Complex 2D Matrix Model and Its Application to *Nc*-dependence of Hadron Structures

Kanabu Nawa[1], Sho Ozaki[2], Hideko Nagahiro[3], Daisuke Jido[4], and Atsushi Hosaka[5]

[1]*RIKEN*, [2]*Yonsei University*, [3]*Nara Women's University*, [4]*Tokyo Metropolitan University*, [5]*RCNP*

PTEP 083D01 (2013).

Abstract

We study the **parameter dependence** of the **internal structure of resonance states**.
The geometry with "exceptional points" on the complex-parameter space is useful to discuss the parameter dependence within real-parameter subspace. By applying the model to hadron physics, we examine the ***Nc*-dependence of hadron structures** from the **geometry on the complex-*Nc* plane**.

1.Introduction

*How do characters of quantum states **change** with **variation of a parameter** which specifies the property of the system or of the environment where the system is placed ?*

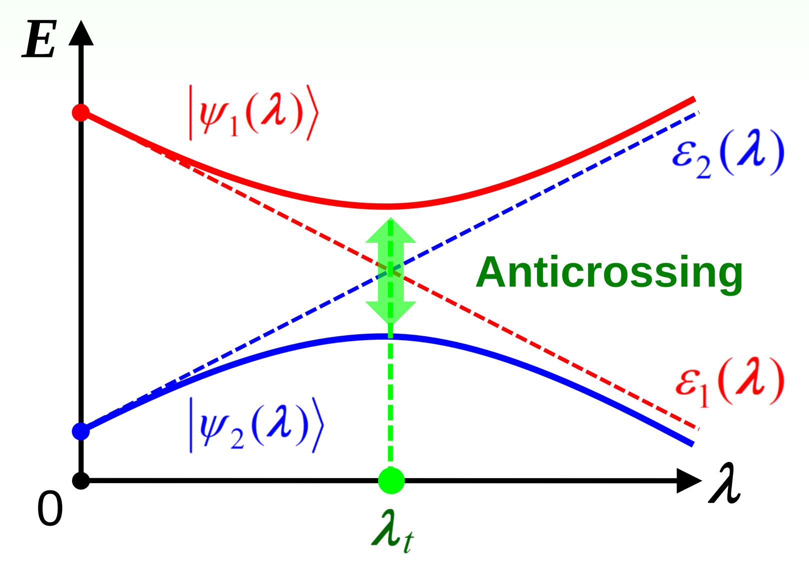
[L.D. Landau and E.M. Lifshitz]

$\hat{\mathcal{H}}(\lambda)$, $\lambda \in \mathbf{R}$

2 level model :

$$\mathrm{H}(\lambda) = \begin{pmatrix} \langle\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_1\rangle & \langle\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_2\rangle \\ \langle\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_1\rangle & \langle\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_2\rangle \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

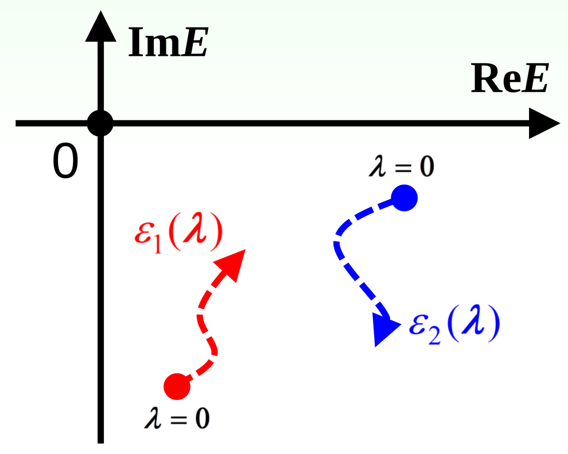
$|\phi_i\rangle$: eigenstates at $\lambda = 0$, having clear characters.



"Nature Transiton"

$|\psi_1(\lambda)\rangle$ and $|\psi_2(\lambda)\rangle$ exchange their characters at anticrossing point $\lambda = \lambda_t$.

How about "resonances" ?



$\varepsilon_1(\lambda)$ and $\varepsilon_2(\lambda)$ can move on complex-E plane !!!

