Structure functions of hadrons at small x in holographic QCD

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KEK theory center workshop on J-PARC hadron physics in 2014
February 10-12, 2014 @ Tokai, Ibaraki, Japan

Based on A. Watanabe and K. Suzuki, PRD 86, 035011 (2012) and arXiv:1312.7114 [hep-ph].

Abstract

We study the nucleon structure functions at the small Bjorken-*x* in the framework of holographic QCD. Using the BPST kernel for the Pomeron exchange and calculating its coupling to the target hadron in the AdS space, we obtain F_2 and F_L structure functions. Results are consistent with experimental data of the deep inelastic scattering. We find that the resulting longitudinal-to-transverse ration of the structure functions, $R=F_L^p/F_T^p$, depends on both of Bjorken-*x* and the probe energy scale Q^2. Furthermore, we show that structure functions of other hadrons can also be considered in this framework.

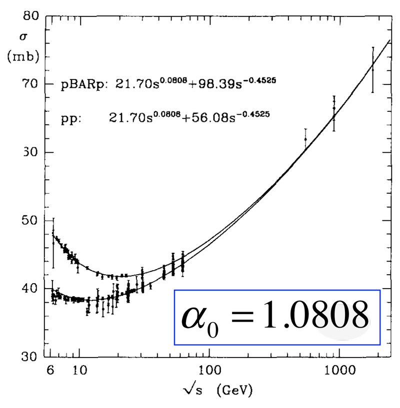
1. Transition from soft- to hard-Pomeron

High energy hadron scattering is dominated by the **Pomeron** exchange.

$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

multi gluons in QCD

s : energy square α_0 : Pomeron intercept

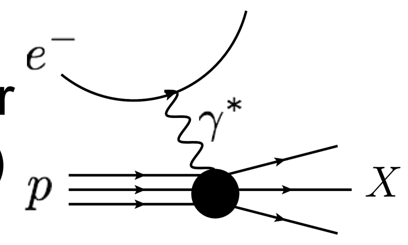


Soft Pomeron [1]

★ "Soft" elastic hadron-hadron scattering is described very well by the "soft" Pomeron intercept [1].

α_0 ~1.1 (soft process)

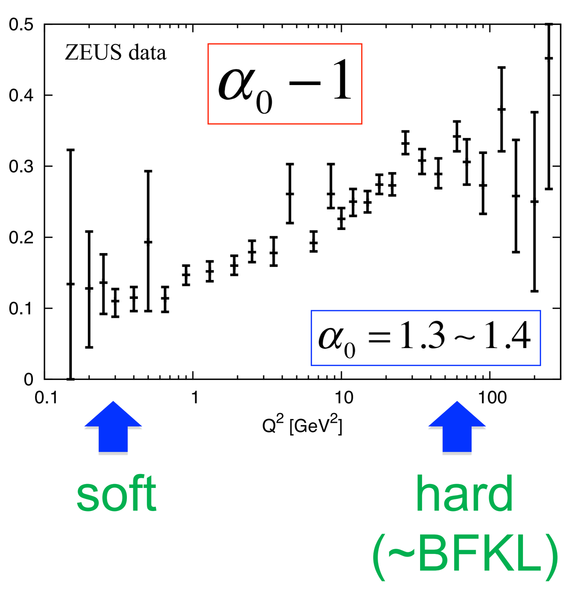
★ "Hard process" (deep inelastic scattering, diffractive vector meson production, etc. at small Bjorken-x, $x= Q^2/s < 0.01$) is reproduced with



α_0 ~1.4 (hard process)

e.g. DIS structure function

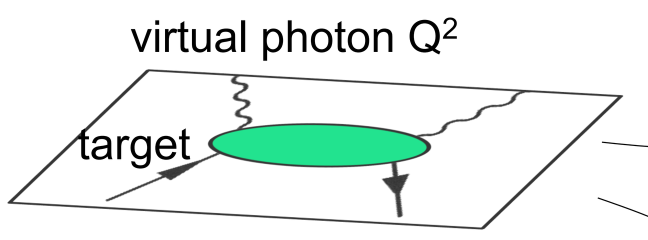
$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha}\sigma_{tot} \sim x^{1-\alpha_0}$$

