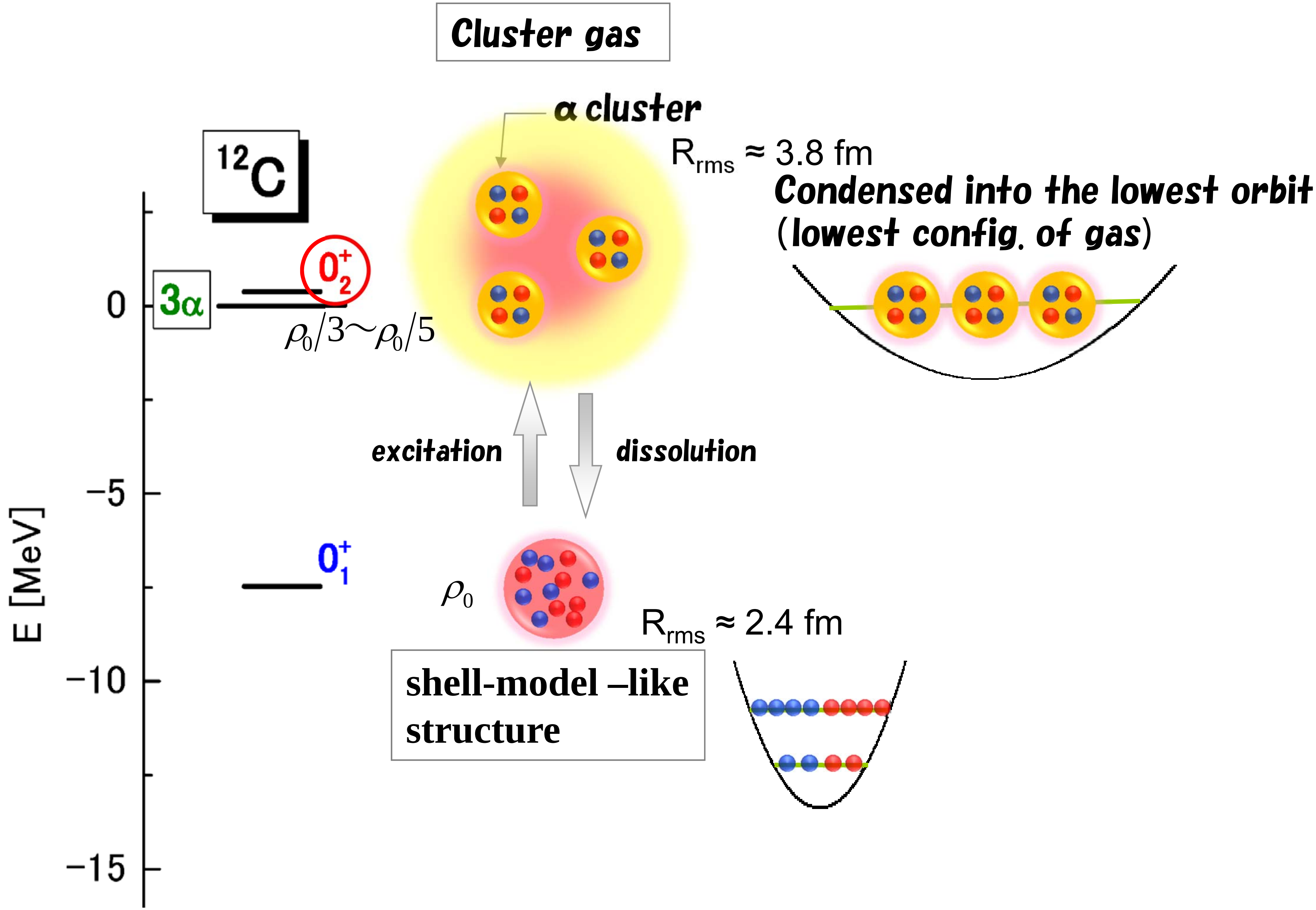
A new theoretical approach to study light hypernuclei

Yasuro Funaki (RIKEN)

KEK theory center workshop on J-PARC hadron physics in 2004, @ KEK, Tokai, February 10-12, 2014.

Alpha condensate state



THSR wave function : Alpha condensate-type wave function

Particle number projected BCS w.f.

$$\langle \boldsymbol{r}_1, \cdots, \boldsymbol{r}_{2n} | \mathrm{BCS} \rangle = \mathcal{A}\Big\{ \Phi(\boldsymbol{r}_1, \boldsymbol{r}_2)\Phi(\boldsymbol{r}_3, \boldsymbol{r}_4)\cdots\Phi(\boldsymbol{r}_{2n-1}, \boldsymbol{r}_{2n}) \Big\}$$

n α condensate w.f.

$$\langle \boldsymbol{r}_1, \cdots, \boldsymbol{r}_{4n} | \Phi_{n\alpha} \rangle = \mathcal{A}\Big\{ \Phi(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4)\Phi(\boldsymbol{r}_5, \boldsymbol{r}_6, \boldsymbol{r}_7, \boldsymbol{r}_8)\cdots\Phi(\boldsymbol{r}_{4n-3}, \boldsymbol{r}_{4n-2}, \boldsymbol{r}_{4n-1}, \boldsymbol{r}_{4n}) \Big\}$$

Variational ansatz (only one parameters B, or with deformation, B_x=B_y, B_z)

(THSR ansatz) A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke et al., PRL **87**, 192501 (2001).

$$\Phi(\boldsymbol{r}_{4i-3}, \cdots, \boldsymbol{r}_{4i}) = e^{-\frac{2}{B^2}(\boldsymbol{X}_i - \boldsymbol{X}_G)^2} \phi_\alpha(\boldsymbol{r}_{4i-3}, \cdots, \boldsymbol{r}_{4i})$$

$$\phi_\alpha \propto e^{-\frac{1}{8b^2}\sum_{k<l}(\boldsymbol{r}_k - \boldsymbol{r}_l)^2}$$

c.o.m. of *i*-th α particle

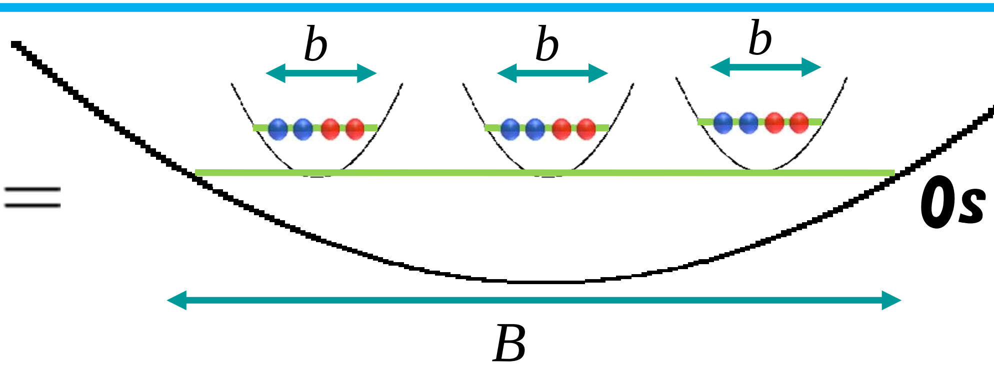
$$\boldsymbol{X}_i = \frac{\boldsymbol{r}_{4i-3} + \cdots + \boldsymbol{r}_{4i}}{4}$$

Total c.o.m.

$$\boldsymbol{X}_G = \frac{\boldsymbol{r}_1 + \cdots + \boldsymbol{r}_{4n}}{4n}$$

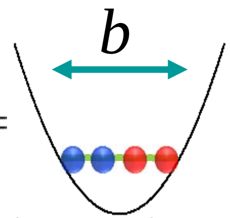
n=3 case

$$\Phi_{3\alpha}(B, b) =$$



b: fixed (≒1.4 fm)

$$\phi_\alpha(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_4) \doteq$$



$(0s)^4$ configuration

Two limits

B =b: Shell model w.f.

B >>b: Gas of independent α-particles

For ^{8}Be **Full 2α RGM solution, which is given by superposing many Brink w.fs, is completely equivalent to a single THSR w.f. (99.99%)**

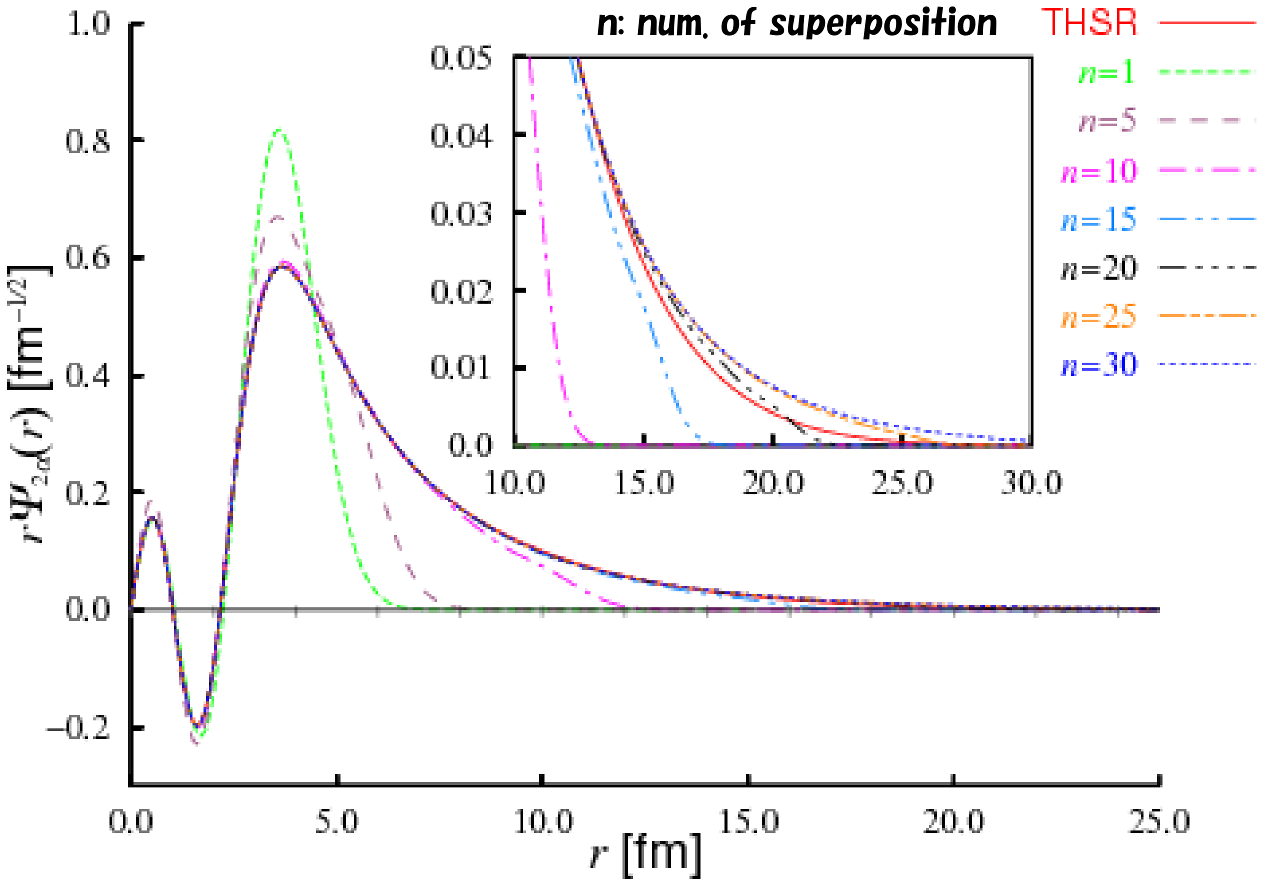
Y. F. et al., PTP 108, 297 (2002); PRC 80, 064326 (2009).

2α RGM eq.

