D meson in nuclear medium from QCD sum rules

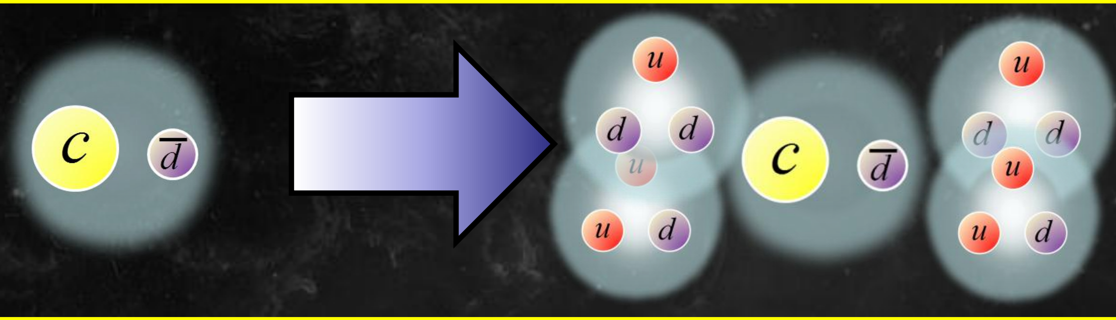
Kei Suzuki (*Tokyo Institute of Technology*)

Philipp Gubler (*RIKEN*)
Makoto Oka (*TITech*)

Overview of Our Work

- Subject

  D meson is modified in dense nuclear matter



- Purpose

  To investigate D meson spectral function

- Method

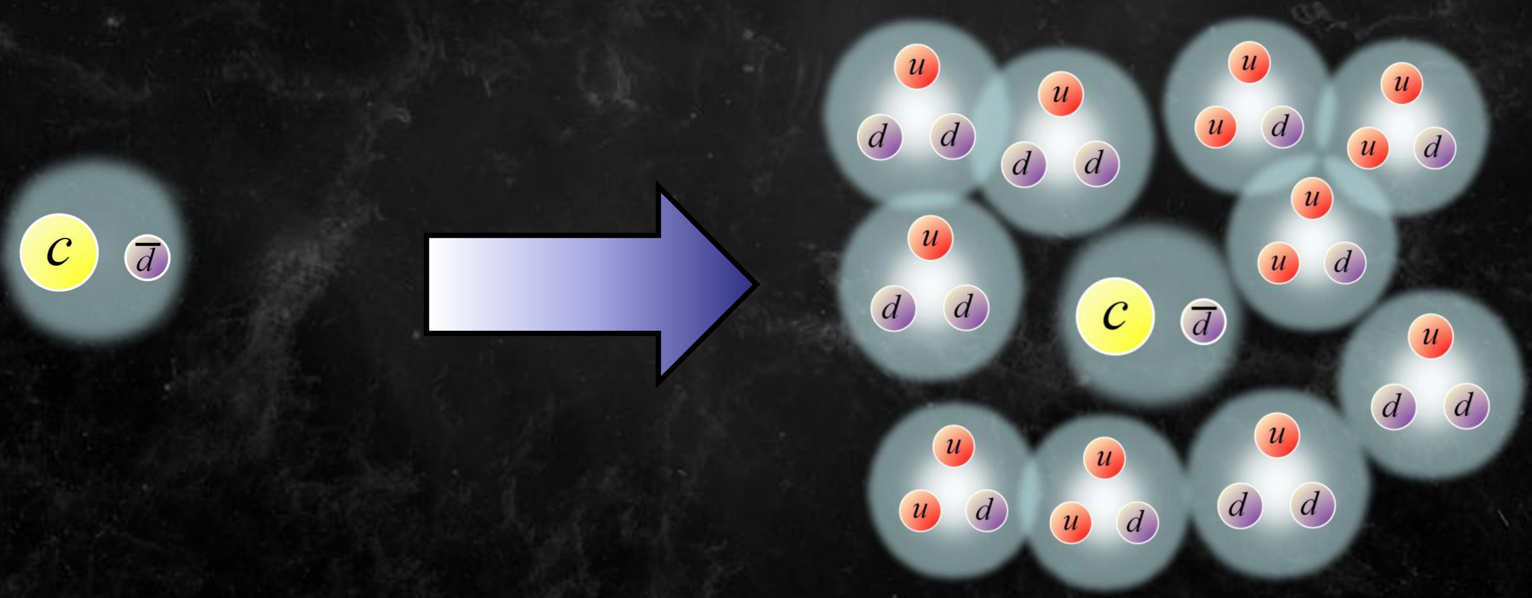
  QCD sum rules and **MEM**

1. Introduction

D meson in nuclear medium

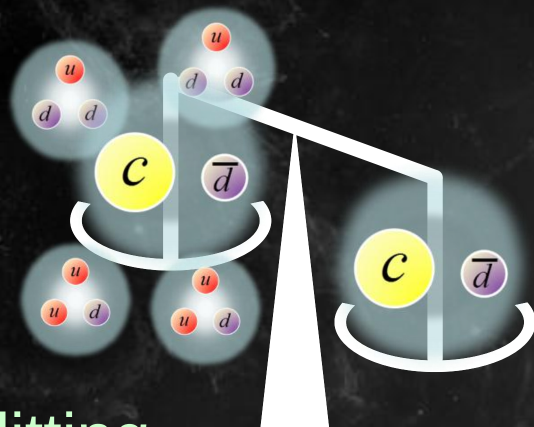
If a D meson is put into nuclear medium, what will happen ?

vacuum **in nuclear medium**

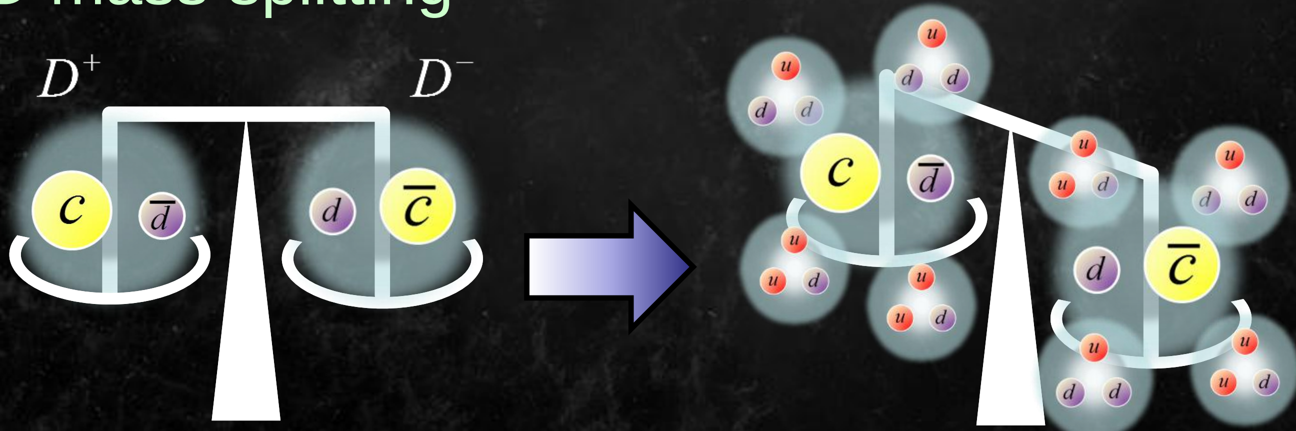


D meson properties in nuclear medium

- Mass shift (partial restoration of chiral symmetry etc.)



