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Hadron properties in nuclear matter within the chiral soliton approach (from nucleons to nuclei)

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Motivation –"Simple Model of Nuclei and their Constituents"

- How well the idea of baryons as topological solutions?
- Whether is it possible to describe
  - the single hadrons properties in separate state,
  - in the community of their partners (interactions, existence as an individual…),
  - as well as the properties of that whole community in same footing?
- Can we construct some simple model to answer those questions, at least qualitatively?
- How far can we go in that direction?
- If it is far enough then how well?

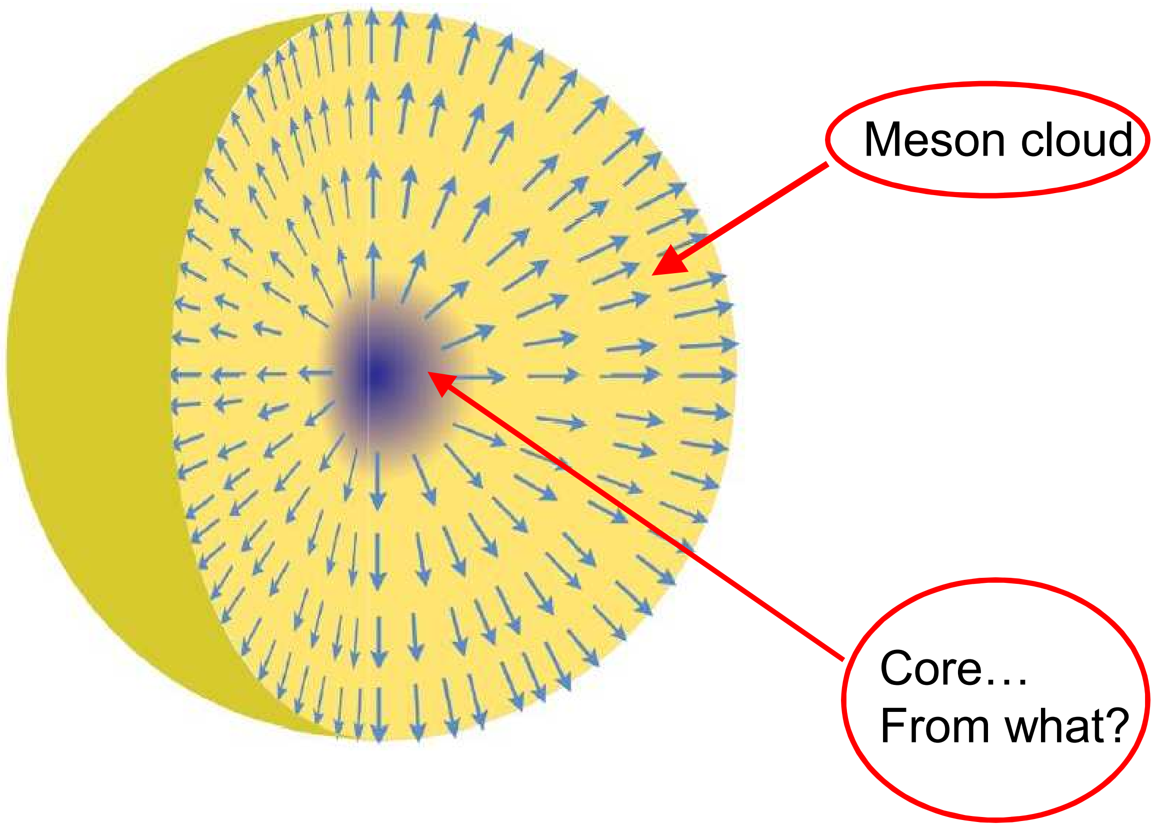
Content

- Topological solitons
  - Prototype Lagrangian
- In-medium modification - I
  - Hadron properties in symmetric nuclear medium
- In-medium modification - II
  - Symmetric nuclear matter properties
- In-medium modification – III
  - Asymmetric nuclear matter properties
- Summary

Topological models and Soliton

STRUCTURE

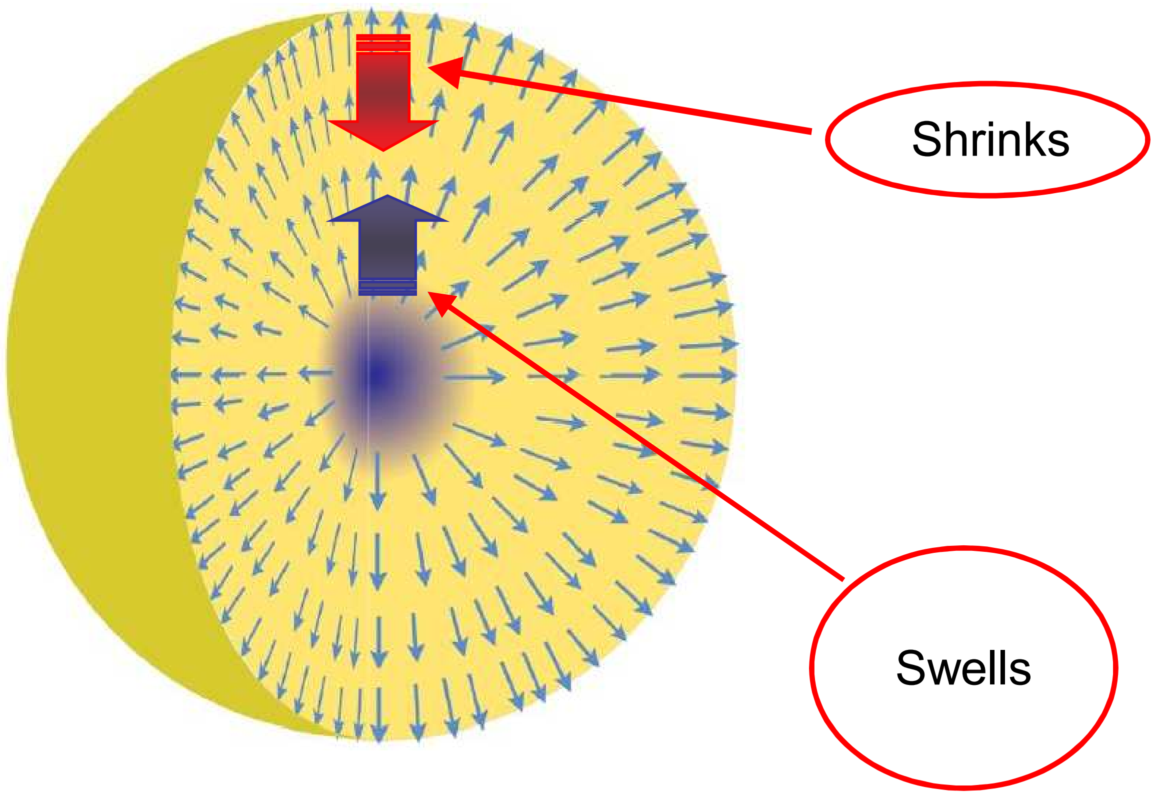
- What is a nucleon and, in particular, its core?
- At large number of colors it still has the mesonic content.



