Baryon Interactions from Lattice QCD

Takumi Doi

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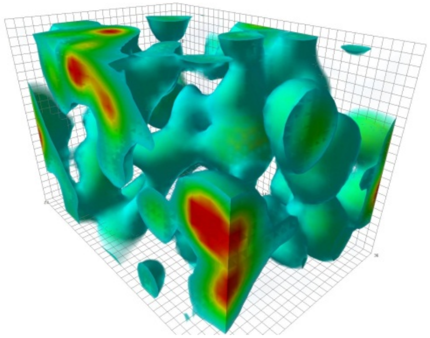
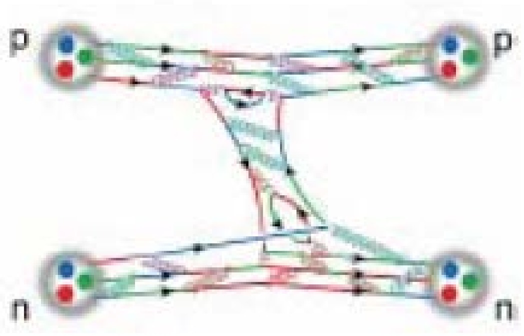
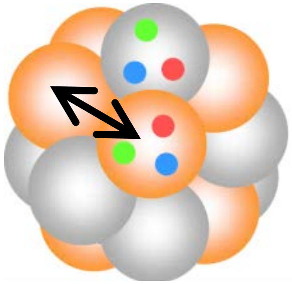


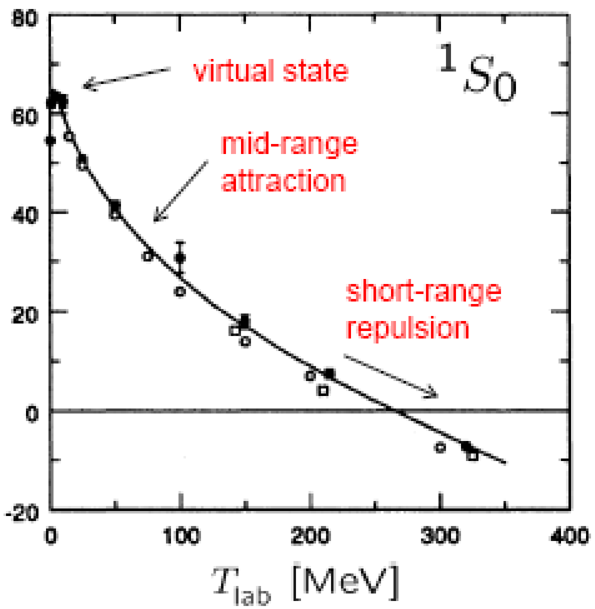




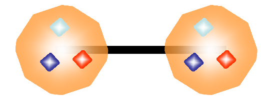
- **Baryon-baryon Interactions**
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics
  - **Frontier:** 1st principles calc by Lattice simulations

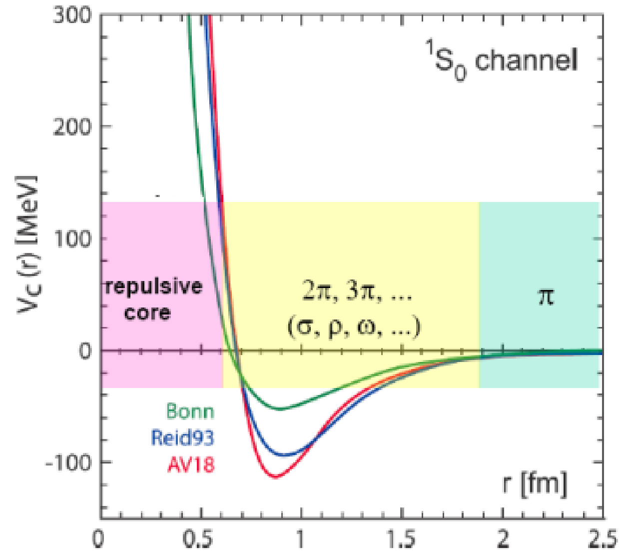
- Outline
  - Introduction
  - Theoretical framework: Interactions on the lattice
  - Challenges in multi-baryons & solutions
  - Lattice QCD results at heavier quark masses
  - Prospects toward physical quark mass point
    - ➔ **Nuclear Physics on the Lattice**
  - Summary

