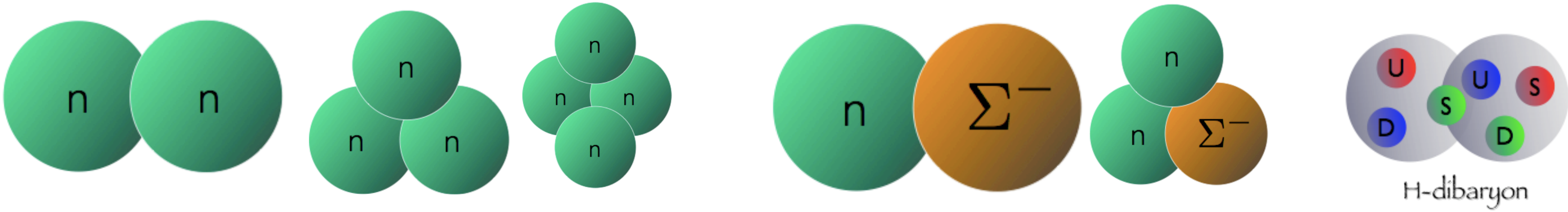
Nuclear Physics with Lattice QCD

make predictions of the structure and interactions of nuclei using LQCD

Baryonic interactions from Lattice QCD

Workshop on 'Future Prospects of Hadron Physics at J-PARC and Large Scale Computational Physics'

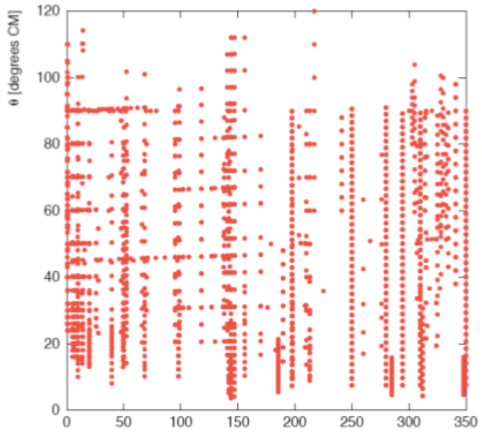


Why we should use Lattice QCD for the study of hadronic processes in nuclear physics?

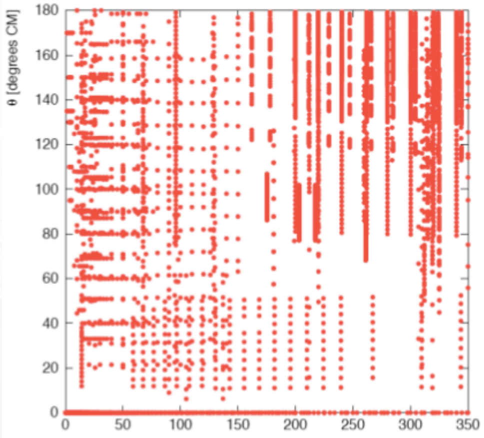
improve our understanding of low-energy QCD
understanding nuclear processes from the underlying theory of strong interactions
first principle calculation
uncertainties can be quantified

Lüscher 's formalism

proton-proton



neutron-proton



Strange sector ~ 35 data points
(many pre-1971) with large errors

Λ p	# = 12	6.5 MeV < T_{lab} < 50 MeV
Σ^- p → Σ^- p Λ n Σ^0 n	# = 6 # = 6 # = 6	9 MeV < Tlab < 12 MeV
Σ^+ p	# = 4	9 MeV < Tlab < 13 MeV + 3 data from KEK-E289

©Rob Timmermans

Additional information:

YN → Light hypernuclei:

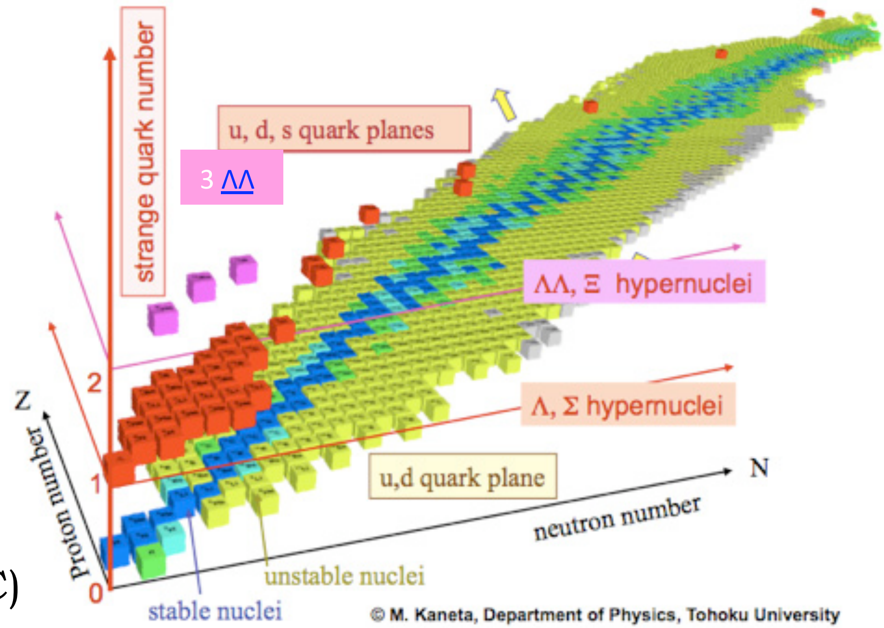
$^3H_\Lambda$, $^4He_\Lambda$, $^4H_\Lambda$, $^5He_\Lambda$

YY → $^6He_{\Lambda\Lambda}$, $^{10}Be_{\Lambda\Lambda}$, $^{13}B_{\Lambda\Lambda}$ …

39 Λ
1 Σ

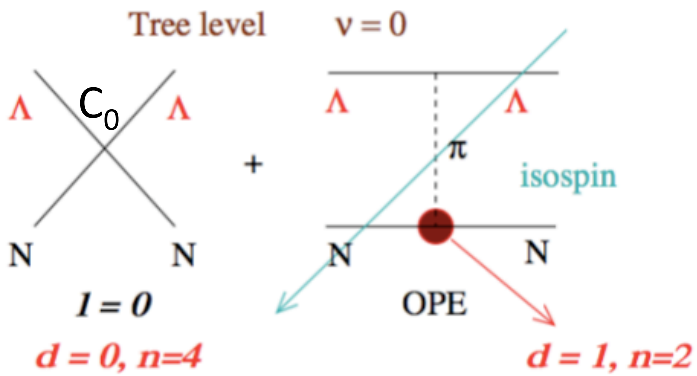
$pp \to K^+ \Lambda p$ (COSY, Jülich)

$\gamma d \to K^+ n \Lambda$ (CEBAF, ELSA, JLAB, MAMI-C)

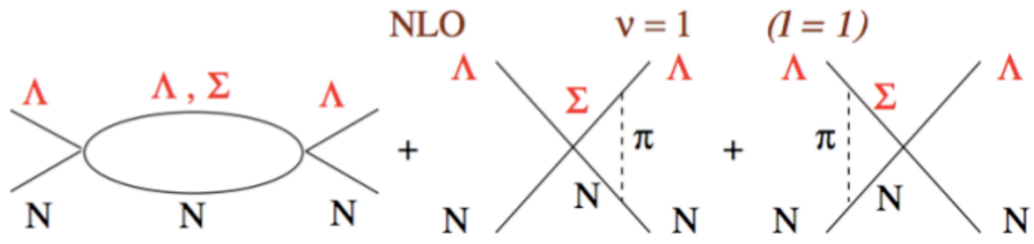


Effective Field Theory for the Low-Energy ΛN interaction

Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027

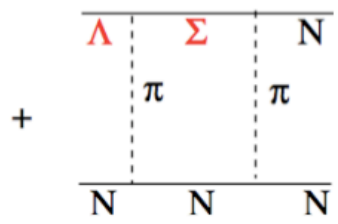


s-wave scattering



2 body diagrams (Weinberg)

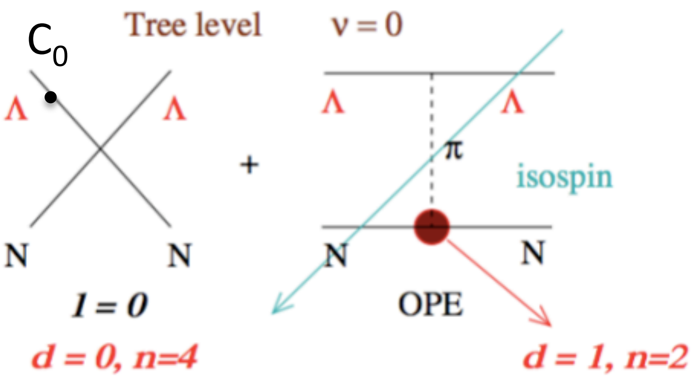
$$\nu = 2L + \sum_i V_i [d_i + \frac{1}{2} n_i - 2]$$

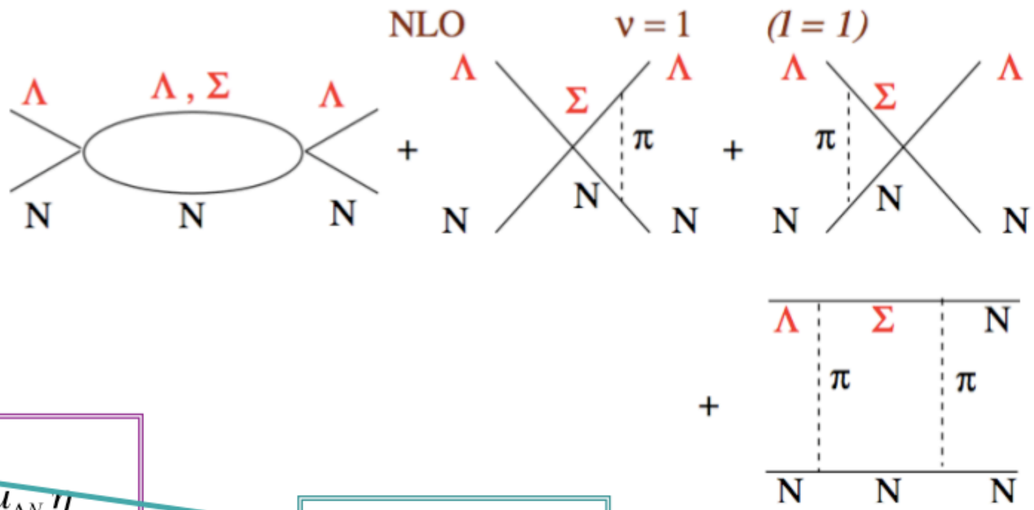


$SU(2)_L \times SU(2)_R$ two-flavor χ PT

Effective Field Theory for the Low-Energy ΛN interaction

Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027





$$a^{(^1S_0)} = -\frac{\mu_{\Lambda N}}{2\pi}\Bigg[\, {}_{\Lambda\Lambda}C_0^{(^1S_0)} - \frac{3}{4\pi}\left({}_{\Sigma\Lambda}C_0^{(^1S_0)}\right)^2 \mu_{\Lambda N}\,\eta + {}_{\Sigma\Lambda}C_0^{(^1S_0)}\,\frac{3 g_{\Sigma\Lambda} g_A \mu_{\Lambda N}}{2\pi f^2}\,\frac{\eta^2+\eta m_\pi+m_\pi^2}{\eta+m_\pi} - \frac{3 g_{\Sigma\Lambda}^2 g_A^2 \mu_{\Lambda N}}{4\pi f^4}\,\frac{2\eta^3+4\eta^2 m_\pi+6\eta m_\pi^2+3m_\pi^3}{2(\eta+m_\pi)^2}\Bigg]$$

Extract LECs

Result of the LQCD simulation

$$r^{(^1S_0)} = -\frac{1}{\mu_{\Lambda N}\pi}\left[\frac{2\pi}{{}_{\Lambda\Lambda}C_0^{(^1S_0)}}\right]^2 \Bigg[\, \frac{3}{8\pi}\left({}_{\Sigma\Lambda}C_0^{(^1S_0)}\right)^2 \frac{\mu_{\Lambda N}}{\eta} + {}_{\Sigma\Lambda}C_0^{(^1S_0)}\,\frac{3 g_{\Sigma\Lambda} g_A \mu_{\Lambda N}}{2\pi f^2}\,\frac{3\eta^2+9\eta m_\pi+8m_\pi^2}{6(\eta+m_\pi)^3} - \frac{3 g_{\Sigma\Lambda}^2 g_A^2 \mu_{\Lambda N}}{4\pi f^4}\,\frac{6\eta^3+23\eta^2 m_\pi+28\eta m_\pi^2+7m_\pi^3}{12(\eta+m_\pi)^4}\Bigg]$$