Topological models and Soliton

STABILIZATION

- Soliton has finite size and finite energy
- One needs at least two contrterms in the effective Lagrangian



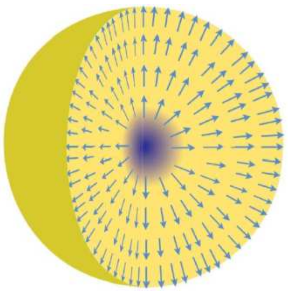
Topological models and Soliton

- Nonlinear chiral effective meson (pionic) theory (prototype - Skyrme Model)

$$\mathcal{L} = \frac{F_\pi^2}{16}\mathrm{Tr}\left(\partial_\alpha U\right)\left(\partial^\alpha U^+\right) + \frac{1}{32e^2}\mathrm{Tr}\left[U^+\partial_\alpha U, U^+\partial_\beta U\right]^2$$

Shrinks ← → Swells

- Hedgehog soliton (nontrivial mapping)



$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\left\{i\bar{\tau}\,\bar{n}F(r)\right\}$$

Prototype Lagrangian

$$\mathcal{L}_{\text{free}} = \frac{F_\pi^2}{16}\text{Tr}\left(\partial^\alpha U\right)\left(\partial_\alpha U^+\right) + \frac{1}{32e^2}\text{Tr}\left[U^+\partial_\alpha U, U^+\partial_\beta U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\text{Tr}\left(U + U^+ - 2\right)$$

- Nontrivial mapping
- It has topologically nontrivial solitonic solutions in different topological sectors with corresponding conserved topological number A
- Nucleon is quantized state of the classical soliton-skyrmion

$$U = \exp\{i\vec{\tau}\,\vec{\pi}/2F_\pi\} = \exp\{i\vec{\tau}\,\vec{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

$$H = M_{cl} + \frac{\vec{S}^2}{2I} = M_{cl} + \frac{\vec{T}^2}{2I},$$

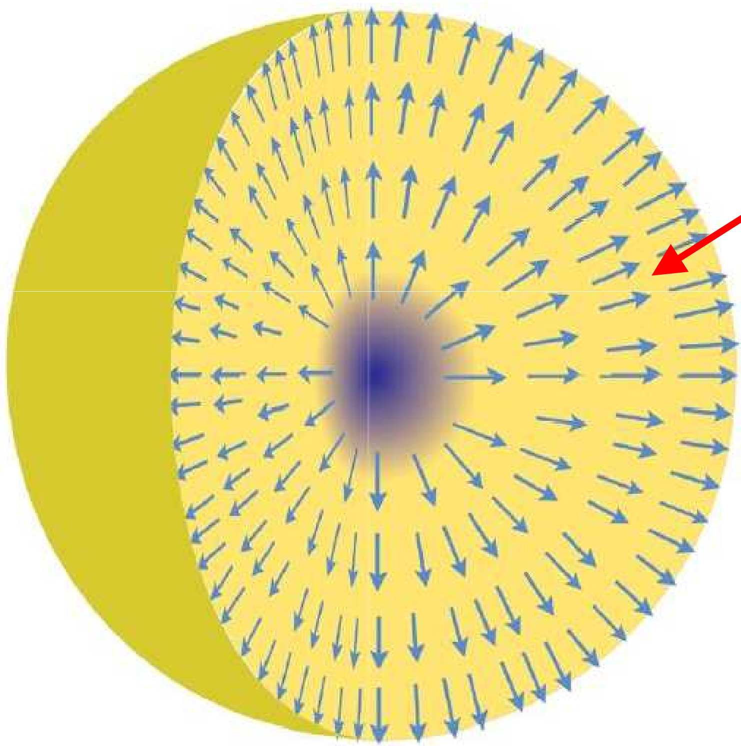
$$|\, S = T, s, t >= (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