(1) Build a foundation for nuclear physics

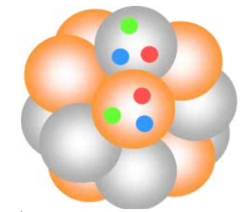


NN phase shifts
from experiments

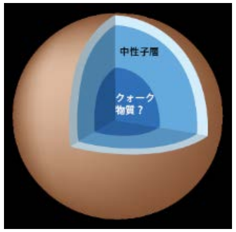




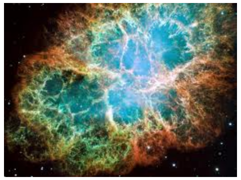
Phenomenological
Nuclear Forces



Nuclei



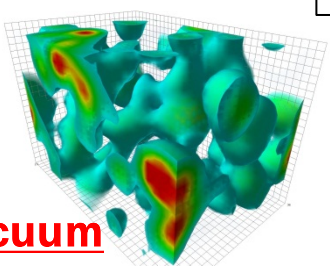
Neutron Stars



Super Novae

Various applications

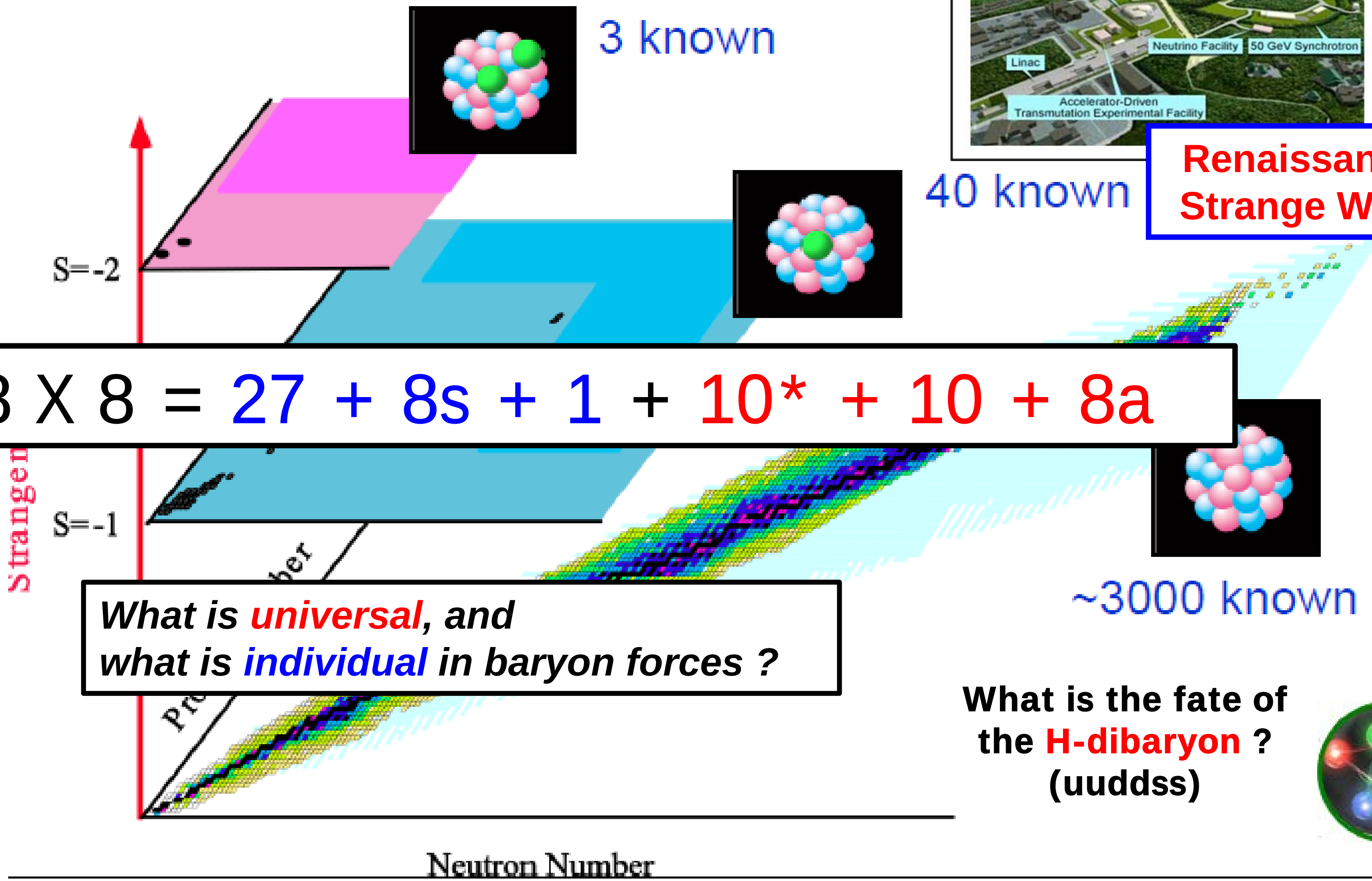
- ***Nuclear Forces*** ***play crucial roles***
  - *Yet, no clear connection to QCD so far*

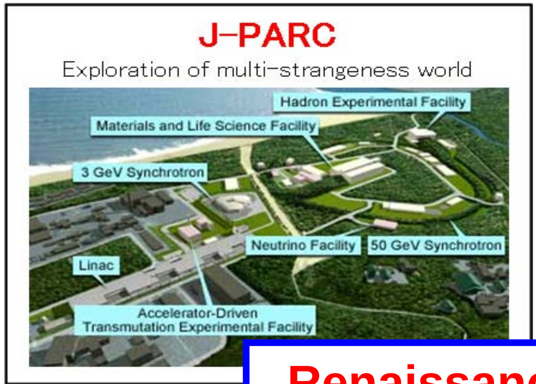


QCD Vacuum

(2) Predict Unknown Interactions (YN, YY, NNN)

3D Nuclear Chart



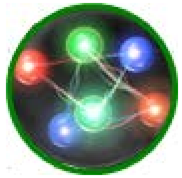


Renaissance in Strange World !