For example, for the singlet channel

$\eta = \sqrt{2\mu_{\Lambda N}\Delta_{\Lambda\Sigma}}$

$g_A \sim 1.27$,

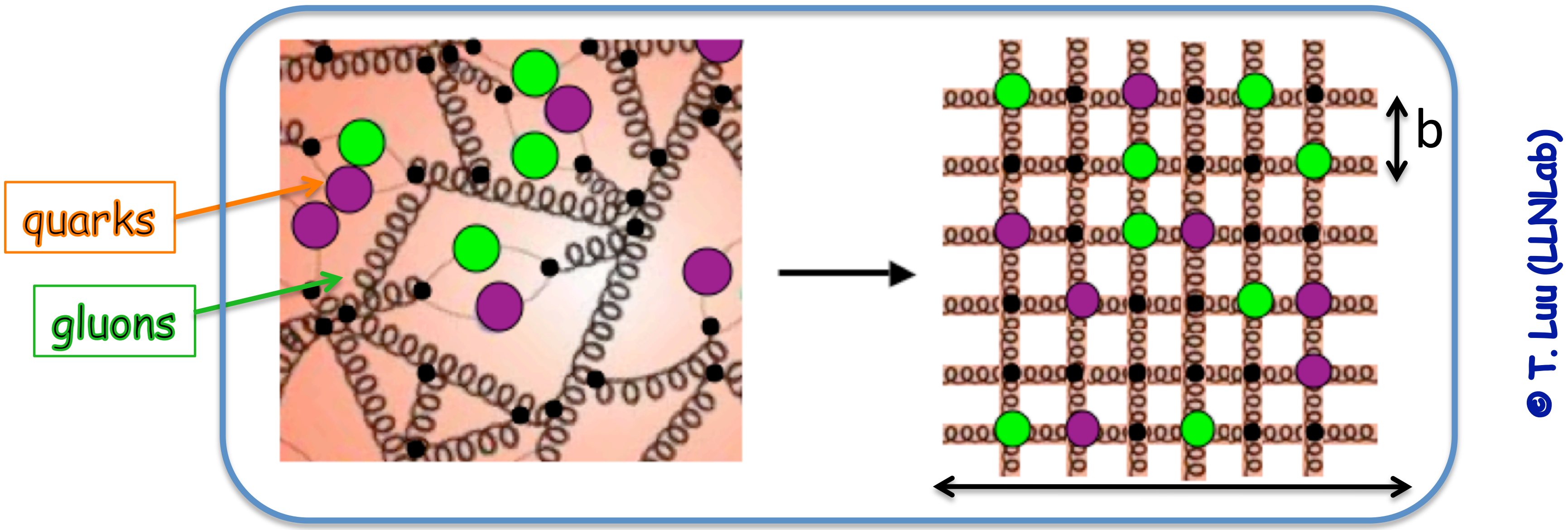
$g_{\Lambda\Sigma} \sim 0.60$,

$0.30 \le g_{\Sigma\Sigma} \le 0.55$

Lattice QCD provides a well-defined approach to calculate observables non-perturbatively, starting from the QCD Lagrangian.

One can simulate the theory on a computer, using methods analogous to the ones used in Statistical Mechanics.

These simulations allow us to calculate correlation functions of hadronic operators and matrix elements of any operator between hadronic states in terms of the fundamental quark and gluon dof.



(periodic boundary conditions)

$$L >> \text{relevant scales} >> b \qquad \left(\frac{1}{L} << m_\pi << \Lambda_\chi << \frac{1}{b}\right)$$

4-D discrete space-time

$N_s \times N_s \times N_s \times N_t$

Lattice QCD provides a well-defined approach to calculate observables non-perturbatively, starting from the QCD Lagrangian.

One can simulate the theory on a computer, using methods analogous to the ones used in Statistical Mechanics.

These simulations allow us to calculate correlation functions of hadronic operators and matrix elements of any operator between hadronic states in terms of the fundamental quark and gluon dof.

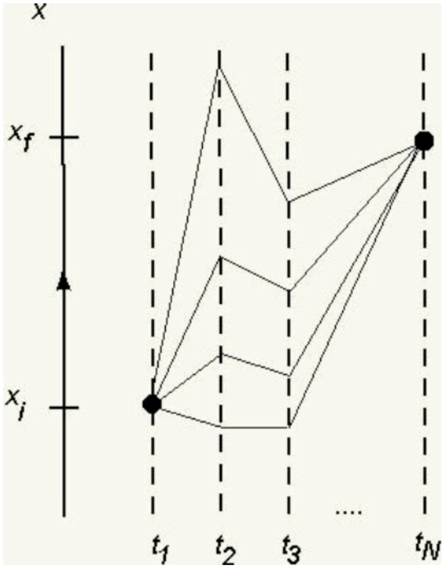
LQCD is a non-perturbative implementation of Field Theory, which uses **the Feynman path-integral approach** to evaluate transition matrix elements

PATH INTEGRAL
Feynman, 1948

$$A = \int D(q) \exp\left(i \int_i^f dt\, L(q(t)) \right)$$

A_i

each path contributes a phase given by the classical action



The quantum propagation is expressed as a weighted sum over paths. The weight is a complex phase factor given by the exponential of i times the classical action *S*.

By rotating to Euclidean time: $t \rightarrow i\tau$

$$x_0 \equiv t \rightarrow -ix_4 \equiv -i\tau$$
$$p_0 \equiv E \rightarrow ip_4$$

$$x_E^2 = \sum_{i=1}^{4} x_i^2 = \vec{x}^2 - t^2 = -x_M^2$$
$$p_E^2 = \sum_{i=1}^{4} p_i^2 = \vec{p}^2 - E^2 = -p_M^2$$

THE EUCLIDEAN PATH IS REAL
(basis of numerical simulations)

Lattice QCD

In continuous Euclidean space:

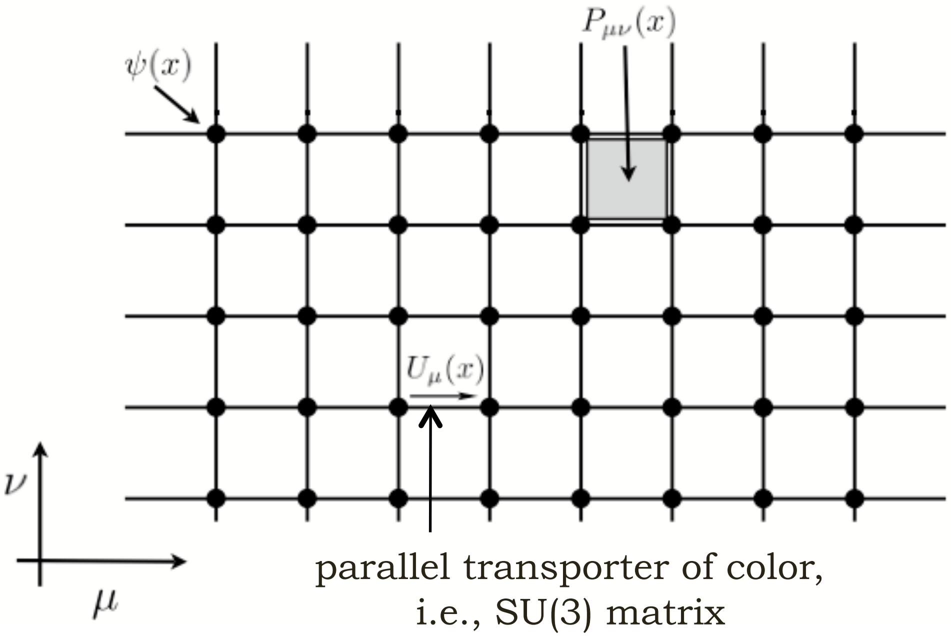
$$Z = \int Dq\, D\bar{q}\, DA_\mu\, e^{-S[q,\bar{q},A_\mu]}$$

$$\langle O \rangle = \frac{1}{Z}\int Dq\, D\bar{q}\, DA_\mu\, O(q,\bar{q},A_\mu)\, e^{-S[q,\bar{q},A_\mu]}$$

Discrete EUCLIDEAN space-time

Evaluate a path ordered exponential between neighboring sites

$$S[q,\bar{q},A_\mu] = S_g(U) + S_f(q,\bar{q},U),$$
$$U_\mu(x) = e^{-ibA_\mu(x+\hat{\mu}/2)}$$
$$S_g(U) = \beta \sum_{x,\nu\mu}\left(1 - \frac{1}{3}\mathrm{Re}(Tr(P_{\nu\mu}(x)))\right)$$
$$P_{\nu\mu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^+(x+\hat{\nu})U_\nu^+(x)$$
$$S_f(q,\bar{q},U) = \bar{q}D(U)q$$



Procedure Configurations ⟹ Compute quark propagators ⟹ Compute correlators

Compute the quark propagator in each gauge-field configuration

$$S_F(y,j,b;x,i,a) = \left(M^{-1}\right)_{x,i,a}^{y,j,b}$$

site, spin, color

(for a given flavor)

On each configuration, the location of the propator source point is chosen randomly (multiple propagators are generated on each configuration)

The use of Gaussian-smeared sources optimizes the overlap onto the ground-state hadrons (for the sinks we use either local or smeared operators)

We use anti-periodic BC in the time direction and periodic BC in the spatial directions

Anisotropic lattices: $N_t >> N_s$ (probe high frequency modes, i.e., better resolve the ground state)

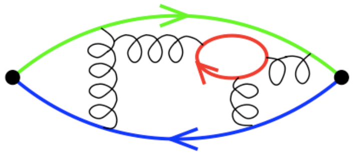
Dynamical $N_f = 2+1$ simulations:

$$Z = \int DA_\mu \exp(-S) = \int DA_\mu \det\left[M_f(A)\right] \exp(-S_{gluon})$$

$$(S = S_{gluon} + S_f)$$

very demanding

$$S_f = \overline{\psi} M_f(A) \psi$$


(B) Full QCD

Hadron masses are two-point correlation functions

$$C(\Gamma,\vec{p},t)=\sum_{\vec{x}_2} e^{-i\vec{p}\vec{x}_2}\Gamma\langle 0|T\left\{\psi(\vec{x}_2,t_2)\psi(\vec{x}_0,t_0)\right\}|0\rangle$$

projects onto zero momentum

spin tensor

sink

source

interpolating operators

The validity of lattice QCD can be tested through the successful calculation of low-lying hadron masses

for mesons $\bar{\psi}_i\psi_j, \bar{\psi}_i\gamma_5\psi_j, \bar{\psi}_i\gamma_k\psi_j$

$$\Gamma_\pm=\frac{1}{2}(1\pm\gamma_0)$$

positive/negative energy (parity) states

for baryons $O_{(\alpha\beta)\gamma}=\left(\psi^T_{i,\alpha}C\gamma_5\psi_{j,\beta}\right)\psi_{k,\gamma}\,\varepsilon^{ijk}$

with $O_{(\beta\alpha)\gamma}=-O_{(\alpha\beta)\gamma}$ $\quad C=\gamma_2\gamma_0$

$$J^\pi=\left(\frac{1}{2}\right)^+$$

$$p_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)$$

$$\Sigma^+_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)$$

$$\Lambda_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)$$

$$\Xi^0_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)$$

charge conjugation matrix

Masses of (colourless) QCD bound states

(locate the source at t=0)

$$C(t) = \langle 0|\phi(t)\phi^\dagger(0)|0\rangle \xrightarrow[\phi(t)=e^{Ht}\phi\, e^{-Ht}]{} \langle\phi|e^{-Ht}|\phi\rangle$$

Insert a complete set of energy eigenstates:

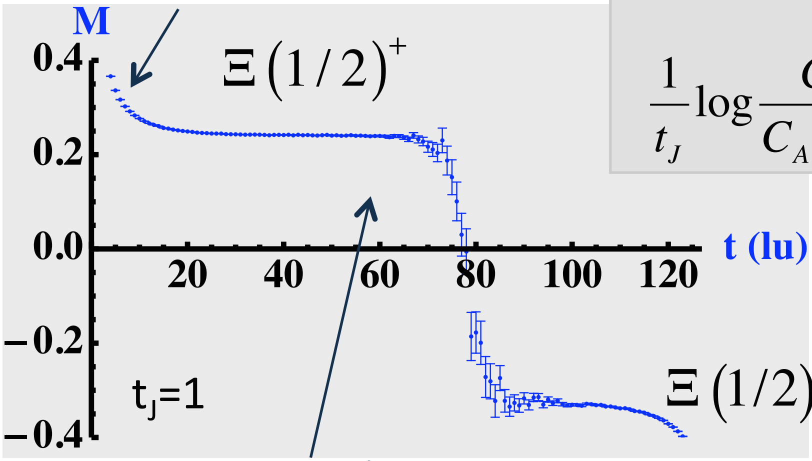
$$C(t) = \sum_n \langle\phi|e^{-Ht}|n\rangle\langle n|\phi\rangle = \sum_n |\langle\phi|n\rangle|^2 e^{-E_n t} \xrightarrow{t\to\infty} Z e^{-E_0 t}$$

mass

i.e. one can obtain the energy of the state provided we see the large time exponential fall-off of the correlation function (Euclidean time evolution suppresses excited states)

n_f=2+1 b_s=0.1227±0.0008 fm b_s/b_t = 3.5 L ~ 2.5 fm m_π~390 MeV m_K~546 MeV

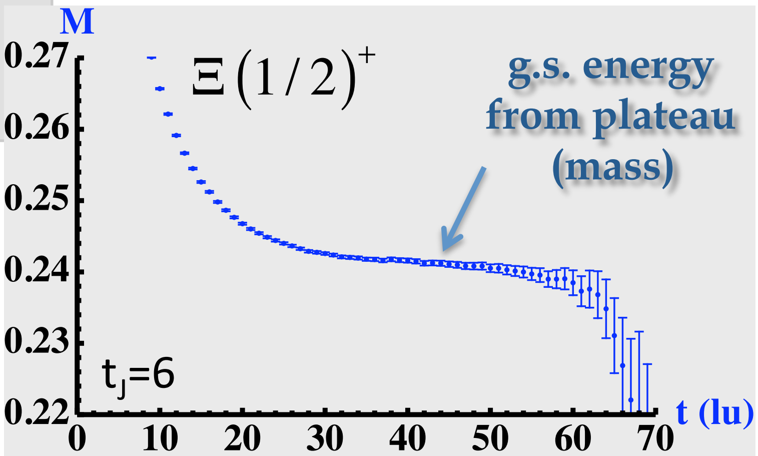
contamination from excited states



statistical noise

effective mass plot

$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)} = m_A$$



(anti-periodic bc in time)

$$C(t)=\sum_n \langle\phi|e^{-Ht}|n\rangle\langle n|\phi\rangle=\sum_n |\langle\phi|n\rangle|^2 e^{-E_n t} \xrightarrow{t\to\infty} Z e^{-E_0 t}$$