Effective Lagrangian-I
Hadron properties in symmetric nuclear medium

Soliton in nuclear medium

Modification

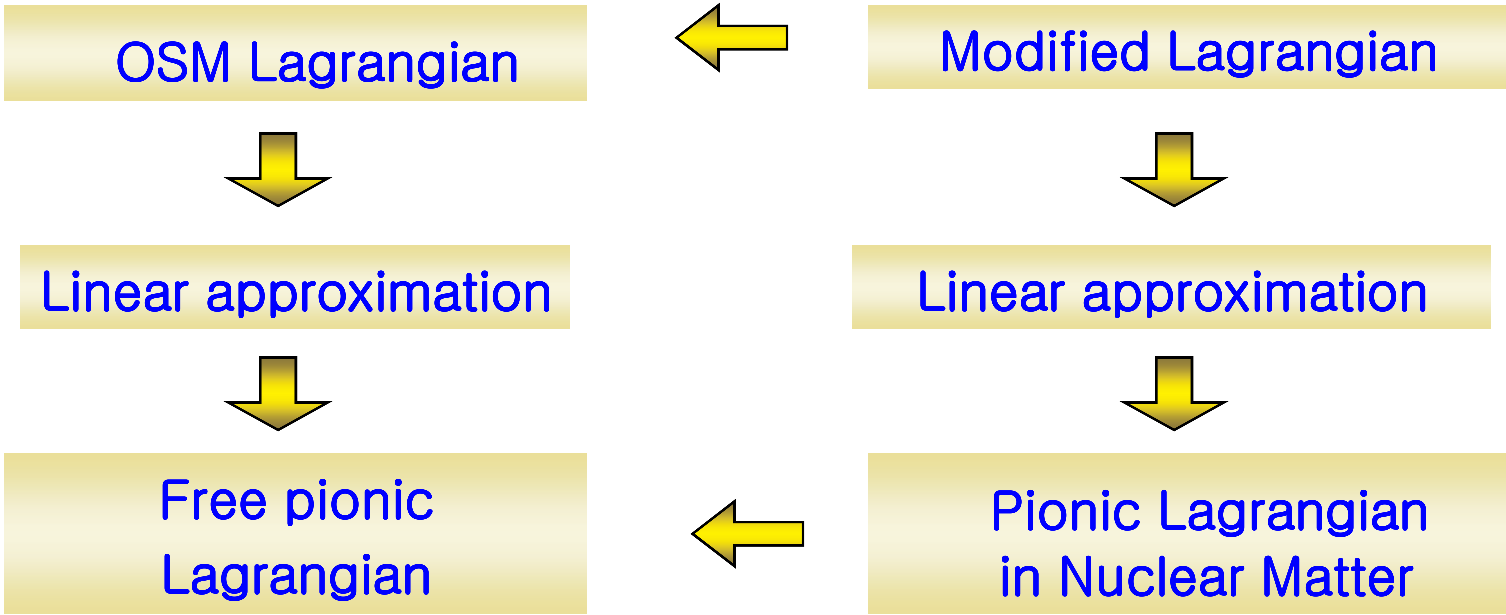
- Outer shell modifications



Meson cloud modifications in nuclear medium

Medium modifications

- Modification in the mesonic sector modifies the baryonic sector



- How to modify the mesonic sector?

Medium modifications

- Pion physics in nuclear matter (Optic potential approach):

  - Equation for in-medium pions

$$\left(\partial^{\mu}\partial_{\mu} + m_{\pi}^{2} + \hat{\Pi}\right)\overline{\pi} = 0$$

  - Structure of the optic potential

$$\hat{\Pi} = 2\omega U_{opt} = \chi_{s} + \vec{\nabla}\cdot\chi_{p}\vec{\nabla}$$

In-medium modified Lagrangian-I

[Rakhimov *et al,* PRC58, 1998]

- Medium modified Lagrangian (outer shell modifications)

$$\mathcal{L}^{*}_{\text{stat}} = -\frac{F_\pi^2}{16}\mathrm{Tr}\left(\vec{\nabla}U\right)\left(\vec{\nabla}U^+\right)\alpha_p + \frac{1}{32e^2}\mathrm{Tr}\left[L_i, L_j\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\mathrm{Tr}\left(U + U^+ - 2\right)\alpha_s$$

  - The medium functionals depend on S and P wave scattering lengths and volumes, and nuclear density

$$\alpha_s = 1 - \frac{4\pi\varsigma b_0 \rho}{m_\pi^2} \qquad \alpha_p = 1 - \frac{4\pi c_0 \rho / \zeta}{1 + 4\pi g' c_0 \rho / \zeta} \qquad \zeta = 1 + m_\pi / M_N$$

  - Scattering lengths (two parameters) are fitted from low energy pion-nucleus scattering data and external in our framework

In-medium modified Lagrangian-I

[A.Rakhimov *et al,* PRC58, 1998]

- How to treat the medium changes?

$$\mathcal{L}^{*}_{\mathrm{stat}} = -\frac{F_\pi^2}{16}\mathrm{Tr}\left(\vec{\nabla}U\right)\left(\vec{\nabla}U^+\right)\alpha_p + \frac{1}{32e^2}\mathrm{Tr}\left[L_i, L_j\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\mathrm{Tr}\left(U + U^+ - 2\right)\alpha_s$$

- Effective pion decay constant

$$F_\pi^* = F_\pi \sqrt{\alpha_p(\rho)}$$

- Effective mass of the pion

$$m_\pi^* = m_\pi \sqrt{1 + \frac{\alpha_s(\rho)}{\alpha_p(\rho)}}$$

- Pion physics in nuclear medium

$$\left(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}\right)\overline{\pi} = 0$$

Hadron properties in nuclear matter

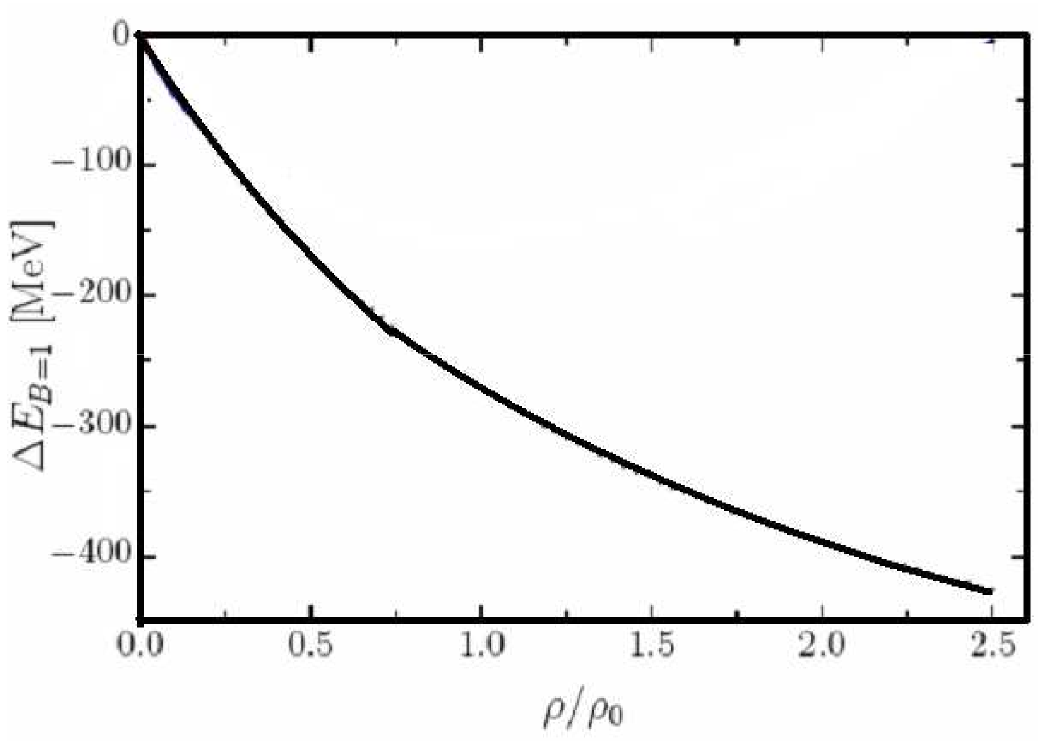
[A.Rakhimov *et al*, PRC58, 1998]