✻ ***What is the simple criterion of nature transition for resonances?***

✻ ***How do poles move two-dimensionally on complex-E plane?***

2.Complex 2D Matrix Model

biorthogonal representation: $|\phi) \equiv |\phi\rangle,\ (\phi| \equiv \langle\phi^*|$

[N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)]
[T. Berggren, Nucl. Phys. A109, 265 (1968)]

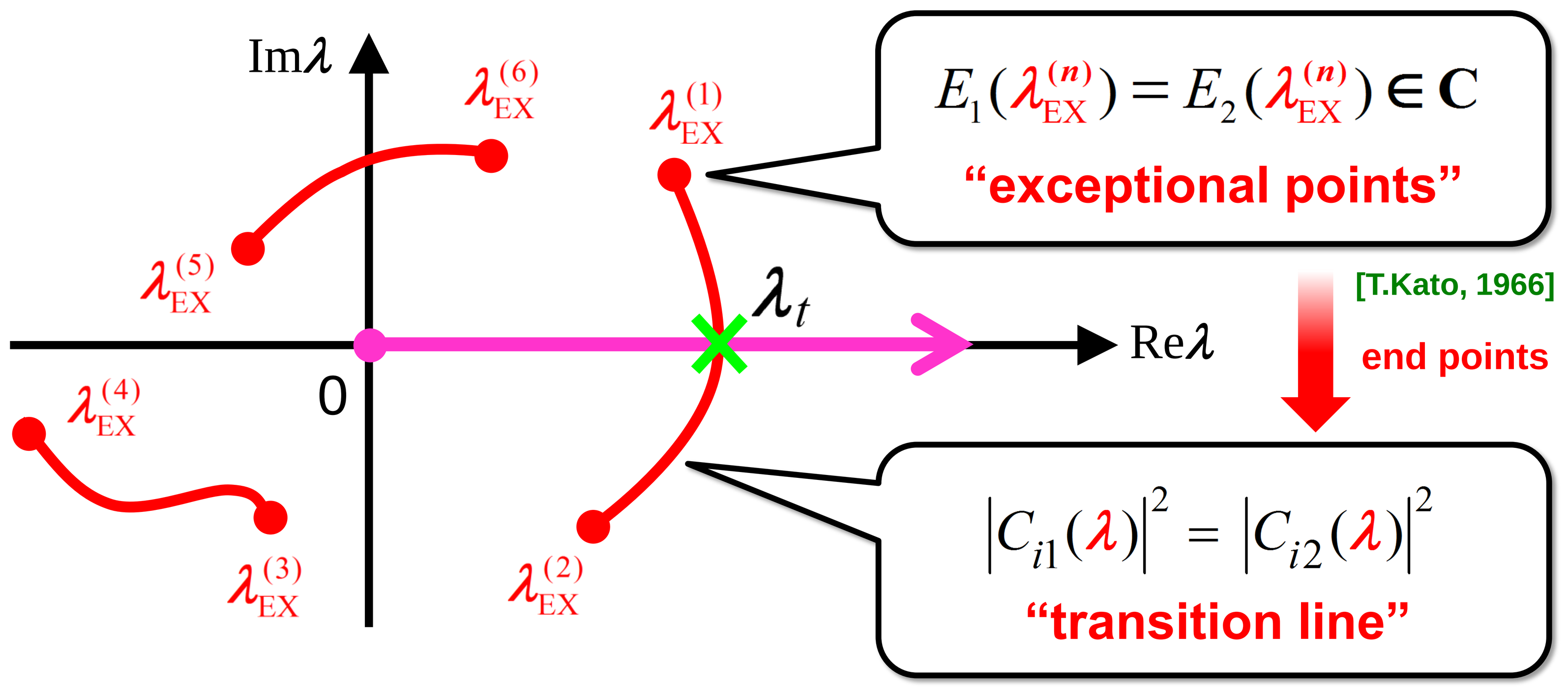
$$\mathrm{H}(\lambda) = \begin{pmatrix} (\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_1) & (\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_2) \\ (\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_1) & (\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_2) \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

$$\varepsilon_i, V_{ij} \in \mathbf{C}\ ,\ \lambda \in \mathbf{R}$$

C_{ij} carry the information about internal structure of eigenstates

$$|\psi_1(\lambda)) = C_{11}(\lambda)|\phi_1) + C_{12}(\lambda)|\phi_2)$$
$$|\psi_2(\lambda)) = C_{21}(\lambda)|\phi_1) + C_{22}(\lambda)|\phi_2)$$

Transition condition: $|C_{i1}(\lambda)|^2 = |C_{i2}(\lambda)|^2$



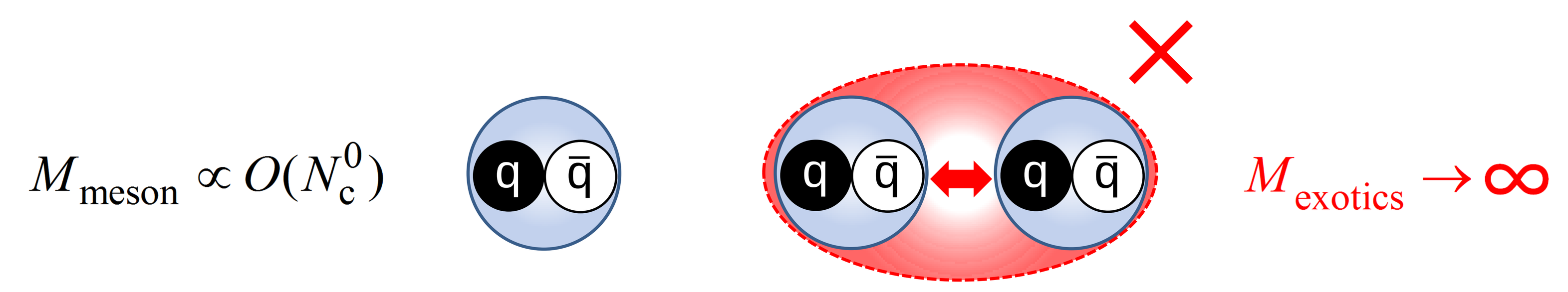
[K.N., et.al., 2013]