Lepage, 1989

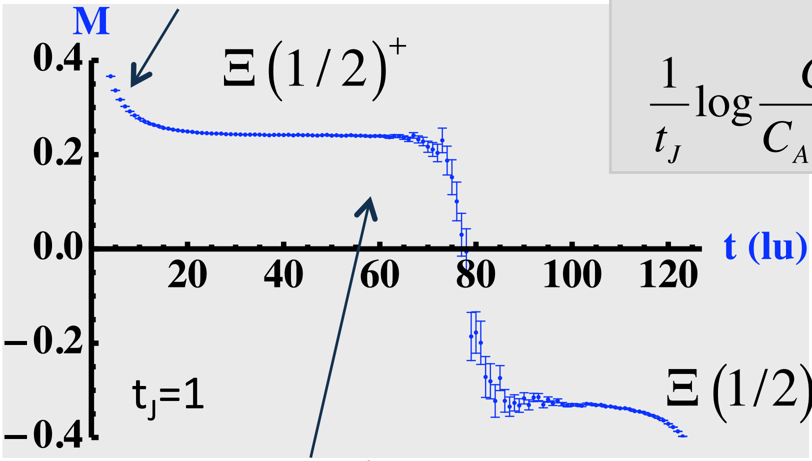
exponential degradation of the signal with time

$$\sigma^2(C)=\langle CC^+\rangle-|\langle C\rangle|^2$$

for one nucleon: $\dfrac{\langle C\rangle}{\sigma}\sim\exp\left\{-\left(M_N-\dfrac{3m_\pi}{2}\right)t\right\}$ $\rightarrow$ A nucleons: $\dfrac{\langle C\rangle}{\sigma}\sim\exp\left\{-A\left(M_N-\dfrac{3m_\pi}{2}\right)t\right\}$

n_f=2+1 b_s=0.1227±0.0008 fm b_s/b_t = 3.5 L ~ 2.5 fm m_π~390 MeV m_K~546 MeV

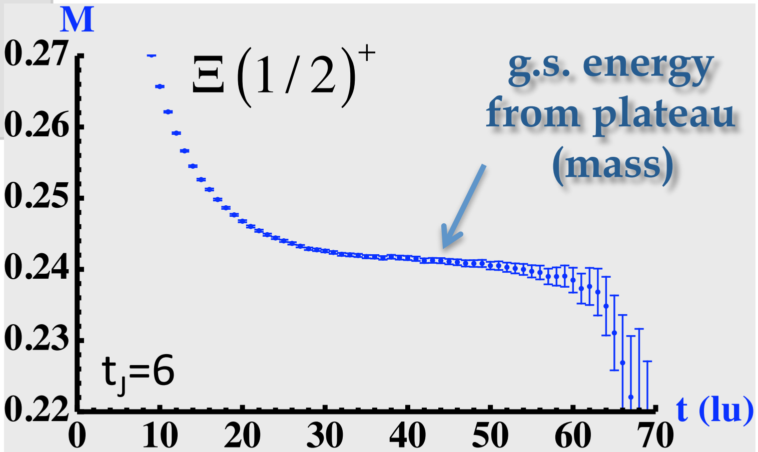
contamination from excited states



statistical noise

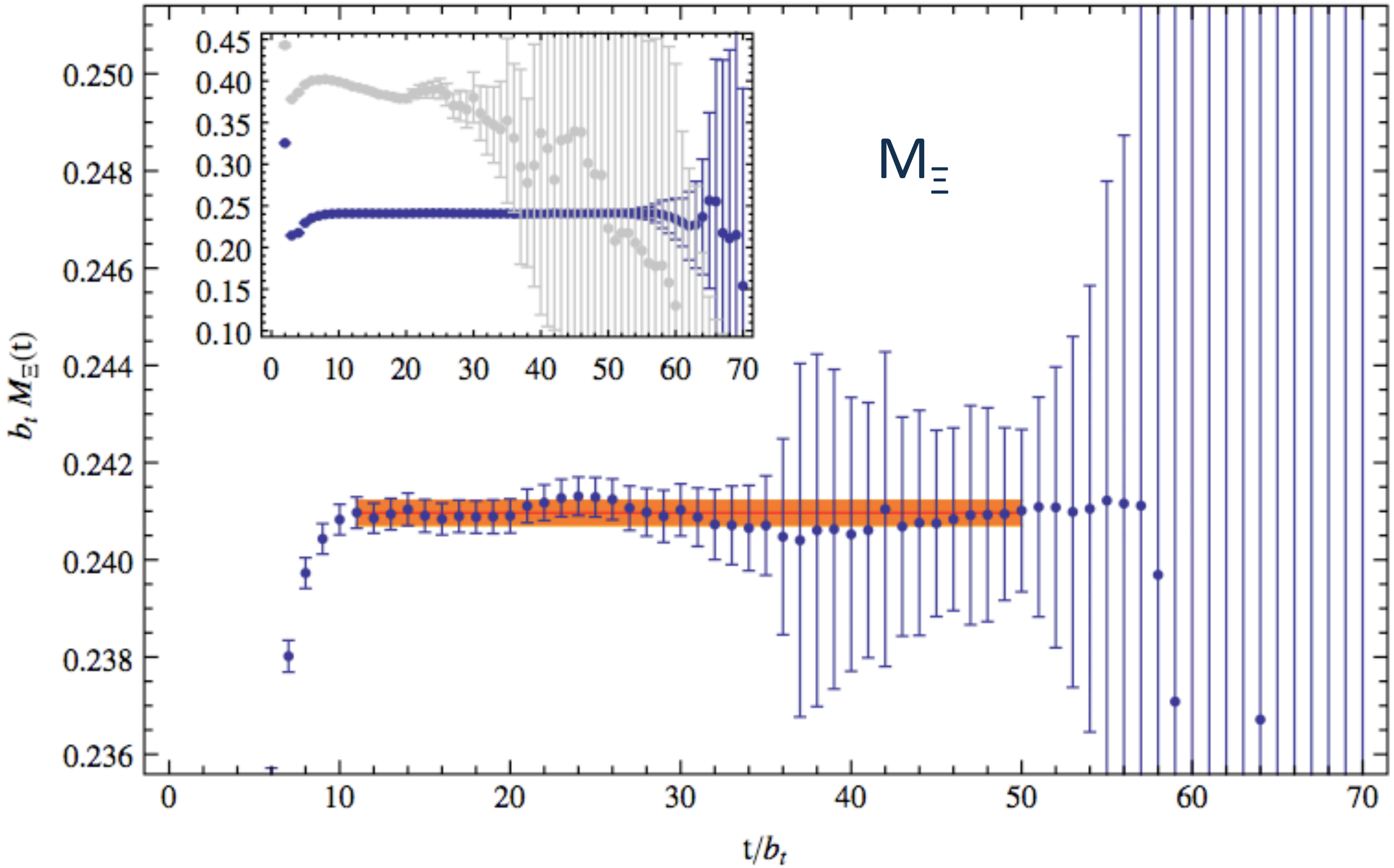
effective mass plot

$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)}=m_A$$



(anti-periodic bc in time)

Tracking sub-leading exponential fall-offs can give us excited states



NPLQCD, Phys. Rev. D 79, 114502 (2009)

Two-particle correlators ⟶ Energy of the interacting 2-particle system

$$C_{H_A H_B,\Gamma}(\vec{p}_1,\vec{p}_2,t)=\sum_{\vec{x}_1\vec{x}_2} e^{i\vec{p}_1\vec{x}_1} e^{i\vec{p}_2\vec{x}_2}\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}\left\langle J_{H_A,\alpha_1}(\vec{x}_1,t)J_{H_B,\alpha_2}(\vec{x}_2,t)\bar{J}_{H_A,\beta_1}(x_0,0)\bar{J}_{H_B,\beta_2}(x_0,0)\right\rangle$$

spin tensor

interpolating operators

source located at t=0

energy-eigenstates

↕

global quantum numbers:
B, I, I_3, J, *s*
+
hyper-cubic transformations

$$\Lambda\Lambda-\Sigma^{\pm,0}\Sigma^{\mp,0}-\Xi N$$
$$\Lambda N-\Sigma N$$

coupled channel analysis

single channel analysis

Channel	I	$\|I_z\|$	s
$pp\ (^1S_0)$	1	1	0
$np\ (^3S_1)$	0	0	0
$n\Lambda\ (^1S_0)$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$n\Lambda\ (^3S_1)$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$n\Sigma^-\ (^1S_0)$	$\frac{3}{2}$	$\frac{3}{2}$	-1
$n\Sigma^-\ (^3S_1)$	$\frac{3}{2}$	$\frac{3}{2}$	-1
$\Sigma^-\Sigma^-\ (^1S_0)$	2	2	-2
$\Lambda\Lambda\ (^1S_0)$	0	0	-2
$\Xi^-\Xi^-\ (^1S_0)$	1	1	-4

Two-particle correlators → Energy of the interacting 2-particle system

$$C_{H_A H_B,\Gamma}(\vec{p}_1,\vec{p}_2,t)=\sum_{\vec{x}_1\vec{x}_2} e^{i\vec{p}_1\vec{x}_1} e^{i\vec{p}_2\vec{x}_2}\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}\left\langle J_{H_A,\alpha_1}(\vec{x}_1,t)J_{H_B,\alpha_2}(\vec{x}_2,t)\bar{J}_{H_A,\beta_1}(x_0,0)\bar{J}_{H_B,\beta_2}(x_0,0)\right\rangle$$

spin tensor

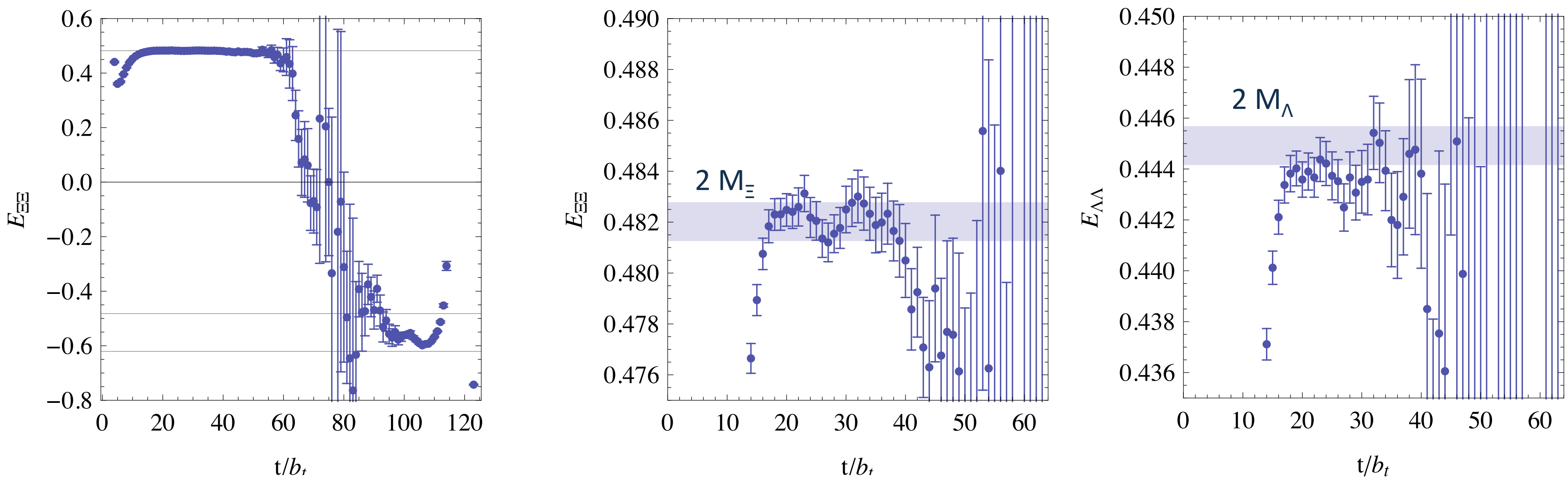
interpolating operators

source located at t=0

away from the source, i.e. at large t

$$C_{H_A H_B}(\vec{p},-\vec{p},t)\sim\sum_n Z^{(i)}_{n;AB}(\vec{p})\,Z^{(f)}_{n;AB}(\vec{p})\,e^{-E^{AB}_n(\vec{0})t}$$