ρ / ρ_0	0	0.5			1.0		
g'	-	0.33	0.6	1	0.33	0.6	1
$g^*_{\pi NN}$	12.49	9.48	9.76	10.08	6.83	7.75	8.66
$M^*_N\ (MeV)$	868	743	756	770	635	675	715
$m^*_\pi\ (MeV)$	140	146	146	146	152	152	152
$\Lambda^*\ (MeV)$	528	484	489	494	448	462	477
$M^*_{\Delta N}\ (MeV)$	243	211	214	218	186	186	206

$$b_0 = b_0^{phen} = -0.024 m_\pi^{-1} \qquad c_0 = c_0^{phen} = -0.15 m_\pi^{-3}$$

Large renormalization: $$\frac{M^*_N(\rho_0)}{M_N^{\text{free}}} \approx 0.78$$

No description of Nuclear Matter!



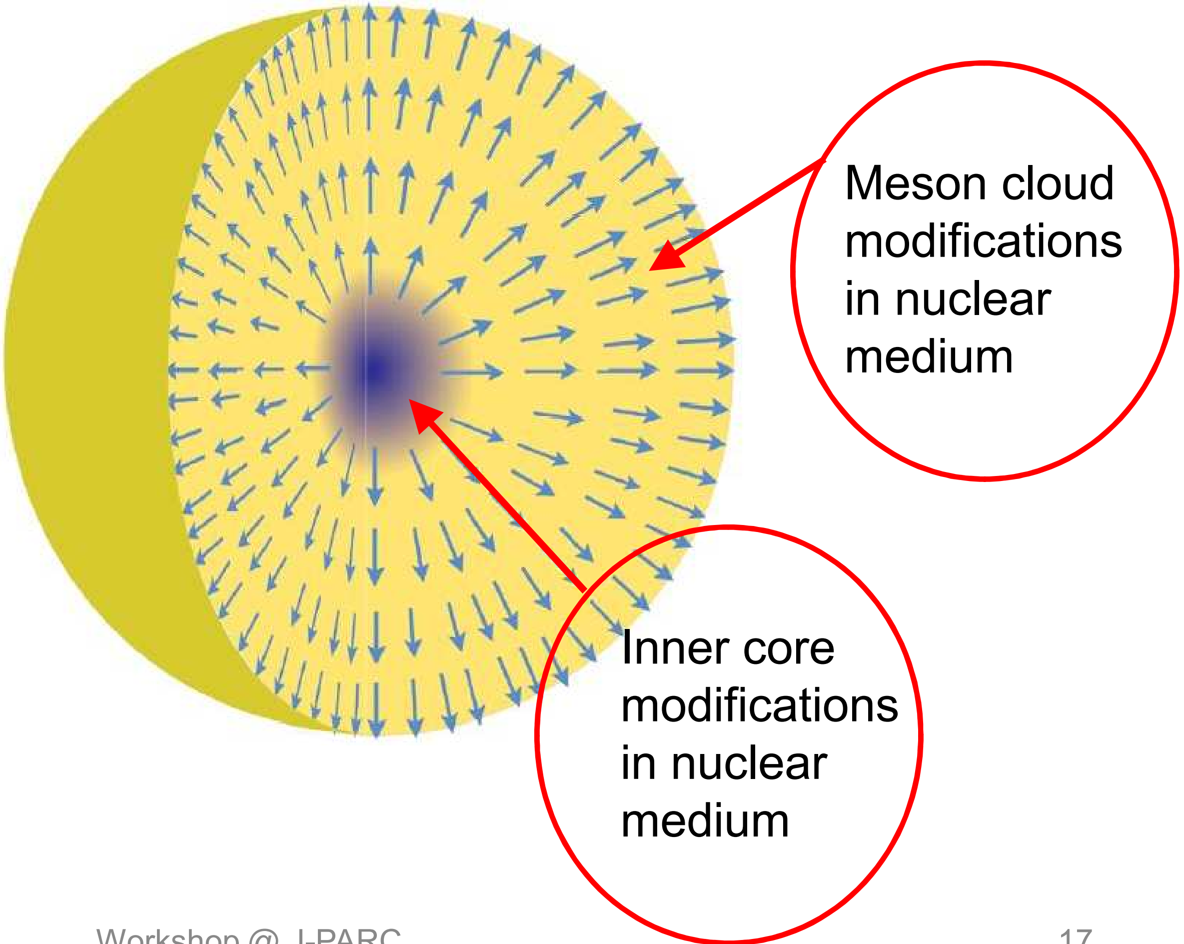
$$\Delta E_{B=1}(\rho) = m_N^*(\rho) - m_N^{\text{free}}$$

Effective Lagrangian-II
Symmetric Nuclear Matter

Soliton in nuclear medium

Modification

- Outer shell plus
- Inner core modifications (in particular at higher densities)



In-medium modified Lagrangian-II

[UY & HC Kim, PRC83, 2011]

- Core modifications - modification of the Skyrme term
  - May be related to vector meson properties in nuclear matter
  - May be related to nuclear matter properties

$$\mathcal{L}_4^* = \frac{1}{32e^{*2}} \mathrm{Tr}\left[L_\alpha, L_\beta\right]^2$$

$$e \to e^* = e\,\gamma^{1/2}(\rho)$$

Binding energy per nucleon (volume term)

