Strategies to determine the X(3872) energy from QCD lattice simulations

R. Molina[1], E. J. Garzon[2], A. Hosaka[1], E. Oset[2]
[1] RCNP-Osaka University, [2] IFIC-Valencia University
molina@rcnp.osaka-u.ac.jp, garzon@ific.uv.es, hosaka@rcnp.osaka-u.ac.jp, Eulogio.Oset@ific.uv.es

Introduction

We develop a method to **determine accurately the binding energy of the X(3872) from lattice data** for the $D\bar{D}^*$ interaction. We show that, because of the ***small difference between the neutral and charged components of the X(3872)***, it is necessary to ***differentiate them in the energy levels*** of the lattice spectrum if one wishes to have a precise determination of the the **binding energy** of the X(3872). The analysis of the data requires the use of coupled channels. Depending on the number of levels available and the size of the box we determine the precision needed in the lattice energies to finally obtain a desired accuracy in the binding energy.

The X(3872) in the continuum limit

Lagrangian $\mathcal{L}_{PPVV} = -\frac{1}{4f^2} Tr\,(J_\mu \mathcal{J}^\mu)$

Currents $J_\mu = (\partial_\mu P)P - P\partial_\mu P$, $\mathcal{J}_\mu = (\partial_\mu \mathcal{V}_\nu)\mathcal{V}^\nu - \mathcal{V}_\nu \partial_\mu \mathcal{V}^\nu$.

Breaking Parameters $m_{8^*} = m_L = 800$ MeV, $m_{3^*} = m_H = 2050$ MeV and $m_{1^*} = m_{J/\psi} = 3097$ MeV, $f = f_\pi = 93$ MeV, and $f = f_D = 165$ MeV.

$$\gamma = \left(\frac{m_{8^*}}{m_{3^*}}\right)^2 = \frac{m_L^2}{m_H^2},\ \psi = \left(\frac{m_{8^*}}{m_{1^*}}\right)^2 = \frac{m_L^2}{m_{J/\psi}^2}$$

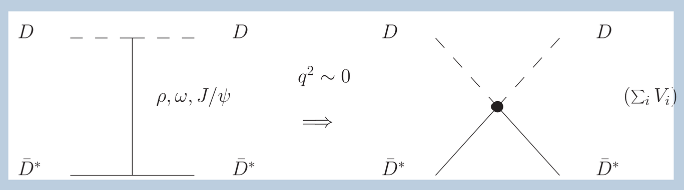

Figure 1: Pointlike pseudos.-vector interaction.

Thus, the amplitude of the process $V_1(k)P_1(p) \to V_2(k')P_2(p')$, is given by

$$V_{ij}(s,t,u) = \frac{\xi_{ij}}{4f_i f_j}(s-u)\,\vec{\epsilon}.\vec{\epsilon}' \quad (1)$$

$$T = (I - VG)^{-1} V\,\vec{\epsilon}.\vec{\epsilon}' \quad (2)$$

For G, dim. regularization formula or cutoff method can be used,

$$G = \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1+\omega_2}{2\omega_1\omega_2} \frac{1}{(P^0)^2 - (\omega_1+\omega_2)^2 + i\epsilon} \quad (3)$$

$\sqrt{s_0} = (3871.6 - i0.001)$ MeV	
Channel	$\lvert g_i \rvert$ [MeV]
$\frac{1}{\sqrt{2}}(K^{*-}K^+ - c.c)$	53
$\frac{1}{\sqrt{2}}(\bar{K}^{*0}K^0 - c.c)$	49
$\frac{1}{\sqrt{2}}(D^{*+}D^- - c.c)$	3638
$\frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 - c.c)$	3663
$\frac{1}{\sqrt{2}}(D_s^{*+}D_s^- - c.c)$	3395

Table 1: Couplings of the pole to the channel i

$$-\sum_i g_i^2 \frac{\partial G}{\partial s} = 1\,, \quad (4)$$

Probability of finding the i ch. in the wave func.,

0.86 for $D^{*0}\bar{D}^0 - c.c$,
0.124 for $D^{*+}D^- - c.c$
and 0.016 for $D_s^{*+}D_s^- - c.c.$

However $(2\pi)^{3/2}\psi(0)_i = g_i G_i$ (wave function at the origin) are nearly equal, and this usually enters the evaluation of observables.