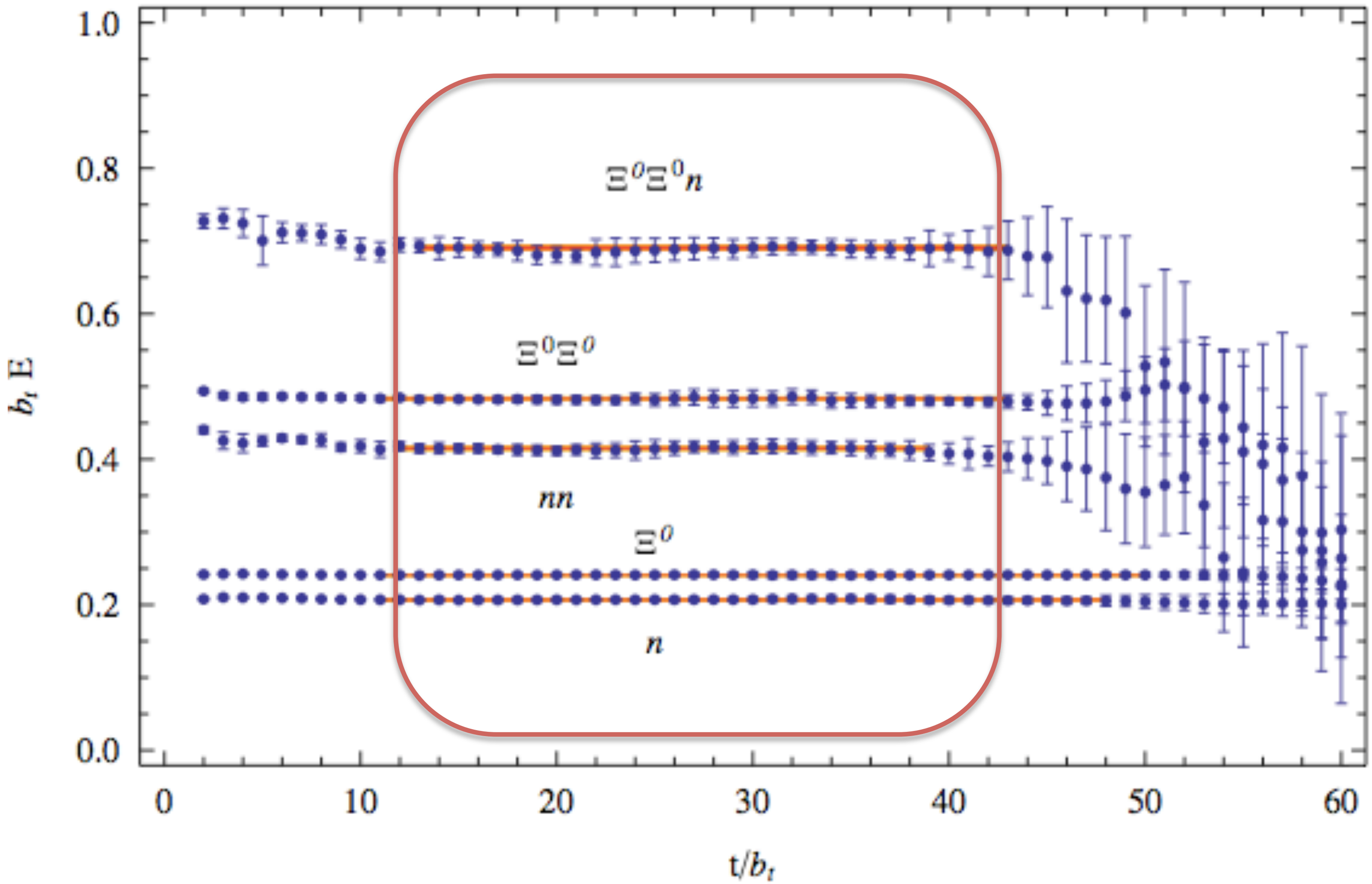
m_π ~390 MeV, L_s ~ (2.5 fm)^3



NPLQCD, Phys. Rev. D80 (2009) 074501

m_π ~390 MeV, L_s ~ (2.5 fm)^3

"Golden window"



Δ E for the BB interaction

mass

One-baryon correlator:

$$C_A(t) = \sum_{\vec{x}} \left\langle A(t,\vec{x})\, A^\dagger(0,\vec{0}) \right\rangle = \sum_n C_A^n\, e^{-E_A^n t} \rightarrow C_A\, e^{-M_A t}$$

2-baryon correlator:

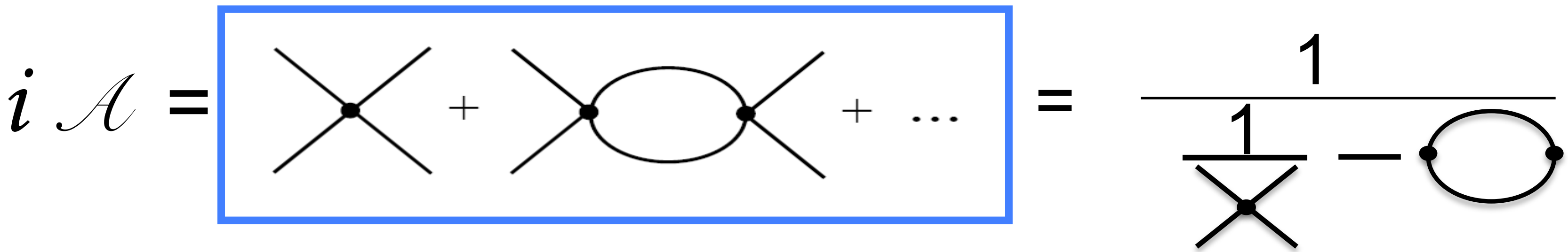
$$C_{AB}(t) = \sum_{\vec{x},\vec{y}} \left\langle A(t,\vec{x})\, B(t,\vec{x})\, B^\dagger(0,\vec{0})\, A^\dagger(0,\vec{0}) \right\rangle = \sum_n C_{AB}^n\, e^{-E_{AB}^n t} \rightarrow C_{AB}\, e^{-E_{AB} t}$$

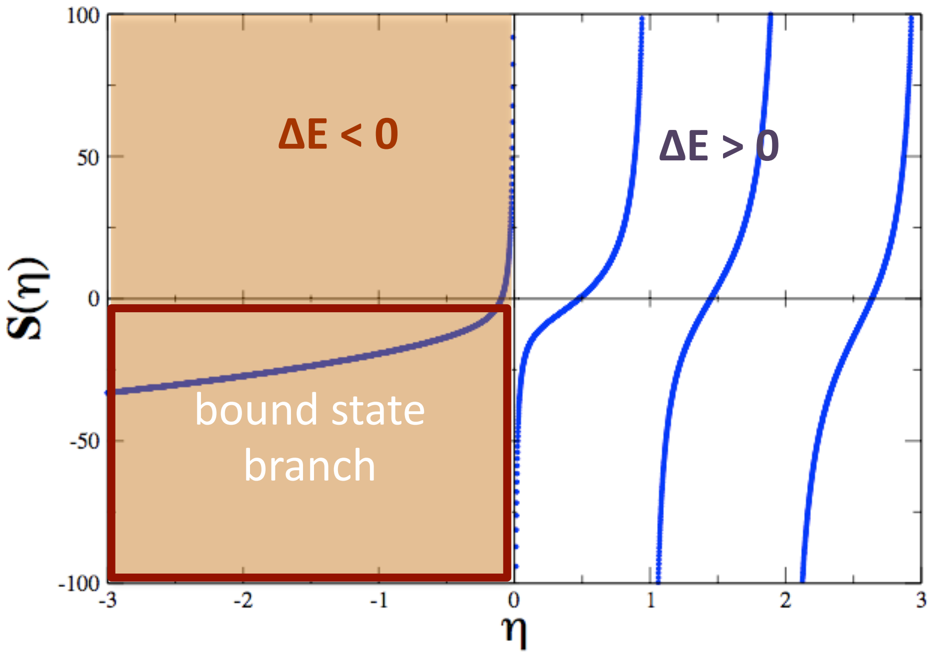
Energy shift: $\Delta E = E_{AB} - M_A - M_B$

$$G_{AB}(t) = \frac{C_{AB}(t)}{C_A(t)\, C_B(t)} = \sum_n C^n e^{-\Delta E^n t} \rightarrow C e^{-\Delta E\, t}$$

Below inelastic thresholds

M. Lüscher, Commun. Math. Phys. 105, 153-188 (1986)
Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 (2004)





$$\Delta E_n^{(AB)} \equiv \Delta E_n^{(AB)}(\vec{0}) \equiv E_n^{(AB)}(\vec{0}) - m_A - m_B$$
$$= \sqrt{q_n^2 + m_A^2} + \sqrt{q_n^2 + m_B^2} - m_A - m_B$$
$$= \frac{q_n^2}{2\mu_{AB}} + \ldots$$

obtained from the simulations

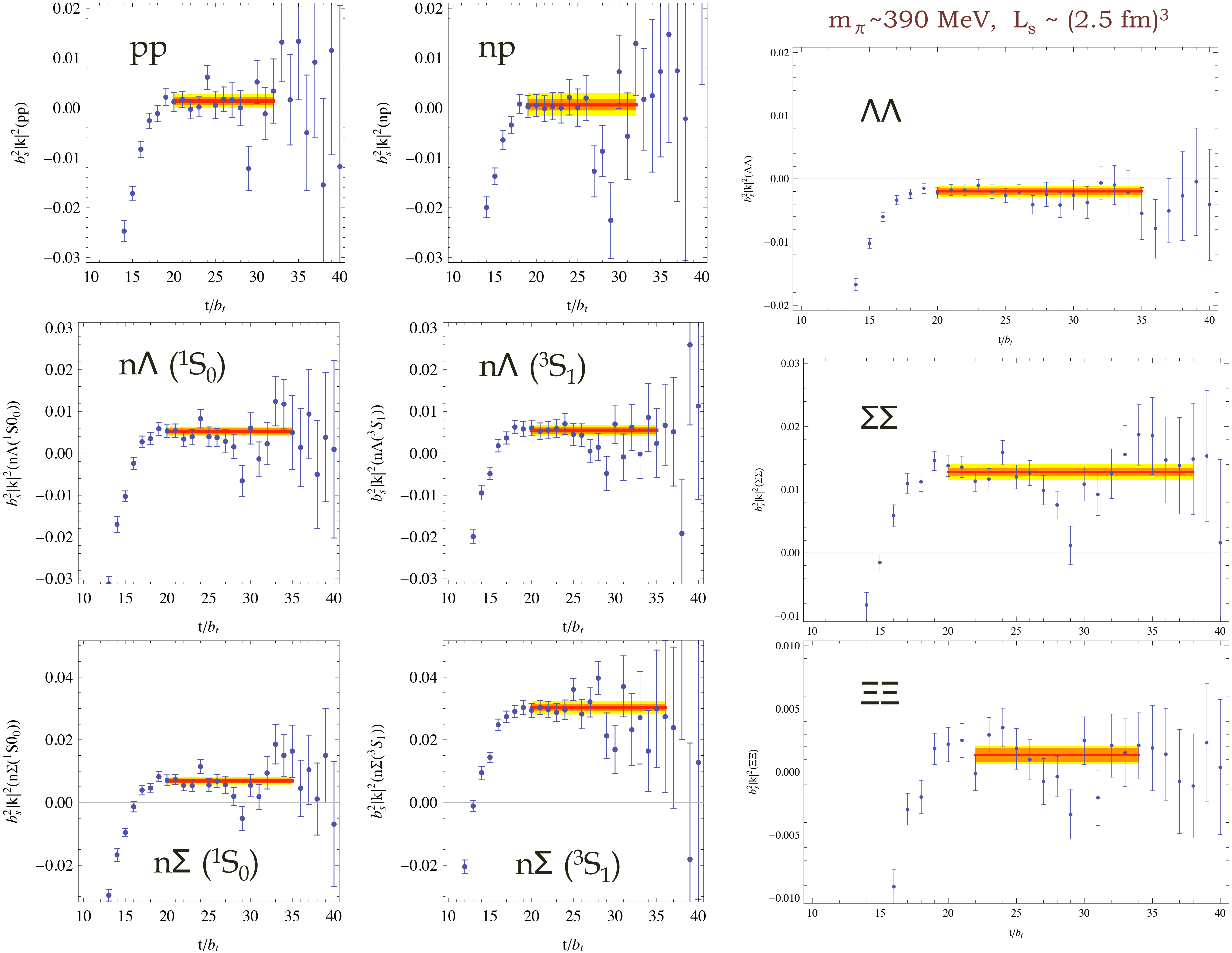
$$q_n \cot\delta(q_n) = \frac{1}{\pi L} S\left(q_n^2 \left(\frac{L}{2\pi} \right)^2 \right)$$

gives us the scattering amplitude at ΔE

$$\equiv \frac{1}{\pi L} \sum_{\vec{j}}^{\Lambda} \frac{1}{|\vec{j}|^2 - \left(\frac{L q_n}{2\pi} \right)^2} - \frac{4\Lambda}{L}$$