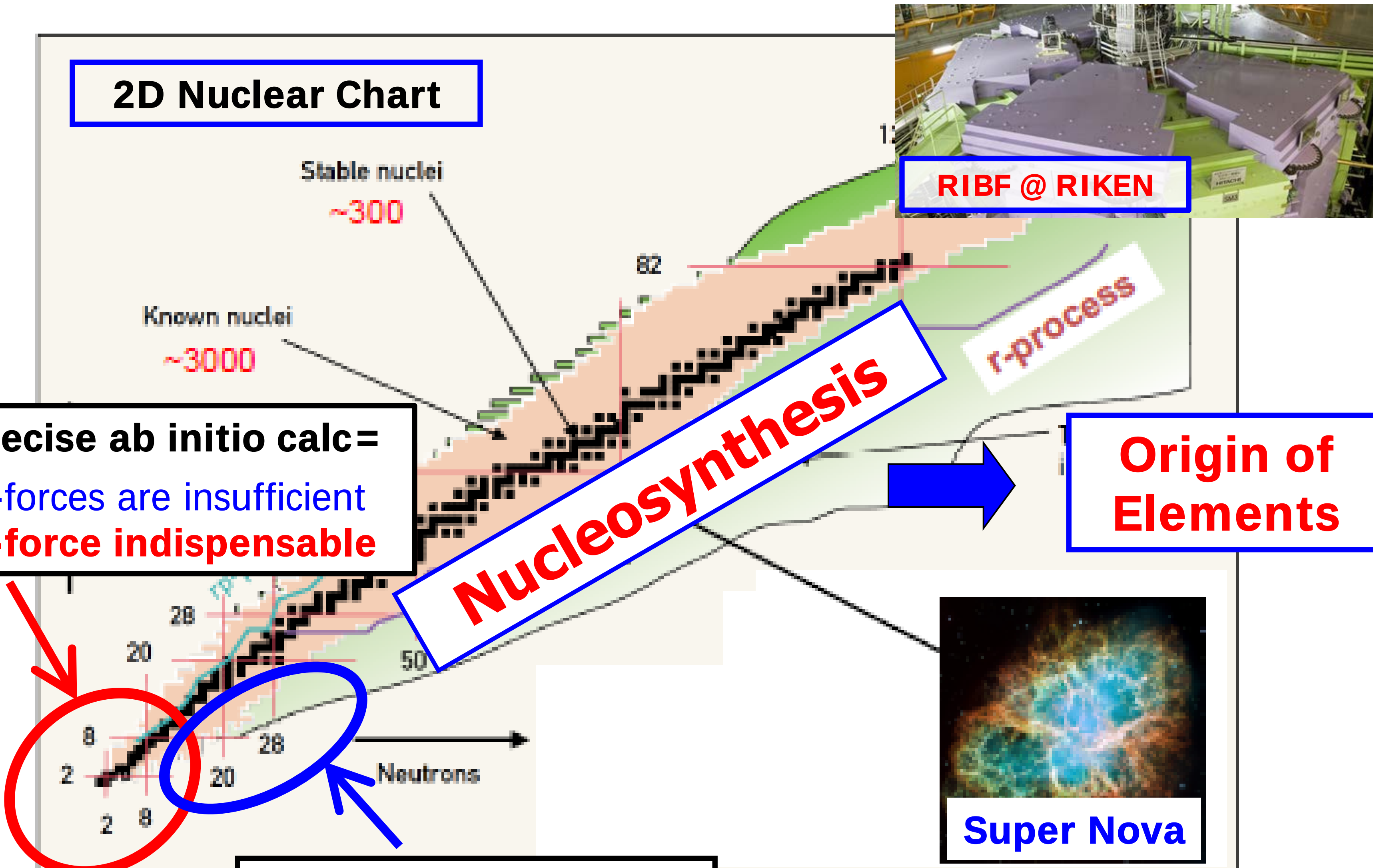
8 X 8 = 27 + 8s + 1 + 10* + 10 + 8a

What is universal, and what is individual in baryon forces ?

What is the fate of the H-dibaryon ? (uuddss)



(2) Predict Unknown Interactions (YN, YY, NNN)



=Precise ab initio calc=

2N-forces are insufficient

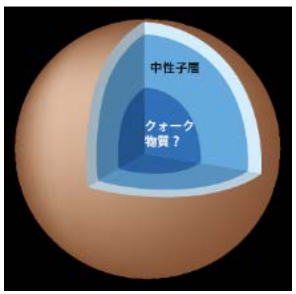
3N-force indispensable

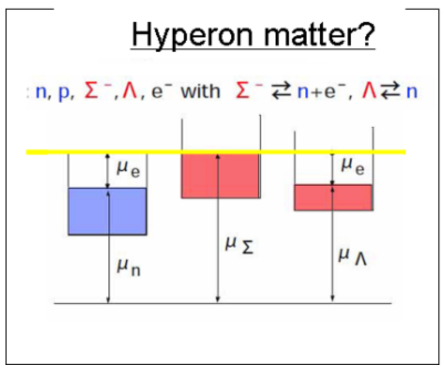
Paradigm Shift in **Unstable Nuclei** (New Magic Numbers !)

← **Important role of 3N-force**

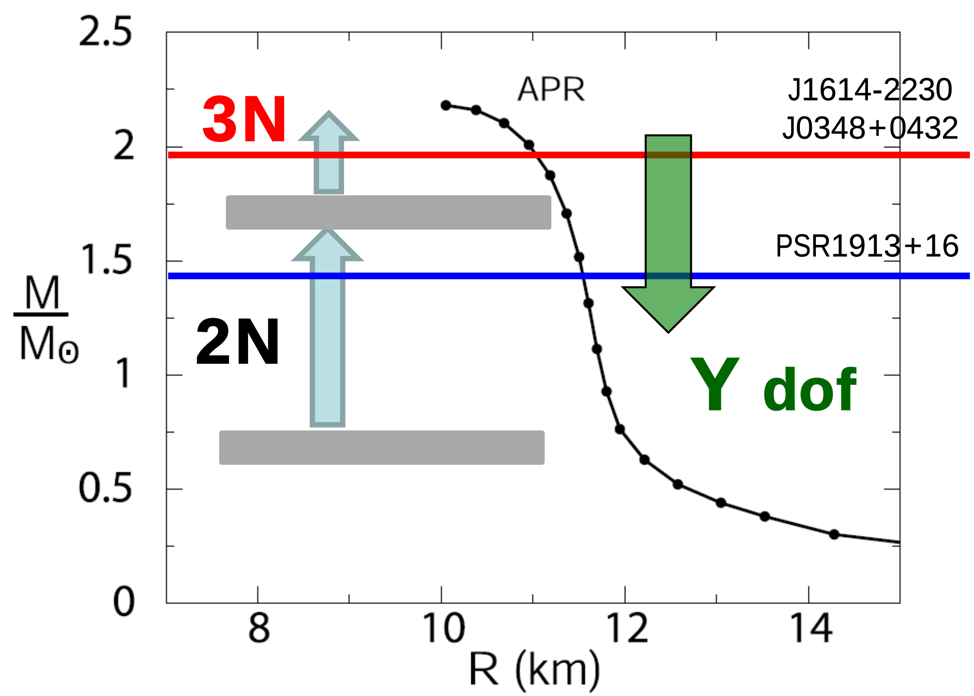
Dense Matter ← Interactions of YN, YY, NNN,… are crucial