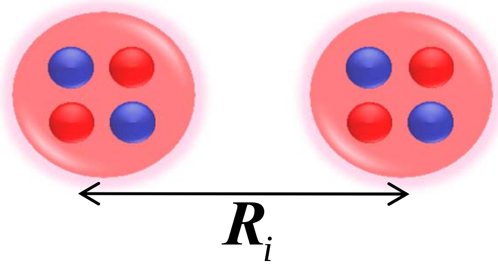
$$(H - EN)\chi = 0 \Longleftrightarrow \left(\frac{1}{\sqrt{N}} H \frac{1}{\sqrt{N}} - E\right)\Psi_{2\alpha} = 0$$

$$\Psi_{2\alpha} = \sqrt{N}\chi = \int d^3\boldsymbol{b}\sqrt{N(\boldsymbol{a},\boldsymbol{b})}\chi(\boldsymbol{b})$$

Relative w.f. between 2α particles



Superposition of dumbells



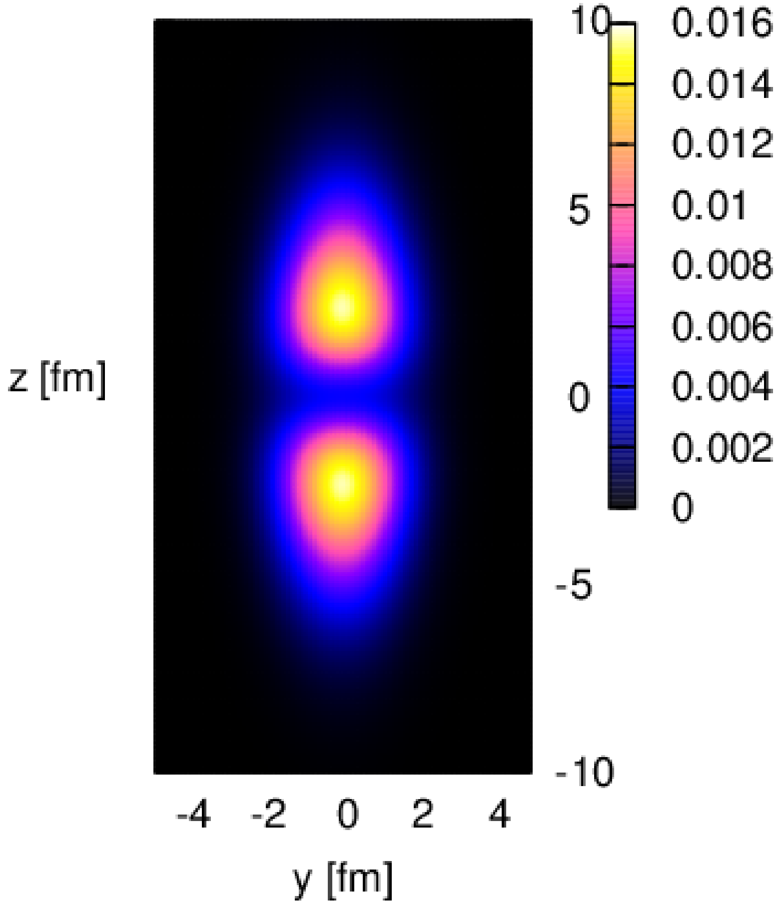
$$\chi^{\mathrm{Brink}}(\boldsymbol{r}) = \sum_{i=1}^{n} f(\boldsymbol{R}_i)\hat{P}_{J=0}\exp\left[-\frac{(\boldsymbol{r}-\boldsymbol{R}_i)^2}{b^2}\right]$$

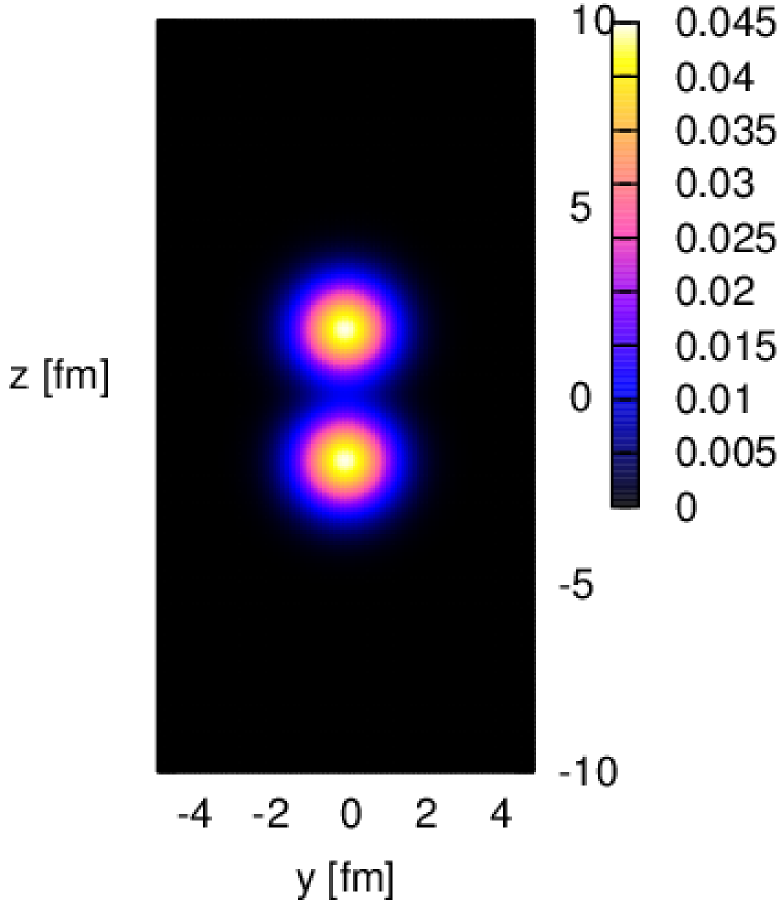
$$\Psi_{2\alpha}^{\mathrm{Brink}} = \sqrt{N}\chi^{\mathrm{Brink}}$$

The superposition of 30 Brink w.fs. coincides with one THSR !

$$\left|\left\langle \Psi_{2\alpha}^{\mathrm{Brink}} \middle| \Psi_{2\alpha}^{\mathrm{THSR}} \right\rangle\right|^2 = 0.9999$$

Intrinsic densities of ^{8}Be via THSR(left) and single Brink (right) wfs.



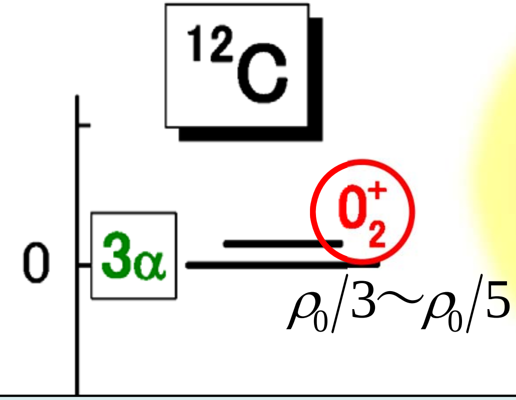


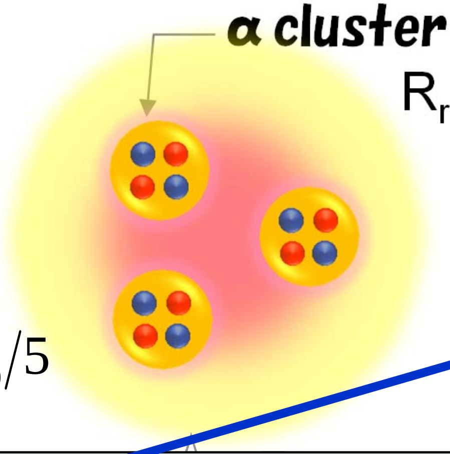
R=3.5 fm

First example of alpha cond. state

Y. F. et al., PRC 67, 051306(2003)

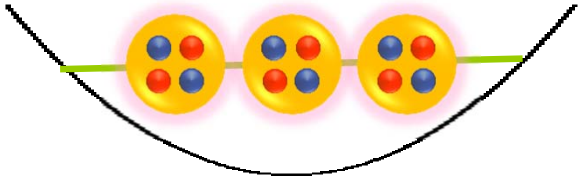
Cluster gas





$R_{rms} \approx 3.8$ fm

Condensed into the lowest orbit (lowest config. of gas)



3α THSR w.f.

RGM $\left\langle \phi_{\alpha_1}\phi_{\alpha_2}\phi_{\alpha_3} \middle| H-E \middle| \mathcal{A}\left\{ \chi_{3\alpha}^{RGM}(\xi_1,\xi_2)\phi_{\alpha_1}\phi_{\alpha_2}\phi_{\alpha_3} \right\} \right\rangle = 0$

$$\hat{\mathcal{P}}_{g.s.}\hat{\mathcal{P}}_{J=0}\mathcal{A}\left\{ \prod_{i=1}^{3} \chi_{3\alpha}^{THSR}(\boldsymbol{X}_i - \boldsymbol{X}_G : B_\perp, B_z)\phi_{\alpha_i} \right\} = \mathcal{A}\left\{ \chi_{3\alpha}^{RGM}(\xi_1,\xi_2)\phi_{\alpha_1}\phi_{\alpha_2}\phi_{\alpha_3} \right\}$$

$$\chi_{3\alpha}^{THSR}\left(X : B_\perp, B_z\right) = e^{-\frac{2}{B_\perp^2}\left(X_x{}^2 + X_y{}^2\right) - \frac{2}{B_z^2}X_z{}^2}$$

($B_\perp$, B_z)=(7.6 fm, 2.5 fm) gives the very Hoyle state w.f.

This gives 99.3 % squared overlap with RGM w.f.   (For ^{8}Be, 99.99 % with 2α RGM w.f.)

c.o.m. of *i*-th α particle

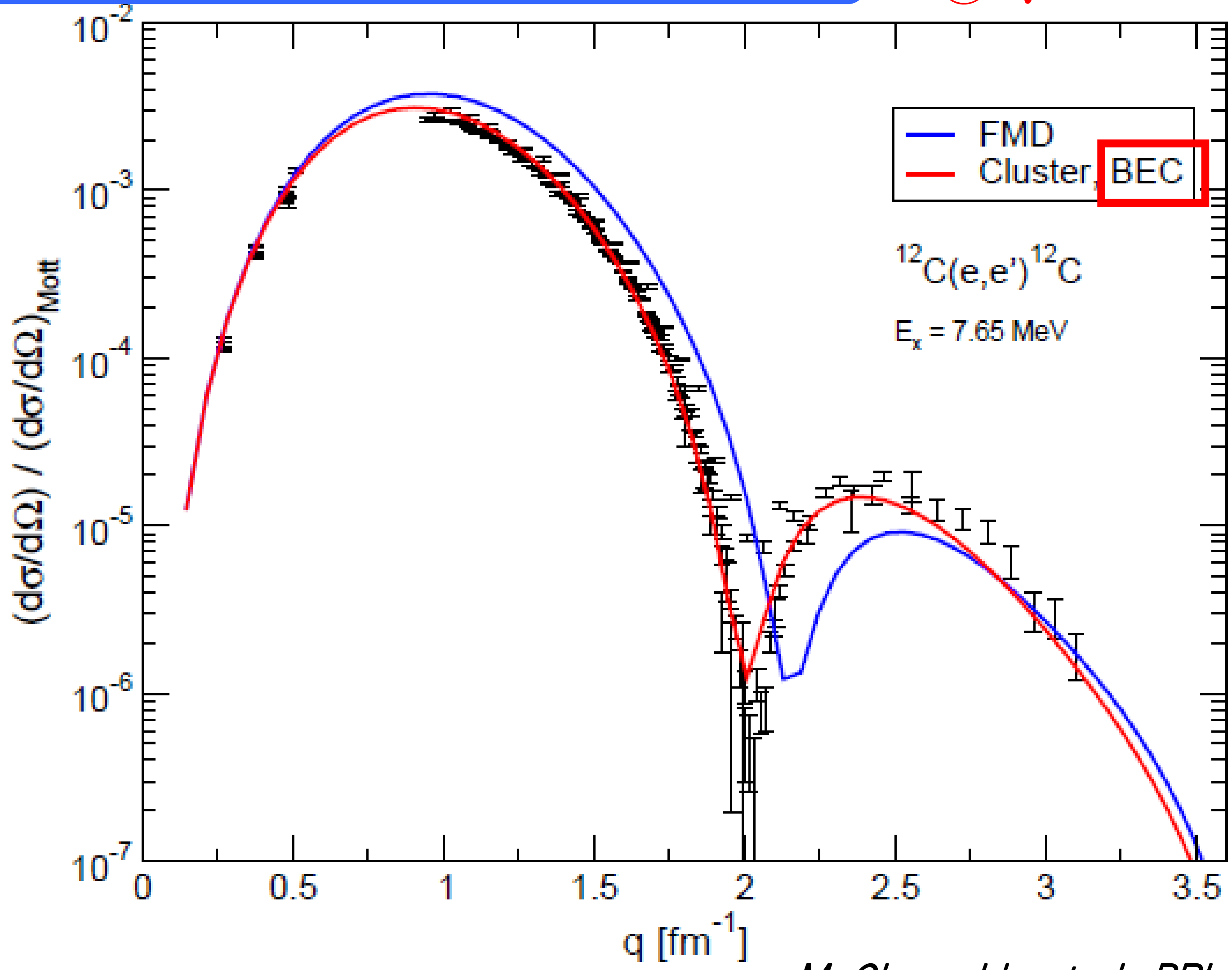
$$\boldsymbol{X}_i = \frac{\boldsymbol{r}_{4i-3} + \cdots + \boldsymbol{r}_{4i}}{4}$$

Total c.o.m.

$$\boldsymbol{X}_G = \frac{\boldsymbol{r}_1 + \cdots + \boldsymbol{r}_{4n}}{4n}$$

Electron Scattering Data ($0_1^+ \rightarrow 0_2^+$)

©T. Neff



M. Chernykh. et al., PRL 98, 032501 (2007)

Very nice reproduction by THSR w.f. (BEC)

Until now, we have thought that the single THSR w.f. is suitable for describing only gas-like cluster states such as represented by the alpha-condensate states.