- D – $\bar{D}$ mass splitting



Mass:1.87GeV different mass?

Previous work (from QCD sum rules)

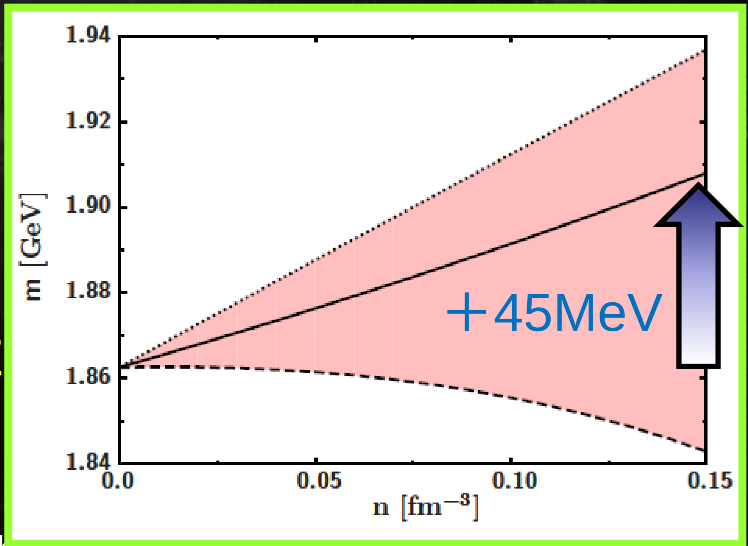
There are many theoretical approachs

A.Hayashigaki, Phys.Lett. B487 (2000) 96

⇒mass shift : $(\Delta m_{D+} + \Delta m_{D-})/2 = -50\text{MeV}$ at ρ_0

T. Hilger, R. Thomas, B. Kampfer, Phys. Rev. C79 (2009) 025202

⇒mass shift : $+45\text{MeV}$ and mass splitting : $(m_{D+} - m_{D-}) = -60\text{MeV}$ at ρ_0



These results depend on phenomenological parameter

⇒We need parameter-independent analysis (→MEM)

2. Methods

Theoretical strategy for hadron in nuclear medium

We study density dependence of <u>spectral function</u>

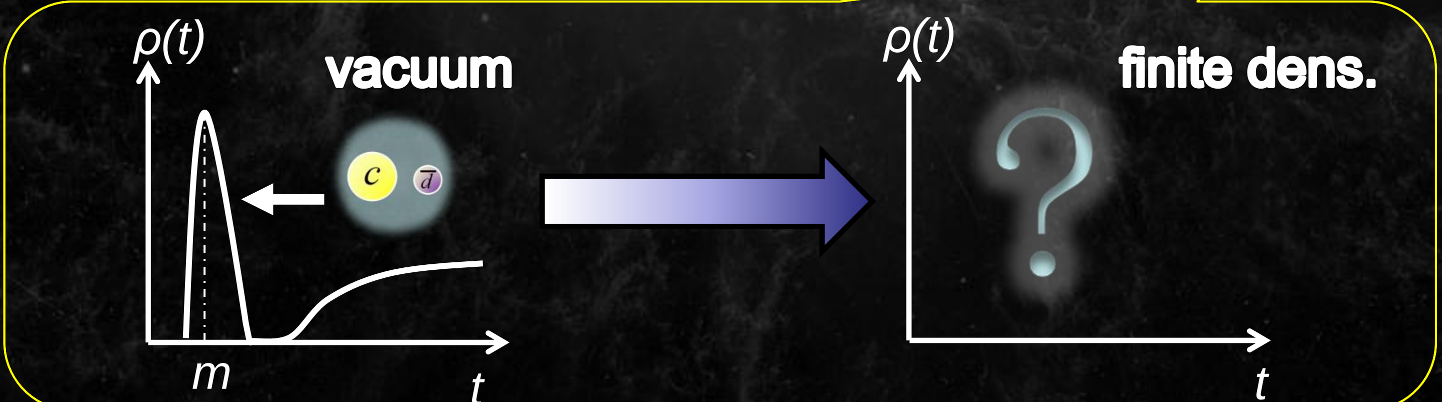
Meson correlation function → **Spectral function of hadron state**

Non-perturbative QCD approach

<u>QCD sum rule</u> etc.

↑ D-dependence input

↓ D-dependence output



QCD sum rule

- QCD sum rule

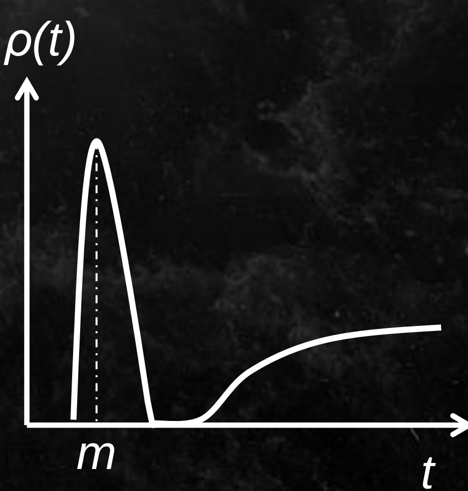
M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. B**147**, 385 (1979); B**147**, 448 (1979)

Relation between <u>operator product expansion (OPE)</u> of correlation function and <u>spectral function of hadron</u>