- Neutron Stars, Super Novae ←→ EoS





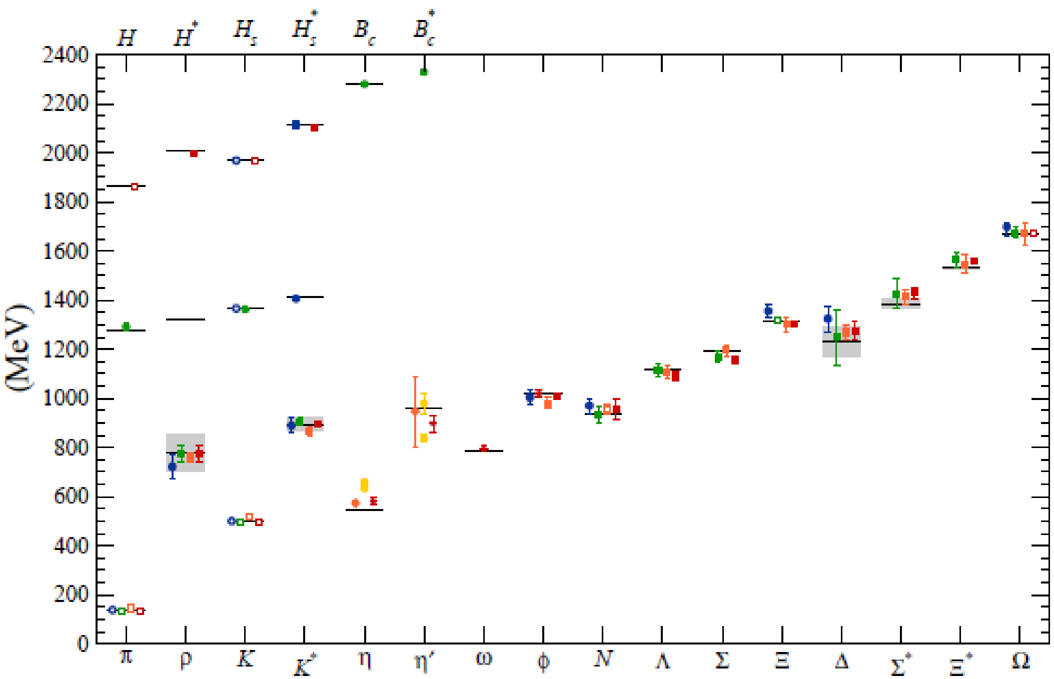
How to sustain a neutron star against gravitational collapse ?

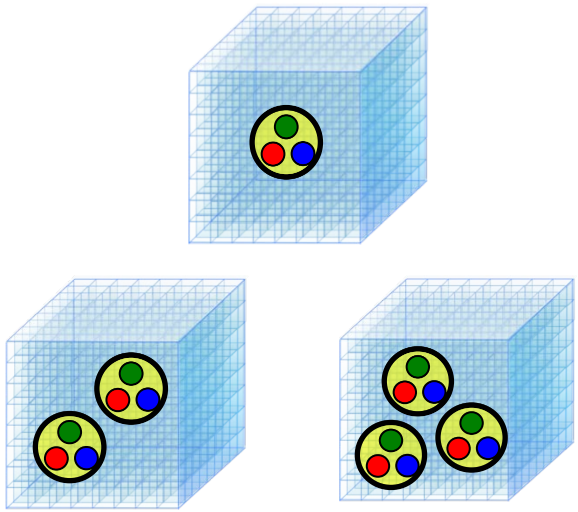


Akmal et al.('98), Nishizaki et al.('02), Takatsuka et al.('08)

Status of Lattice QCD

Hadron spectrum well reproduced !





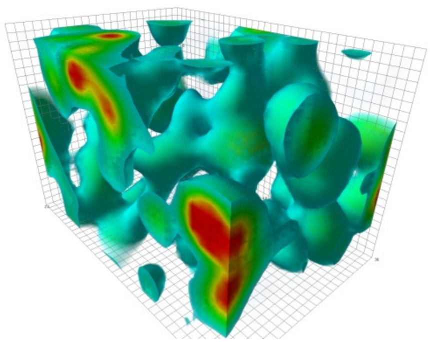
Inputs:

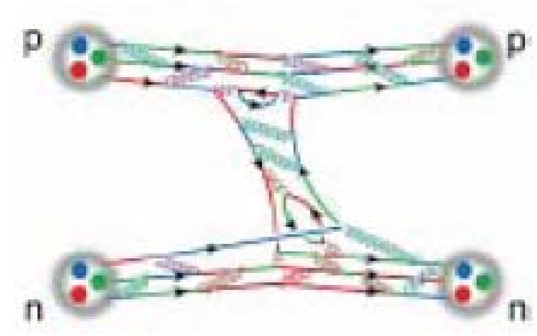
- **quark masses (m_u, m_d, m_s)**
- **coupling constants α_s ($+\alpha_{em}$)**

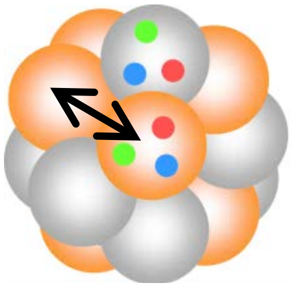
Summary by Kronfeld, arXiv:1203.1204

Fully dynamical (unquenched) QCD simulations
at the physical quark mass point already performed

PACS-CS Coll., PRD81(2010)074503
BMW Coll., JHEP1108(2011)148







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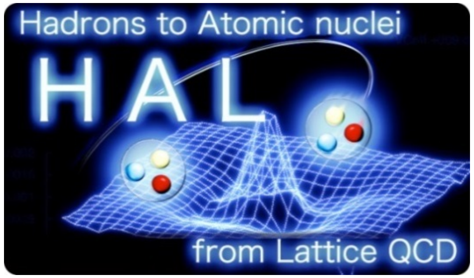
Interactions on the Lattice