For large $(B_\perp, B_z)$

However, this idea is completely misleading.

We pointed It out in new collaboration with Nanjing group.

Non-localized clustering: A new concept in nuclear clustering

Bo Zhou (Nanjing), Zhongzhou Ren (Nanjing), Chang Xu (Nanjing)
Y. Funaki, T. Yamada, A. Tohsaki , H. Horiuchi, P. Schuck, G. Röpke

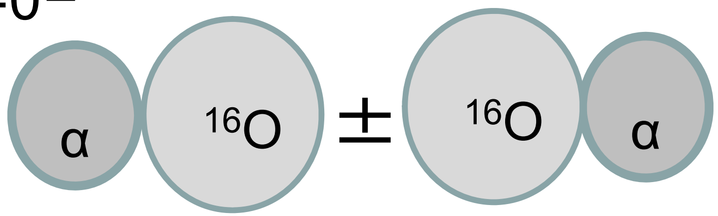
This opened a new horizon for nuclear cluster physics.

B. Zhou, Y. F. et al., PRC86, 014301 (2012); PRL 110, 262501(2013); arXiv:1310.7684(submitted to PRC)

First example is the inversion doublet bands in ^{20}Ne.

Localized clustering picture has been (had been) an important basis to understand them.

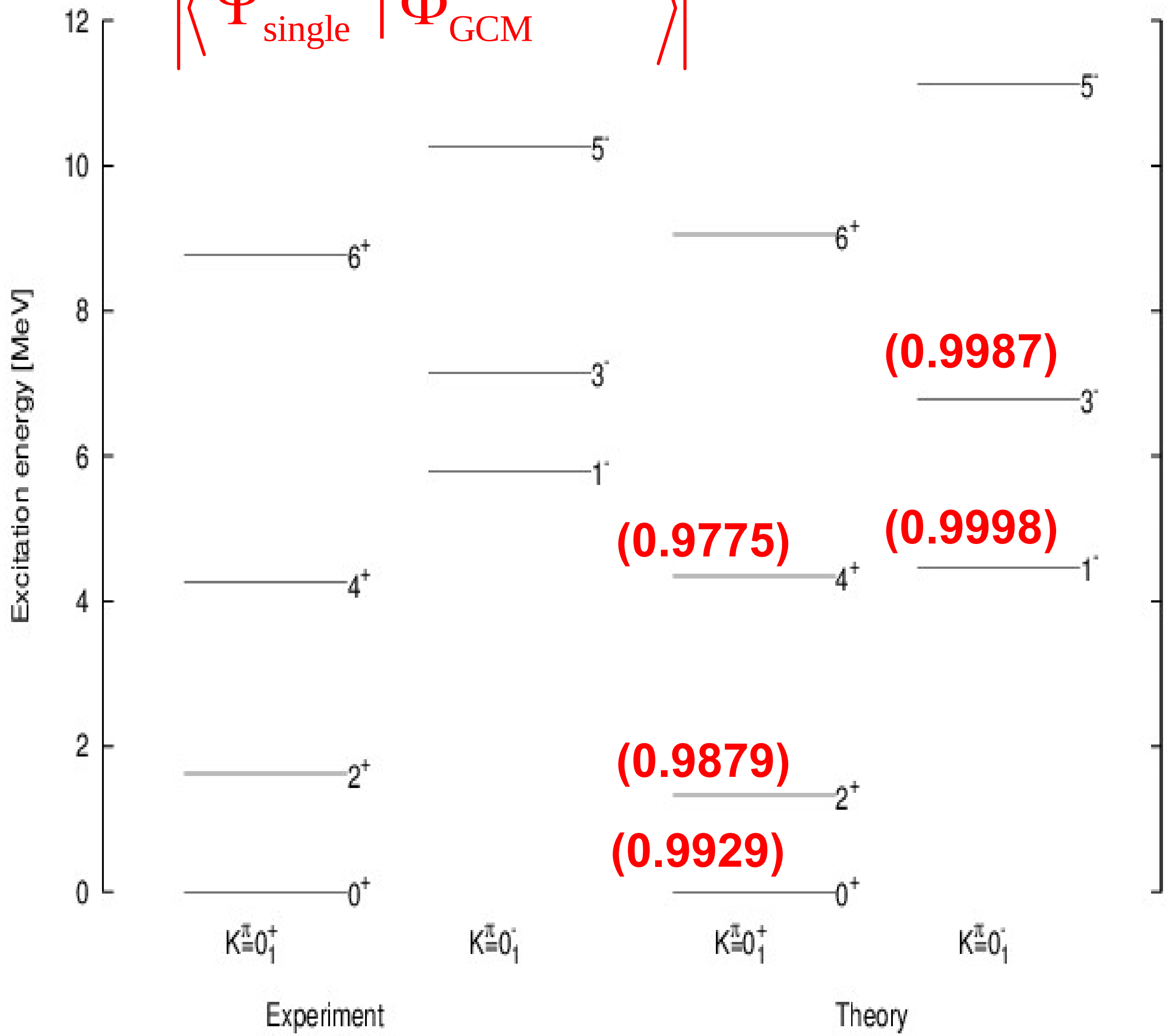
$K^\pi=0^\pm$

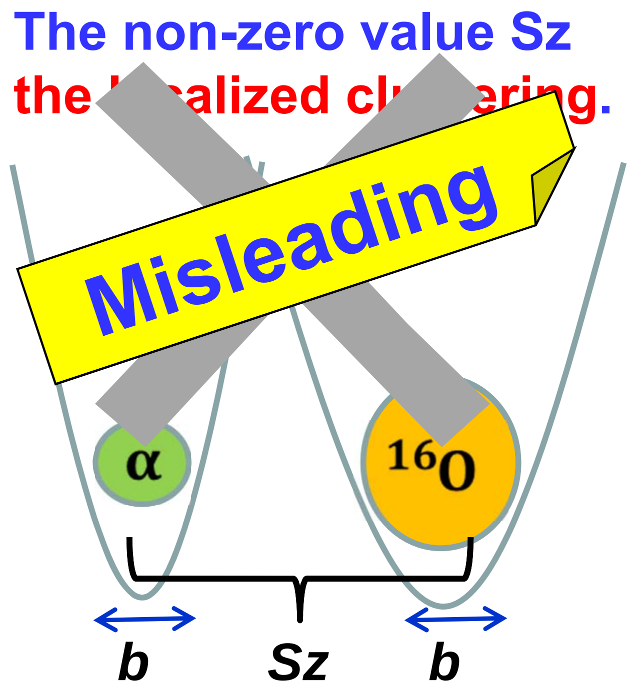


The energy levels of α+^{16}O inversion doublet bands in ^{20}Ne by THSR w.f.

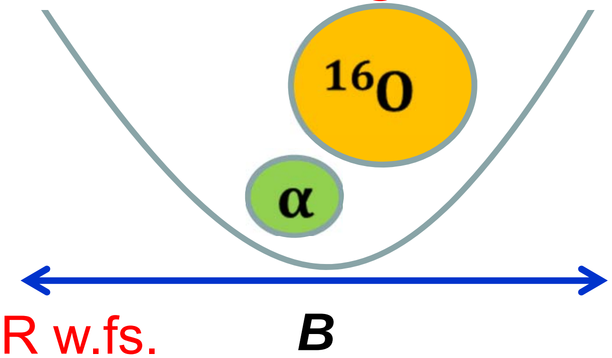
In parentheses

$$\left|\left\langle \Psi^{\mathrm{THSR}}_{\mathrm{single}} \middle| \Phi^{\alpha+{}^{16}\mathrm{O}\ \mathrm{Brink}}_{\mathrm{GCM}} \right\rangle\right|^2$$





The localized (from Pauli principle) and non-localized.
Cluster orbit is generated.

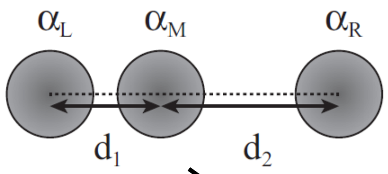


The rotational bands are reproduced using the single THSR w.fs.

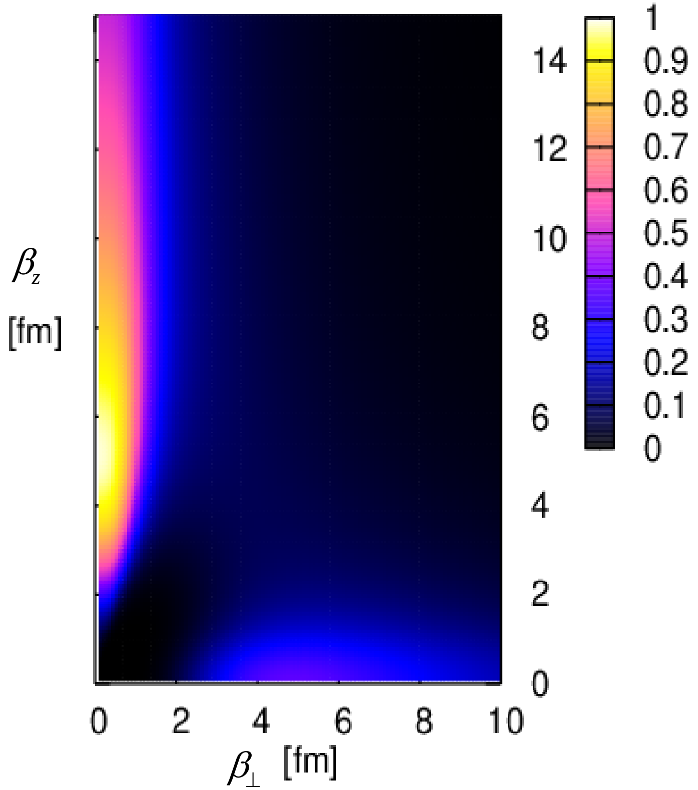
The squared overlap with one-dim. THSR with single parameter

T. Suhara, Y. F. et al., submitted to PRL(arXiv:1310.7684)

$$B_k^2 = b^2 + 2\beta_k^2 \ (k = x, y, z)$$
$$\beta_\perp \equiv \beta_x = \beta_y$$

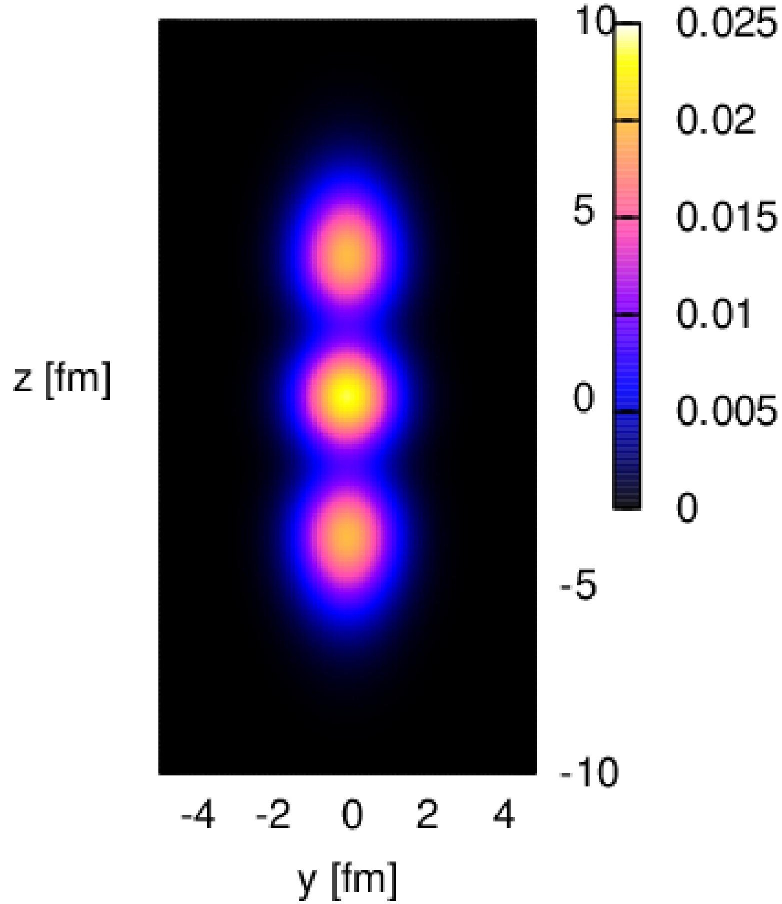


$$O(\beta_\perp, \beta_z) = \left| \left\langle \Phi^{\rm THSR}_{J=0}(\beta_\perp, \beta_z) \middle| \sum_{d_1, d_2} f_\lambda^{(J=0)}(d_1, d_2) \Phi_{J=0}(d_1, d_2) \right\rangle \right|^2$$



The superposition of 100 Brink w.fs. coincides with one one-dim. THSR !