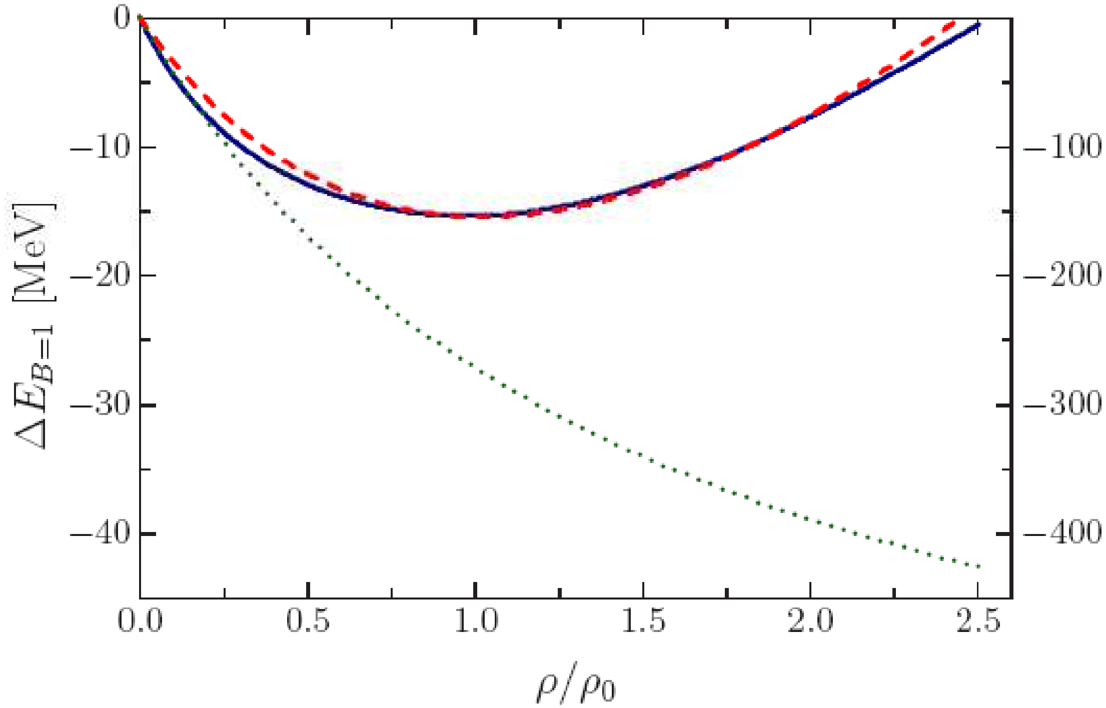
[UY & HC Kim, PRC83, 2011]

$$\Delta E(A,Z) = -a_V A + \ldots$$

$$\Delta E_{B=1} = -a_V = m_N^* - m_N^{\text{free}}$$

$$e \to e^* = e\,\gamma^{1/2}(\rho)$$

$$\gamma = \exp\left\{-\frac{\gamma_{\text{num}}\rho}{1+\gamma_{\text{den}}\rho}\right\}$$



$$b_0 = -0.024 m_\pi^{-1}$$

$c_0 = 0.21 m_\pi^{-3}$ – solid curve

$c_0 = 0.09 m_\pi^{-3}$ – dashed curve

Compressibility of nuclear matter

- Nuclear matter compression modulus and thermodynamic compressibility relation

$$\frac{1}{9}\rho K = \frac{1}{K^{\mathrm{th}}}$$

- Isothermal compressibility is defined as

$$\frac{1}{K^{\mathrm{th}}} = \rho \frac{\partial p}{\partial \rho} = \rho^2 \left(2 \frac{\partial a_V}{\partial \rho} + \rho \frac{\partial^2 a_V}{\partial \rho^2} \right)$$

- At normal nuclear matter density one has expression

$$K = 9\rho^2 \frac{\partial^2 a_V}{\partial \rho^2}\bigg|_{\rho=\rho_0}$$

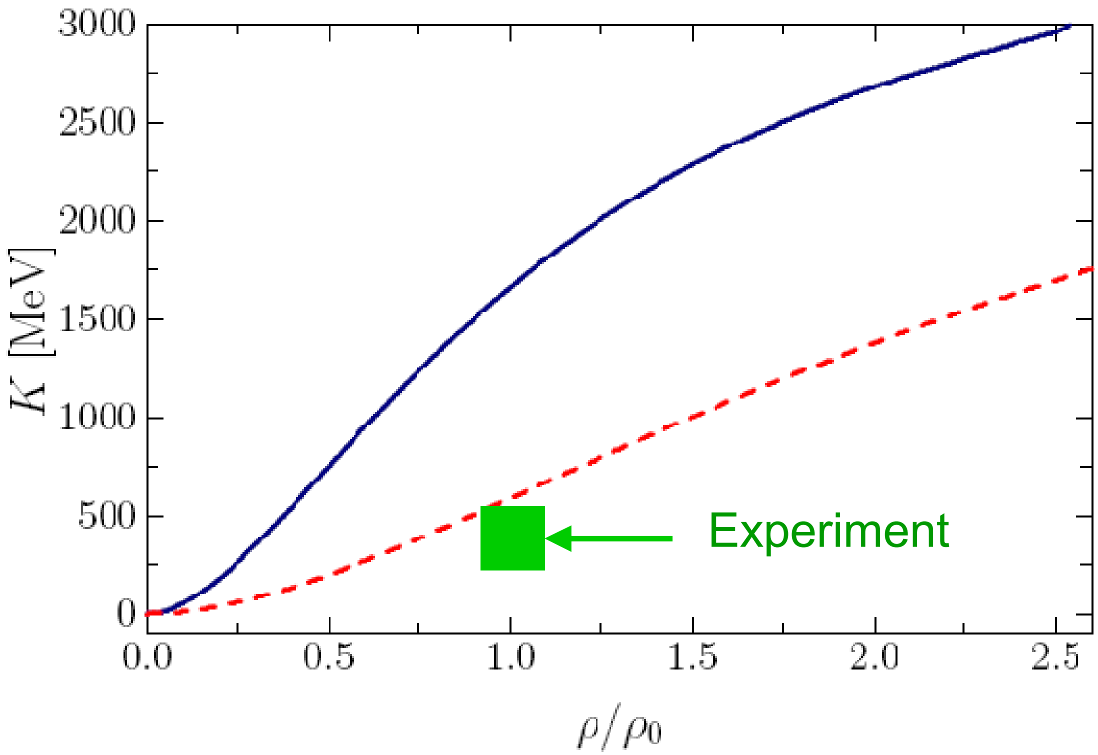
Compressibility of nuclear matter

[UY & HC Kim, PRC83, 2011]

$$\Delta E_{B=1} = m_N^* - m_N^{\text{free}}$$

$$e \to e^* = e\,\gamma^{1/2}(\rho)$$

$$\gamma = \exp\left\{ -\frac{\gamma_{\text{num}}\rho}{1+\gamma_{\text{den}}\rho} \right\}$$