$$\Pi_{\mathrm{OPE}}(q^2) = \int_0^\infty \frac{\rho(t)}{t - q^2} dt$$

transformation

$$\Pi_{\mathrm{OPE}}(M^2) = \int_0^\infty K(t, M^2)\, \rho(t)\, dt$$



Input OPE

Output spectral function → <u>One hadron state</u>

Setup in QCD sum rules

QCD sum rule

$$\Pi_{\mathrm{OPE}}(s,\tau) = \int_0^\infty K(t,s,\tau)\,\rho(t)\,dt$$

Gaussian sum rule

$$\frac{1}{\sqrt{4\pi\tau}} exp\left[\frac{-(t-s^2)^2}{4\tau}\right]$$

D meson OPE

$$\begin{aligned}
G^{\mathrm{even}}(\hat{s},\tau) = {} & \frac{1}{\sqrt{4\pi\tau}}\frac{1}{\pi}\int_{m_h^2}^{\infty} ds e^{-\frac{(s-\hat{s})^2}{4\tau}} \mathrm{Im}\Pi^{\mathrm{pert}}(s) \\
& + \frac{1}{\sqrt{4\pi\tau}} e^{-\frac{(m_h^2-\hat{s})^2}{4\tau}} \Bigg[-m_h\langle\bar{q}q\rangle + \frac{1}{12}\langle\frac{\alpha}{\pi}G^2\rangle - \frac{1}{2}\left(\frac{3m_h^2-2\hat{s}}{4\tau} - \frac{2(m_h^2-\hat{s})^2 m_h^2}{(4\tau)^2}\right) m_h\langle\bar{q}g\sigma Gq\rangle \\
& - \left\{\left(\frac{7}{18}+\frac{1}{3}\ln\frac{\mu^2}{m_h^2}\right)\left(1-\frac{(m_h^2-\hat{s})m_h^2}{2\tau}\right) + \frac{m_h^4 - m_h^2\hat{s} - 2\tau}{3\tau}\right\}\langle\frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2}-\frac{G^2}{4}\right)\rangle \\
& - 2\left(1-\frac{(m_h^2-\hat{s})m_h^2}{2\tau}\right)\langle q^\dagger i\overrightarrow{D}_0 q\rangle \\
& - 4\left(\frac{3m_h^2-2\hat{s}}{4\tau} - \frac{2(m_h^2-\hat{s})^2 m_h^2}{(4\tau)^2}\right) m_h \left[\langle\bar{q}\overrightarrow{D}_0^2 q\rangle - \langle\frac{1}{8}\bar{q}g\sigma Gq\rangle\right]\Bigg]
\end{aligned}$$

$$\mathrm{Im}\Pi^{\mathrm{pert}}(s) = \frac{3}{8\pi}s\left(1-\frac{m_h^2}{s}\right)^2 \times \left(1+\frac{4}{3}\frac{\alpha_s}{\pi}R_0(m_h^2/s)\right)$$

$$R_0(m_h^2/s = x) = \frac{9}{4} + 2Li_2(x) + \ln x\ln(1-x) - \frac{3}{2}\ln\frac{1-x}{x} - \ln(1-x) + x\ln\frac{1-x}{x} - \frac{x}{1-x}\ln x$$

Output spectral function

- maximum entropy method (MEM)

P. Gubler and M. Oka, Prog. Theor. Phys. 124, 995 (2010)

D meson OPE in vacuum

$$\Pi_{\mathrm{OPE}}(M^2) = \text{perturbative term}$$

$$+e^{-m_c^2/M^2}\left[-m_c\langle\bar{q}q\rangle + \frac{1}{2}\left(\frac{m_c^2}{2M^4} - \frac{1}{M^2}\right)m_c\langle\bar{q}g\sigma Gq\rangle\right.$$

$$\left.+\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{16\pi}{27}\frac{1}{M^2}\left(1+\frac{1}{2}\frac{m_c^2}{M^2} - \frac{1}{12}\frac{m_c^4}{M^4}\right)\alpha_s\langle\bar{q}q\rangle^2\right]$$

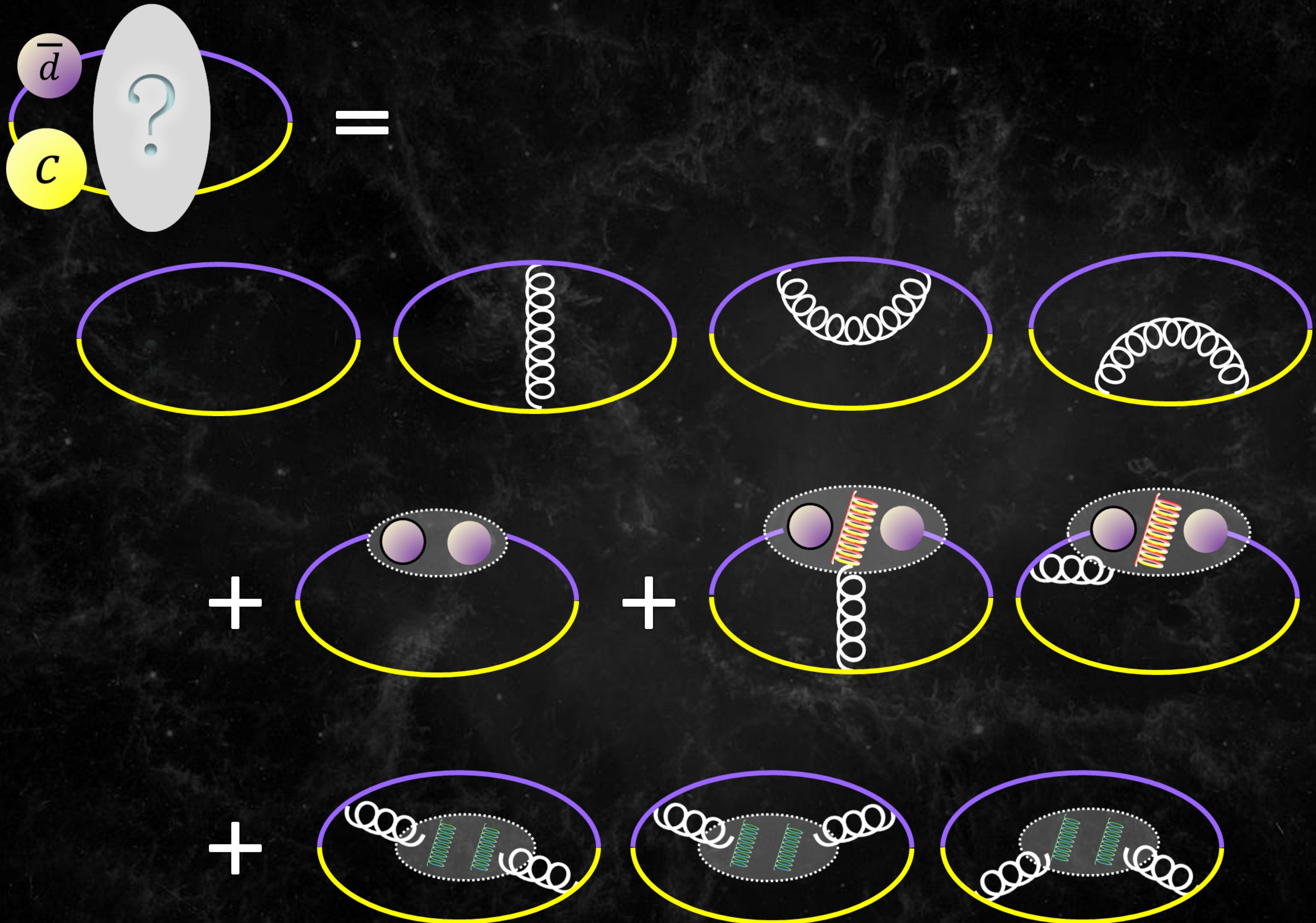
1. Chiral condensate
2. Mixed condensate

Coefficients are proportional to charm quark mass ⇒These terms are enhanced

3. Gluon condensate
4. 4-quark condensate

Other condensates are relatively suppressed

⇒We expect that chiral and mixed condensate are dominant


$\bar{d}$
c
?
=
+
+
+

D meson OPE in nuclear medium

- All of the condensates have density dependence

$\langle \bar{d}d \rangle_n = \langle \bar{d}d \rangle_{vac} + \frac{\sigma_N}{2m_q} n$	$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_n = \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{vac} - \frac{8M_N^0}{9} n$	$\langle \bar{d} g\sigma G d \rangle_n = \lambda^2 \langle \bar{d}d \rangle_n$