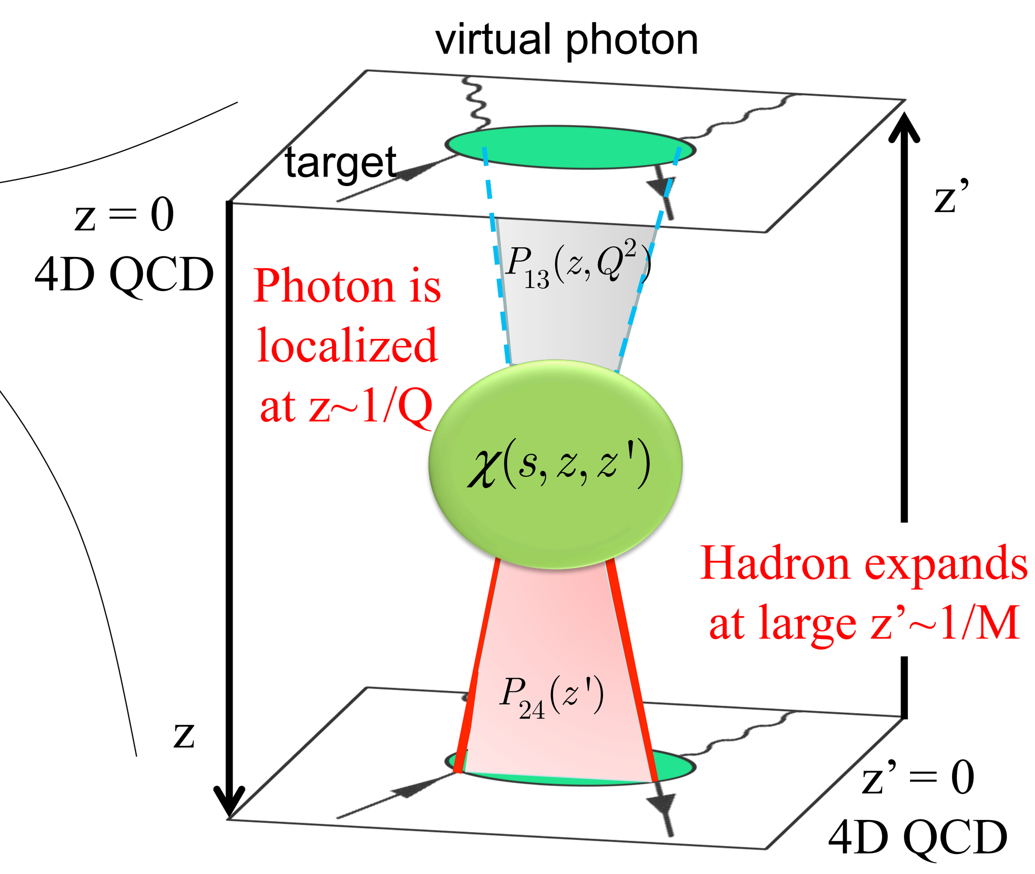


Aim of this work

- **Account for the observed scale dependence of Pomeron intercept based on holographic QCD**
- **Calculate the nucleon structure functions using realistic wave functions in AdS space**

2. Deep inelastic scattering at small-*x* in AdS space





4D DIS can be described by the scattering of photon (z) & hadron (z') with the BPST Pomeron exchange (z, z') in the AdS space

★Pomeron with gauge/string correspondence (~ graviton in large N limit)
Brower-Polchinski-Strassler-Tan (BPST) kernel [2,3]

(Conformal) $$\mathrm{Im}[\chi_c(s,z,z')] \equiv e^{(1-\rho)\tau}\exp\left(\frac{-(\log z - \log z')^2}{\rho\tau}\right)/\tau^{1/2} \quad \tau = \log(\rho z z' s/2), \quad \xi = \sinh^{-1}\left(\frac{b^2+(z-z')^2}{2zz'}\right)$$

To mimic QCD and reproduce the confinement effect, we need a scale to break the conformal symmetry. It can be done by introducing the modified kernel.

(Modified) $$\mathrm{Im}[\chi_{mod}(s,z,z')] \equiv \mathrm{Im}[\chi_c(s,z,z')] + \mathcal{F}(z,z',\tau)\mathrm{Im}[\chi_c(s,z,z_0^2/z')]$$

$$\mathcal{F}(z,z',\tau) = 1 - 2\sqrt{\rho\pi\tau}\,e^{\eta^2}\mathrm{erfc}(\eta)$$

$$\eta = \left(-\log\frac{zz'}{z_0^2} + \rho\tau\right)/\sqrt{\rho\tau}$$

★Structure functions

$$F_i(x,Q^2) = \frac{g_0^2\rho^{3/2}Q^2}{32\pi^{5/2}}\int dz\,dz' P_{13}^{(i)}(z,Q^2)P_{24}(z')(zz')\mathrm{Im}[\chi(s,z,z')] \quad (\text{i = 2 or L})$$

where $P_{13}(z,Q^2)$ and $P_{24}(z')$ are overlap functions of the photon and the hadron
$P_{13}(z,Q^2)$: massless 5D U(1) vector field [4], dual to the electromagnetic current
$P_{24}(z')$: target hadron
g_0^2 and ρ are adjustable parameters of the model