- Luscher's method
  - Phase shift & B.E. from temporal correlation in finite V

M.Luscher, CMP104(1986)177
CMP105(1986)153
NPB354(1991)531

- HAL QCD method
  - Potential from spacial (& temporal) correlation
  - Phase shift & B.E by solving Schrodinger eq in infinite V

Ishii-Aoki-Hatsuda, PRL99(2007)022001, PTP123(2010)89
HAL QCD Coll., PTEP2012(2012)01A105



S. Aoki, K. Murano (YITP)
N. Ishii, H. Nemura, K. Sasaki, M. Yamada (Univ. of Tsukuba)
B. Charron (Univ. of Tokyo)
T. Doi, T. Hatsuda , Y. Ikeda (RIKEN)
T. Inoue (Nihon Univ.)
F. Etminan (Univ. of Birjand)

Nuclear Forces from Lattice QCD [HAL QCD method]

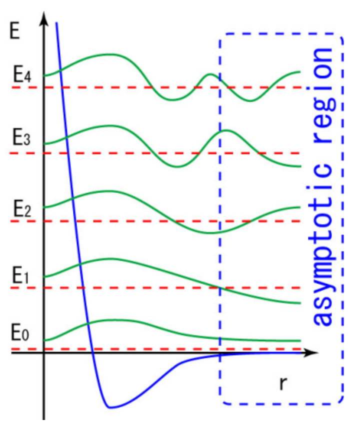
- Potential is constructed so as to reproduce the NN phase shifts (or, S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})|2N\rangle$$

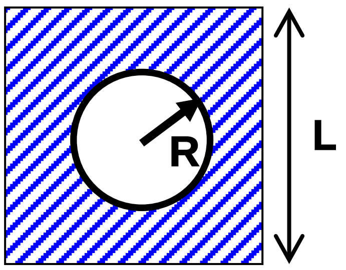
$$E = 2\sqrt{m^2+k^2}$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$



- Wave function ←→ phase shifts

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$



M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504

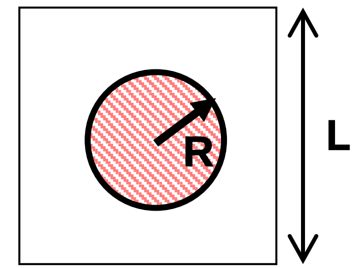
Ishizuka, Pos LAT2009 (2009) 119
S.Aoki et al., PRD88(2013)014036

Extended to multi-particle systems

“Potential” as a representation of S-matrix

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  – U(r,r’): faithful to the phase shift by construction
    - U(r,r’): <u>NOT</u> an observable, but well defined
    - U(r,r’): E-independent, while non-local in general
      – “Proof of Existence”: Explicit form can be given as

$$U(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{m}\sum_{n,n'}^{n_{\rm th}}(\nabla^2_{\boldsymbol{r}} + k_n^2)\psi_n(\boldsymbol{r})\mathcal{N}^{-1}_{nn'}\psi^*_{n'}(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r}\,\psi^*_n(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

      – Non-locality ➔ derivative expansion

Okubo-Marshak(1958)

$$U(\vec{r},\vec{r'}) = \underbrace{V_c(r)}_{\text{LO}} + \underbrace{S_{12}V_T(r)}_{\text{LO}} + \underbrace{\vec{L}\cdot\vec{S}V_{LS}(r)}_{\text{NLO}} + \underbrace{\mathcal{O}(\nabla^2)}_{\text{NNLO}}$$

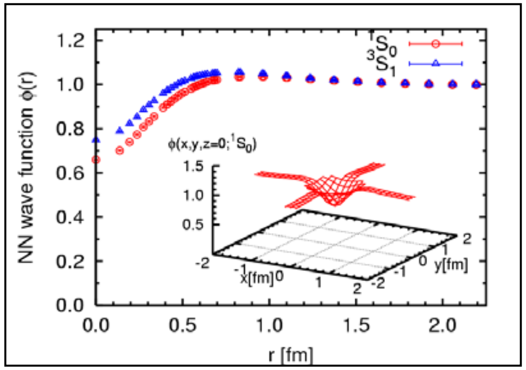
Aoki-Hatsuda-Ishii PTP123(2010)89

Check on convergence: K.Murano et al., PTP125(2011)1225

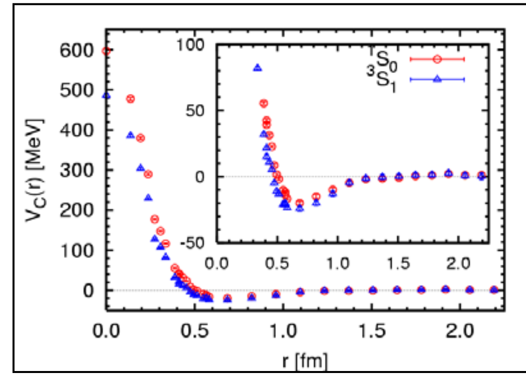
Prescription in HAL QCD method

Lattice QCD

NBS wave func.



Lat Nuclear Force



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq e^{i\delta_l(k)} \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

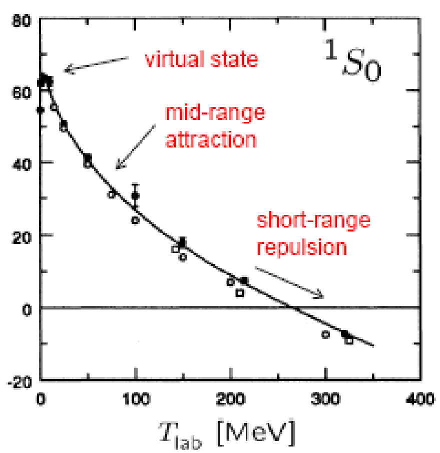
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,')$$