$b_0 = -0.024 m_\pi^{-1}$

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Compressibility of nuclear matter

[UY & HC Kim, PRC83, 2011]

$b_0\ [m_\pi^{-1}]$	$c_0\ [m_\pi^{-3}]$	$\gamma_{\rm num}\ [m_\pi^{-3}]$	$\gamma_{\rm den}\ [m_\pi^{-3}]$	K [MeV]	$m^*_{N-\Delta}$ [MeV]
−0.024	0.21	2.098	1.451	1647.47	105.21
−0.024	0.15	1.448	0.998	1148.18	129.39
−0.024	0.09	0.797	0.496	582.79	170.34
−0.029	0.21	2.106	1.506	1637.16	107.13
−0.029	0.15	1.444	1.031	1142.00	131.59
−0.029	0.09	0.785	0.502	580.03	172.91

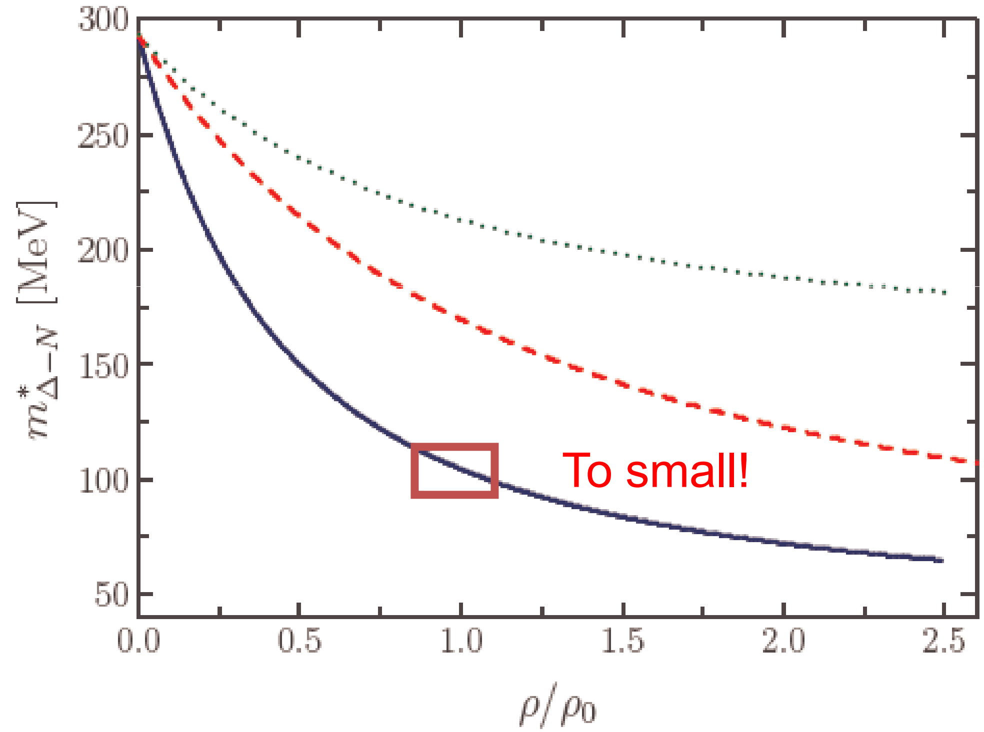
$$\alpha_s(\bar{r}) = 1 - \frac{4\pi\varsigma\, b_0 \rho(\bar{r})}{m_\pi^2}$$

$$\alpha_p(\bar{r}) = 1 - \frac{4\pi\, c_0 \rho(\bar{r})/\zeta}{1 + 4\pi g' c_0 \rho(\bar{r})/\zeta}$$

Symmetry energy in infinite nuclear matter approximation

- Symmetry energy can be related to Δ-N mass difference

$$E_{\mathrm{sym}} = \frac{1}{12} m^{*}_{\Delta-N}$$



Effective Lagrangian-III
Isospin breaking & Symmetry energy

Isospin breaking effects

- Three types of pions treated separately
- In nuclear matter, one considers three types of polarization operators
- There will be some additional parameters which correspond to isospin breaking environment

$$\left(\partial^{\mu}\partial_{\mu} + m^{2}_{\pi^{(\pm,0)}}\right)\vec{\pi}^{(\pm,0)} = 0$$

$$\left(\partial^{\mu}\partial_{\mu} + m^{2}_{\pi^{(\pm,0)}} + \hat{\Pi}^{(\pm,0)}\right)\vec{\pi}^{(\pm,0)} = 0$$

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

In-medium modified Lagrangian-III

- Outer&inner shell modifications + isospin breaking

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm as}\,,$$
$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}_4 + \mathcal{L}^*_{\chi{\rm SB}}\,,$$
$$\mathcal{L}^*_{\rm as} = \Delta\mathcal{L}_{\rm mes} + \Delta\mathcal{L}^*_{\rm env}\,,$$
$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\left\{\left(1 + \frac{\chi_s^{02}}{m_\pi^2}\right)\mathrm{Tr}\,(\partial_0 U \partial_0 U^\dagger) - (1 - \chi_p^0)\,\mathrm{Tr}(\nabla U \cdot \nabla U^\dagger)\right\},$$
$$\mathcal{L}_4 = \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2\,,$$

$$e \to e^* = e\,\gamma^{1/2}(\rho)$$

$$\mathcal{L}^*_{\chi{\rm SB}} = -\frac{F_\pi^2 m_\pi^2}{16}\left(1 + m_\pi^{-2}\chi_s^{00}\right) \times \mathrm{Tr}\left[(U-1)(U^\dagger - 1)\right],$$
$$\Delta\mathcal{L}_{\rm mes} = -\frac{F_\pi^2}{16}\sum_{a=1}^{2}\mathcal{M}_-^2\,\mathrm{Tr}(\tau_a U)\,\mathrm{Tr}(\tau_a U^\dagger),$$
$$\Delta\mathcal{L}^*_{\rm env} = -\frac{F_\pi^2}{16}\sum_{a,b=1}^{2}\varepsilon_{ab3}\frac{\Delta\chi_s + \Delta\chi_p}{2m_\pi} \times \mathrm{Tr}(\tau_a U)\,\mathrm{Tr}(\tau_b \partial_0 U^\dagger),$$