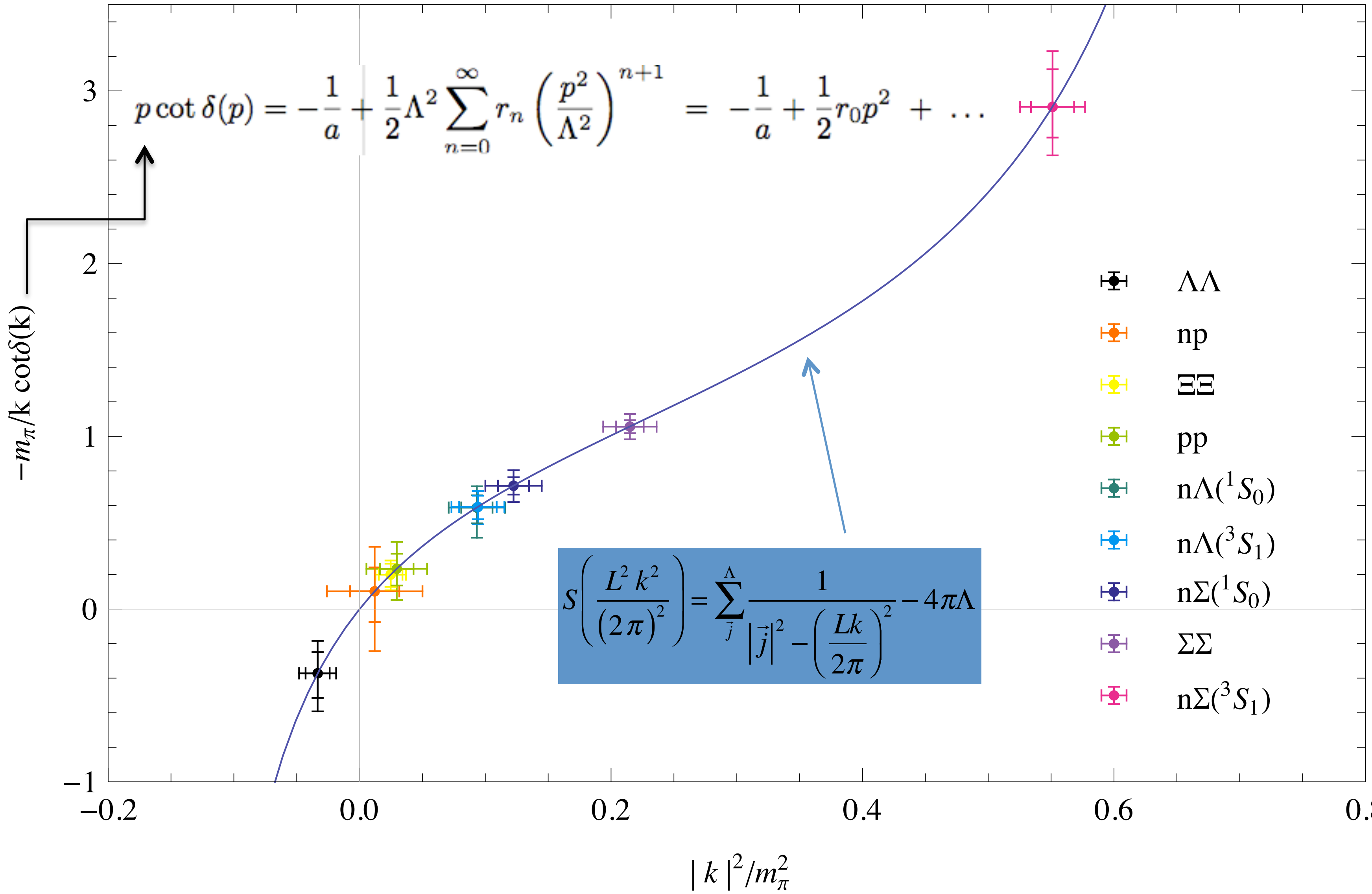
$$p \cot\delta(p) = -\frac{1}{a} + \frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2} \right)^{n+1} = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + \ldots$$

(Λ: UV regulator)

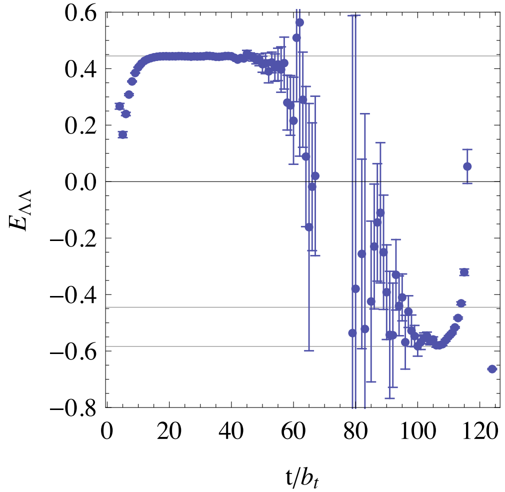


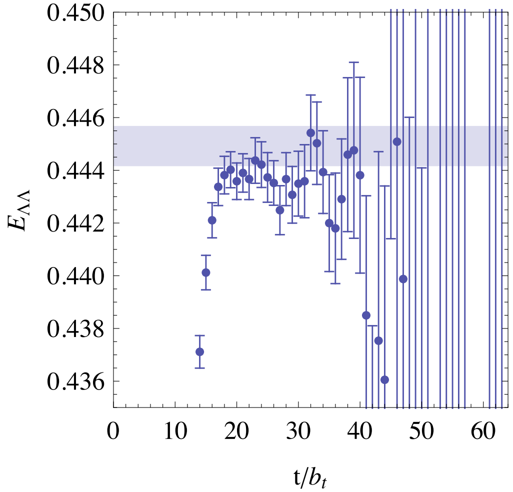
mπ ~390 MeV, Ls ~ (2.5 fm)3
pp
np
ΛΛ
nΛ (1S0)
nΛ (3S1)
ΣΣ
nΣ (1S0)
nΣ (3S1)
ΞΞ
t/bt

m_π~390 MeV, L_s ~ (2.5 fm)^3 *NPLQCD, Phys. Rev. D81 (2010) 054505*



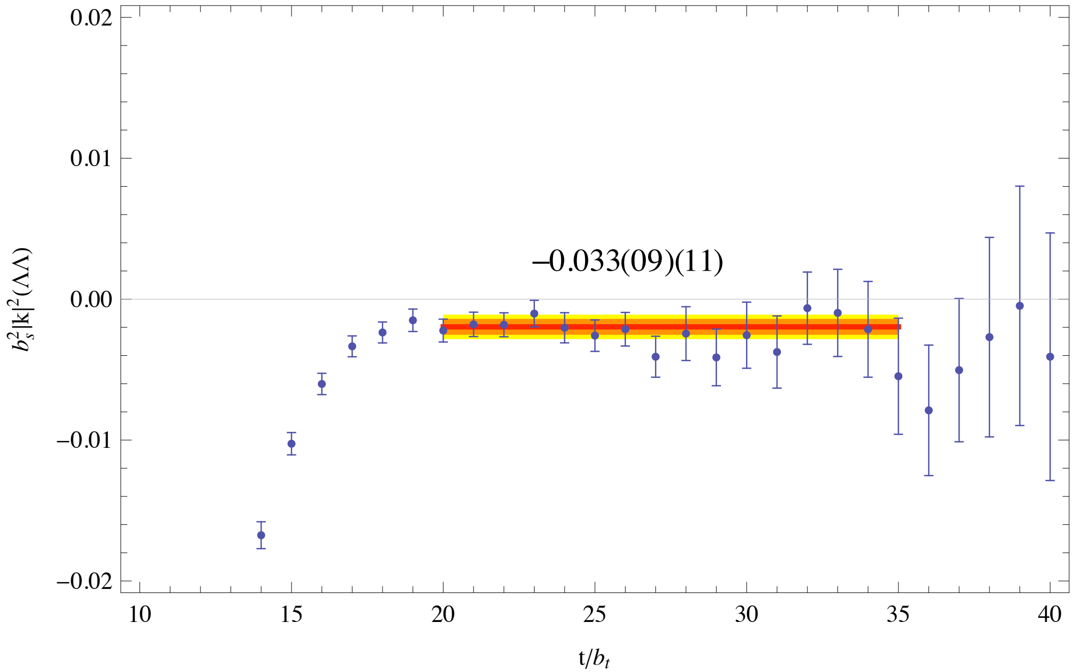
NPLQCD, Phys. Rev. D81 (2010) 054505





m_π ~390 MeV, L_s ~ (2.5 fm)^3

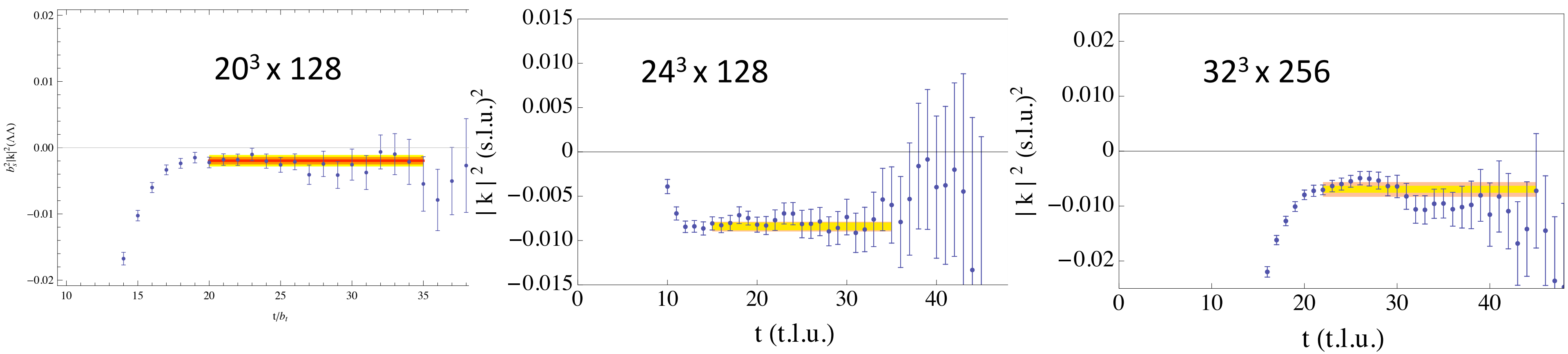
only channel for which a negative energy shift was obtained



$$\Delta E = -4.1(1.2)(1.4)\ \mathrm{MeV}$$

$$-\frac{1}{p\cot\delta} = -0.188^{+0.062}_{-0.072}{}^{+0.072}_{-0.085}\ \mathrm{fm}$$

	L ~ 2 fm	L ~ 2.5 fm	L ~ 3 fm	L ~ 4 fm
E (MeV)	**16^3 x 128**	**20^3 x 128**	**24^3 x 128**	**32^3 x 256**
$E_{\Lambda\Lambda}$	2504.78(5.6)(8.3)	2499.72(6.2)(6.5)	2468.20(5.1)(5.3)	2475.51(2.8)(3.3)
$B_{\Lambda\Lambda} = - \Delta E_{\Lambda\Lambda}$	12.3(1.1)(4.0)	4.5(1.1)(1.3)	16.3(1.2)(1.4)	16.6(1.4)(3.1)
baryon-baryon thresholds (MeV)				
$2M_{\Lambda}$	2526.85(5.07)(8.78)	2504.33(3.04)(4.28)	2484.97(2.25)(4.73)	2482.72(2.59)(3.49)
$M_{\Xi}+M_N$	2543.96(4.62)(8.33)	2520.72(3.42)(4.67)	2501.29(2.65)(3.83)	2500.90(2.36)(3.88)
$2M_{\Sigma}$	2573.57(4.28)(7.54)	2561.29(3.60)(4.84)	2565.69(2.70)(3.49)	2558.37(2.70)(4.84)
mass splitting (MeV)				
$M_{\Xi}+M_N - 2 M_{\Lambda}$	17.11(0.45)(0.45)	16.38(0.39)(0.39)	16.32(0.39)(0.90)	18.18(0.23)(0.39)
$2M_{\Sigma} - 2 M_{\Lambda}$	46.72(0.78)(1.24)	56.96(0.56)(0.56)	80.72(0.45)(1.24)	75.65(0.11)(1.35)



bound states

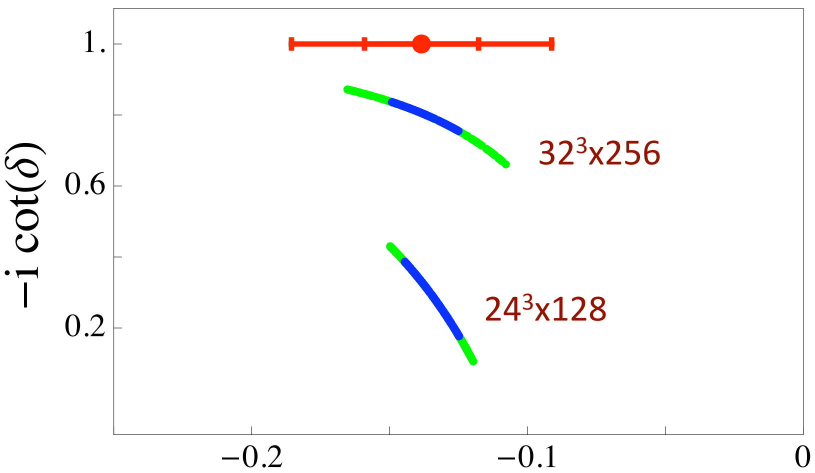
$$k_n \cot\delta(k_n) = \frac{1}{\pi L} S\left(\frac{k_n^2 L^2}{4\pi^2}\right) = \frac{1}{\pi L} \lim_{\Lambda\to\infty}\left(\sum_{\vec{j}}^{|\vec{j}|<\Lambda} \frac{1}{|\vec{j}|^2 - \eta^2} - 4\pi\Lambda\right), \qquad \eta = \frac{k_n^2 L^2}{4\pi^2}$$

$$k_{-1} = \mathrm{i}\,\kappa$$

$$k \cot\delta(k)\big|_{k=i\kappa} + \kappa = \frac{1}{L}\sum_{\vec{m}\neq\vec{0}} \frac{1}{|\vec{m}|} e^{-|\vec{m}|\kappa L} = \frac{1}{L}\left(6e^{-\kappa L} + 6\sqrt{2}e^{-\sqrt{2}\kappa L} + \frac{8}{\sqrt{3}} e^{-\sqrt{3}\kappa L} + \ldots\right)$$

for L large compared to the size of the system, perturbation theory yields

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L} + \ldots\right) \qquad B_\infty^H = \frac{\gamma^2}{M}$$