- New (Lorentz variant) condensates appear

$\langle d^\dagger d \rangle_n = \frac{3}{2} n$	$\left\langle \frac{\alpha_s}{\pi} \left(\frac{(vG)^2}{v^2} - \frac{G^2}{4} \right) \right\rangle_n = -\frac{3}{4} M_N A_2^q(\mu^2) n$
$\langle d^\dagger i D_0 d \rangle_n = \frac{3}{8} M_N A_2^q(\mu^2) n$	$\left[\langle \bar{d} D_0^2 d \rangle_n - \frac{1}{8} \langle \bar{d} g\sigma G d \rangle_n \right] = \frac{\lambda^2 \sigma_N}{2m_q} n$
$\langle d^\dagger g\sigma G d \rangle_n = (-0.33 \mathrm{GeV}^2) n$	$\langle d^\dagger D_0^2 d \rangle_n = -\frac{1}{4} M_N^2 A_3^q(\mu^2) n + \frac{1}{12} \langle d^\dagger g\sigma G d \rangle_n$

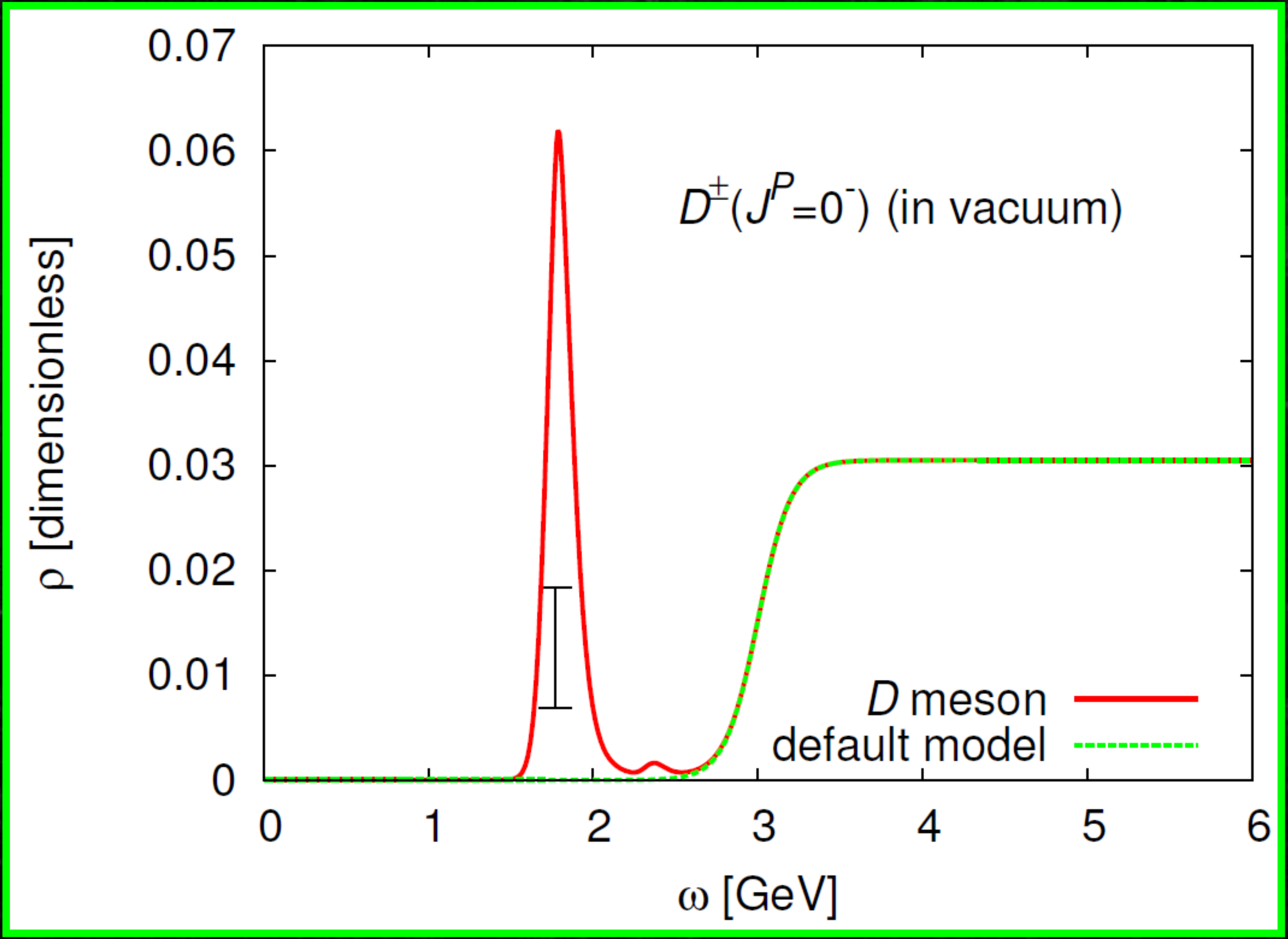
3. Results

3-1. D meson spectral function in vacuum

3-2. D meson spectral function in nuclear matter

3-3. Relation between condensates and hadron

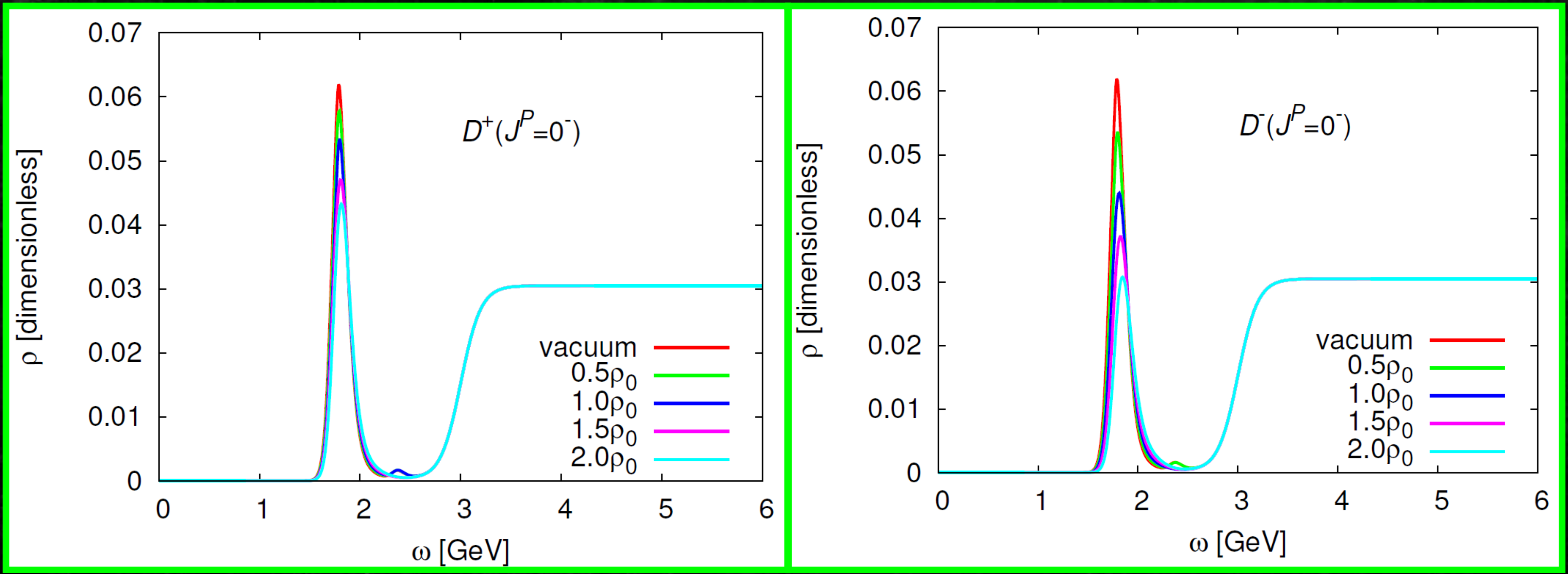
D meson spectral function (in vacuum)