Intrinsic density (0.1, 5.1) for β



Maxima

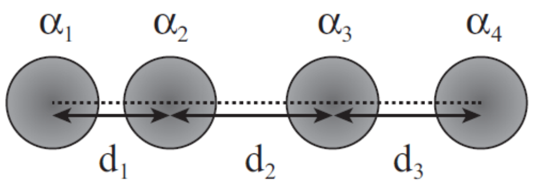
J=0: 0.9873 β=(0.1,5.1)

J=2: 0.9887 β=(0.1,5.4)

J=4: 0.9806 β=(0.1,6.6)

The squared overlap with one-dim. THSR with single parameter

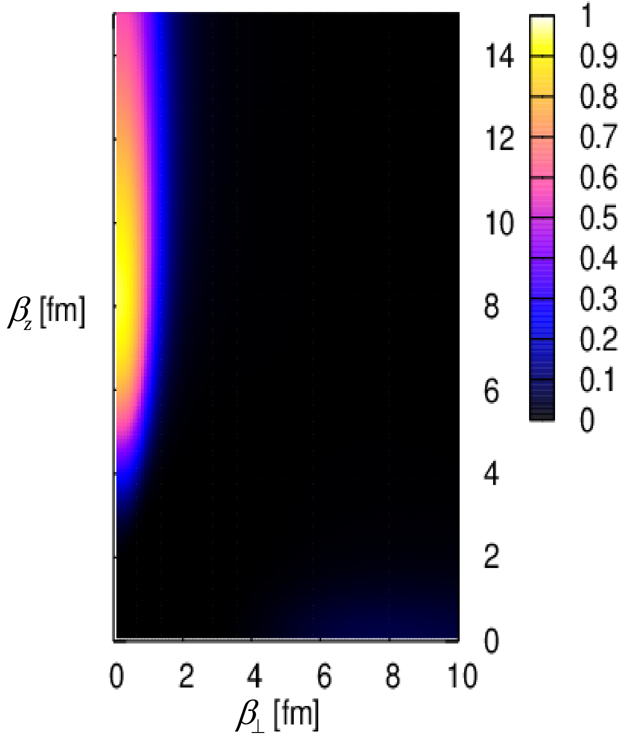
T. Suhara, Y. F. et al., submitted to PRL(arXiv:1310.7684)



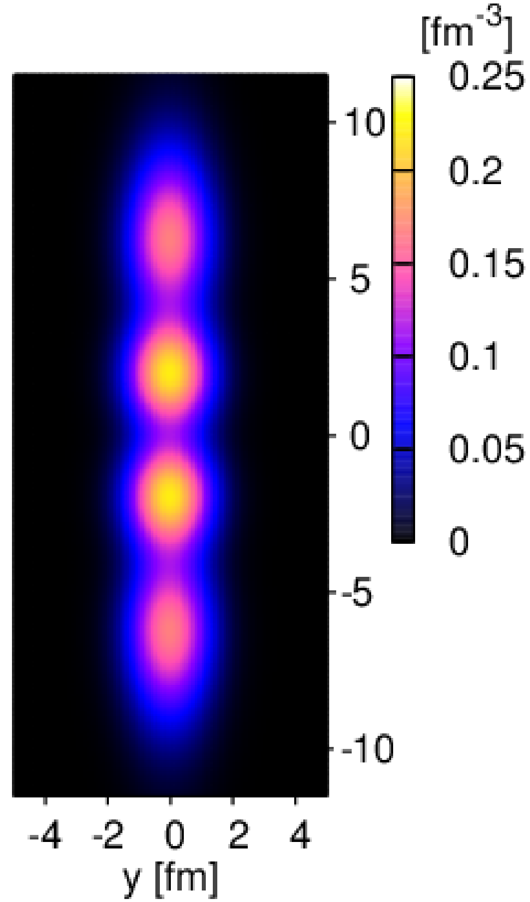
$$B_k^2 = b^2 + 2\beta_k^2 \ (k = x, y, z)$$

$$\beta_\perp \equiv \beta_x = \beta_y$$

$$O(\beta_\perp, \beta_z) = \left| \left\langle \Phi_{J=0}^{\mathrm{THSR}}(\beta_\perp, \beta_z) \right| \sum_{d_1, d_2, d_3} f_\lambda^{(J=0)}(d_1, d_2, d_3) \Phi_{J=0}(d_1, d_2, d_3) \right\rangle \Bigg|^2$$



Intrinsic density (0.1, 8.2) for β



The superposition of 300 Brink w.fs. coincides with one one-dim. THSR !

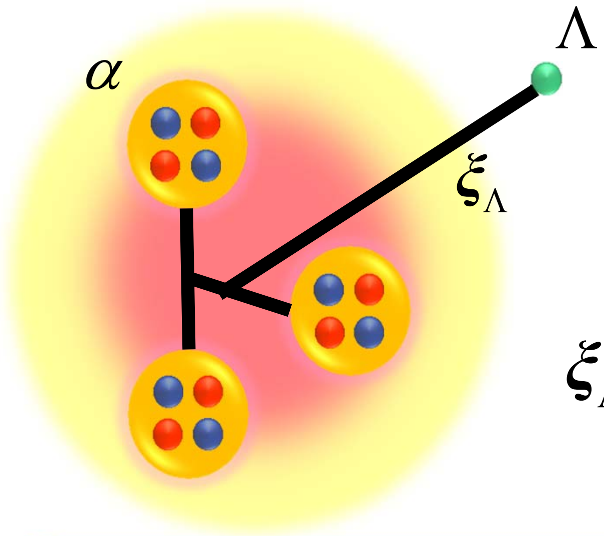
Maxima

J=0: 0.9440 β=(0.1, 8.2)

J=2: 0.9417 β=(0.1,8.4)

J=4: 0.9307 β=(0.1,9.0)

We make use of the THSR w.f. in the hyperon world.



Hyper-THSR, applied to $^{9}_{\Lambda}$Be, $^{13}_{\Lambda}$C, $^{17}_{\Lambda}$O, …

$$\boldsymbol{\xi}_\Lambda = \boldsymbol{r}_\Lambda - \boldsymbol{X}_C \quad \boldsymbol{X}_C = \frac{\boldsymbol{r}_1 + \cdots + \boldsymbol{r}_{4n}}{4n}$$

$\hat{\mathcal{P}}_I$: angular momentum projection operator

$$\Phi^{\text{Hyper-THSR}}_{[I,l]_J}(B_\perp, B_z, \kappa) = \mathcal{A}\left\{\prod_{i=1}^{n} \hat{\mathcal{P}}_I \chi^{THSR}_{3\alpha}(B_\perp, B_z : \boldsymbol{X}_i - \boldsymbol{X}_C)\phi(\alpha_i)\right\} \varphi^{(l)}_\kappa(\boldsymbol{\xi}_\Lambda)$$

$$\chi^{\text{THSR}}(\boldsymbol{X} : B_\perp, B_z) = \exp\left(-\frac{2}{B_\perp^2}(X_x^2 + X_y^2) - \frac{2}{B_z^2} X_z^2\right)$$

$$\varphi^{(l)}_\kappa(\boldsymbol{\xi}_\Lambda) = N_{\kappa,l}\,\xi_\Lambda^{\ l} \exp\left(-\frac{\xi_\Lambda^2}{\kappa^2}\right) Y_{lm}(\hat{\boldsymbol{\xi}}_\Lambda)$$

In the present study, l=0 only taken into account

Spatial shrinkage happens when Λ particle is injected in a nucleus.
The corresponding rearrangement effect can be optimally described.

${}^{9}_{\Lambda}$Be(0^+,2^+, 4^+) Energy spectra

ΛN:YNG (ND) interaction

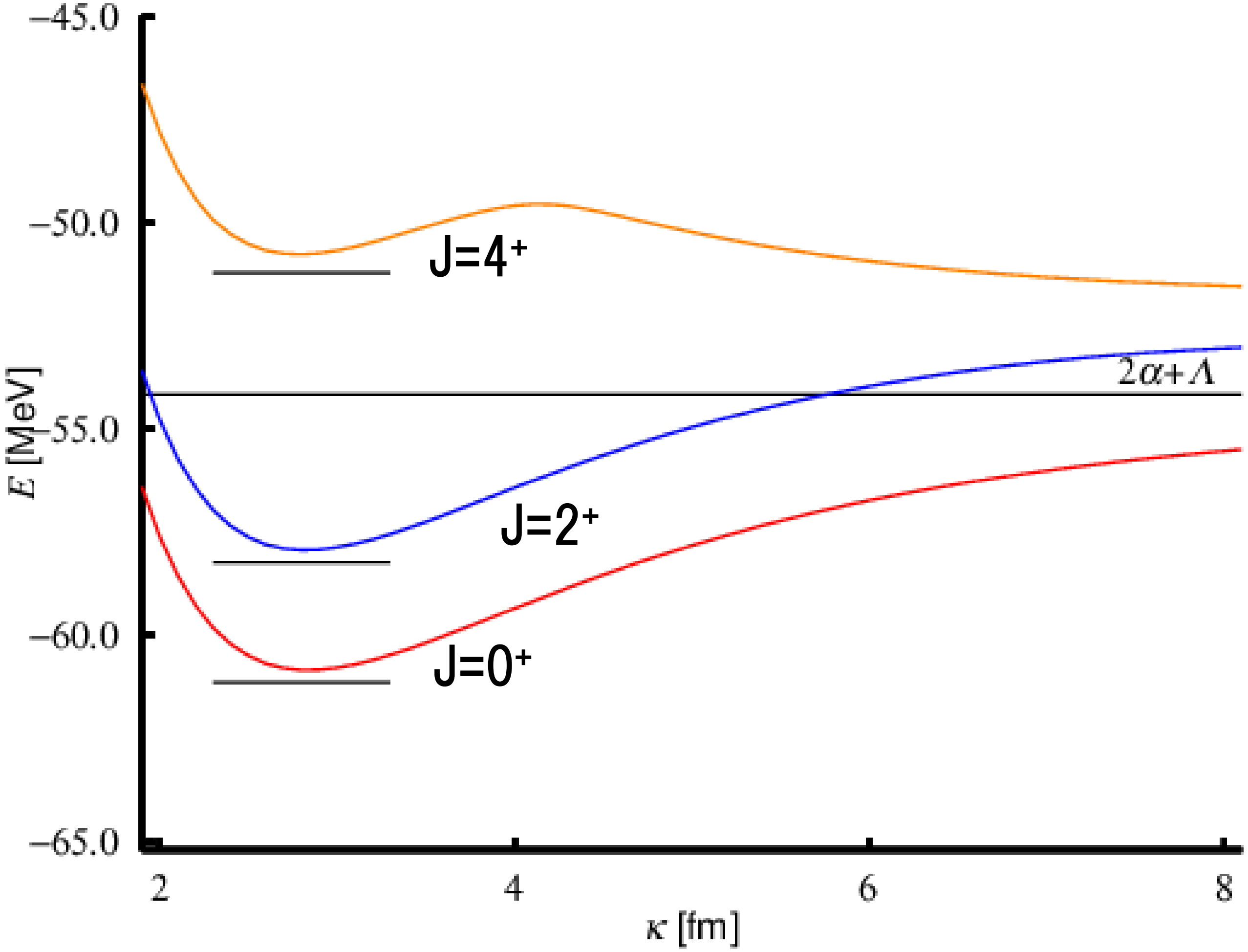
$$\sum_{B'_\perp, B'_z} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda(\kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa) \right\rangle f_\lambda(B'_\perp, B'_z) = 0$$

$$\Lambda \text{ particle w.f.}: \varphi_\kappa^{(l=0)}(\xi_\Lambda) = N_{\kappa,l=0} \exp\left(-\frac{\xi_\Lambda^2}{\kappa^2}\right) Y_{00}(\hat{\xi}_\Lambda)$$

