NG modes on inhomogeneous phases

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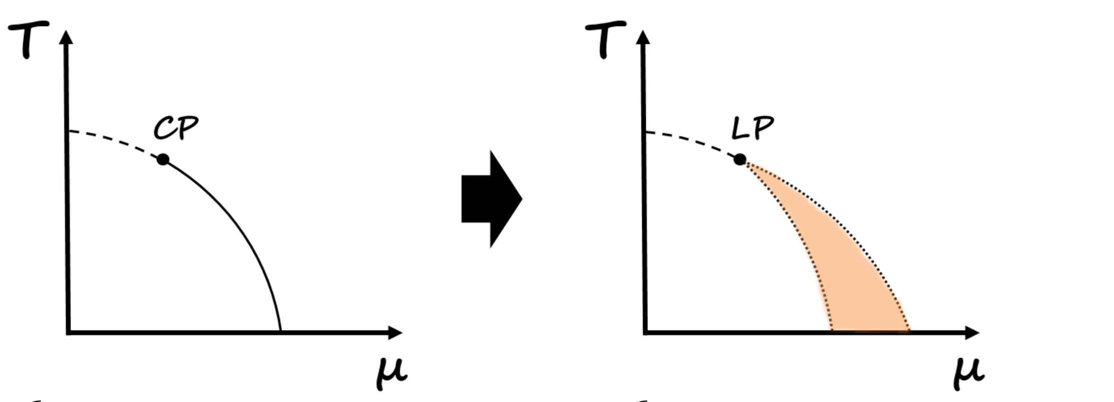
1. INTRODUCTION

◎ Chiral phase transition and its boundary

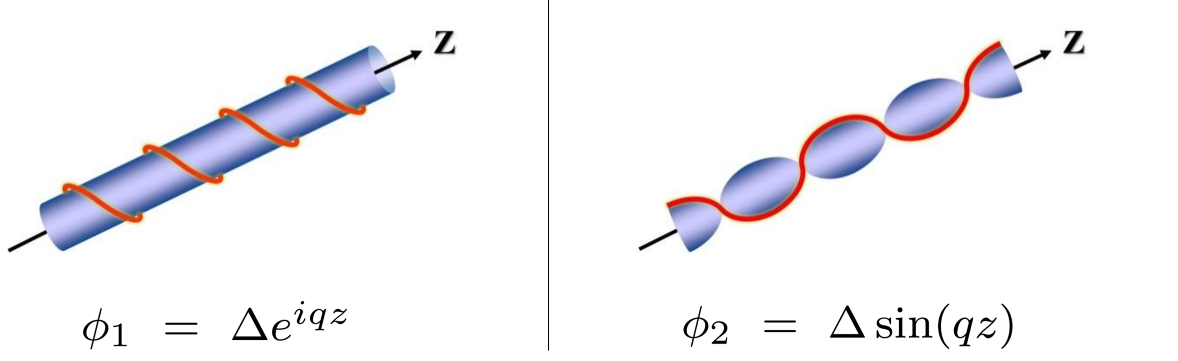
▶ *Conventional case* ▶ *Energetically favored case*



Uniform condensates / First-order phase transition / Critical end point ⇒ Nonuniform condensates / Inhomogeneous phase / Lifshitz point Nickel(2009) [1]

How does the order parameter modulate in the inhomogeneous phase?
⇒ There are two typical forms of condensates with 1-D (z-direction) modulation.

▶ *FF-type ground state* Flude-Ferrell (1964) ▶ *LO-type ground state* Larkin-Ovchinnikov (1965)