$(k/m_\pi)^2$ m_π~390 MeV

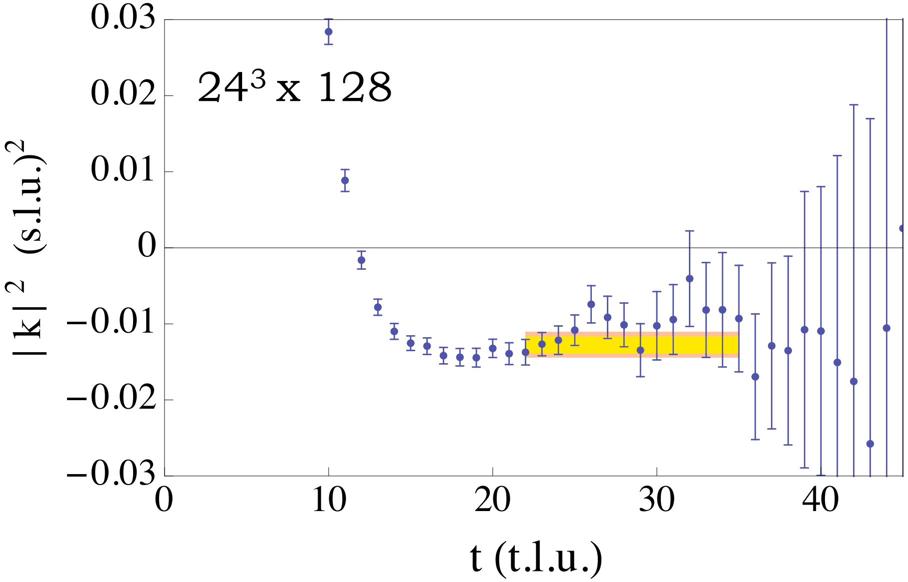
$$\cot\delta(k)\big|_{k=i\gamma} = +\mathrm{i}$$

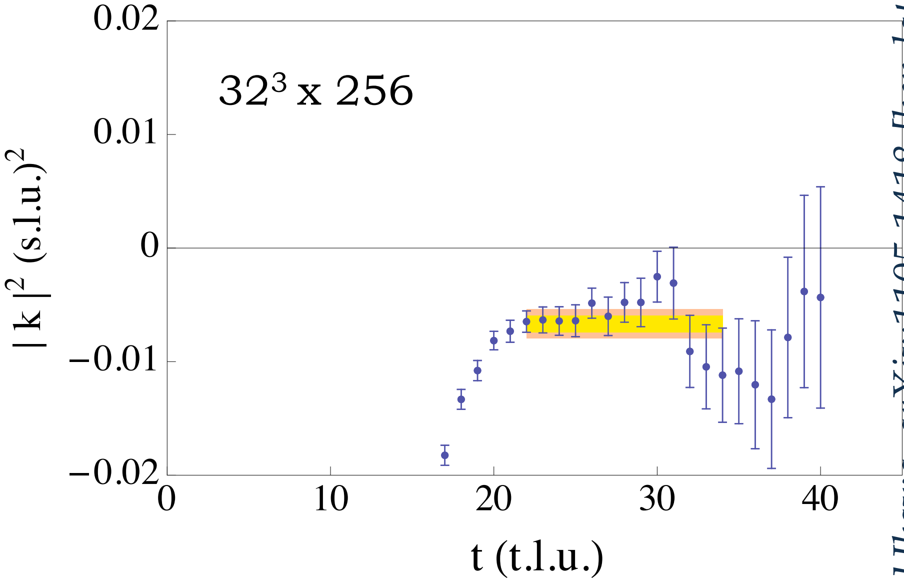
$$\left(f(\Omega) = \frac{1}{k\cot\delta(k) - ik} \quad \to \infty \ \ \text{(b.s.)} \quad \text{-i}\ \cot\delta(k) = 1\right)$$

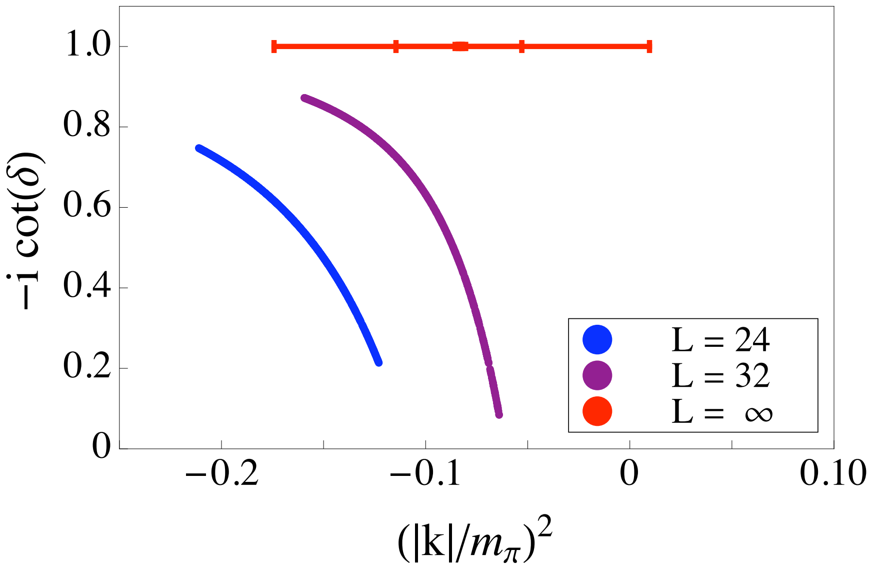
$$B_\infty^H = 13.2 \pm 4.4 \ \text{MeV}$$