Lat potential is faithful to phase shift by construction

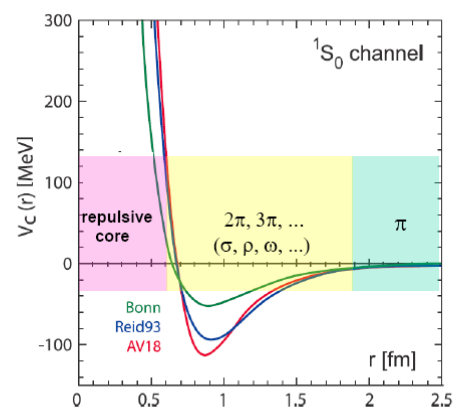
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential

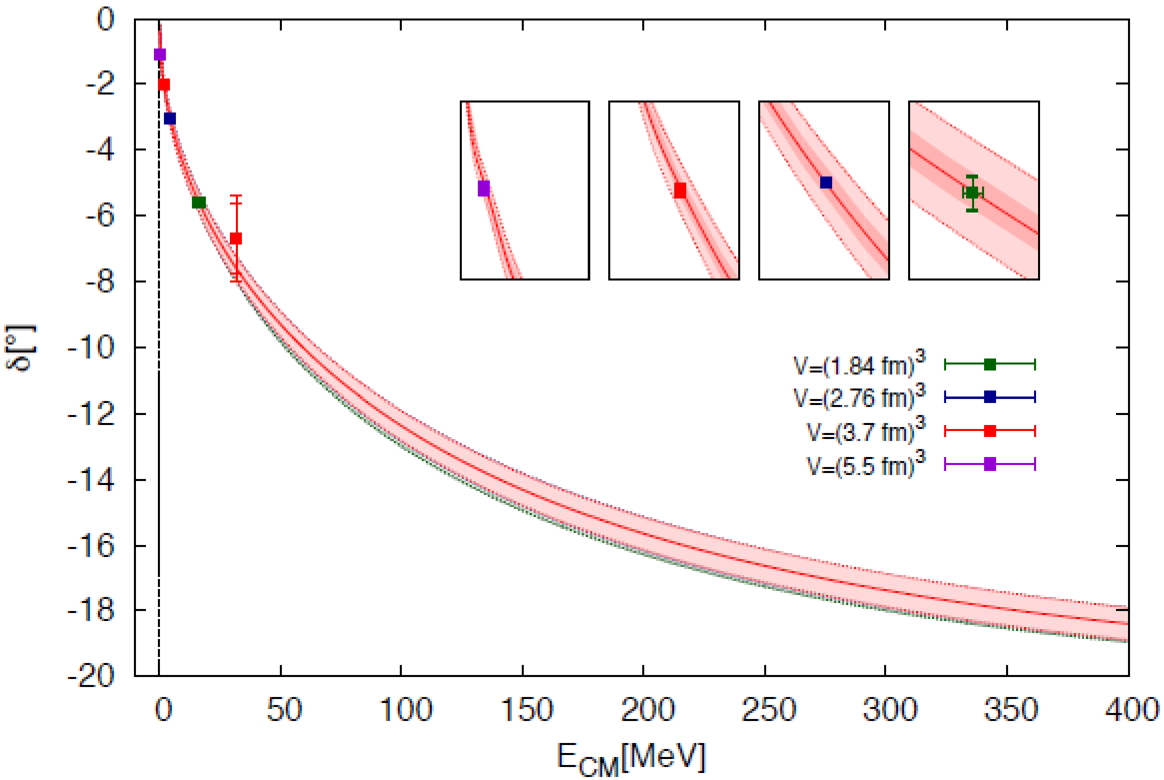


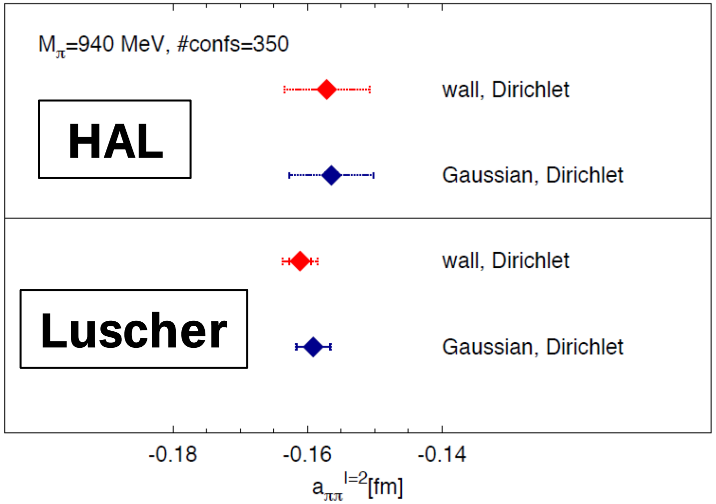
Luscher's method vs. HAL method

Beautiful Agreement !

I=2 $\pi\pi$ system

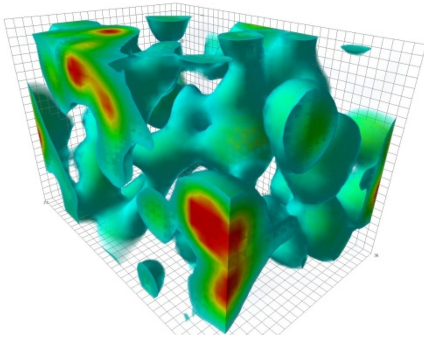
Best S/N on the lattice
G.S. saturation can be achieved in this case

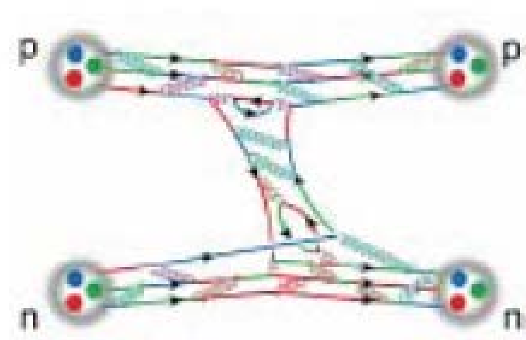


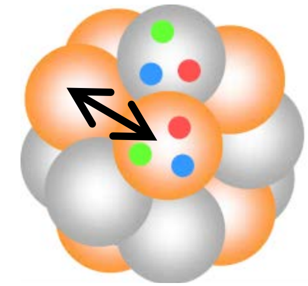


(HAL = "time-dependent" HAL method)