Improvements

- Nucleon in finite nuclei (including isospin asymmetric environment)
  - Volume energy
  - Surface term
  - Coulomb term ← Studies in future
  - Symmetry energy ← Present talk
- Applications
  - Mirror nuclei
  - Neutron matter (astrophysics) ← Elsewhere

Symmetric nuclear matter

- Volume term in the binding energy formula takes form

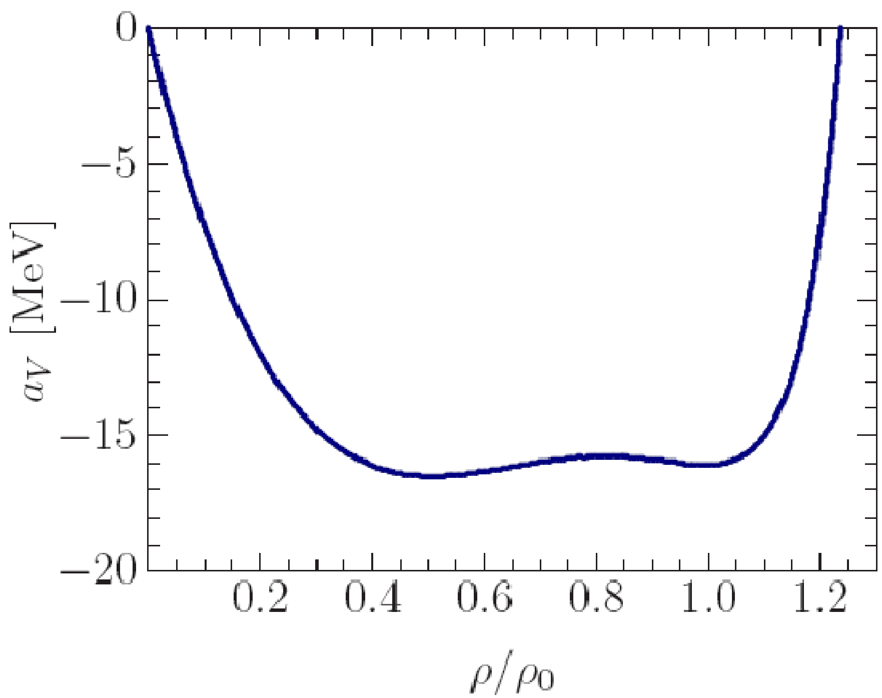
$$\Delta E_V = \left(\frac{m_{\rm p}^* + m_{\rm n}^*}{2} - \frac{m_{\rm p} + m_{\rm n}}{2} \right) A \equiv a_V(\rho) A$$

  - In general, due to isospin breaking effects the masses of nucleons are different
  - Due to medium modifications the masses are density dependent

Binding energy per nucleon

[UY, JKPS 60, 2012]

- Volume term coefficient as a function of normalized nuclear matter density

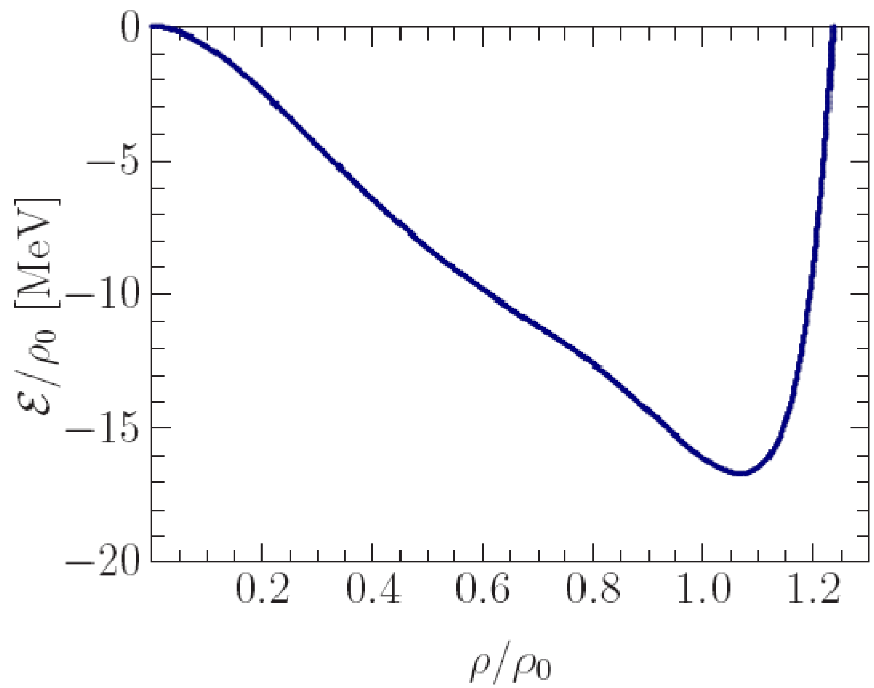


Binding energy per nucleon

[UY, JKPS 60, 2012]

- Fraction of the binding energy per unit volume to normal nuclear matter density as a function of normalized density