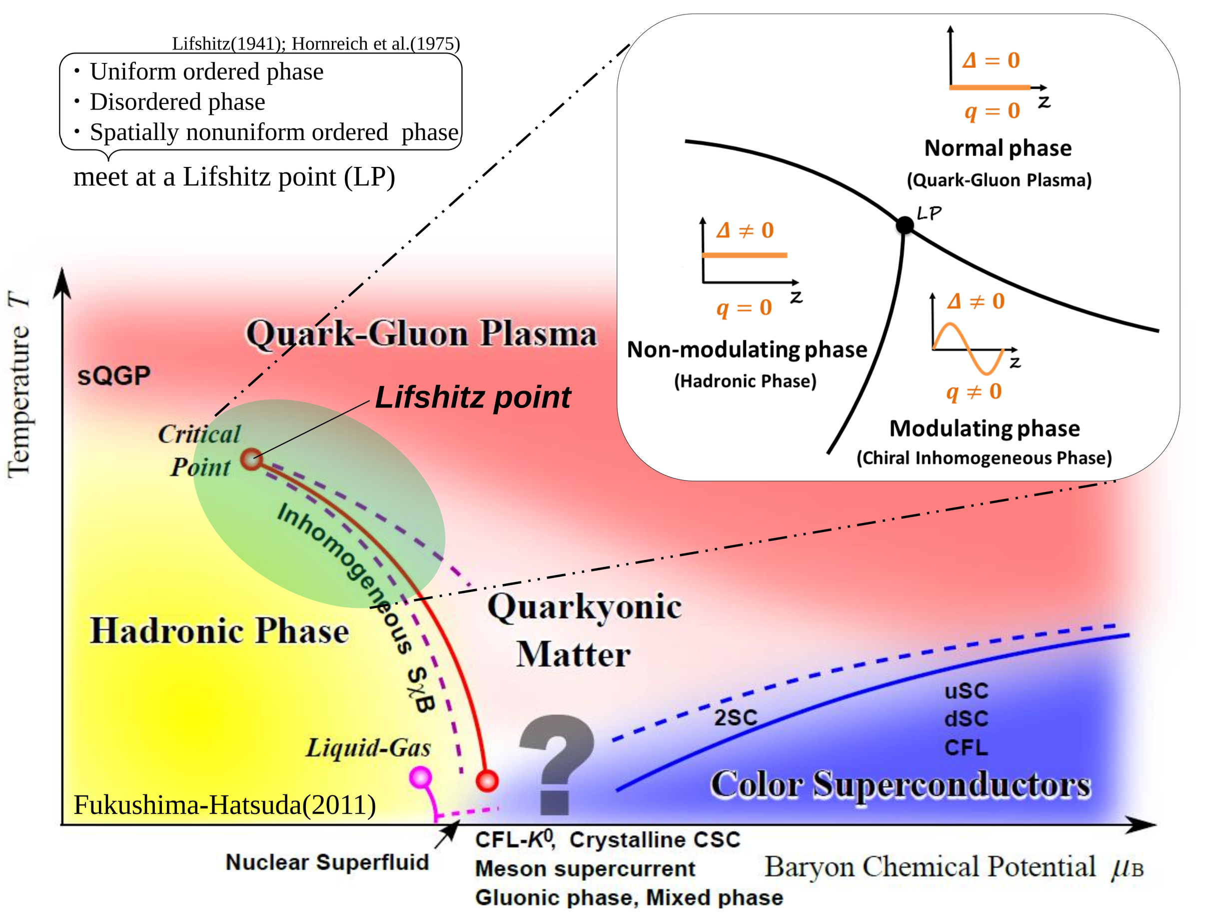
$$\phi_1 = \Delta e^{iqz} \qquad \phi_2 = \Delta \sin(qz)$$

(Δ : constant amplitude, q : wave vector of modulation)

Spiral-type complex ground state where the amplitude is constant, while the phase ($\theta_{(z)}=qz$) modulates with q.
e.g.) Dual Chiral Density Wave, Quarkyonic Chiral Spiral, etc.
Nakano-Tatsumi (2005) [2]
Kojo-Hidaka-McLerran-Pisarski (2010)

Sinusoidal-type real ground state where the potential energy $|\phi_2|$ itself modulates with q.
e.g.) Real Kink Crystal, etc.
Basar-Dunne-Thies (2009)
Nickel (2009) [1]

◎ Recent conjectured QCD phase diagram with Lifshitz point



Motivation

If such inhomogeneous phases actually exist, an elementary excitation on the ground state there should be observed experimentally. Then, what is the physical degrees of freedom emerging from the inhomogeneous phase?

⇒ Therefore, we investigate the low-energy collective excitations, i.e., Nambu-Goldstone excitations, on the inhomogeneous phase in the vicinity of LP.

⇒ This may lead to insights for probing the intermediate-density regime in QCD.