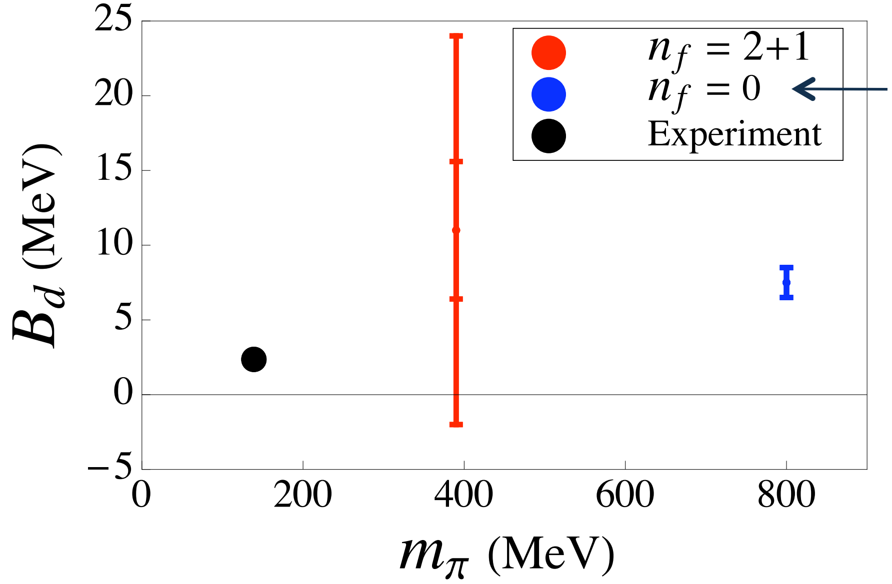
updated from PRL 106 (2011) 162001

np



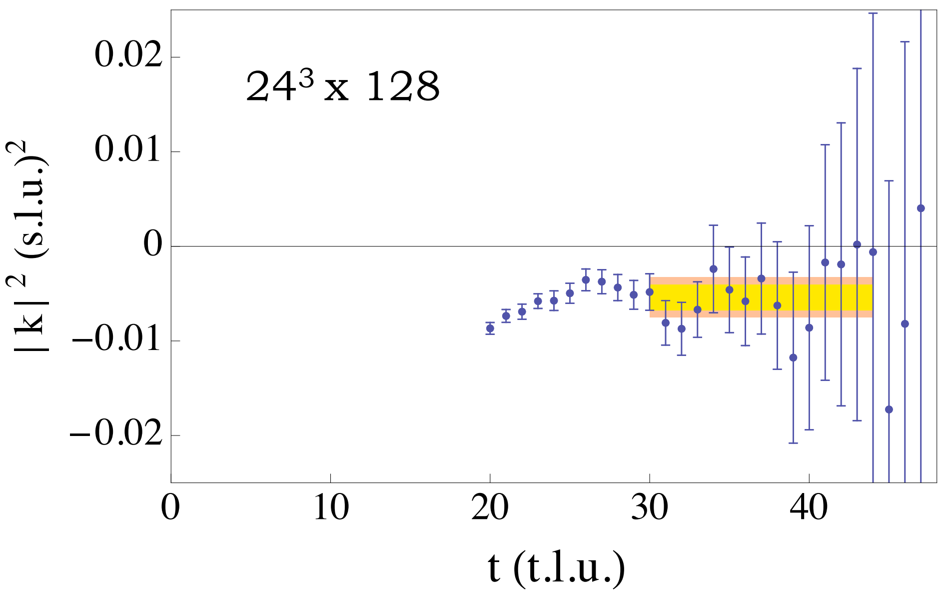


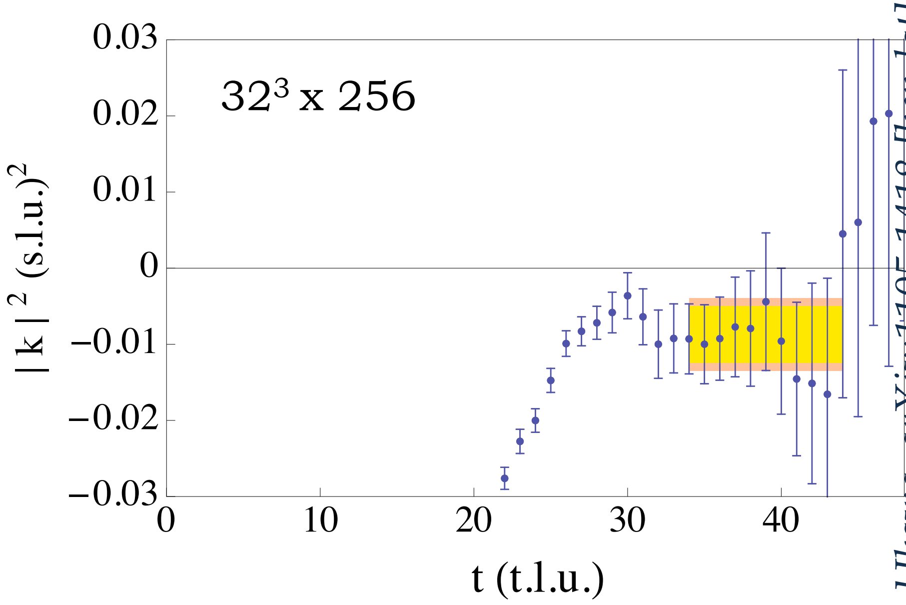


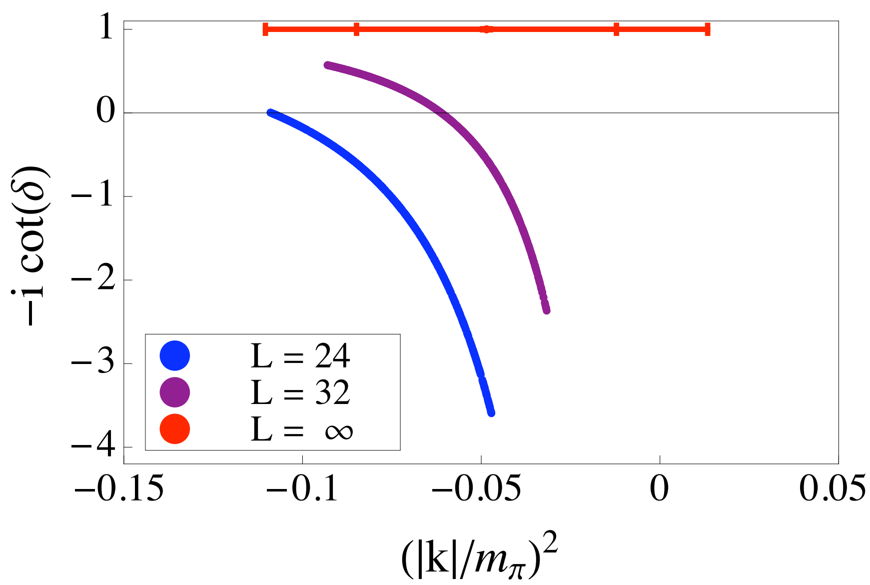


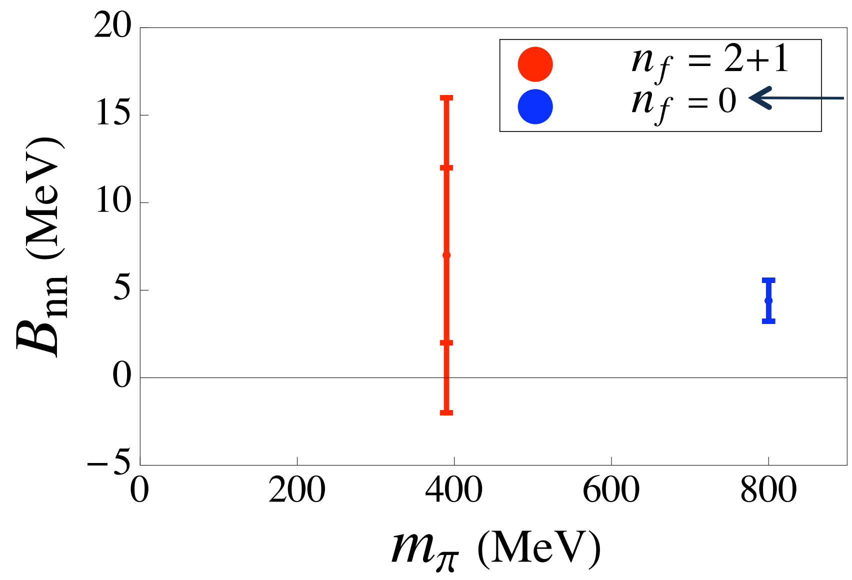
T. Yamazaki, Y. Kuramashi, A. Ukawa, arXiv:1105.1418 [hep-lat]

nn



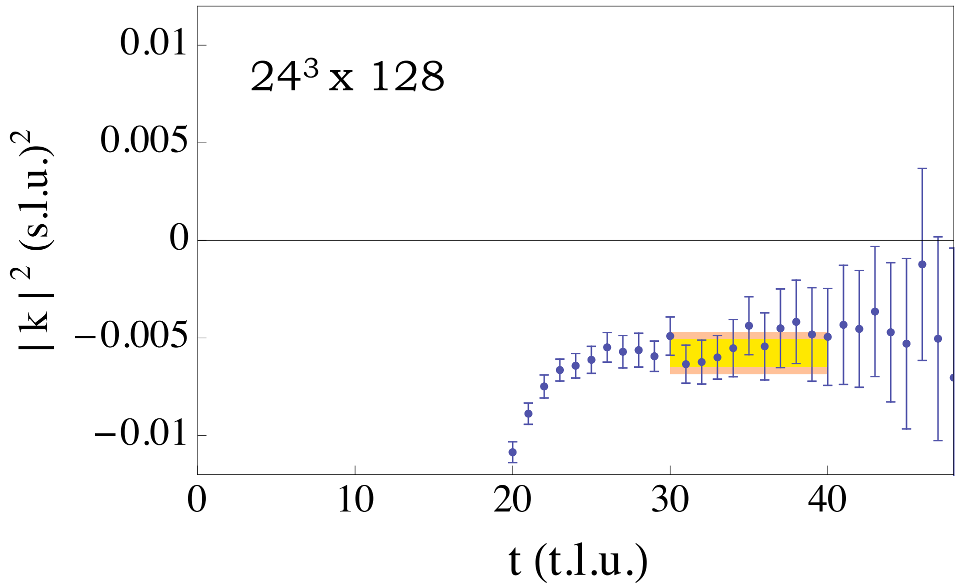




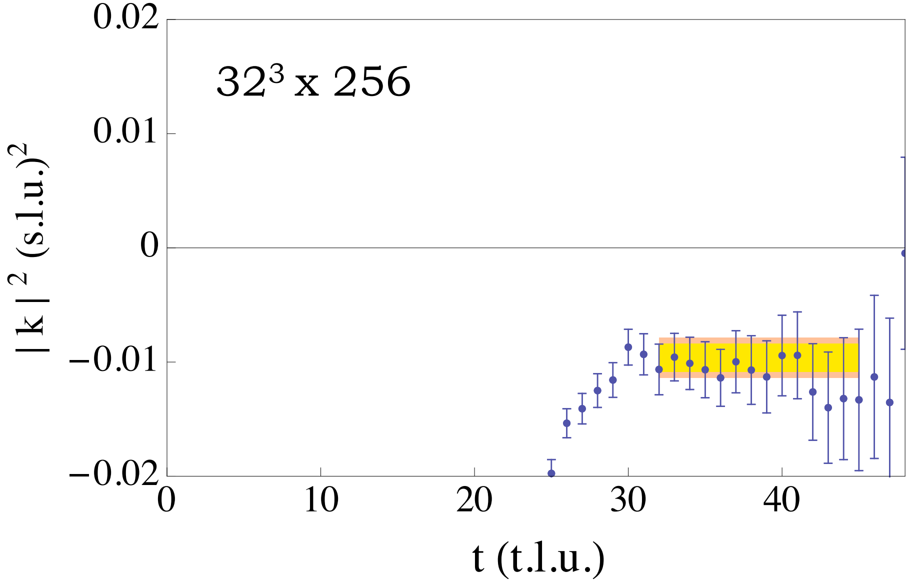


T. Yamazaki, Y. Kuramashi, A. Ukawa, arXiv:1105.1418 [hep-lat]

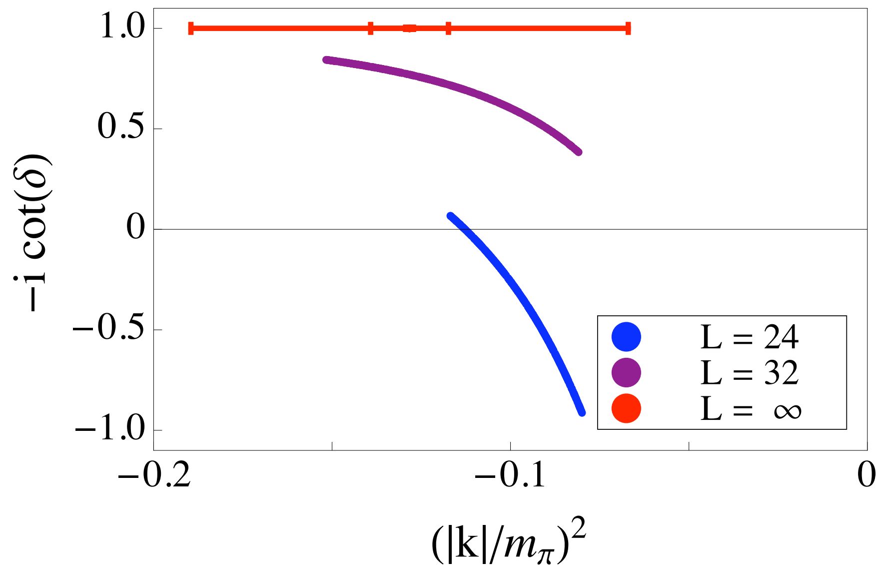
$\Xi^- \Xi^-$



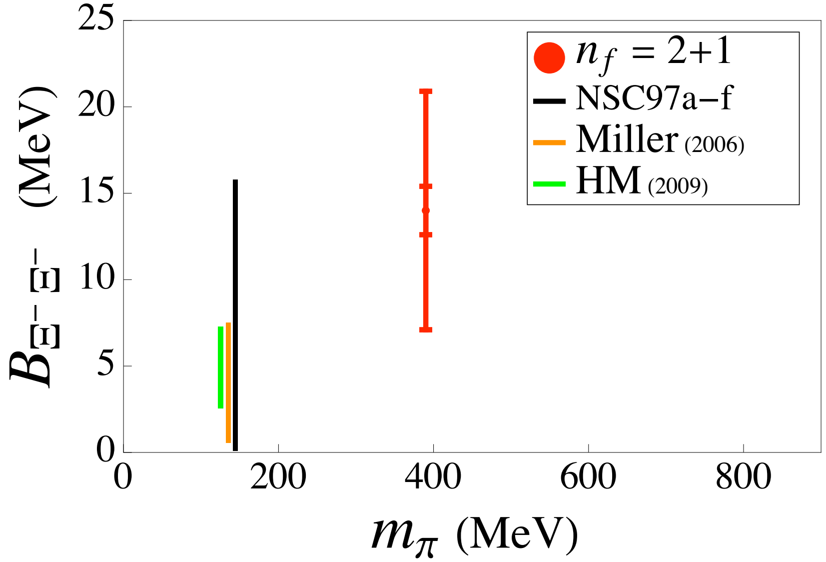
24^3 x 128
0.01
0.005
0
−0.005
−0.01
$|k|^2$ (s.l.u.)^2
0
10
20
30
40
t (t.l.u.)



32^3 x 256
0.02
0.01
0
−0.01
−0.02
$|k|^2$ (s.l.u.)^2
0
10
20
30
40
t (t.l.u.)



1.0
0.5
0
−0.5
−1.0
$-i\cot(\delta)$
−0.2
−0.1
0
$(|k|/m_\pi)^2$
L = 24
L = 32
L = ∞



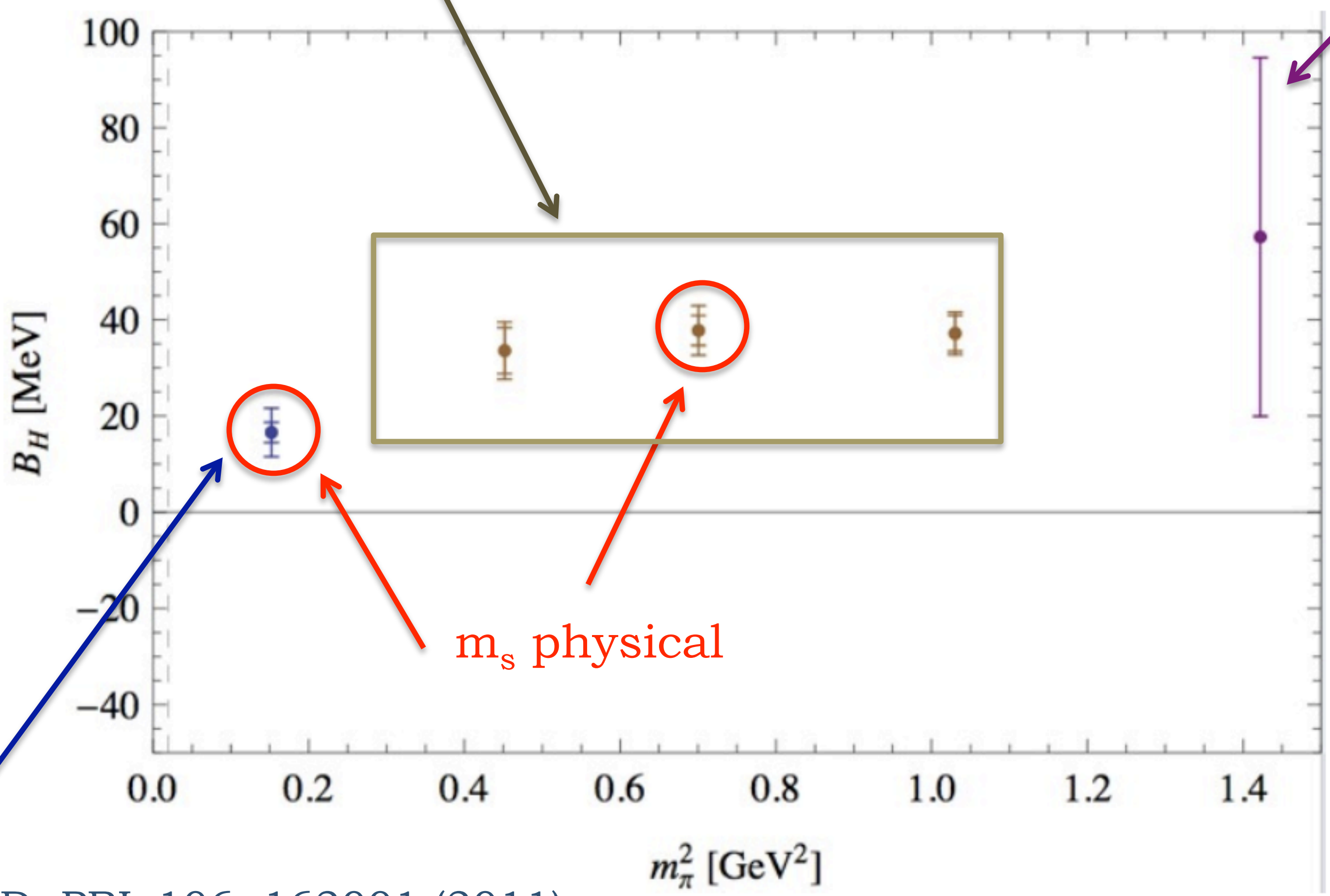
25
20
15
10
5
0
$B_{\Xi^-\Xi^-}$ (MeV)
0
200
400
600
800
m_π (MeV)
n_f = 2+1
NSC97a−f
Miller (2006)
HM (2009)