Mass : 1.78GeV Exp. : 1.87GeV

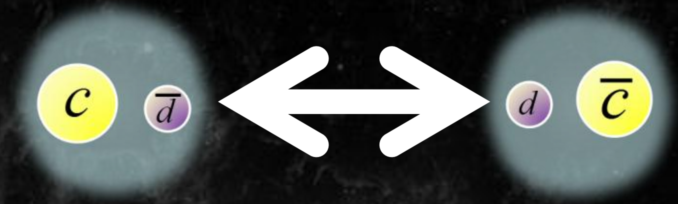
D meson spectral function (in medium)

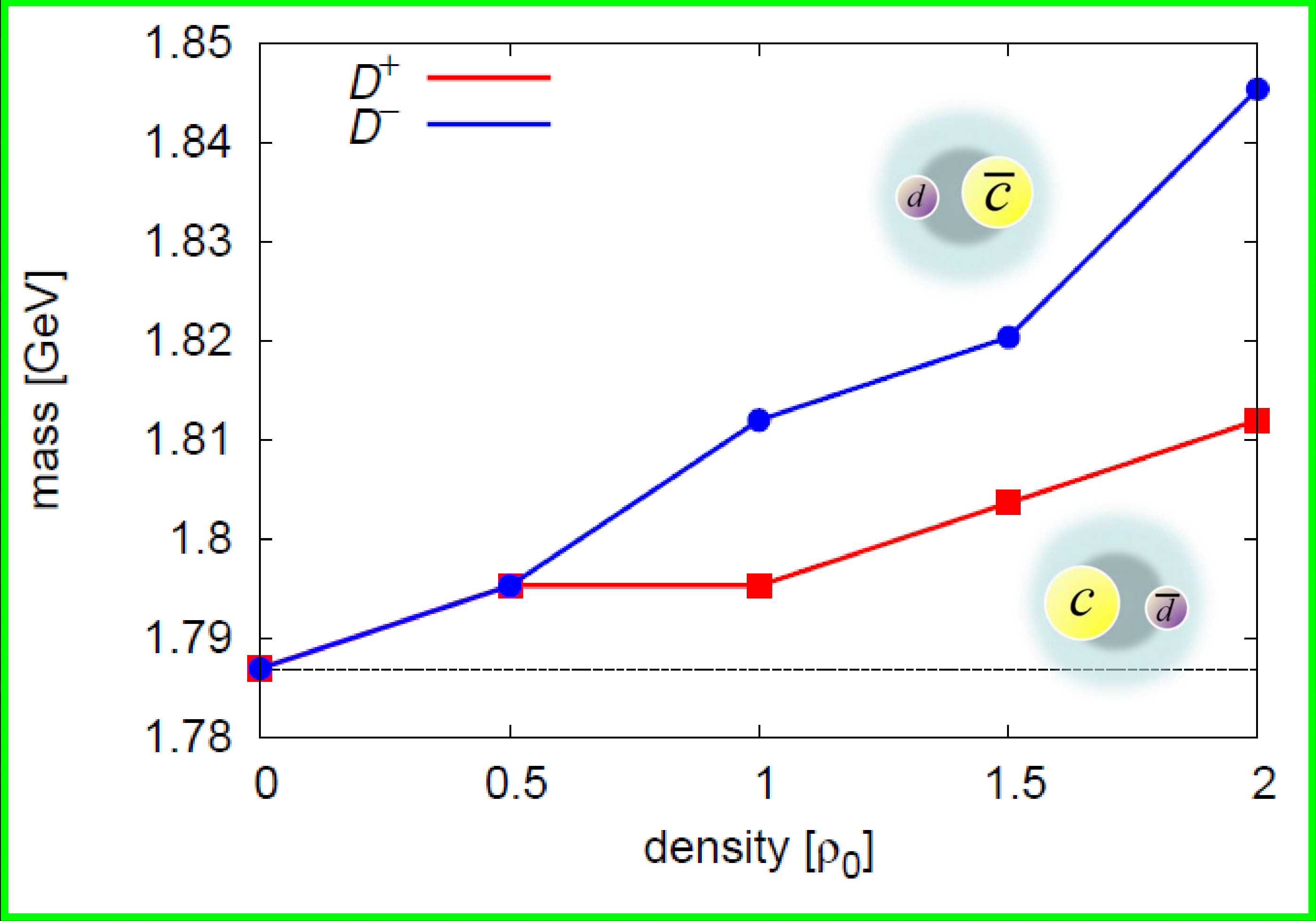
D+ $c\bar{d}$ D− $d\bar{c}$



⇒Peak position in $D^{\pm}$ shifts to higher energy side with increasing density (D^+: ~10MeV D^-: ~30MeV at ρ_0)