Formalism in finite volume

$G \to \tilde{G}$:
$$\tilde{G}(P^0) = \frac{1}{L^3}\sum_{\vec{q}_i} \frac{\omega_1(\vec{q}_i)+\omega_2(\vec{q}_i)}{2\omega_1(\vec{q}_i)\omega_2(\vec{q}_i)} \frac{1}{(P^0)^2 - (\omega_1(\vec{q}_i)+\omega_2(\vec{q}_i))^2} = \frac{1}{L^3}\sum_{\vec{q}_i} I(P^0,\vec{q})$$

where $\omega_i = \sqrt{m_i^2 + |\vec{q}_i|^2}$ and the momentum $\vec{q}$ is quantized as $\vec{q}_i = \frac{2\pi}{L}\vec{n}_i$, $|\vec{q}_i| = \frac{2\pi}{L}\sqrt{m_i}$, $n_{x,i}^2 + n_{y,i}^2 + n_{z,i}^2 = m_i$ and $n_{max} = \frac{q_{max}L}{2\pi}$ (symmetric box).

$$\tilde{G} = G^{DR} + \lim_{q_{max}\to\infty}\left(\frac{1}{L^3}\sum_{q<q_{max}} I(P^0,\vec{q}) - \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} I(P^0,\vec{q})\right) \equiv G^{DR} + \lim_{q_{max}\to\infty} \delta G \quad (5)$$

$T \to \tilde{T}$ $\quad \tilde{T} = (I - V\tilde{G})^{-1}V \quad$ Energy levels in the box: $det(I - V\tilde{G}) = 0$

Single channel case: **(a)$D^0\bar{D}^{*0}$** **(b)D^+D^{*-}** **Two channel case:**

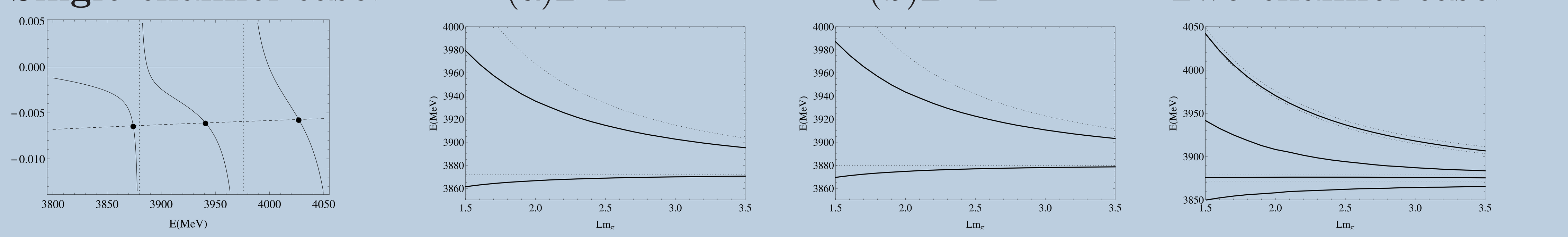

Figure 2: Left: $\tilde{G}$(solid) and V^{-1}(dashed) energy dependence of D^+D^{*-} for $Lm_\pi = 2.0$. Dotted lines are the free energies; (a), (b) and right: L dependence of the energies for a single channel and two channels respectively.

The inverse problem

QCD lattice data can be used to determine bound states of the $D\bar{D}^*$ system. We assume that the lattice data are some discrete points on the energy trajectories (synthetic data). We want to determine the potential and evaluate the pole position of the X(3872) in infinite volume. A set of data of 5 points in a range of $Lm_\pi = [1.5,\ 3.5]$ for each level (four levels with $n = 0$ and 1) with uncertainties, moving randomly by 1 MeV the centroid assigning an error of 2 MeV, are generated.

