Chiral dynamics in a magnetic field from the functional renormalization group

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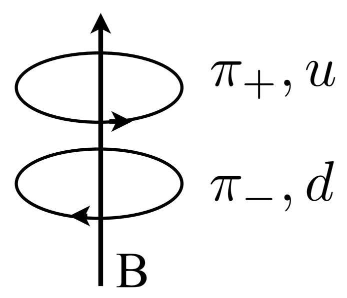
Based on

arXiv:1312.312 K. Kamikado, T. Kanazawa, to appear in JHEP

Chiral phase transition in strong magnetic field

Magnetic catalysis



$$E_n^2 = p_z^2 + m_\pi^2 + |eB|(2n+1);\ \text{spin } 0$$

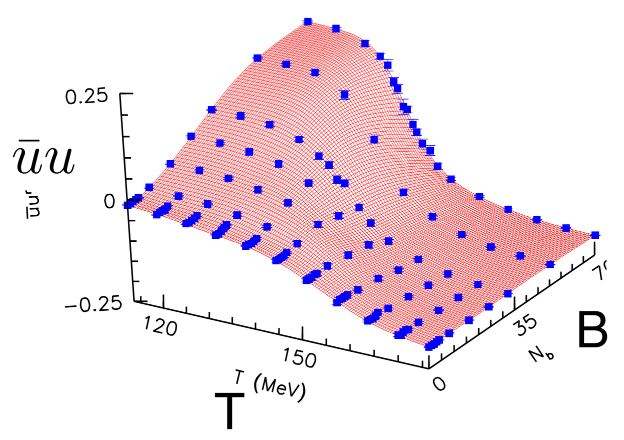
$$E_n^2 = p_z^2 + m_q^2 + |eB|2n;\ \text{spin } 1/2$$

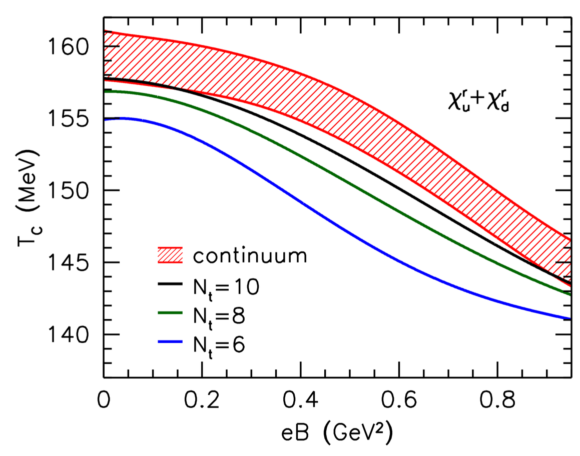
Due to the quasi-one dimensionality of the quark lowest Landau level (n=0), symmetric vacuum is unstable for any small B and interaction.

$$\bar\psi\psi \sim \sqrt{eB}\exp[-a/G]$$

Inverse Magnetic catalysis

Lattice QCD simulation



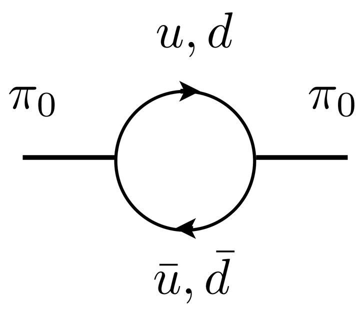


G.S. Bali et al. (2012)

At finite temperature, chiral condensate "**decreases**" with B critical temperature also "**decreases**" with B

Discrepancy with many chiral model analyses

Neutral Pion in magnetic field



$$\sim Z^\perp p_\perp^2 + Z^\parallel p_\perp^2 + m_\pi^2$$

Transverse velocity

Hidaka, Fukushima(2013)

$$\nu_\perp^2 \equiv Z^\perp / Z^\parallel \sim 1/(eB)$$

Neutral mesons get an strong anisotropy via quark loops.
This feature may realise the inverse magnetic catalysis?