⇒ This also would contribute to the development of Nambu-Goldstone theorem in non-Lorentz invariant systems because both internal and space-time symmetries can be coupled in the inhomogeneous phase. Watanabe-Murayama(2012); Hidaka(2013) [3]

2. GINZBURG-LANDAU ANALYSIS

General U(1) symmetric Lagrangian density:

$$\mathcal{L} = a_3\left[i\phi^*\dot\phi - i\dot\phi^*\phi\right] + a_4|\dot\phi|^2 + \tilde a_5\left[i\dot\phi^*\ddot\phi - i\ddot\phi^*\dot\phi\right] + a_5\left[i\nabla\phi^*\nabla\dot\phi - i\nabla\dot\phi^*\nabla\phi\right] + \tilde a_6|\ddot\phi|^2 + a_6|\nabla\dot\phi|^2 - \mathcal{V}$$

GL potential:

$$\mathcal{V} = a|\phi|^2 + b|\phi|^4 + c|\nabla\phi|^2 + d|\nabla^2\phi|^2 + e|\nabla\phi|^2|\phi|^2 + f|\phi|^6 - h\phi$$

The GL expansion is taken up to 6th order in space-time derivatives $(\partial t, \nabla)$ and complex order- parameter field $\phi(x)$ in order to stabilize the potential when dealing with the modulating order-parameter.

≫ time reversal symmetry : $a_3, \tilde a_5, a_5 > 0$
≫ higher-order time-derivatives : $\tilde a_5 = \tilde a_6 = 0$
≫ normarization : $a_4 = 1$
≫ explicit breaking term : $h = 0$ (chiral limit)

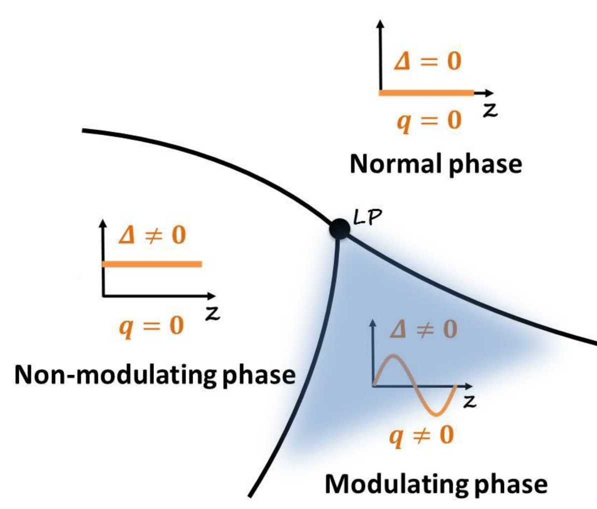
In non Lorentz invariant systems, the surface term, i.e., 1st-order time-derivative term with $\dot\phi \equiv \partial_t\phi$, is allowed. Leutwyler (1994)

FF-type case (e.g., *DCDW*)

We first consider FF-type ground state.
The GL potential for ϕ_1 reads $v(\phi_1) = a\Delta^2 + b\Delta^4 + cq^2\Delta^2 + dq^4\Delta^2 + eq^2\Delta^4 + f\Delta^6$,
where the potential-stability requires $v_{6th}(\phi_1) = dq^4\Delta^2 + eq^2\Delta^4 + f\Delta^6 > 0$,
i.e., $d > 0$ $(f > 0)$ and $df - e^2/4 > 0$.
Values of q and Δ are determined from the potential v.

The stationary conditions (gap equations) for q and Δ ;

$$\frac{\partial v(\phi_1)}{\partial q} = q\Delta^2(c + 2dq^2 + e\Delta^2) = 0$$
$$\frac{\partial v(\phi_1)}{\partial \Delta} = \Delta(a + cq^2 + dq^4 + 2(b + eq^2)\Delta^2 + 3f\Delta^4) = 0$$



▶ **Normal phase:**
$q = \Delta = 0$

▶ **Non-modulating phase:**
$q = 0, \Delta \neq 0$ (Under $q = 0$, $\to \Delta^2 = \frac{-b \pm \sqrt{b^2 - 3af}}{3f}$)

▶ **Modulating phase:**
$q \neq 0, \Delta \neq 0$

Under $q^2 = -(c + e\Delta^2)/2d$, $\to \Delta^2 = -\frac{2(ce - 2bd) \pm \sqrt{3(c^2 - 4ad)(4df - e^2) + 4(ce - 2bd)^2}}{3(4df - e^2)}$

➢ Under a specific coefficient set, the energetically favored phase is realized, where coefficients implicitly depend on thermodynamic environment.