$$\mathcal{E} \equiv \frac{\Delta E}{V} = a_V \frac{A}{V} = a_V \rho$$

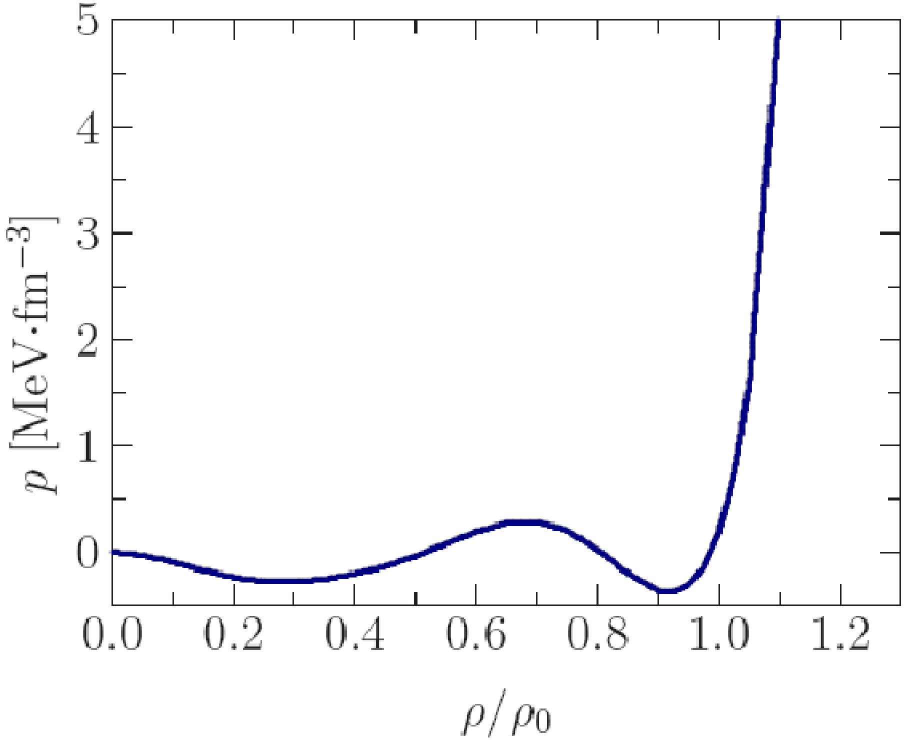


Pressure

[UY, JKPS 60, 2012]

- Pressure as a function of normalized density

$$p = \rho\frac{\partial\mathcal{E}}{\partial\rho} - \mathcal{E} = \rho^2\frac{\partial a_V}{\partial\rho}$$



Compressibility

[UY, JKPS 60, 2012]

TABLE I: Compression modulus K of nuclear matter. The variational parameters $\gamma_{\rm num}$ and $\gamma_{\rm den}$ are chosen in such a way that at $\rho = \rho_0$ the minimum of binding energy per nucleon $\Delta E/A \simeq -16$ MeV is reproduced correctly.

ρ_0 [fm^{-3}]	$\gamma_{\rm num}$ [m_π^{-3}]	$\gamma_{\rm den}$ [m_π^{-3}]	K [MeV]
0.159	2.038	0.165	931
0.156	2.030	0.157	732
0.152	2.025	0.151	317

Asymmetric matter

- Hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{sym}} - \left(a^* - \frac{\Lambda^*_{\mathrm{env}}}{\Lambda^*_{\omega\Omega,33}} \right) \hat{T}_3$$

- Effective mass

$$m^*_{\mathrm{n,p}} = m^*_{\mathrm{N,S}} + T_3 \, \Delta m^*_{\mathrm{np}}$$

- Nucleus mass

$$\begin{aligned} M(A,Z) &= Z \left(m^*_{\mathrm{N,S}} - \frac{\Delta m^*_{\mathrm{np}}}{2} \right) \\ &\quad + (A-Z) \left(m^*_{\mathrm{N,S}} + \frac{\Delta m^*_{\mathrm{np}}}{2} \right) \\ &= A m^*_{\mathrm{N,S}} + \left(\frac{A}{2} - Z \right) \Delta m^*_{\mathrm{np}}. \end{aligned}$$

Symmetry energy

- Binding energy per nucleon

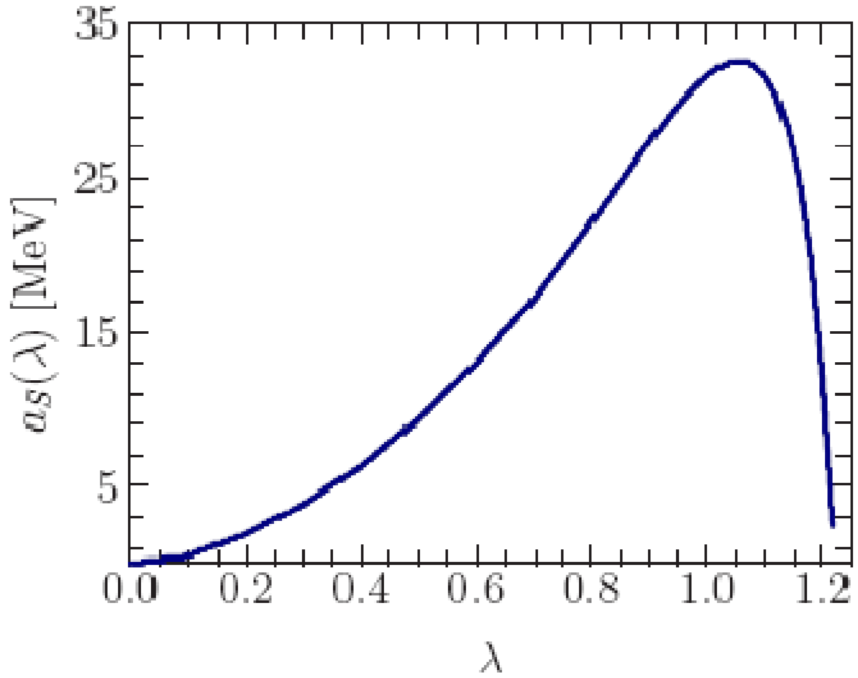
$$E_{A=1} = -\left(m_{\mathrm{N,S}}^{\mathrm{free}} - m_{\mathrm{N,S}}^{*}\right) + \frac{N-Z}{2A}\left(\Delta m_{\mathrm{np}}^{*} - \Delta m_{\mathrm{np}}^{\mathrm{free}}\right)$$

- Symmetry energy term

$$\frac{N-Z}{2A}\left(\Delta m_{\mathrm{np}}^{*} - \Delta m_{\mathrm{np}}^{\mathrm{free}}\right) \equiv a_S \beta^2$$

Symmetry energy

- Symmetry energy term coefficient as a function of normalized nuclear matter density



Symmetry energy

- Symmetry energy term coefficients (in MeV)

$$a_S(\lambda) = a_S(1) + \frac{L_S}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 + \ldots$$

	$a_S(1)$	L_S	K_S
The result	31.65	91.21	-3700
„Experiment"	~ 30-35	~ 70-90	~ -500

Summary

- How well the idea of baryons as topological solutions? It seems, not bad…
- Whether is it possible to describe
  - the single hadrons properties in separate state,
  - in the community of their partners (interactions, existence as an individual…),
  - as well as the properties of that whole community in same footing? It seems, yes.
- Can we construct some simple model to answer those questions, at least qualitatively? It seems, yes.
- How far can we go in that direction? We will see…
- If it is far enough how well is that direction? …

Thank you very much!