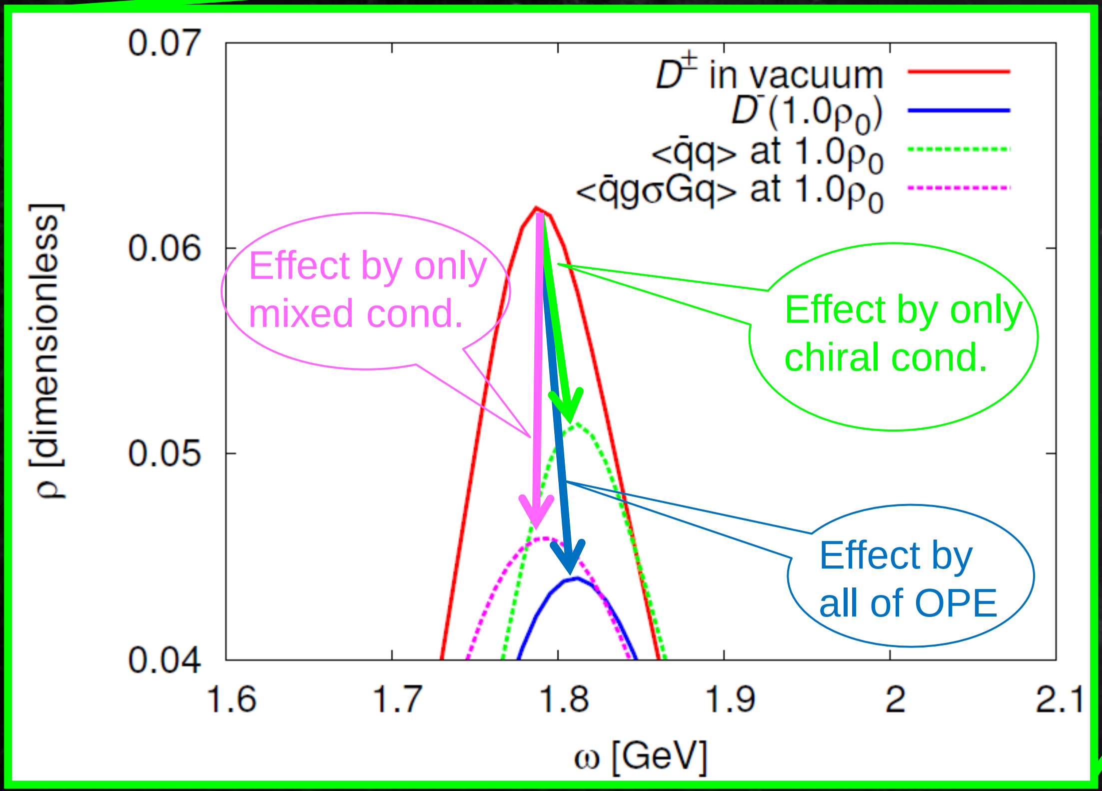
Comparison of D^+ and D^-

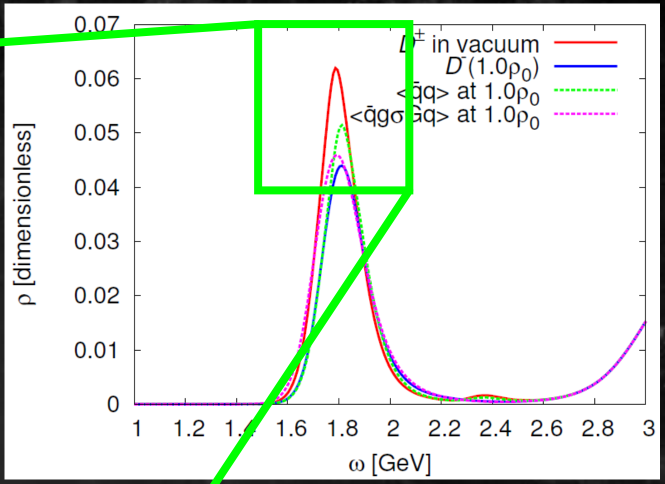




⇒ D^+-D^- mass splitting is about <u>20 MeV</u> at ρ_0

Contribution of vacuum condensates





⇒Most dominant contribution of mass shift to higher energy is D-dependence of chiral condensate

Summary

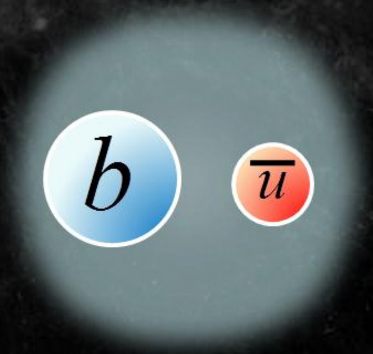
- We extracted D meson spectral functions in nuclear medium from QCD sum rules and MEM
- We obtained mass shift and D-$\bar{D}$ mass splitting

D^+ mass shift	D^- mass shift	D^+-D^- splitting
~+10MeV	~+30MeV	~20MeV

- Mass shift comes from density dependence of chiral condensate

Outlook

- B meson, D_s meson, D^* meson…
- Momentum dependence



Backup

What is difference between 2 previous works from QCD sum rules?

A. Hayashigaki, Phys.Lett. B487 (2000) 96

including condensates up to dim.4 $\langle \bar{q}q \rangle, \langle G^2 \rangle, \langle q^\dagger i D_0 q \rangle_n, \left\langle \frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2} - \frac{G^2}{4}\right)\right\rangle_n$

⇒mass shift : $(\Delta m_{D+} + \Delta m_{D-})/2 = -50$MeV at ρ_0

T. Hilger, R. Thomas, B. Kampfer, Phys. Rev. C79 (2009) 025202

including condensates up to dim.5 and q_0-odd terms

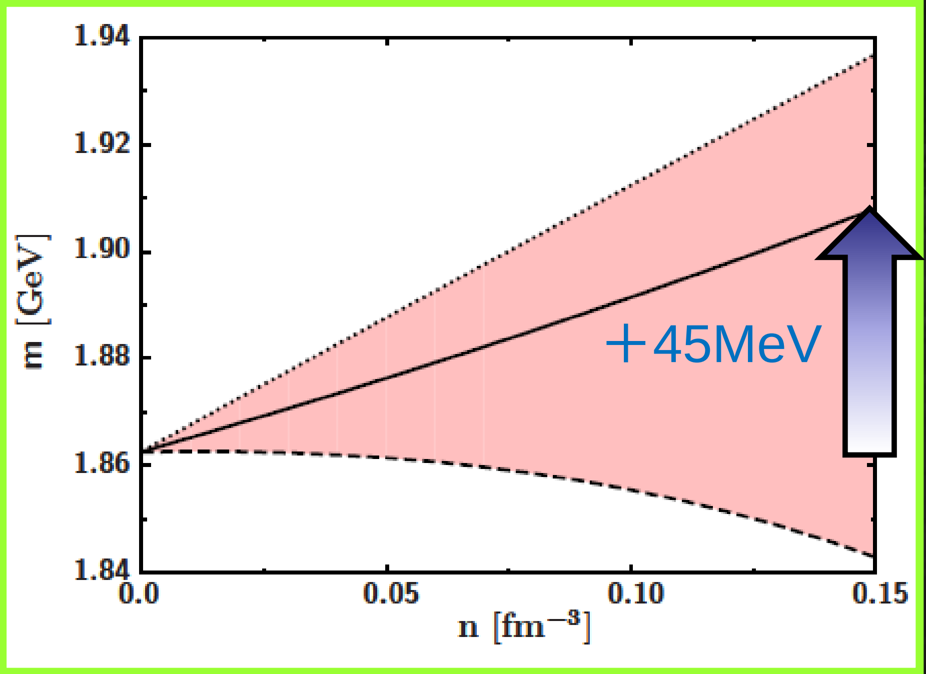
$\langle \bar{q} g \sigma G q \rangle \ldots$ $\langle d^\dagger d \rangle_n \ldots$