➢ Above all, we focus on **the modulating phase** with ϕ_1 and investigate the low-energy collective excitations on there.

3. SSB AND NG MODES

Spontaneously Symmetry Breaking
in the inhomogeneous ϕ_1-phase with finite q and Δ

➢ U(1) symmetry
➢ Translation symmetry in z direction
} Both symmetries are uniform and global.

$$\Delta e^{iqz} \mapsto \Delta e^{iq(z+s(t,\vec x))+i\tilde\alpha(t,\vec x)}$$

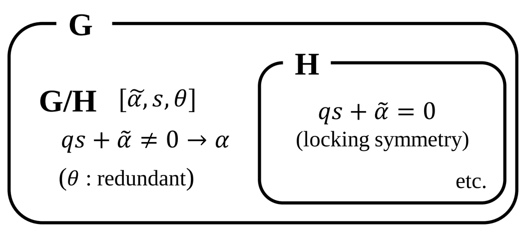
($\tilde\alpha(t,\vec x)$: U(1) rotation, $s(t,\vec x)$: translation operation)

⇒ ϕ_1 is invariant under $qs(t,\vec x) + \tilde\alpha(t,\vec x) = 0$.
(i.e., $\Delta e^{iqz+i(qs(t,\vec x)+\tilde\alpha(t,\vec x))} \mapsto \Delta e^{iqz}$)

Two unique linear combinations

・$qs(t,\vec x) + \tilde\alpha(t,\vec x) = 0 \to$ H (Unbroken symmetry)
U(1)-translation locking symmetry

・$qs(t,\vec x) + \tilde\alpha(t,\vec x) \neq 0 \to$ G/H (Broken symmetry)
NG mode $\alpha(t,\vec x)$ $(\equiv qs(t,\vec x) + \tilde\alpha(t,\vec x))$
which is described by out-of-phase operation; $qs(t,\vec x) + \tilde\alpha(t,\vec x) \neq 0$



➢ Spatial rotation symmetry (x-axis and y-axis rotations)

$$\Delta e^{iqz} \mapsto \Delta e^{iq(z\cos\theta(t,\vec x) + y\sin\theta(t,\vec x))}$$

($\theta(t,\vec x)$: angle operator, Rotation symmetry is nonuniform in space)

However…

⇒ The global operation, i.e., NG mode $\alpha(t,\vec x)$, can completely describe the local operation $\theta(t,\vec x)$.

⇒ $\theta(t,\vec x)$: Redundant NG mode Low-Manohar (2002) Watanabe-Murayama(2013); Hayata-Hidaka(2013)

▶ A single NG mode in ϕ_1-phase with FF-type condensate

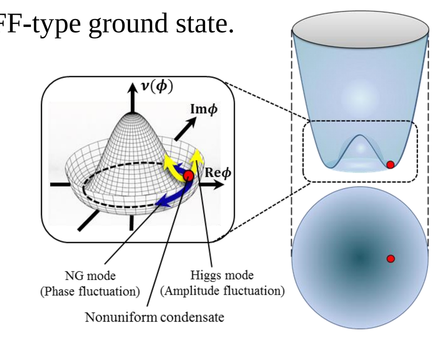
$$dim(G/H) = 1 \;\Rightarrow\; \alpha(t,\vec x) \quad \text{(out-of-phase } U(1) \text{ rotation)}$$