k_f=0.962 fm^{-1}

J=0^+

	Exp.	Cal.
B_Λ :	6.71 MeV	6.69 MeV



NN:Volkov No.1 M=0.56
b=1.36 fm

${}^{9}_{\Lambda}$Be(0⁺,2⁺, 4⁺) Energy spectra

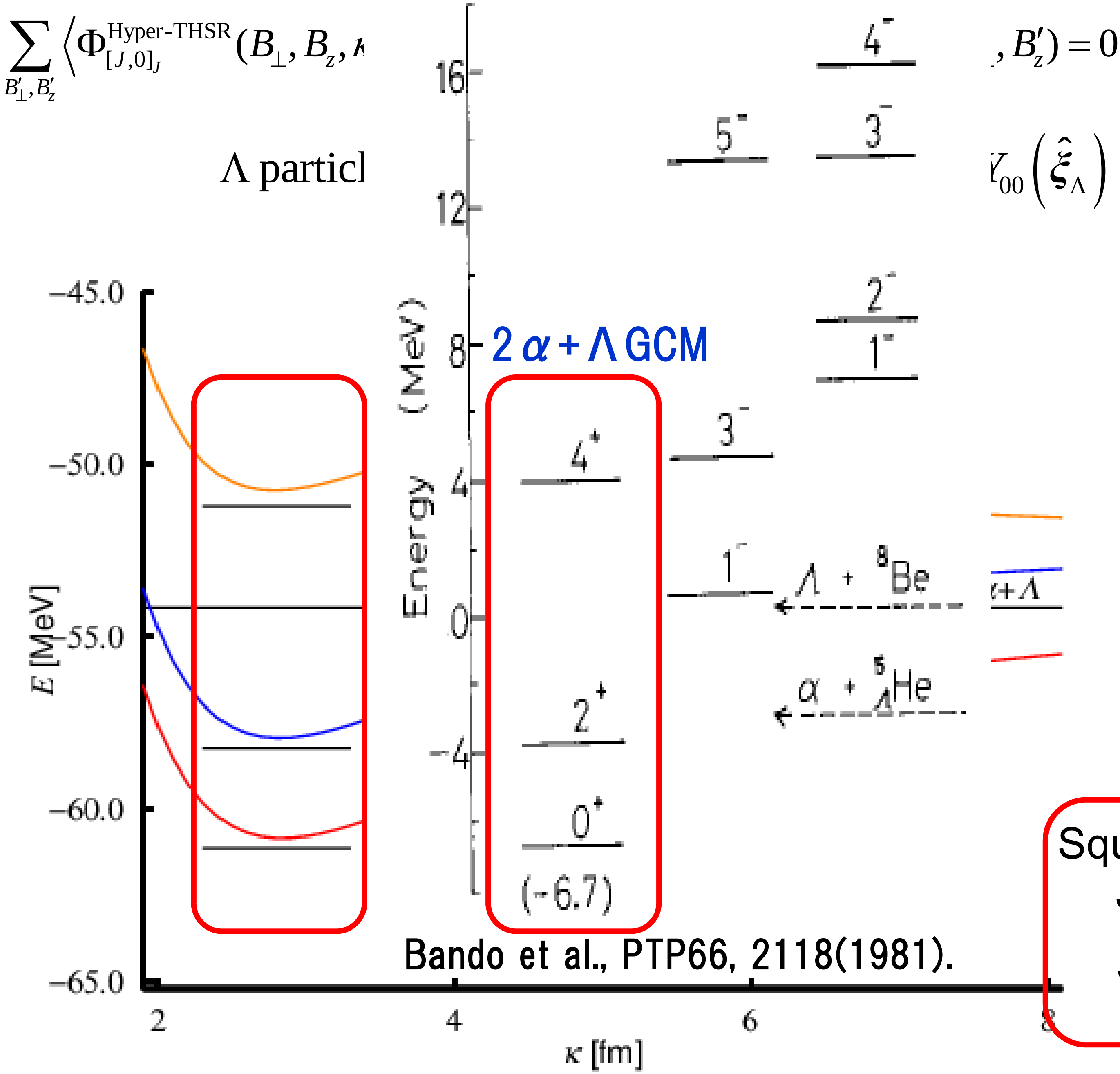
ΛN:YNG (ND) interaction

k_f=0.962 fm^{-1}

J=0⁺

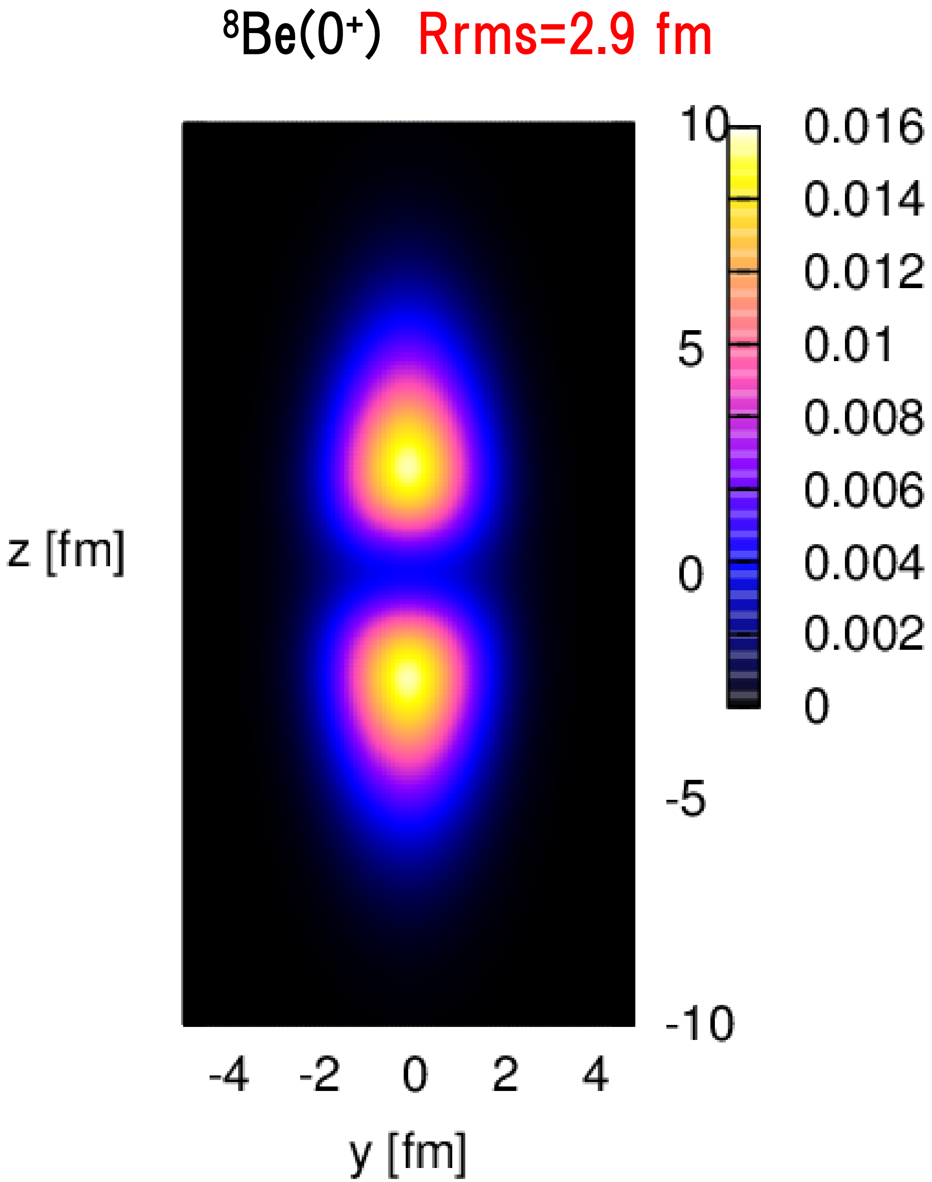
	Exp.	Cal.
B_Λ :	6.71 MeV	6.69 MeV

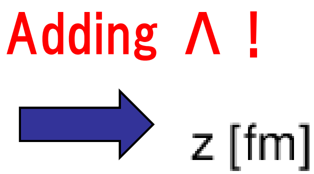
NN:Volkov No.1 M=0.56
b=1.36 fm

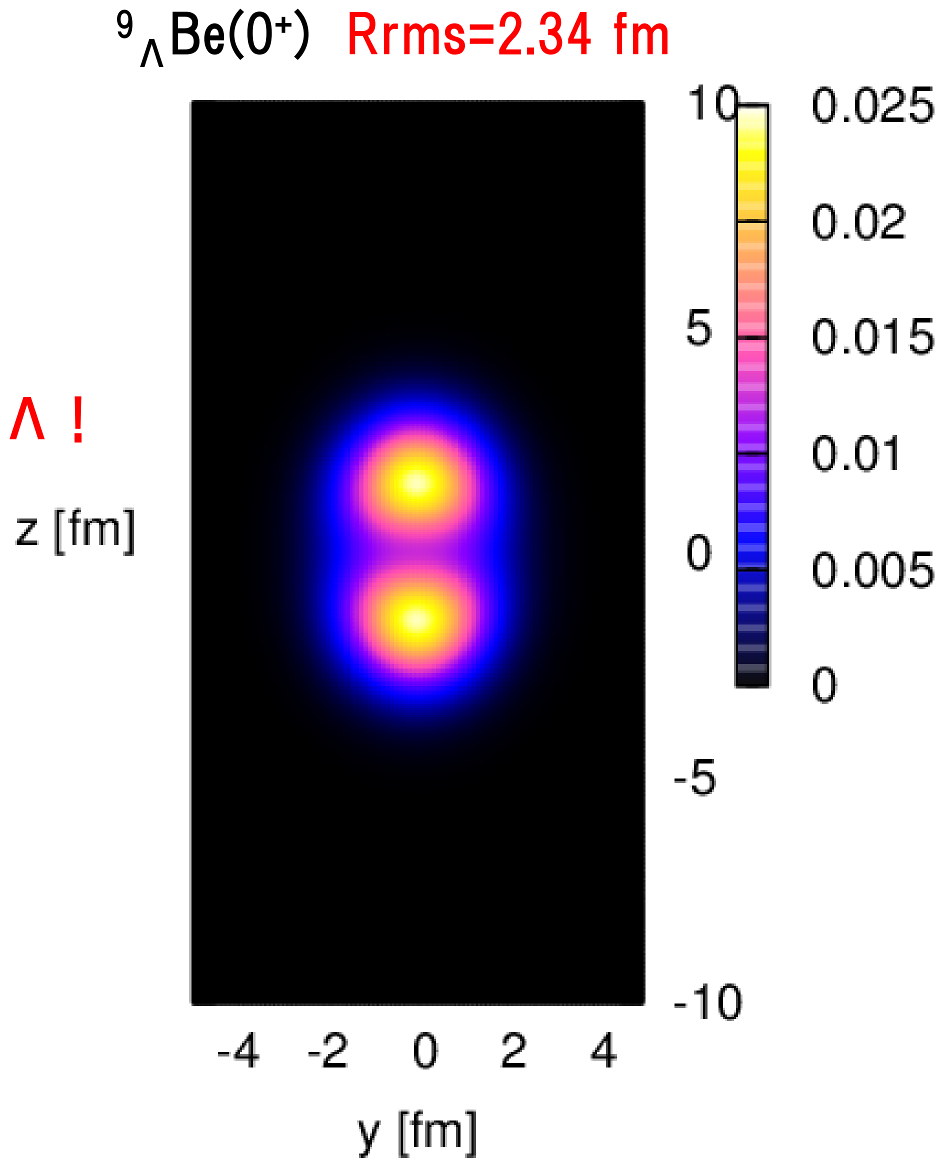


Squared overlap with 2α+Λ GCM
J=0: 0.992 β=(0.1,3.3)
J=2: 0.994 β=(0.1,3.0)

Comparison of intrinsic density between $^{8}Be(0^{+})$ & $^{9}_{\Lambda}Be(0^{+})$







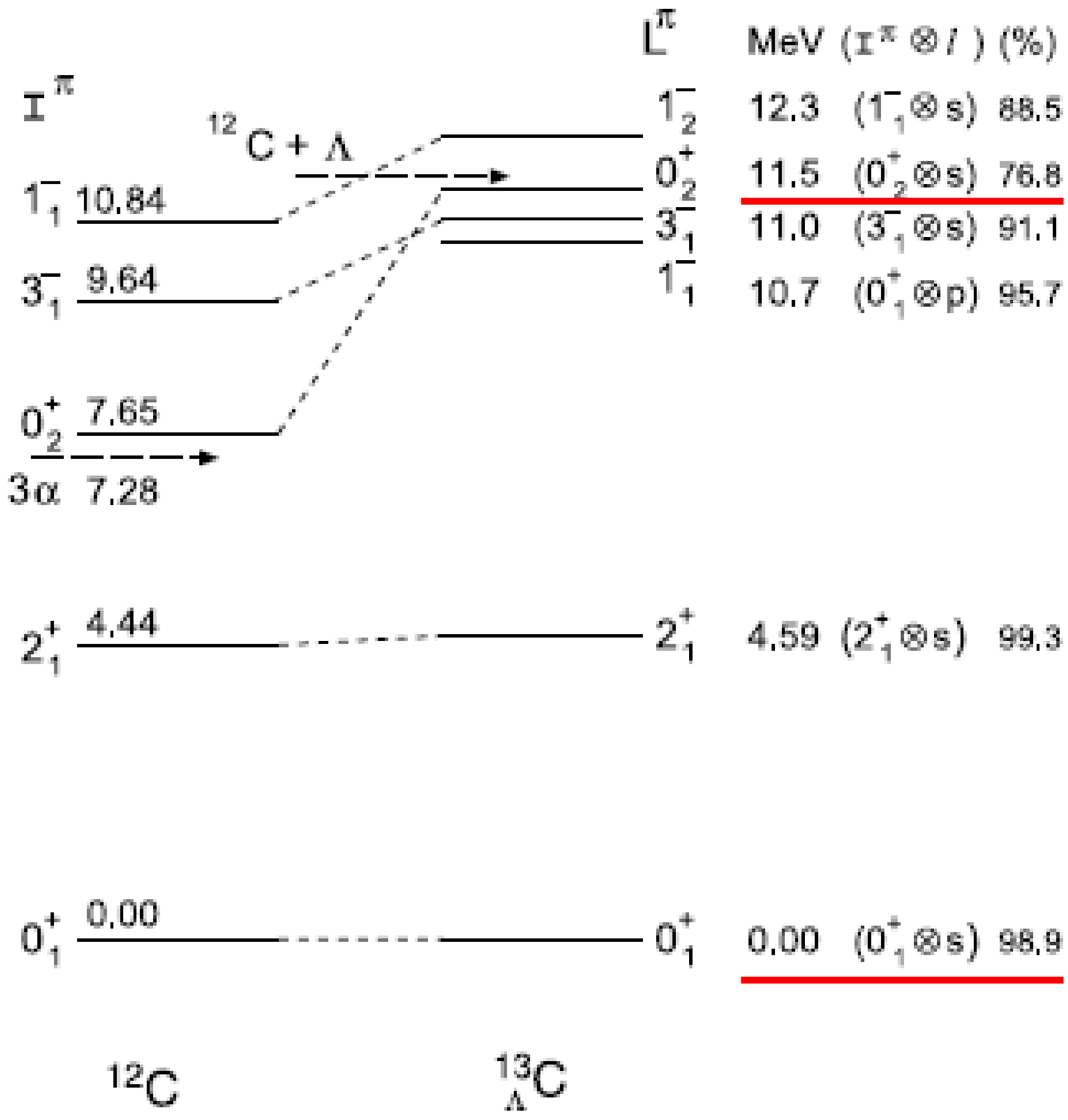
2α structure still survives in the normal density !