Kurth et al., JHEP1312(2013)015







- **<u>Outline</u>**
  - Introduction
  - Theoretical framework: Interactions on the lattice
  - Challenges in multi-baryons & solutions
    - (1) S/N issue (2) computational cost
  - Lattice QCD results at heavier quark masses
  - Prospects toward physical quark mass point
  - Summary

Challenges in multi-baryons on the lattice (1)

- Signal / Noise estimate

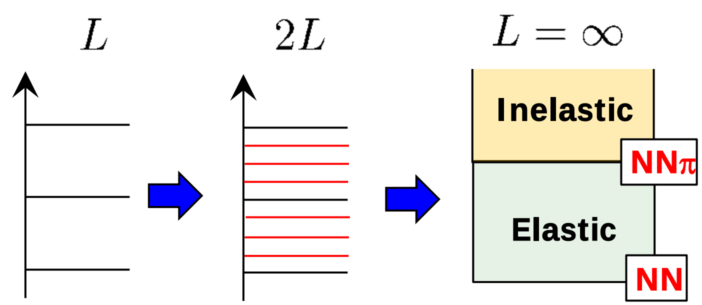
  Lepage(1989)

  - Traditional Lat calc
    ➔ Ground State (G.S.) saturation is necessary
  - S/N gets worse
    for larger mass number A & light quark mass & $t \to \infty$

    $$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2 m_\pi}) \times \mathbf{t}]$$

  - Larger spectral density
    ➔ larger t required

$$\Delta E \simeq \frac{\vec{p}^2}{m_N} \simeq 15\mathrm{MeV} \quad \text{for } L = 10\mathrm{fm}$$



G.S. saturation becomes more and more difficult for larger V & lighter mass

$S/N \sim 10^{-42}$!?

Solution (only) in HAL method
Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

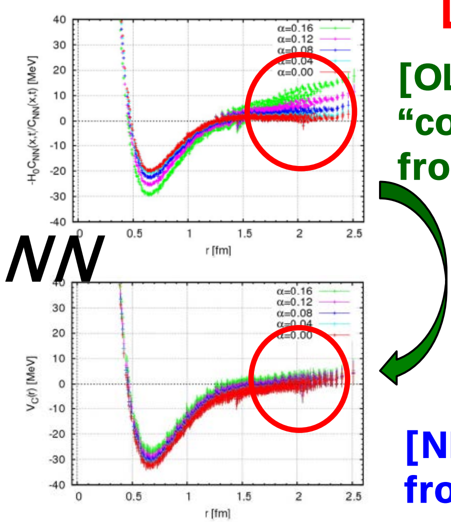
➔ **Time-dependent Schrodinger Eq.**

$$2\sqrt{m^2 + k_n^2} = E_n = -\frac{\partial}{\partial_t}$$

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0 \right) R(\boldsymbol{r}, t) = \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R(\boldsymbol{r}', t)$$

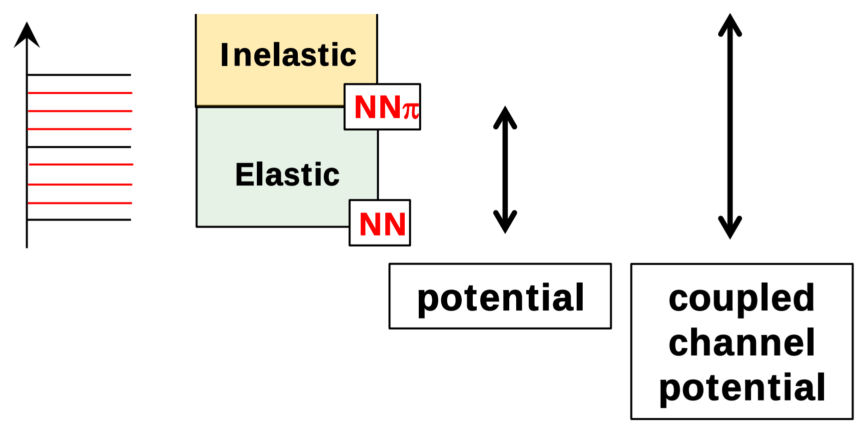
$$R(\boldsymbol{r}, t) \equiv C_{NN}(\boldsymbol{r}, t)/C_N(t)^2$$

$$\boxed{\textit{Ground State (G.S.) saturation is NOT necessary !}}$$



[OLD] "contaminations" from excited states

[NEW] "signals" from excited states



Challenges in multi-baryons on the lattice (2)

- Enormous computational cost for correlators
  - # of Wick contraction (permutation)

    $N_{\rm perm} = N_u! \times N_d! \sim [\,(\frac{3}{2}A)!\,]^2$ for mass number A

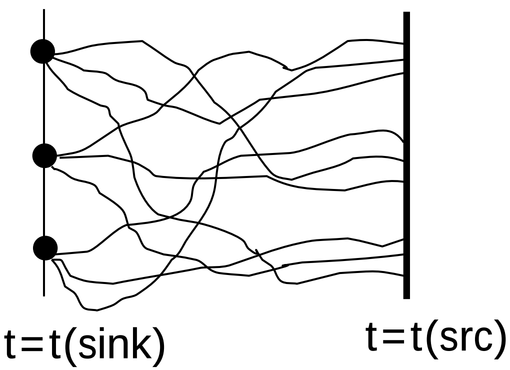
    (🡐 can be reduced by 2^A by inner-baryon exchange)

  - # of color / spinor contractions

    $N_{\rm loop} = 6^A \cdot 4^A$ or $6^A \cdot 2^A$

    (color) (spinor)