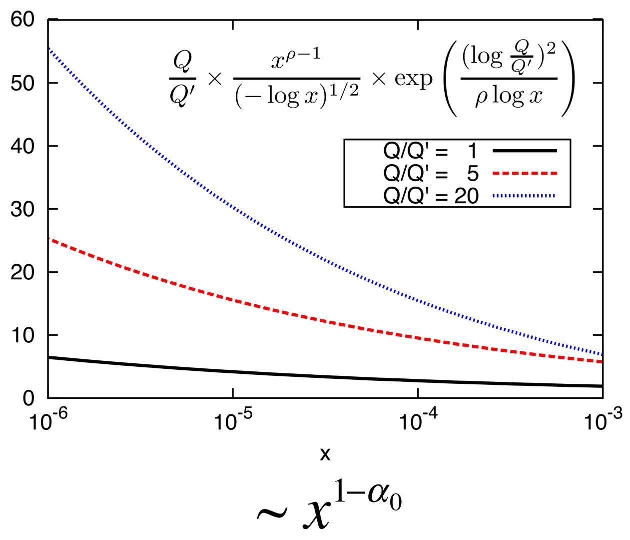
★Toy model : ***"super local" (delta function) approximation [3]***

$$P_{13}(z,Q^2) \approx \delta(z-1/Q)$$
$$P_{24}(z') \approx \delta(z'-1/Q')$$

both distributions are localized in z, z' space

$$\Rightarrow F_2(x,Q^2) \propto \frac{Q}{Q'} \times \frac{x^{\rho-1}}{(-\log x)^{1/2}} \times \exp\left(\frac{\log^2\frac{Q}{Q'}}{\rho\log x}\right)$$

this log(Q/Q') factor clearly enhances F_2



Toy model shows

small Q/Q' → α_0 small (~1.1) : soft Pomeron
large Q/Q' → α_0 large (~1.4) : hard Pomeron

Since Q,Q' correspond to 'localization' in AdS space, behavior of realistic WFs in AdS space is a key to understand the Pomeron properties.

For example, if we want to acquire the large Pomeron intercept, we need a "gap" between the peak positions of $P_{13}(z)$ and $P_{24}(z')$.

3. Hadronic wave functions and Pomeron couplings

Calculate hadronic wave functions in the AdS space with the metric,

$$ds^2 = g_{MN}dx^M dx^N = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$ with $\varepsilon \le z < \infty$ ($\varepsilon \to 0$) [soft-wall model]

★ 5D classical action describing the nucleon [5]

$$S_F = \int d^5x\sqrt{g}e^{-\kappa^2 z^2}\left(\frac{i}{2}\bar{\Psi}e_A^N\Gamma^A D_N\Psi - \frac{i}{2}(D_N\Psi)^\dagger\Gamma^0 e_A^N\Gamma^A\Psi - (M+\kappa^2 z^2)\bar{\Psi}\Psi\right)$$

The nucleon is described as a normalizable mode of the 5D Dirac equation.

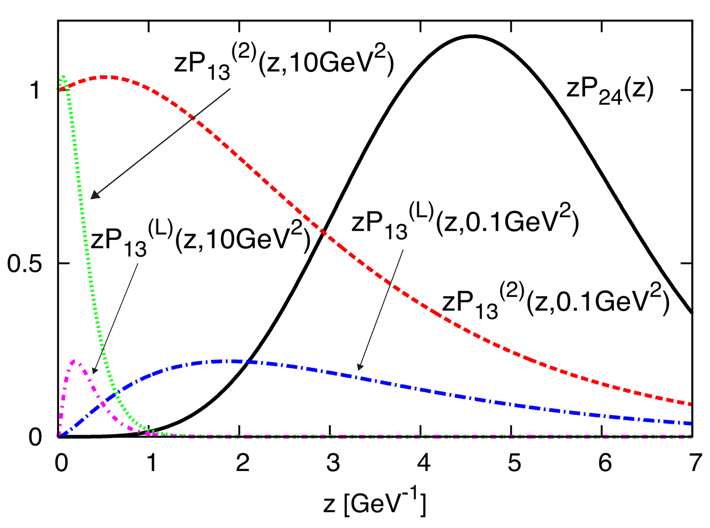
$$\Psi_{R,L}(p,z) = z^\Delta \Psi_{R,L}^0(p) f_{R,L}(p,z)$$

By taking the limit $\kappa \to 0$, one can consider the hard-wall version, where the AdS geometry is sharply cut off at z_0.