⇒mass shift : +45MeV and mass splitting : $(m_{D+} - m_{D-}) = -60$MeV at ρ_0

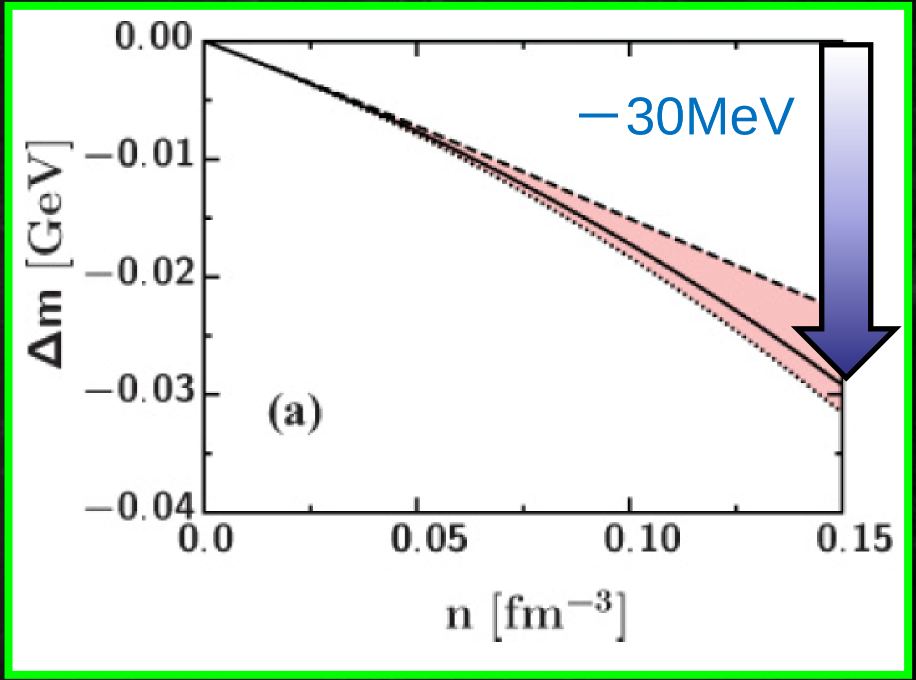
Previous work (from QCD sum rules)

T. Hilger, R. Thomas, B. Kampfer, Phys. Rev. C79 (2009) 025202

Mass shift $(\Delta m_{D+} + \Delta m_{D-})/2$



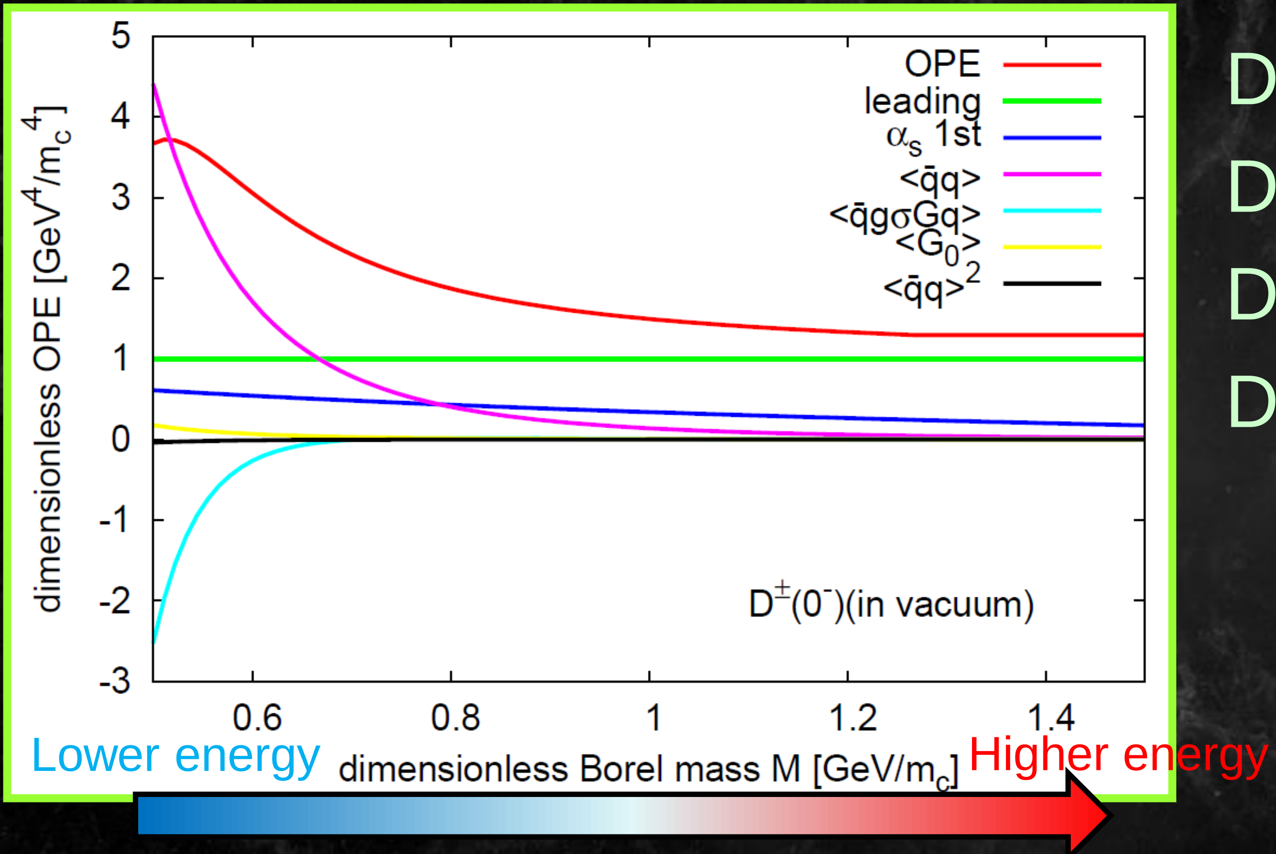
mass splitting $(m_{D+} - m_{D-})/2$



- These results depend on phenomenological parameter

⇒We need parameter independent analysis (=MEM)

How to determine criterion of Borel window ?



Dim3: enhanced

Dim4: suppressed

Dim5: enhanced

Dim6: suppressed

- Dim 6 (4-quark) condensate is strongly suppressed

⇒Should we regard the highest term as 4-quark con.?

Separation of D^+ and D^-

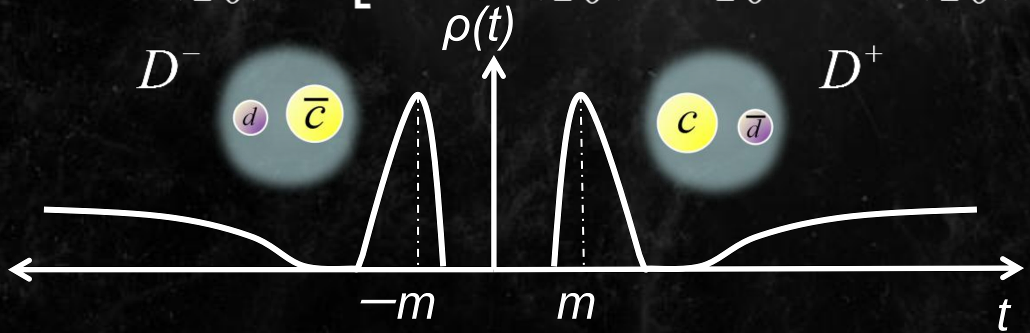
- In finite density, we have to construct q_0 sum rule instead of q_0^2