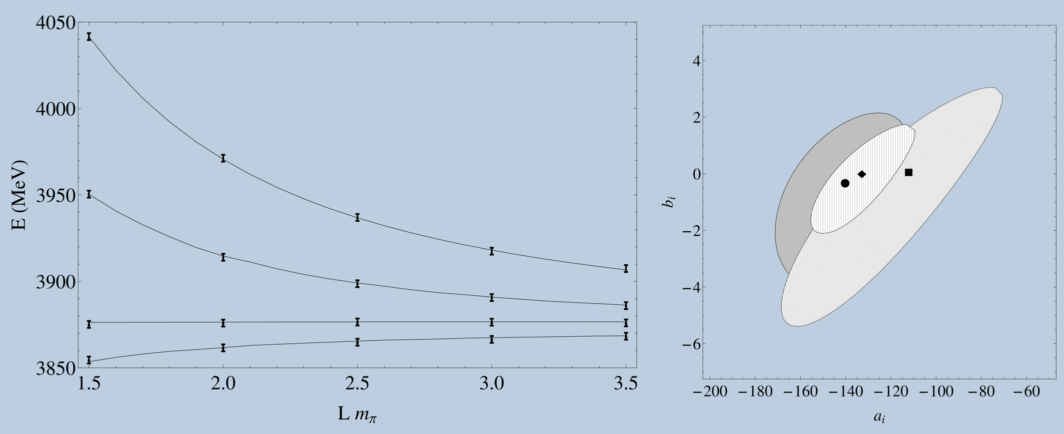

Figure 3: Left: Fit to the data. Dots: synthetic data. Solid lines: with potential fitted. Right: Contour plot for the χ^2 representing $\chi^2 \leq \chi^2_{min} + 1$. Points correspond to values of the parameters in the χ^2 minimum. (Circle and grey area: a_1 and b_1, Square and diagonal lined area: a_2 and b_2 and Diamond and vertical lined area: a_3 and b_3.)

Potential (six parameters to fit) :
$$V_i = a_i + b_i\left(\sqrt{s} - \sqrt{s^{th}}\right);\quad i = 1: D^+D^{*-},\ i = 2: D^0\bar{D}^{*0}:\ i = 3: \text{nondiagonal}$$

The χ^2 function is **minimized**. The binding energy is essentially **independent** of the choice of α.

Results

(B,P,ΔE,ΔC)	a_1	a_2	a_3	b_1	b_2	b_3	χ^2	Pole	Mean Pole	σ
(4,5,2,1)	-140.18	-112.08	-132.81	-0.310	0.074	0.012	2.32	3871.51	3871.49	0.07
(4,5,5,2)	-140.18	-112.08	-132.81	-0.310	0.074	0.012	0.79	3871.51	3871.25	0.38
(4,3,2,1)	-133.01	-131.92	-124.60	-0.242	0.048	-0.075	1.02	3871.44	3871.49	0.18
(4,3,5,2)	-120.09	-98.19	-150.94	-0.377	-0.075	0.102	0.28	3871.41	3871.15	0.49
(2,5,2,1)	-176.08	-154.11	-89.26	9.92	7.01	-8.72	0.259	3871.70	3871.47	0.30
(2,5,5,2)	-158.49	-152.15	-103.23	4.56	6.58	-6.74	0.982	3871.34	3871.30	0.43
(2,3,2,1)	-132.74	-176.62	-105.53	3.23	0.84	-3.36	0.074	3870.51	3870.48	0.61
(2,3,5,2)	-226.57	-194.51	-32.74	31.81	13.28	-18.89	0.942	3869.49	3870.37	1.06

It is necessary to distinguish between the levels of D^+D^{*-} and $D^0\bar{D}^{*0}$. With errors in the data of 5 MeV, one can obtain the binding energy with 1 MeV precision, and two levels are enough to have an accurate value. To have a very high precision in the binding energy (~ 0.2 MeV), requires high precision in the data. In addition, **we can know about the nature of the X(3872)**. If the X(3872) was genuine, we can generate it using a potential containing a CDD pole

$$V = V_M + \frac{g^2_{\rm CDD}}{s - s_{\rm CDD}} \quad (Castillejo - Dalitz - Dyson) \quad (V_M = 1/10V'_M)\,, \quad (6)$$

$\sqrt{s_{\rm CDD}}$ 20 MeV above the threshold, $g_{\rm CDD} = 4620$ MeV. Taking the two lower levels, we obtain $-\sum_{i=1}^2 g_i^2 \frac{\partial G_i}{\partial s} = 1 - Z = 0.51$. This tell us that the state has a large genuine component $Z \simeq 0.5$. On the contrary, if $V_M = V'_M$, $-\sum_{i=1}^2 g_i^2 \frac{\partial G_i}{\partial s} = 1 - Z = 0.97$.

[1] E. J. Garzon, R. Molina, A. Hosaka and E. Oset, Phys. Rev. D 89, 014504 (2014).