"Geometry" with **"exceptional points"** and **"transition lines"** on complex parameter space is important to judge the existence of nature transition within the real parameter subspace :

- ***Dense exceptional points show quantum chaos.***
- ***Phase singular point of the Berry phase.***

3. *Nc*-dependence of Hadron Structures

By extending *Nc* to an arbitrary number, 1/*Nc*-expansion provides a systematic perturbative treatment.

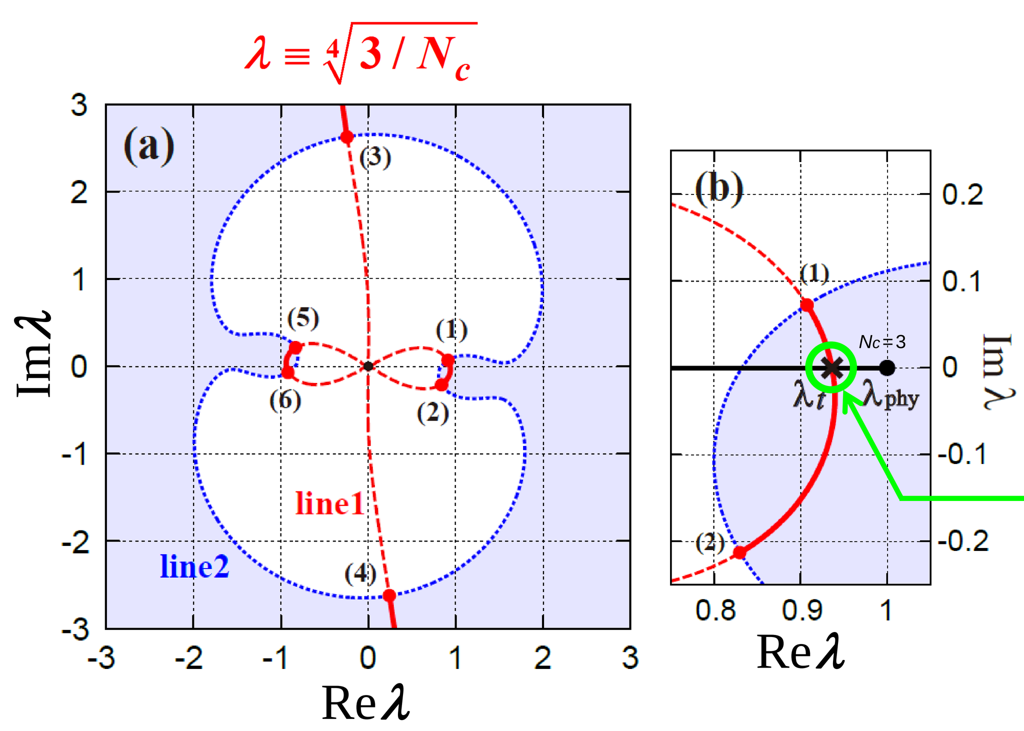


***Exotics** are probably not entirely absent in the real world, but they are **certainly suppressed** – they are certainly not conspicuous in phenomenology. The only known field theoretic reason for this suppression is the **1/Nc expansion** .* [E. Witten, NPB160,57(1979)]

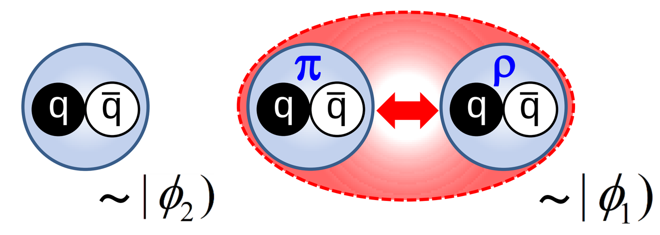
Internal structure of hadrons should depend on *Nc*.

(It can drastically change due to the development of hadron dynamics scaled by 1/*Nc*.)

How does the internal structure of hadrons change from *Nc* = ∞ to *Nc* = 3 ? → **1/*Nc* ~ λ**



a_1 meson :



Nature transition occurs at critical color number:

Nc ~ 4.0

Summary

- **Geometry on complex-parameter space** controls the parameter-dep. within real-parameter subspace.
- **Critical color number** can be calculated from the **geometry on complex-*Nc* plane**.