Formalism

Functional-RG

$$k\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB} + \Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF} + \Gamma_k^{(2,0)}[\varphi]}\right]$$

C. Wetterich (1993)

Scale (k) dependent effective action

$$\Gamma_{k=\Lambda}[\phi] = S[\phi] \quad \text{UV: classical}$$

$$\downarrow$$

$$\Gamma_{k=0}[\phi] = \Gamma[\phi] \quad \text{IR: quantum}$$

R_k is arbitrary cutoff function. Our choices are

$$R_{kB} = (k^2 - \vec{p}_z^{\ 2})\theta[k^2 - \vec{p}_z^{\ 2}],\ R_{kF} = ik\frac{\vec{\not p}_z}{|\vec{p}_z|}\theta[k^2 - \vec{p}_z^{\ 2}]$$

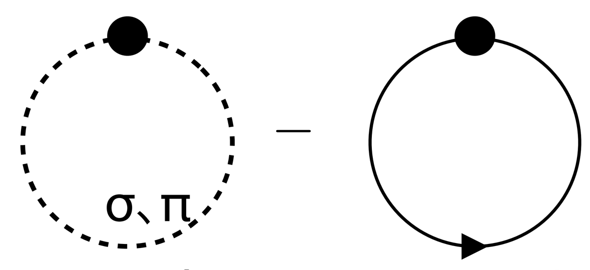
D. Litim (2000)

Anzats for scale dependent action for one flavour Quark-meson (σ+π) model

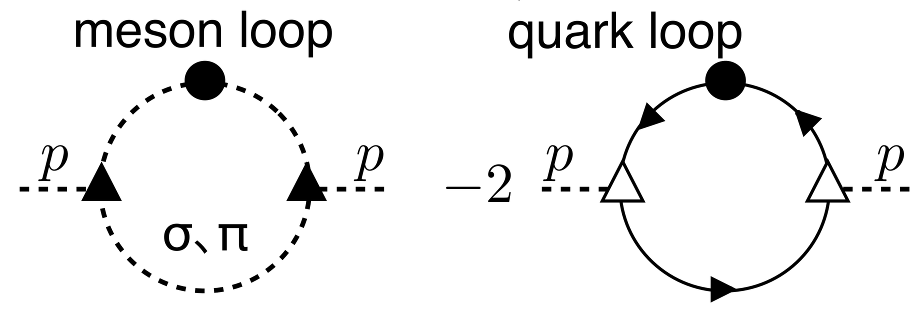
$$\Gamma_k[\psi,\sigma,\pi] = \int_0^\beta dx_4 \int d^3x \Big[\bar\psi\left[\gamma_\mu D_\mu + g(\sigma + i\gamma\pi)\right]\psi + U_k(\sigma^2+\pi^2) - h\sigma$$
$$+ \frac{Z_k^\perp}{2}\left((\partial_\perp\sigma)^2 + (\partial_\perp\pi)^2\right) + \frac{Z_k^\parallel}{2}\left((\partial_\parallel\sigma)^2 + (\partial_\parallel\pi)^2\right)\Big]$$

Flow equations for U_k and Z_k

$$\partial_k U_k = \frac{1}{2}\ \text{(meson loop)} - \text{(quark loop)}$$



$$\partial_k \Gamma^{(2,0)}(p) \sim \text{(meson diagram)} - 2\ \text{(quark diagram)}$$



$$\sim \partial_k Z_k p^2 + O(p^4)$$

With scale dependent propagators

$$\Gamma_k^{(0,2)} = Z_k^\parallel p_\parallel^2 + Z_k^\perp p_\perp^2 + U_k'' + R_{KB} \qquad \Gamma_k^{(2,0)} = \not\partial + e\not A + g\sigma + R_{kF}$$

$$\nabla \times A = B\vec{e}_z$$

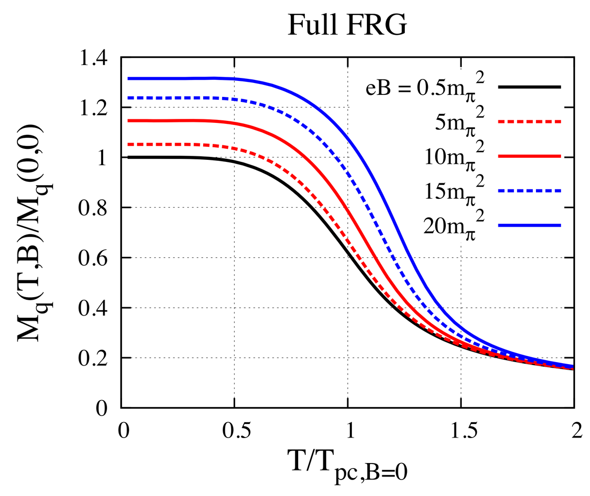
Gap equation for order parameter (k=0)

$$\left.\frac{\partial U_{k=0}}{\partial\sigma}\right|_{\sigma=\sigma_{\min}} = 0$$