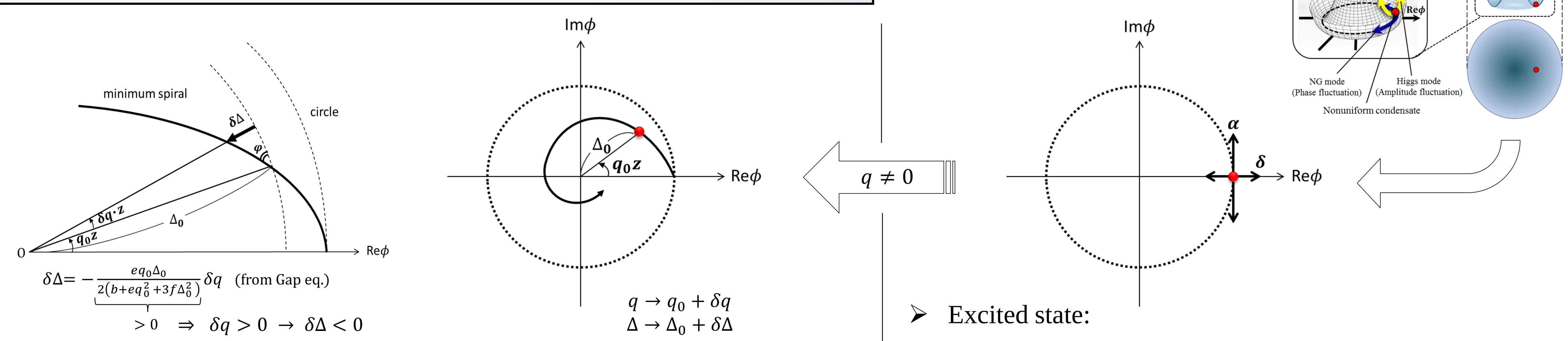
Then, what kind of dispersion does the zero mode α have?
⇒ We investigate the low-energy excitations on ϕ_1-phase.

4. COLLECTIVE EXCITATIONS

Fluctuations on inhomogeneous phases

➢ We consider fluctuations on the FF-type ground state.





$$\delta\Delta = -\frac{eq_0\Delta_0}{2(b + eq_0^2 + 3f\Delta_0^2)}\delta q \quad \text{(from Gap eq.)}$$

$> 0 \;\Rightarrow\; \delta q > 0 \to \delta\Delta < 0$

➢ Excited state:

$$\phi(x) = (\Delta + \delta(x))e^{iqz + i\alpha(x)}$$

$\alpha(x)$: phase-fluctuation (phason) field
$\delta(x)$: amplitude-fluctuation (amplitudon) field
x : abbreviated space-time variable $\{t, x, y, z\}$

➢ Euler-Lagrange equations for $\alpha(x)$ and $\delta(x)$:

$$\frac{\delta L}{\delta\alpha(x)} = \frac{\delta\phi(x)}{\delta\alpha(x)}\frac{\delta L}{\delta\phi(x)} + \frac{\delta\phi^*(x)}{\delta\alpha(x)}\frac{\delta L}{\delta\phi^*(x)} = 0$$
$$\frac{\delta L}{\delta\delta(x)} = \frac{\delta\phi(x)}{\delta\delta(x)}\frac{\delta L}{\delta\phi(x)} + \frac{\delta\phi^*(x)}{\delta\delta(x)}\frac{\delta L}{\delta\phi^*(x)} = 0$$

(GL action: $L = \int d^4x\,\mathcal{L}$)

In the non-modulating case with $q = 0$:

$$\left[\partial_t{}^2 - (c + e\Delta^2)\vec\nabla^2 + d(\vec\nabla^2)^2\right]\alpha(x) = 0$$
$$\left[\partial_t{}^2 - (c + e\Delta^2)\vec\nabla^2 + d(\vec\nabla^2)^2 + m^2\right]\delta(x) = 0$$

(mass gap; $m^2 = 4(b + 3f\Delta^2)\Delta^2$)

The corresponding dispersion relations:

$$\omega_\alpha(k) = \sqrt{c + e\Delta^2}|\vec k| + O(|\vec k|^3) \quad \text{Goldstone (phason) mode } (\Rightarrow \omega \propto |\vec k|)$$

$$\omega_\delta(k) = m + \frac{c + e\Delta^2}{2m}|\vec k|^2 + O(|\vec k|^4) \quad \text{Higgs (amplitudon) mode}$$

In the modulating phase with $q \neq 0$, the Eular-Lagrange equations for $\alpha(x)$ and $\delta(x)$:
(up to the linear order in fluctuations in the Fourier spaces $k = (\omega, \vec k)$)

$$\begin{pmatrix} G_k\omega^2 - E_k\omega - C_k & 2i(A_k + B_k\omega + F_k\omega^2) \\ -2i(A_k + B_k\omega + F_k\omega^2) & G_k\omega^2 - E_k\omega - C_k - M^2 \end{pmatrix}\begin{pmatrix} \Delta\alpha(k) \\ \delta(k) \end{pmatrix} = 0$$

⇒ α-δ mixing — Fluctuations α and δ are coupled.