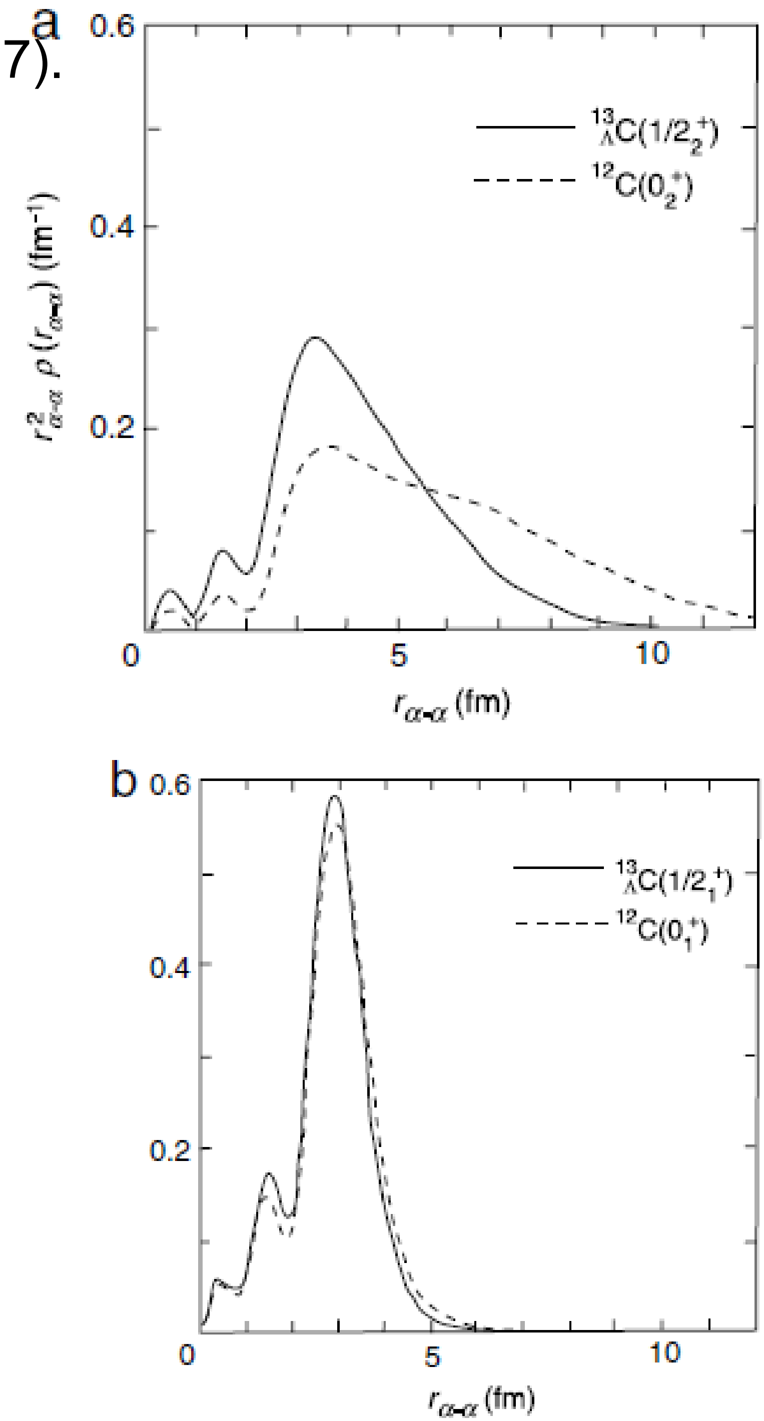
Λ particle does not disturb the strong Pauli repulsion of α-α

Pauli principle plays a crucial role in producing clusters.

3α+Λ OCM by Hiyama et al. YNG (JA) interaction

E. Hiyama et al., PTP 97, 881 (1997).

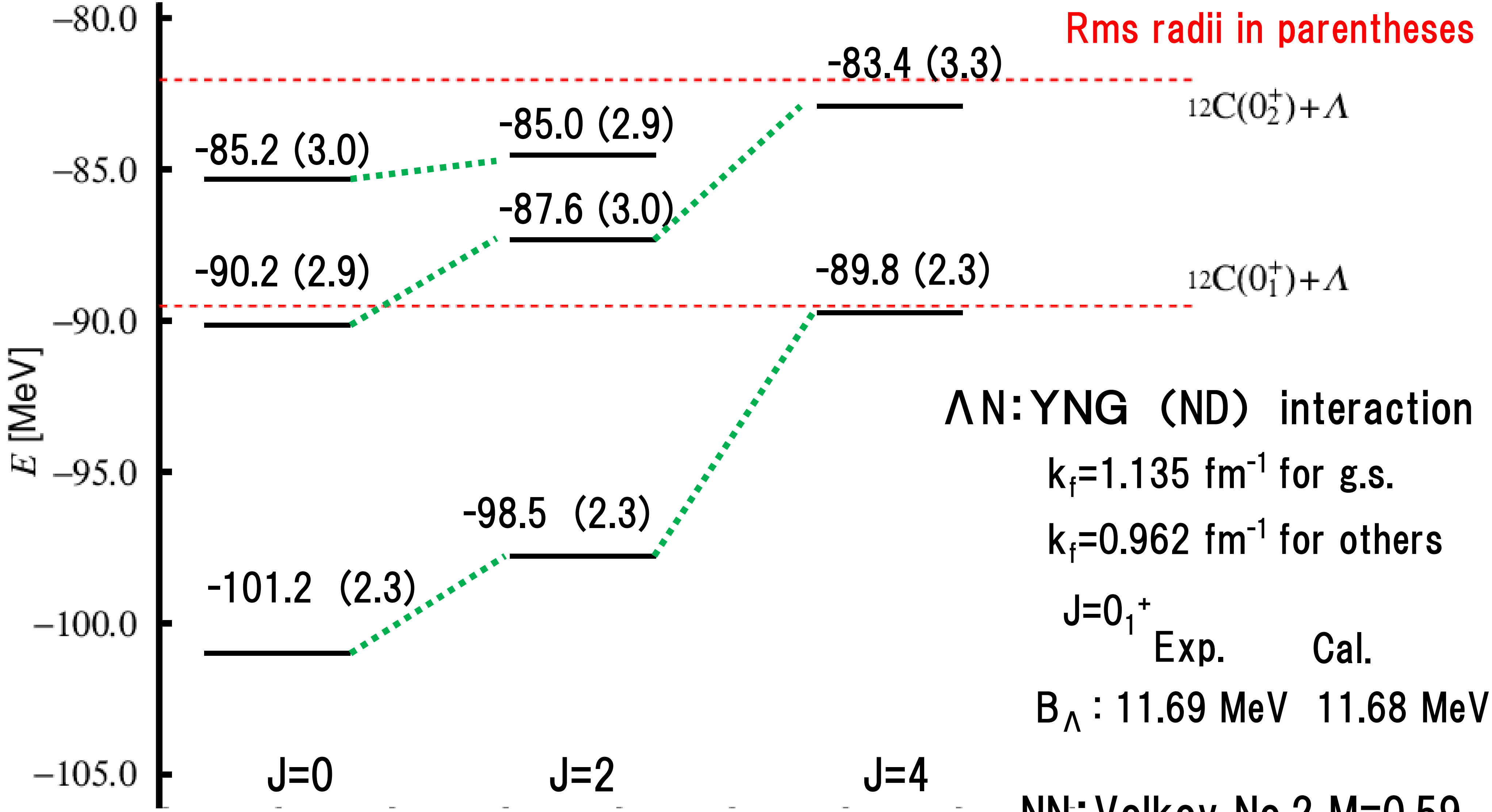




Spatial shrinkage is seen.

Energy of $^{13}{}_{\Lambda}C(0^+, 2^+, 4^+)$

$$\sum_{B'_\perp, B'_z, \kappa'} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| H - E_\lambda \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa') \right\rangle f_\lambda(B'_\perp, B'_z, \kappa') = 0$$



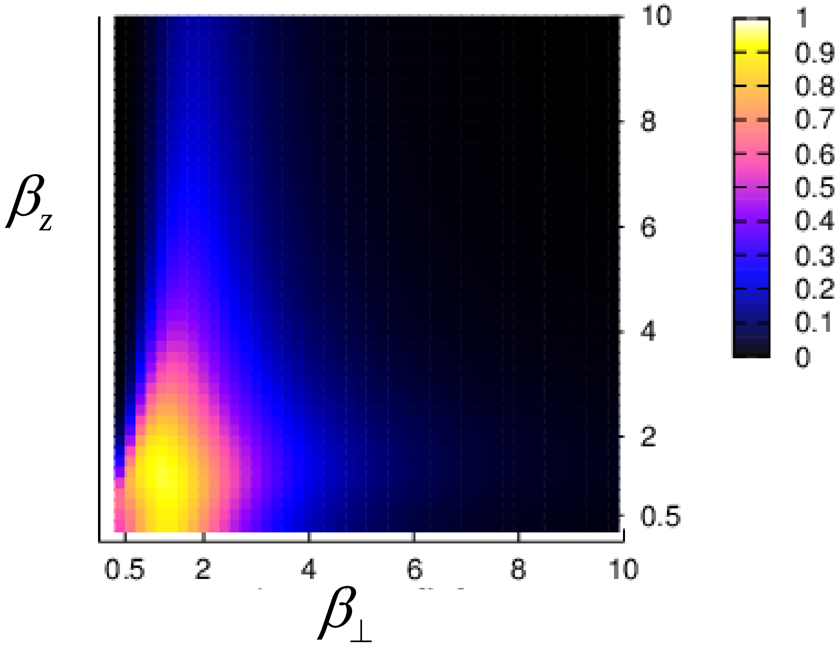
NN: Volkov No.2 M=0.59

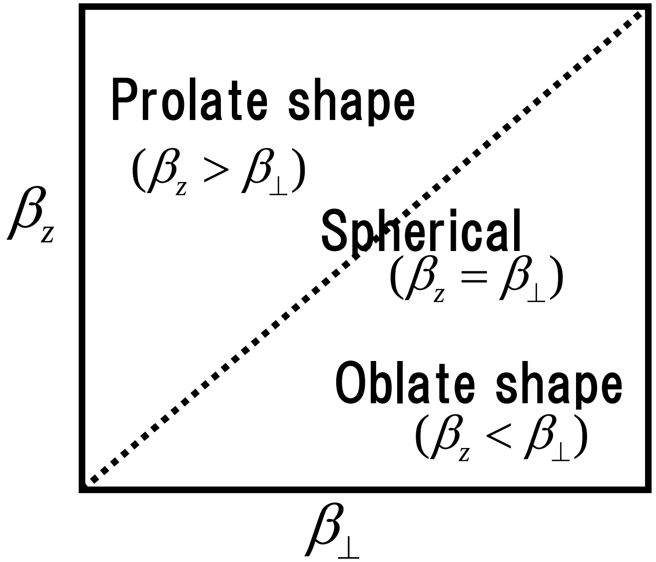
b=1.35 fm

Squared overlap surfaces for 0_1^+, 2_1^+, 4_1^+

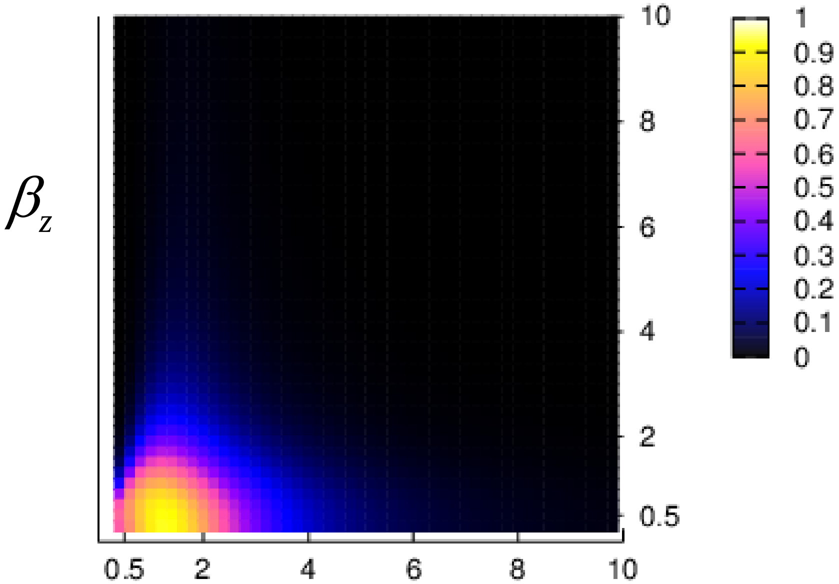
$$O(\beta_\perp, \beta_z, \kappa) = \left| \sum_{B'_\perp, B'_z} \left\langle \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B_\perp, B_z, \kappa) \middle| \Phi^{\text{Hyper-THSR}}_{[J,0]_J}(B'_\perp, B'_z, \kappa) \right\rangle f_\lambda(B'_\perp, B'_z, \kappa) \right|^2$$