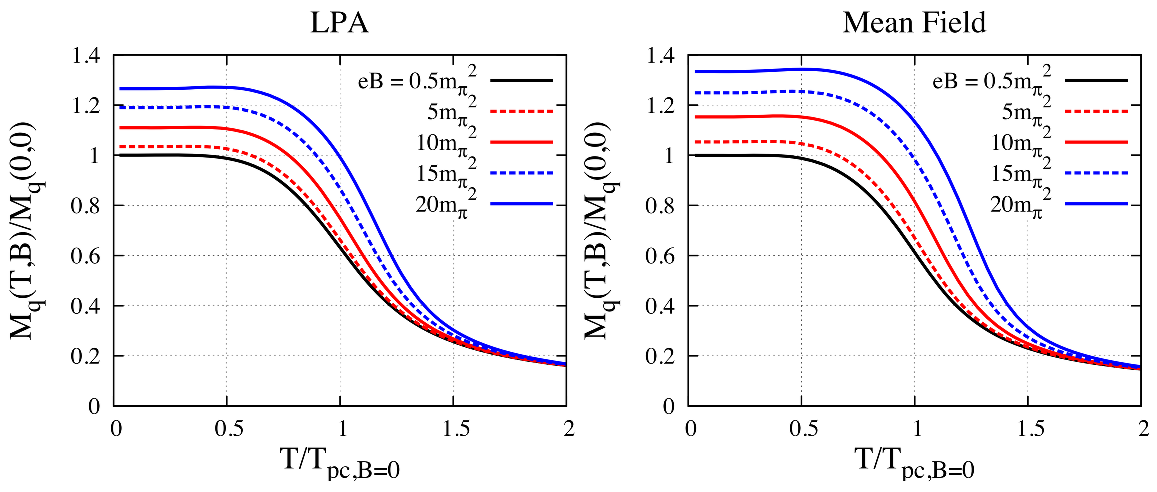
Results

Order parameter vs T and B

$$M_q = g\sigma_{\min}$$

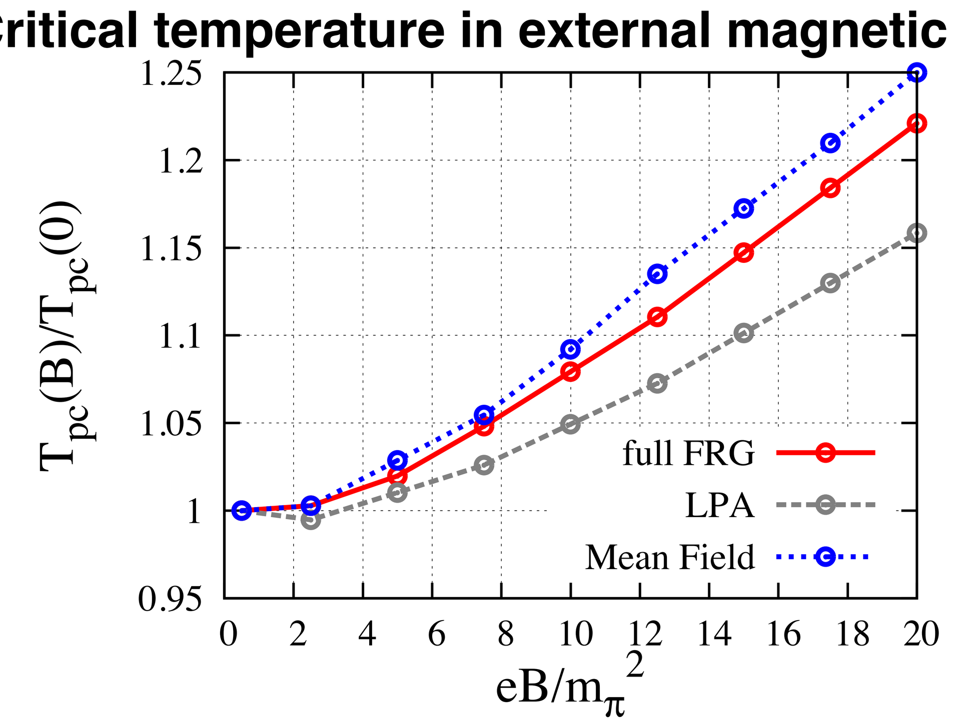


LPA: no anisotropy of mesons (Z=1)
Mean field: neglect meson loop



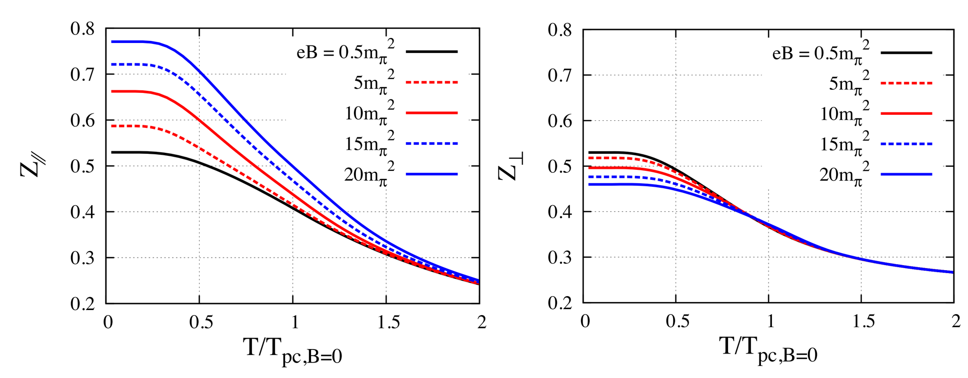
Order parameter monotonically increases with B for all temperature.
Inverse magnetic catalysis is not realized.

Critical temperature in external magnetic field

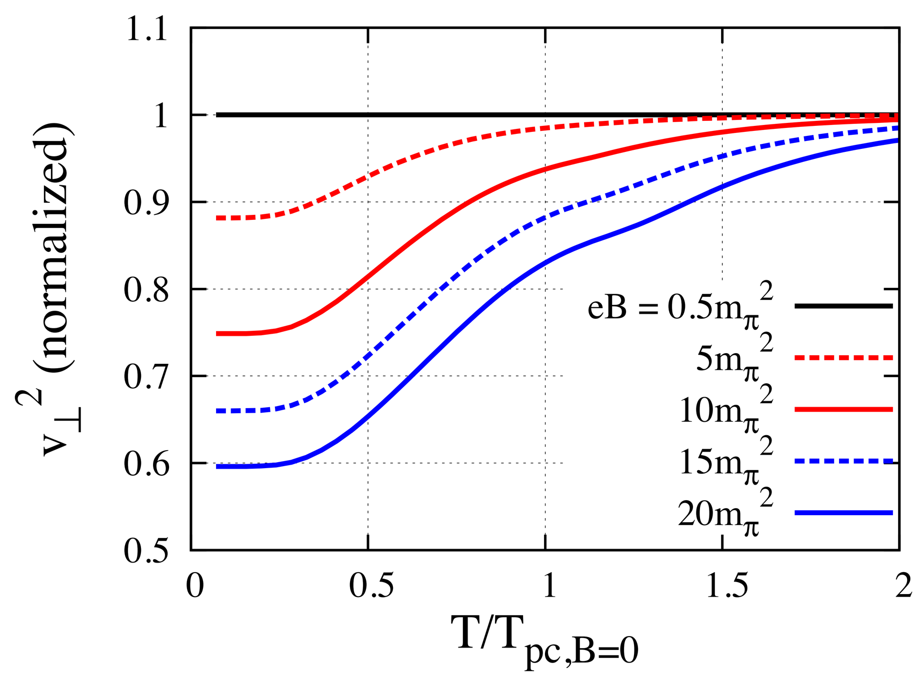


T_c also increases with B.

Z are suppressed at finite temperature then the full FRG results are close to the MF



Pion transverse velocity at finite T and B



$$\nu_\perp^2 \equiv Z^\perp / Z^\parallel$$

Transverse velocity decrease with external magnetic field.
Anisotropy of the pion is suppressed at high temperature

Conclusion

- We have studied chiral phase transition under strong magnetic field by using the functional-RG method.
- We have used a truncation which enable us to include the anisotropy of the neutral pion.
- Even we include the anisotropy, the inverse magnetic catalysis is not realised.
- Chiral model approach still miss the origin of the inverse magnetic catalysis.