

Complex plane QCD sum rules

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Abstract

- New sum rules for hadronic spectrum are constructed
- In this sum rules complex parameter are available
- They are analyzed with maximum entropy method (MEM)

Construction of complex plane QCD sum rules

1. Dispersion relation allows its parameter $q^2 (= z)$ to be complex

$$\Pi(z) = \int_0^\infty \frac{\rho(s)}{s - z} ds + \text{polynomial in } z$$

2. Analytic continuation of OPE is available in the region where the error resulting from the truncated terms can be neglected

$$\Pi^{\text{OPE}}(z) = \int_0^\infty \frac{\rho(s)}{s - z} ds + \text{polynomial in } z \quad - (\star)$$

3. Polynomial terms can be eliminated by the Borel transformation with the variable $|z|$:

$$\hat{B}_{|z|} = \lim_{\substack{|z|, n \rightarrow \infty \\ |z|/n = M^2}} \frac{|z|^n}{(n-1)!} \left(-\frac{\partial}{\partial |z|} \right)^n$$

4. Operated by the Borel transformation, the sum rules (\star) can be written as below :

complex plane QCD sum rules

$$G^{\text{OPE}}(\mathcal{M}^2) = \frac{1}{\mathcal{M}^2} \int_0^\infty e^{-s/\mathcal{M}^2} \rho(s) ds$$

where \mathcal{M}^2 is complex number. K^R and K^I are damped oscillation.

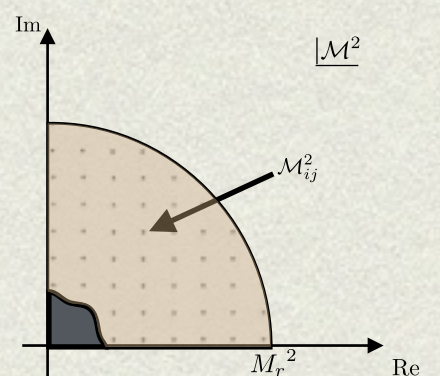
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$$\begin{cases} \text{Re}[G^{\text{OPE}}(\mathcal{M}^2)] &= \int_0^\infty K^R(\mathcal{M}^2; s) \rho(s) ds, \\ \text{Im}[G^{\text{OPE}}(\mathcal{M}^2)] &= \int_0^\infty K^I(\mathcal{M}^2; s) \rho(s) ds, \end{cases}$$

5. The choice of Complex Borel mass

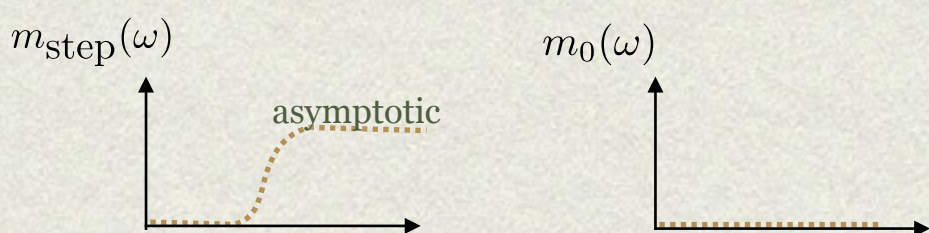
- The Borel transformation does not interchange with the integral in the second and third quadrants
- The sum rules in the forth quadrant are identical with ones in the first
- Considering the convergence of OPE, the region near the origin is prohibited

※ In practical MEM analyses , complex Borel masses only inner the outer radius M_r^2 are used and discretized.



Analysis with MEM (ϕ meson)

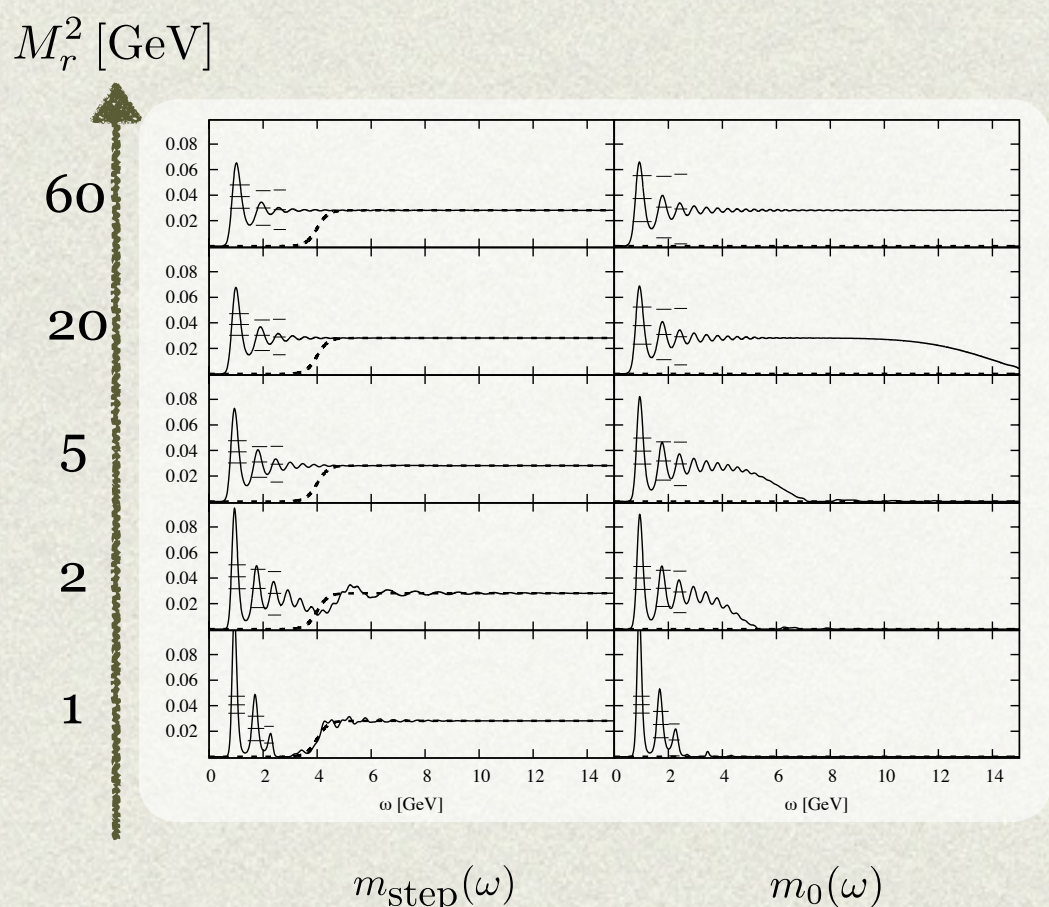
- Two type of default models are used



- Five values of outer circle radiuses are used

$$M_r^2 = 1, 2, 5, 20, 60 \text{ [GeV]}^2$$

The ten kinds of analyses results are shown right.
Solid lines are spectral functions and dotted lines are default model



Conclusion

- We have constructed complex plane QCD sum rules and applied them to MEM
- The positions of the first peaks are all about 0.93 [GeV], which agree with the experimental value 1.02 [GeV]
- Moreover, it is found to be able to produce **continuum state** not by default model