⇒ Δ-shrink — Zero mode direction is changed from minimal circle to minimal spiral.

where
$A_k = q(2d\vec k^2 + e\Delta^2)k_z$
$B_k = -a_3 - a_5(\vec k + q^2)$
$C_k = 4dq^2k_z^2 + d(\vec k^2)^2$
$E_k = -4a_5qk_z$
$F_k = -a_6qk_z$
$G_k = 1 + a_6(\vec k + q^2)$
$M^2 = 4(b + eq^2 + 3f\Delta^2)\Delta^2$

➢ Dispersion relations for the above eigen modes:

$$\omega^2 = \begin{cases} v_t^2\vec k_t^2 + v_{z-}^2k_z^2 + O(k^4) \\ M^2 + v_t^2\vec k_t^2 + v_{z+}^2k_z^2 + O(k^4) \end{cases}$$

where
$a_3 = a_5 = a_6 = 0$ (for simplicity)
$\vec k^2 = (k_x^2 + k_y^2) + k_z^2 = \vec k_t^2 + k_z^2$
$v_t^2 = c + 2dq^2 + e\Delta^2 = 0$ (Gap eq.)
$v_{z\pm}^2 = 4q^2(dM^2 \pm e^2\Delta^4)/M^2$

U(1)-phonon mode

$$\omega^2 = v_{z-}^2k_z^2 + d(k_t^2)^2 + \cdots$$

Direction-dependent dispersion
➢ z direction: linear dispersion
➢ x-y direction: quadratic dispersion

5. REMARKS AND PROSPECTS

Missions

➢ To clarify the low-energy effective degrees of freedom in inhomogeneous phases
⇒ Effective field theory constructed from coset space G/H (NG modes)

➢ To understand fluctuations or NG excitations on density wave ground states
⇒ Based on zero temperature arguments due to Landau-Peierls instability [4]
(1-D modulation instability against thermal fluctuations)

Results and Remarks

✓ Coupling of inner and outer spaces is realized: U(1)-Translation locking symmetry (inner and spacetime symmetries are locked)
✓ NG mode identified as out-of-phase U(1) rotation emerged from ϕ_1-phase
✓ α-δ mixing mode is an Eigenmode in FF-type modulating phase
✓ Zero-mode moves in a direction toward symmetry restoration when $\delta q > 0$ on minimal spiral
✓ NG mode has an anisotropic dispersion relation (x-y: Quadratic, z: Linear)

Ongoing work and Prospects

☐ $SU(2)_L \otimes SU(2)_R$ chiral model (or chiral O(4) model)
⇒ Nonuniform chiral condensate $\langle\bar\psi e^{i\gamma_5\tau_3 qz}\psi_q\rangle$
FF-type ground state ($\phi_1 = \Delta\exp(iqz)$; DCDW) Flavor-Translation locking
LO-type ground state ($\phi_2 = \Delta\sin(qz)$; RKC) Only phonon(phason)-like mode?
Possible general ground state ($\phi_{FF+LO} = \lambda\left(\frac{2\sqrt{\nu}}{1+\sqrt{\nu}}\right)sn\left(\frac{2\lambda x}{1+\sqrt{\nu}};\nu\right)$; DCDW+RKC)

☐ Interaction between NG modes at tree level

☐ Construction of the low-energy effective theory for chiral inhomogeneous phases

☐ Finite temperature arguments
Possibility of Kosterlitz-Thouless-like transition, stability of 2-D modulation, etc.
Kosterlitz-Thouless (1973) e.g., Carignano-Buballa (2012)

☐ Phenomenological implications
e.g., Coupling of inhomogeneous phases (e.g., DCDW) and magnetic fields
⇒ longitudinal and transverse excitations may be possible along the magnetic field. (Spin wave-like?)
Besides, experimental signals of chiral inhomogeneous phases in HICs, e.g., J-PARC, etc.

6. REFERENCES

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