★ Hadron coupling with graviton

We need the hadron-graviton-hadron three point function for our purpose.

➡ *Introduce the perturbation to the metric tensor $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$*
&
extract hΨΨ terms



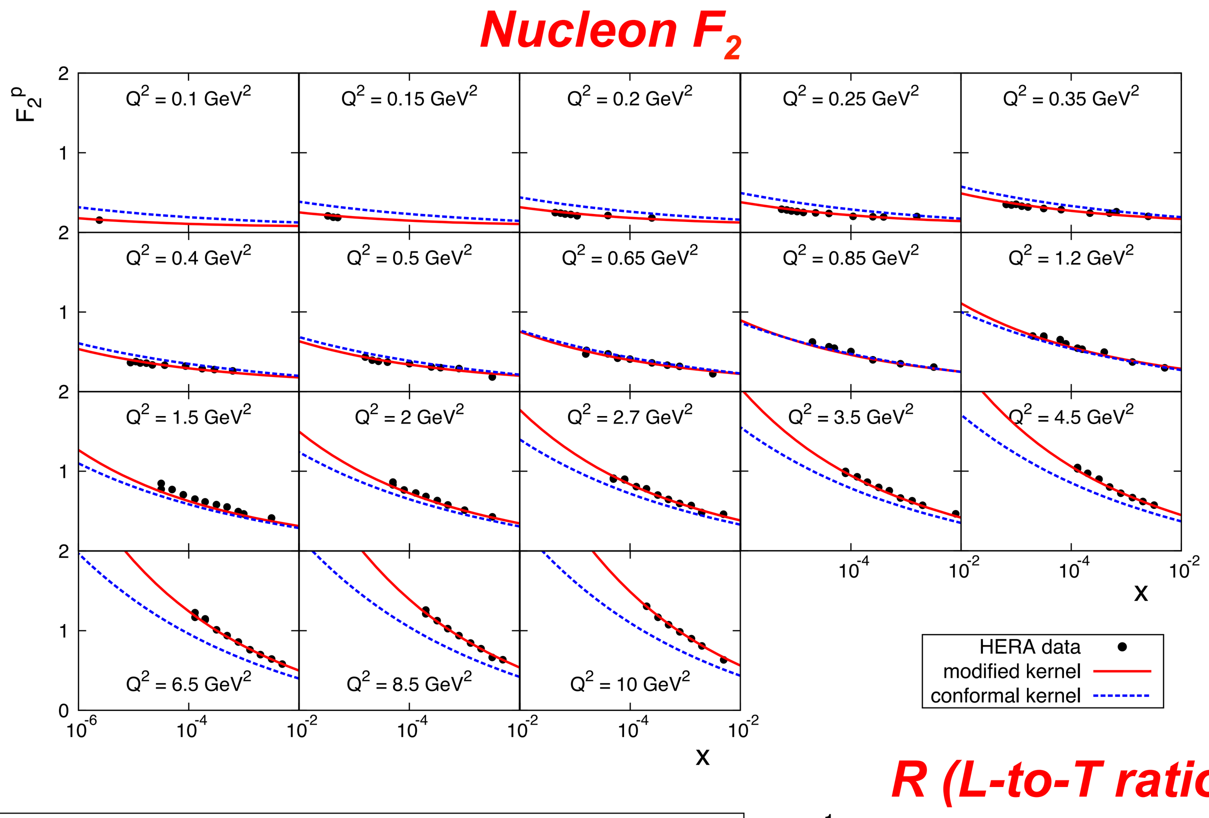
Note:
- Satisfy required normalization condition
- Nucleon overlap function localized at large z
- Photon shows flat distribution at Q~0, while a sharp peak around z=0 for larger Q^2~10GeV²

We expect large intercept α_0 for larger Q^2!