0_1^+



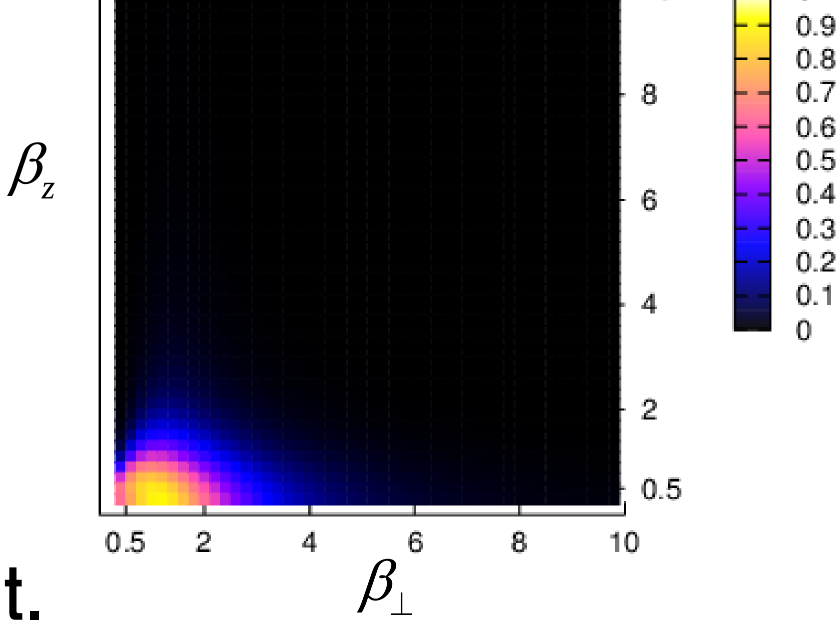


2_1^+



$\beta_\perp$

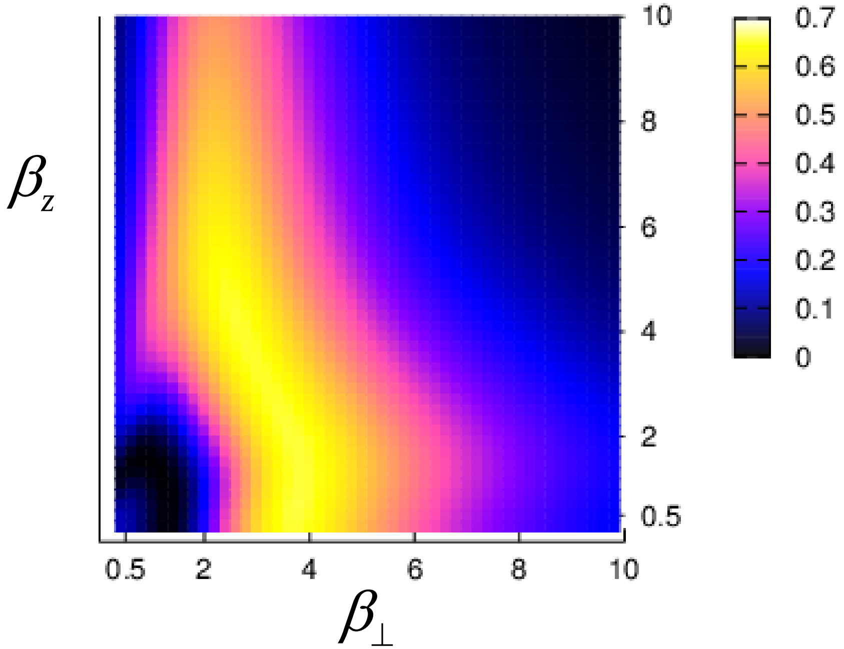
4_1^+

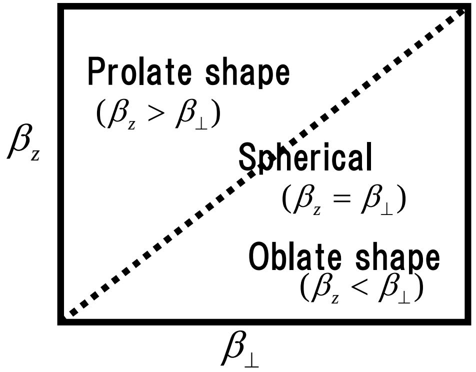


All are compact.

Family of the Hoyle state

0_2^+

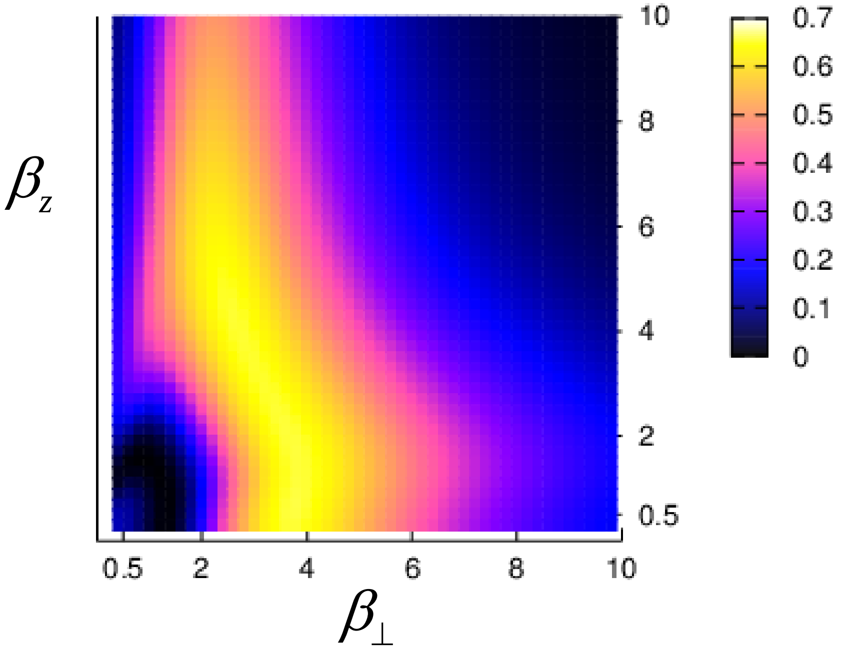




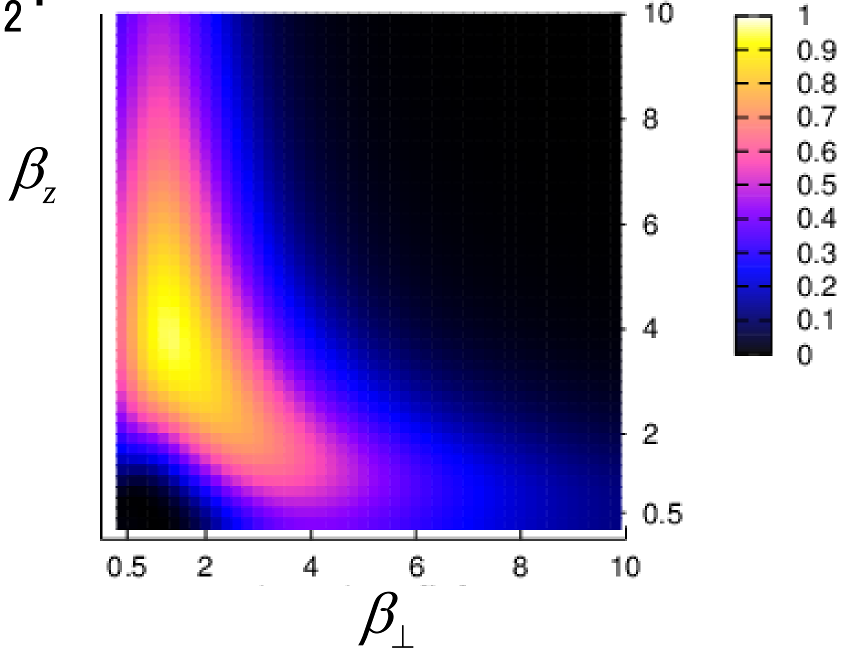
Dilute density like a gas
All do not have definite shape.

Note: The Hoyle state band is not yet confirmed in ^{12}C.

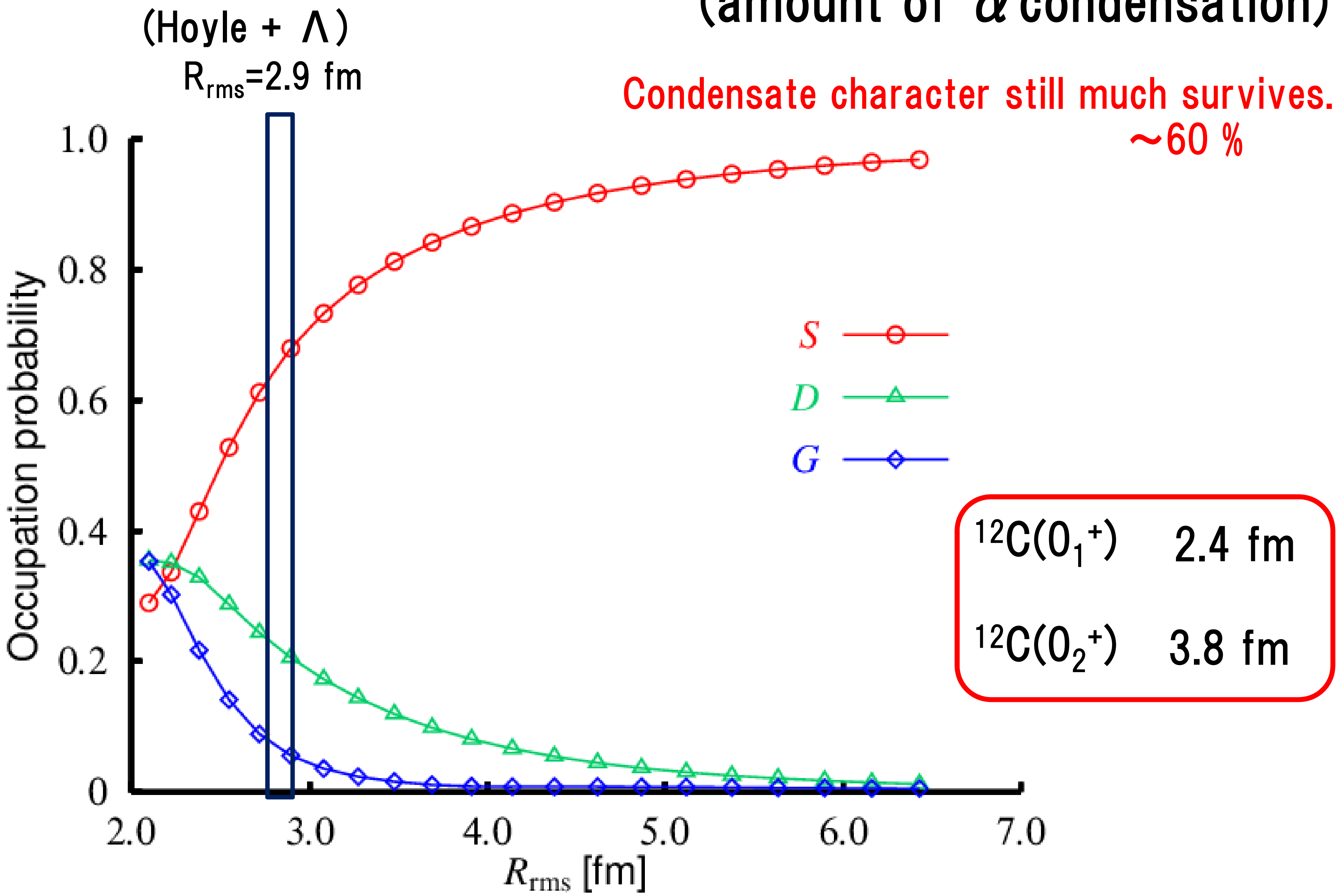
2_2^+



4_2^+



Size dependence of occupation probability
(amount of α condensation)