$$\Pi(q_0) = \Pi^{\text{even}}(q_0^2) + q_0 \Pi^{\text{odd}}(q_0^2)$$

- Moreover, we separate positive and negative energy on q_0 axis

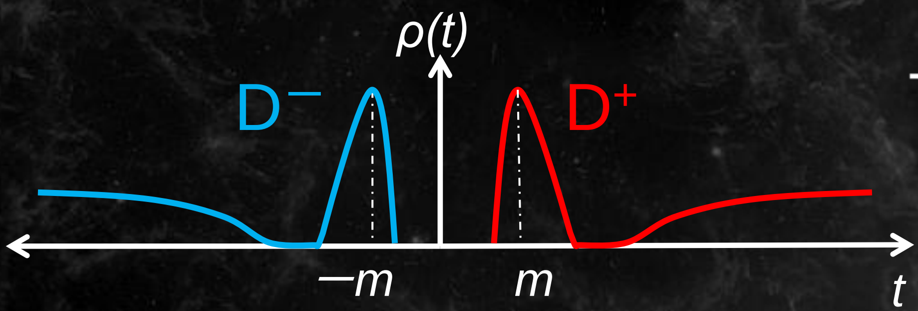
$$\Pi^+(q_0) = \left[\Pi^{\text{even}}(q_0^2) + q_0 \Pi^{\text{odd}}(q_0^2)\right]^{\text{old}}$$

$$\Pi^-(q_0) = \left[\Pi^{\text{even}}(q_0^2) - q_0 \Pi^{\text{odd}}(q_0^2)\right]^{\text{old}}$$

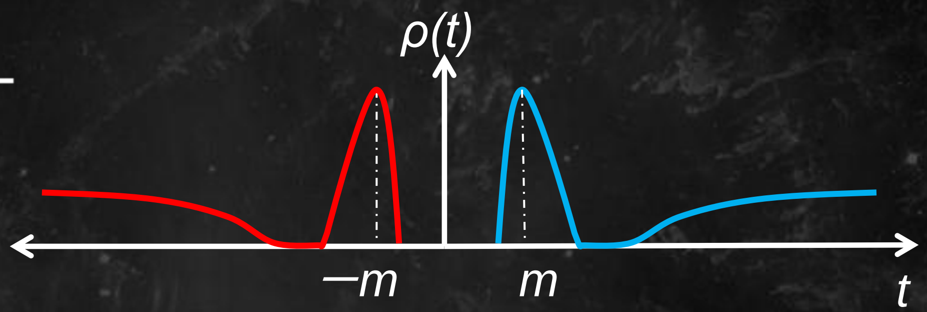


Separation of D^+ and D^-

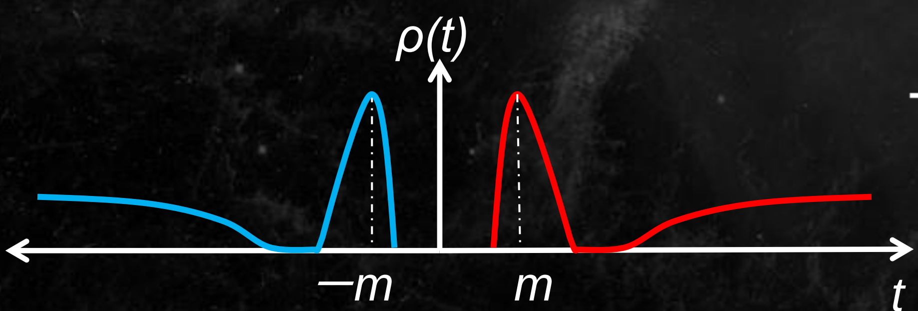
$$\Pi^{\text{even}}(q_0^2) =$$



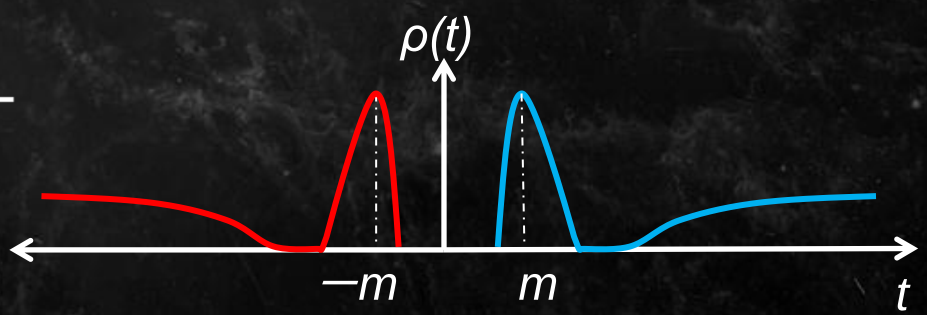
+



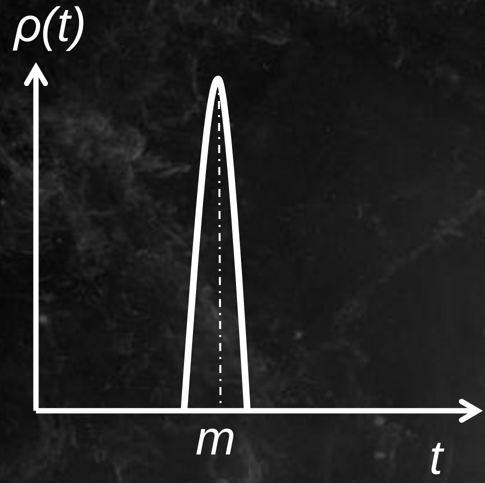
$$q_0 \Pi^{\text{odd}}(q_0^2) =$$



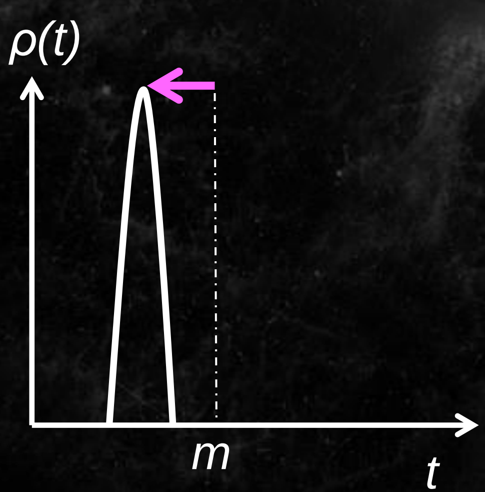
−



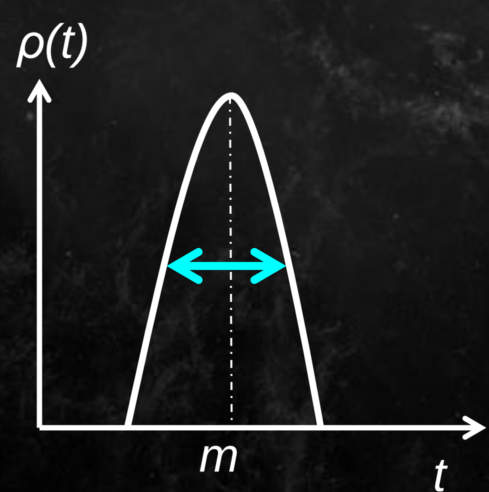
3 possibility of spectral modification



1. Mass shift



2. width broadening



3. residue reduction