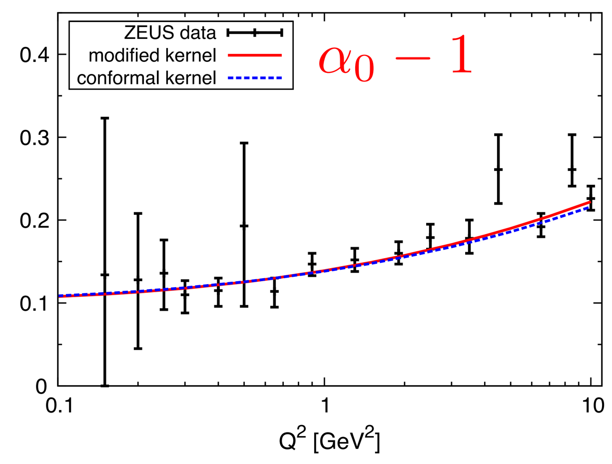
4. Numerical results

Model parameters, g_0^2, ρ, and z_0, are fixed to reproduce the F_2^p data.

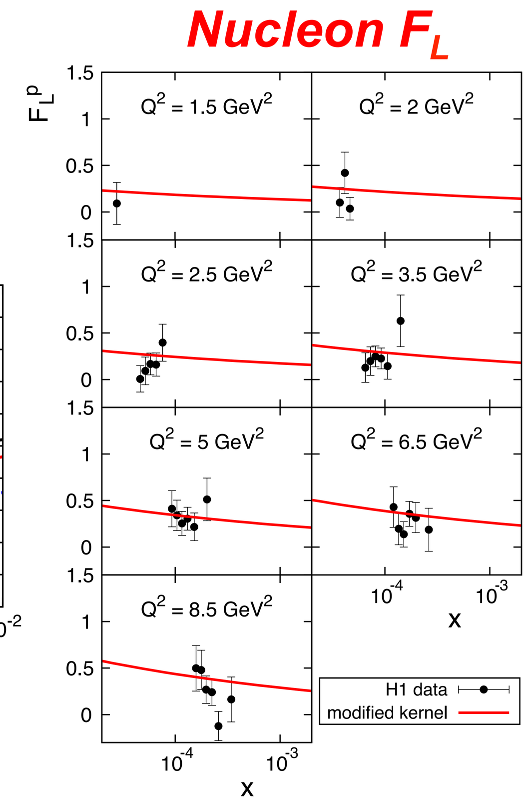
Nucleon F_2



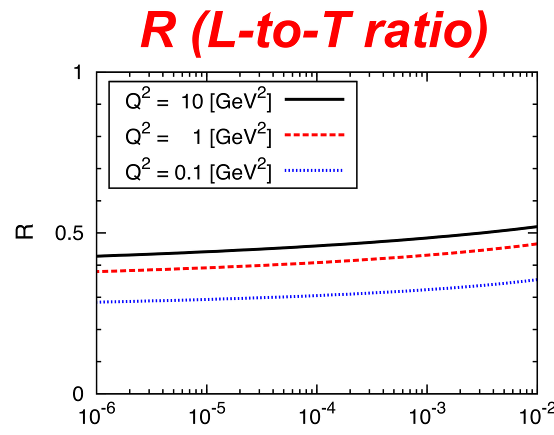
Pomeron intercept



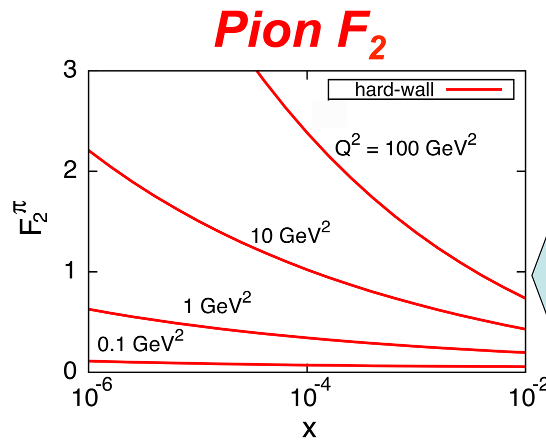
Nucleon F_L



R (L-to-T ratio)



Pion F_2



- Magnitudes of nucleon F_2 are fairly agree with data (with three parameters).
- Q^2 dependence of the Pomeron intercept is well reproduced.
- Results for longitudinal structure function F_L are also consistent with the experiment.
- Resulting longitudinal-to-transverse ratio depends on both of *x* and Q^2.
- Pion case is suppressed by about 30% compared with the nucleon case.

Giving the Pomeron-pion coupling [6] calculated via a holographic model of mesons [7], one can obtain F_2^π in a similar way.

5. Summary

- We have calculated the nucleon and pion structure functions at the small *x* in terms of holographic QCD, which are in good agreement with the data.
- By giving the appropriate Pomeron couplings, one can calculate structure functions of other hadrons at the small *x* in this framework.
- Applications to other high energy scattering processes may be possible.

6. References

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