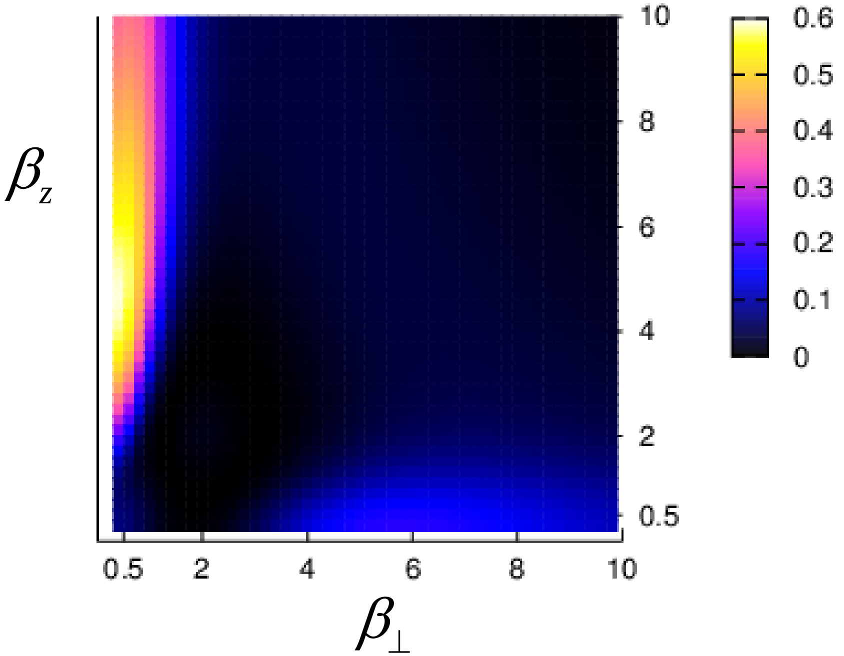


R_{rms} < 2.5 fm: Alpha's are resolved due to the antisymmetrization.

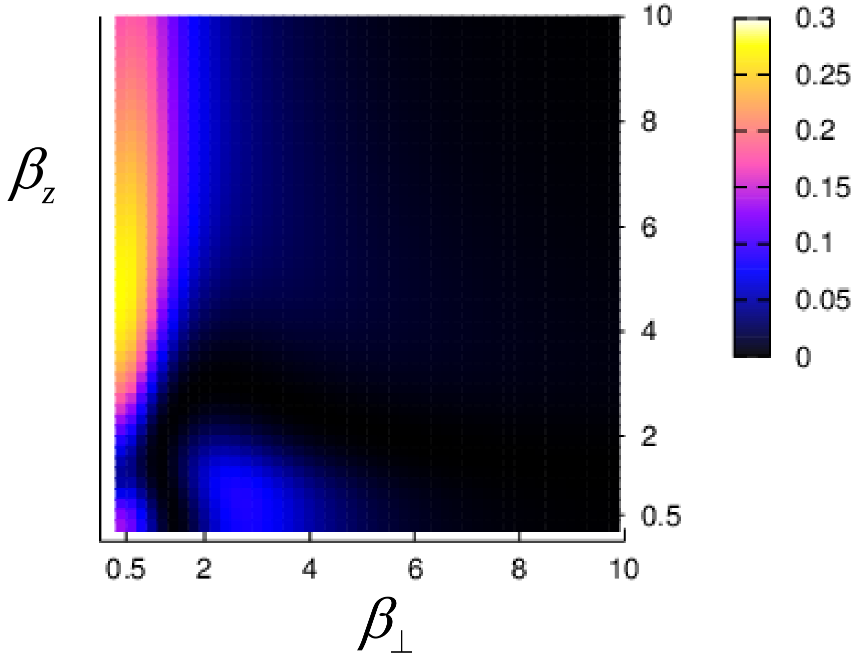
R_{rms} → large: Alpha's occupy a single *S*-orbit only.

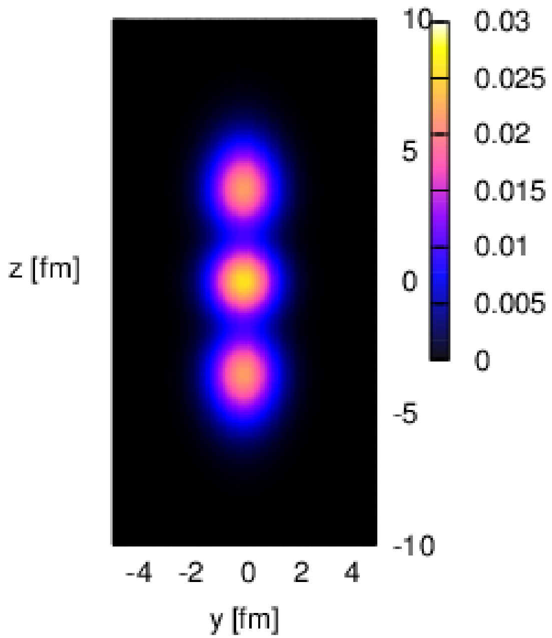
1 dim.-like linear-chain band

0_3+

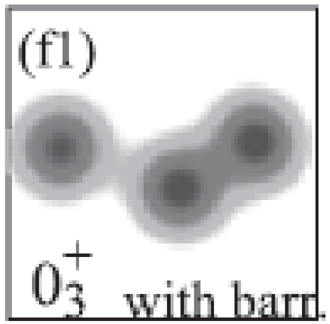


2_3+





^{12}C: 0_3+ state (or 0_4+)



AMD by En'yo, (PTP117, 655(2007).
and FMD by T. Neff.

Summary

Based on the fact that THSR w.f. succeeded in describing gas-like states (^{8}Be, ^{12}C) and even for ordinary cluster states (^{20}Ne and g.s. ^{12}C)

→ The linear chain states which we have imagined are one-dim. gases of alphas to be described by THSR.

→ Hyper-THSR w.f. is introduced to apply it to Λ hypernuclei.

very promising way of describing light hypernuclei

- $^{9}{}_{\Lambda}Be$: The ground rotational band is successfully reproduced.

  Large shrinkage effect: 2 alpha structures still survive.
  Powerful effect of Pauli principle.

- $^{13}{}_{\Lambda}C$: One dimensional gas of three alphas, as the 0_3^+, 2_3^+ states.
  More straightly aligned than in $^{12}C(0_3^+)$

Thanks

to my Collaborators

Bo Zhou (Nanjing U.)

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Peter Schuck (IPN, Orsay)

Gerd Röpke (Rostock U.)

Shigeo Ohkubo (Kochi women U.)

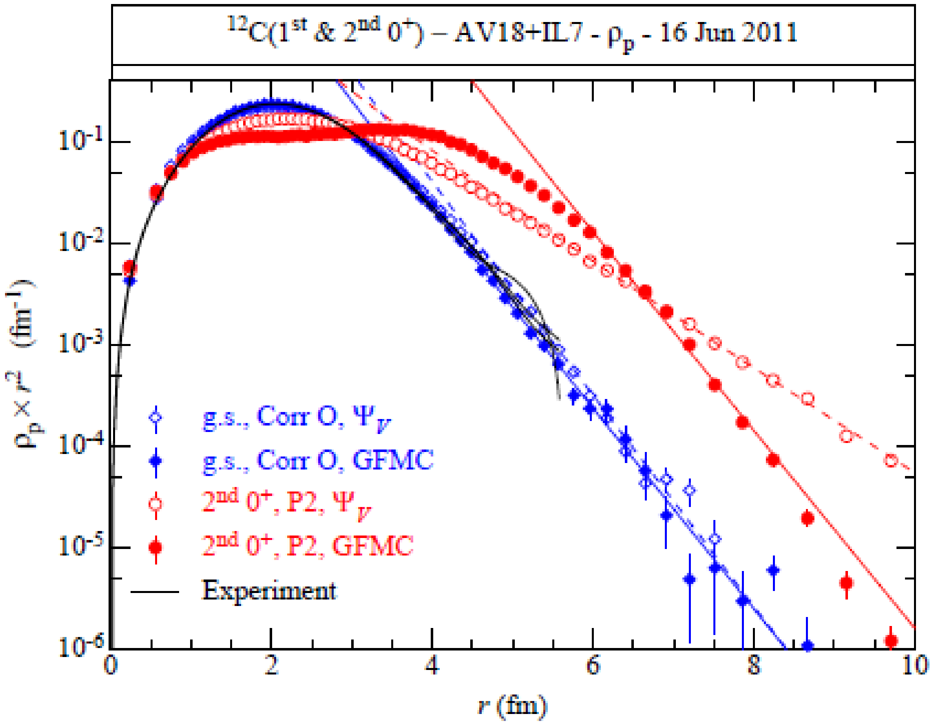
Emiko Hiyama (RIKEN)

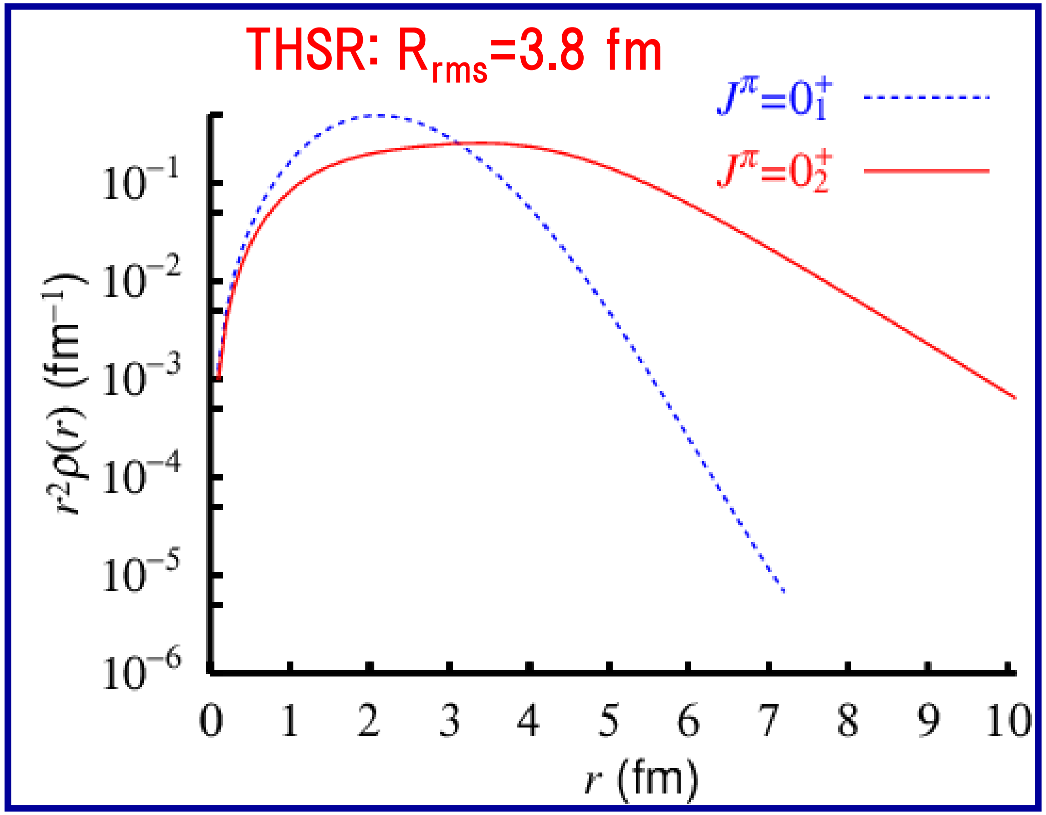
Kiyomi Ikeda (RIKEN)

and for your attention.

GFMC (slide from Wringa) and Comparison with THSR

1^{st} AND 2^{nd} (HOYLE) 0^+ STATES IN ^{12}C – PRELIMINARY





Very significant computer & computer science resources required for this

- UNEDF SciDAC grant to develop general-purpose load-balancing li
runs with ~ 85% efficieny under MPI on 32,768 nodes with OpenM
- INCITE grant of Argonne BG/P time used for ^{12}C calculations

Lusk, Pieper, & Butler, SciDAC Review Spring 2010

For Hoyle state

R_{rms}=3.0 fm ∼ 3.5 fm

For Hoyle state

AV18+IL7: 10.4 MeV
Exp: 7.65 MeV