Channel	I	$\lvert I_z \rvert$	s
$pp\ (^1S_0)$	1	1	0
$np\ (^3S_1)$	0	0	0
$n\Lambda\ (^1S_0)$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$n\Lambda\ (^3S_1)$	$\frac{1}{2}$	$\frac{1}{2}$	-1
$n\Sigma^-\ (^1S_0)$	$\frac{3}{2}$	$\frac{3}{2}$	-1
$n\Sigma^-\ (^3S_1)$	$\frac{3}{2}$	$\frac{3}{2}$	-1
$\Sigma^-\Sigma^-\ (^1S_0)$	2	2	-2
$\Lambda\Lambda\ (^1S_0)$	0	0	-2
$\Xi^-\Xi^-\ (^1S_0)$	1	1	-4

B.E. (MeV)	Deuteron	Di-neutron	H-dibaryon	$\Xi^-\Xi^-$
	11(05)(12)	7.1(5.2)(7.3)	13.2(1.8)(4.8)	14.0(1.4)(6.7)

HALQCD, PRL 106, 162002 (2011)
n_f=3, b_s = 0.12 fm, L: 2, 3, 3.9 fm
m_π = 670, 830, 1015 MeV

Luo, Loan & Liu arXiv:1106.1945
n_f=0, 0.2 < b < 0.4 fm,
m_π > 1 GeV, L: 2.4 – 4.8 fm

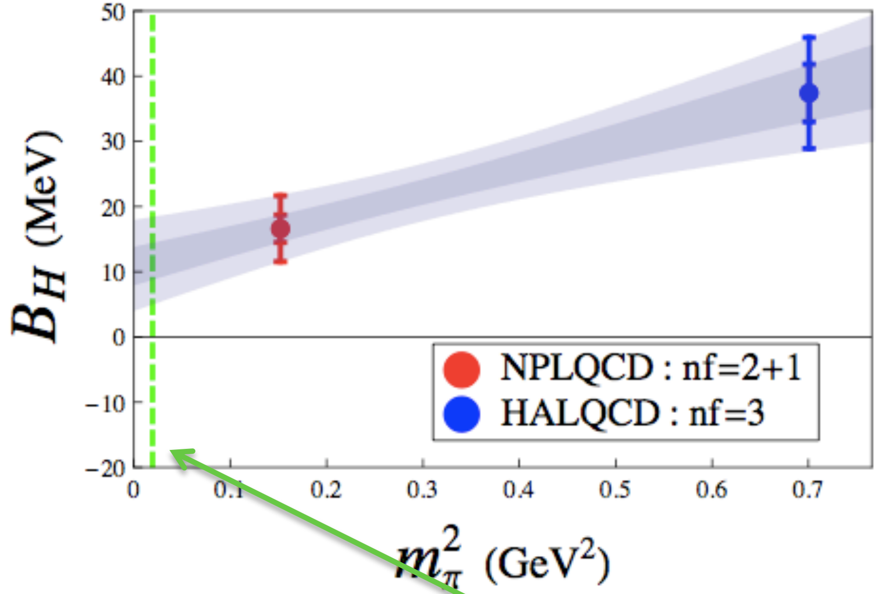


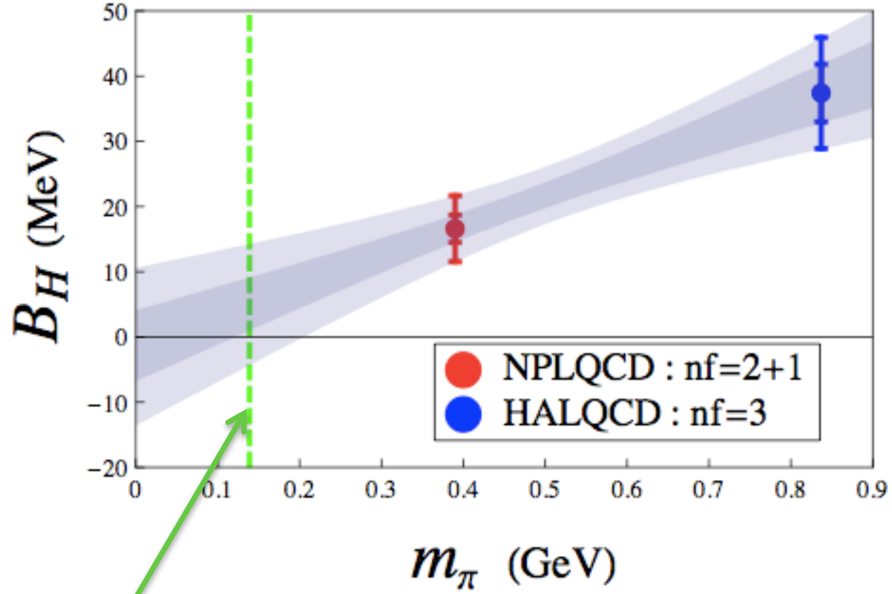
m_s physical

NPLQCD, PRL 106, 162001 (2011)
n_f=2+1, b_s = 0.12 fm, L: 2, 2.5, 3, 3.9 fm
m_π = 390 MeV

Extrapolation of the lattice results

Mod. Phys. Lett. A26 (2011) 2587





physical pion mass

$$B(m_\pi) = B_0 + d_1\, m_\pi^2$$

$$B_H^{quad} = +11.5 \pm 2.8 \pm 6 \text{ MeV}$$

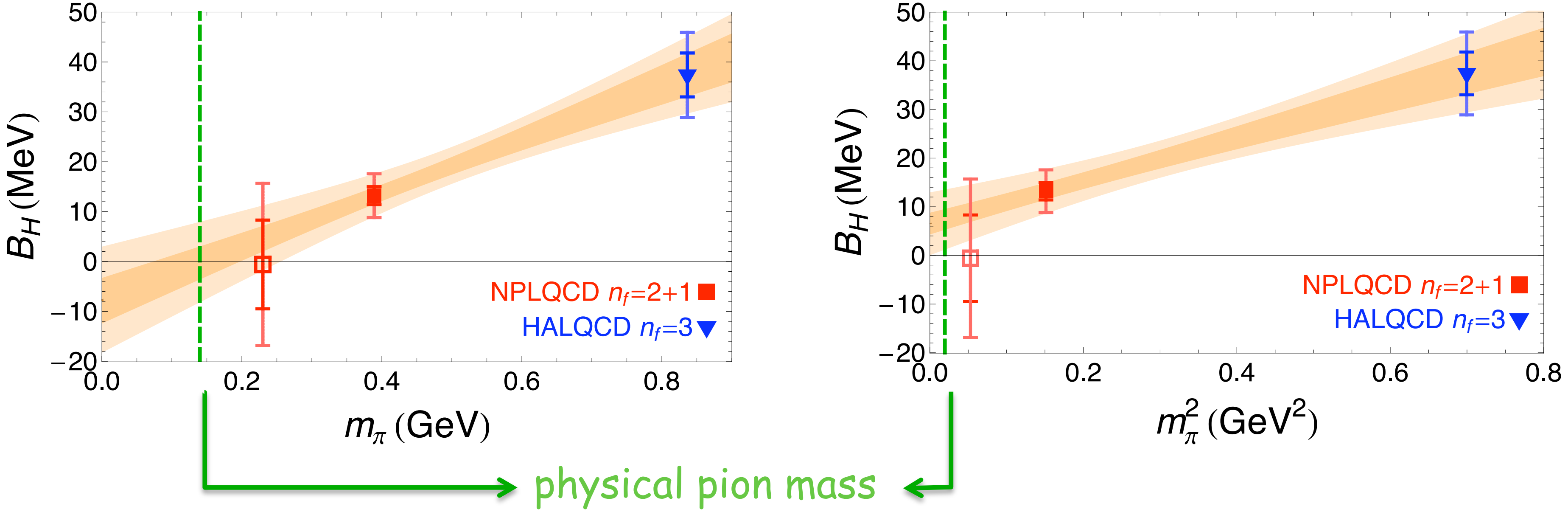
$$B(m_\pi) = B_0 + c_1\, m_\pi$$

$$B_H^{lin} = +4.9 \pm 4.0 \pm 8.3 \text{ MeV}$$

no definite conclusion of the H b.e.

... more simulations at lighter quark masses (m_π~200–250 MeV)

H-dibaryon $(\Lambda\Lambda)_{J=0,\ I=0}$



Mod. Phys. Lett. A26 (2011) 2587

... more simulations at lighter quark masses (m_π~200–250 MeV)

Quark mass dependence of the H-dibaryon b.e. in the framework of Chiral EFT and effects of SU(3) breaking

Haidenbauer, Meissner, Phys. Lett. B 706 (2011)

They studied the ΛΛ $^1S_0$ assuming a loosely bound H-dibaryon

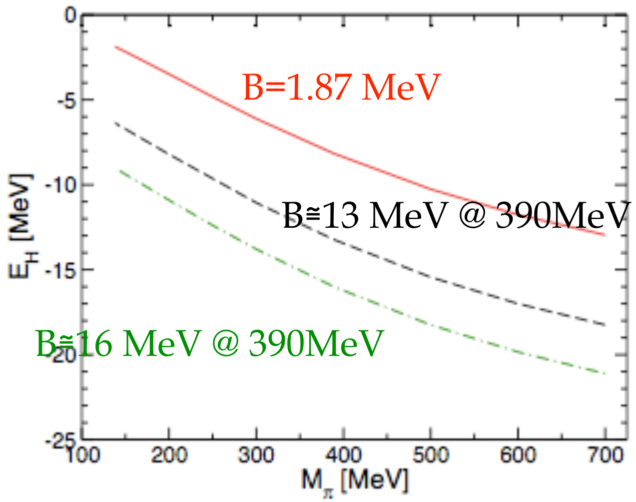
$SU_f(3)$ symmetry

"any attraction supplied by the flavor-singlet channel contributes with a much larger weight to the ΞN interactions instead than to ΛΛ"

$$V^{\Lambda\Lambda\to\Lambda\Lambda} = \frac{1}{40}\left(27C^{27} + 8C^{8_S} + \boxed{5C^1}\right) \qquad V^{\Xi N\to\Xi N} = \frac{1}{40}\left(12C^{27} + 8C^{8_S} + \boxed{20C^1}\right)$$

$$\delta_{\Lambda\Lambda}(0) - \delta_{\Lambda\Lambda}(\infty) = 0 \qquad \delta_{\Xi N}(0) - \delta_{\Xi N}(\infty) = 180^\circ$$

They also found that for pion masses below 400 MeV, the dependence of the binding energy of the H is linearly decreasing as one approaches the physical mass



HAL QCD $\quad m_{ps} = 673$ MeV $\quad m_B = 1485$ MeV $\xleftrightarrow[C_1]{} E_H = -35$ MeV

$\rightarrow$ the b.s. disappears

NPLQCD $\quad m_N = 1151.3$ MeV $\quad m_\Lambda = 1241.9$ MeV $\quad m_\Sigma = 1280.3$ MeV

$m_\Xi = 1349.6$ MeV $\quad m_\pi = 389$ MeV $\xleftrightarrow[C_1]{} E_H = -13.2$ MeV

$\rightarrow$ the b.s. moves upwards but remains below the ΞN threshold

Lattice QCD is a field rapidly growing that can play an important role in the determination of quantities relevant to nuclear physics processes

We have performed high statistics dynamical simulations of BB systems @ $m_\pi \sim 390$ MeV and at L ~ 2 , 2.5 , 3 , 4 fm

We found evidence of bound ΞΞ ($^1S_0$) and ΛΛ systems

It is necessary to undertake simulations at lighter quark masses to constrain the chiral extrapolations

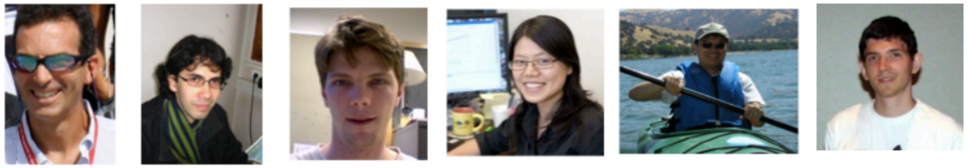
At present:

- Study of the ΛN and ΣN systems
- Simulations @ 230 MeV of pion mass

Nuclear Physics involve more than 2 particles. We are developping techniques and algorithms to study systems with multiple baryons

Acknowledgments and credits

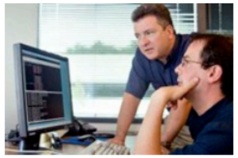
NPLQCD Collaboration



Silas Beane, New Hampshire · Emmanuel Chang, Barcelona · William Detmold, Willam+Mary · Huey-Wen Lin, U. of Washington · Tom Luu, LLNL · Saul Cohen, U. of Washington

Kostas Orginos, William+Mary · Assumpta Parreno, Barcelona · Marton Savage, U. of Washington · Aaron Torok, Indiana · Andre Walker-Loud, LBNL

+



R. Edwards, B. Joo
JLab

Resources

+

March-Oct 2012

GOBIERNO DE ESPAÑA · MINISTERIO DE CIENCIA E INNOVACIÓN

Generalitat de Catalunya
Departament d'Economia i Coneixement

Thank you