  - Total cost: $N_{\rm perm} \times N_{\rm loop}$
    - ^{2}H : 9 x 144 = 1 x 10^3
    - ^{3}H : 360 x 1728 = 6 x 10^5
    - ^{4}He : 32400 x 20736 = 7 x 10^8



c.f. T.Yamazaki et al., PRD81(2010)111504

$N_{\rm perm} = 1107$ for ^{4}He in the isospin limit

Solution: Unified contraction algorithm

TD, M.Endres, Comput. Phys. Comm.184(2013)117

- Traditional algorithm

color/spinor contractions (ξ')

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \underbrace{\mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)}_{\text{color } \epsilon_{abc},\ \text{spinor } (C\gamma_5),\ \text{etc.}}$$

Permutations

- New algorithm

[impose the same spacial label at source]

  - Permutation applies to color/spinor indices at “Coeff”

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

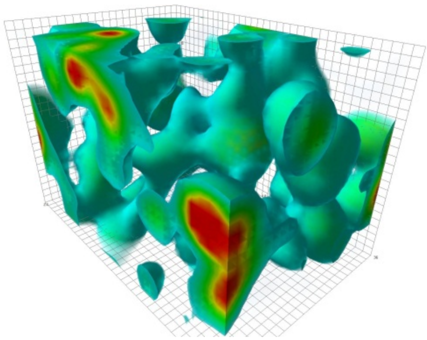
  - **Permutation DONE beforehand**
    - (Wick contraction and color/spinor contractions are unified)
  - Significant improvement

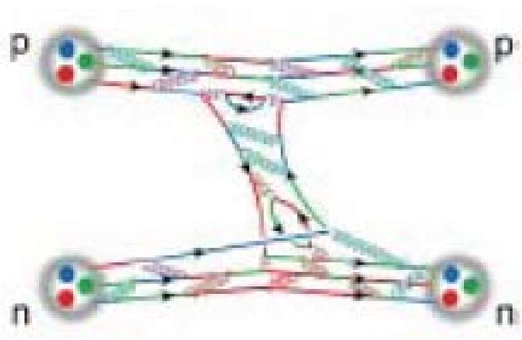
$$\boxed{\times 192 \text{ for } {}^3\mathrm{H}/{}^3\mathrm{He},\ \times 20736 \text{ for } {}^4\mathrm{He},\ \times 10^{11} \text{ for } {}^8\mathrm{Be}}$$

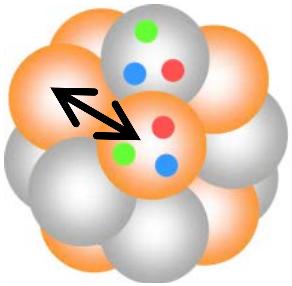
(x add'l. speedup)

See also subsequent works:

Detmold et al., PRD87(2013)114512
Gunther et al., PRD87(2013)094513







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Exotic states as Tcc: poster by Y.Ikeda

  - Prospects toward physical quark mass point
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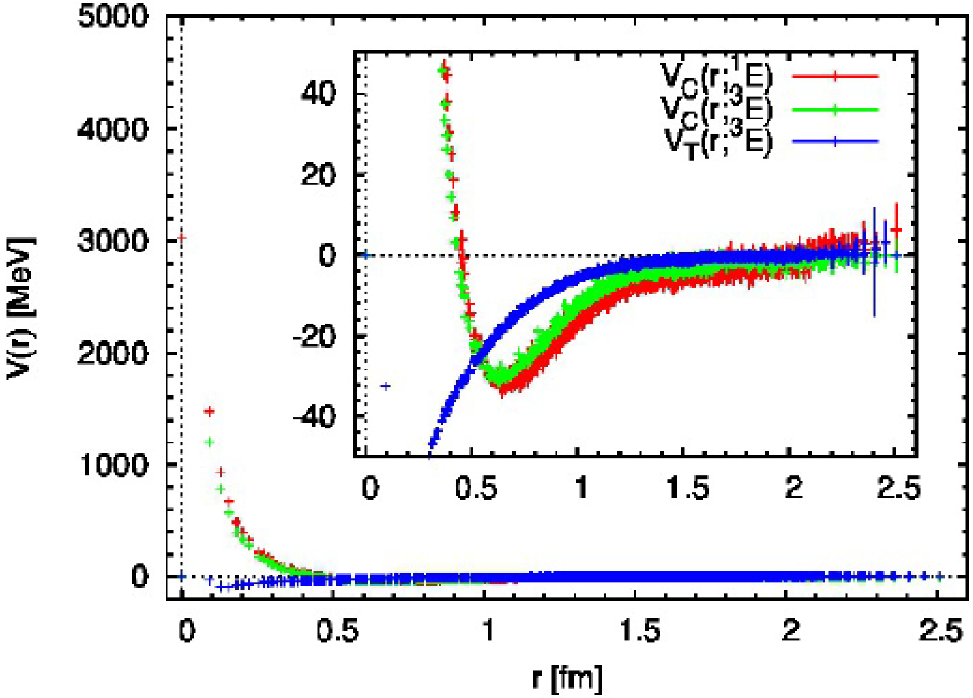
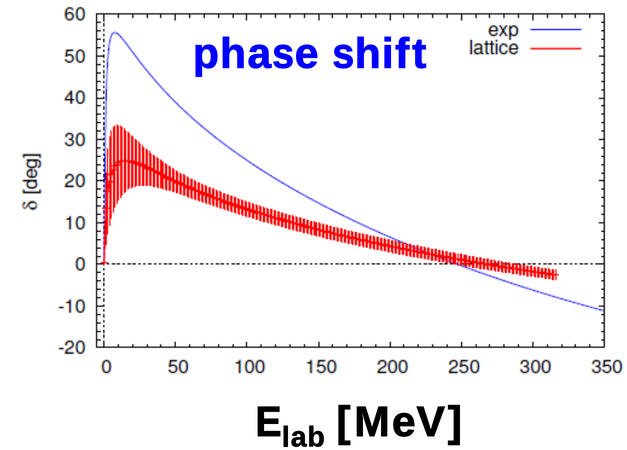
(1) NN potential on the lattice (positive parity)

$^{2S+1}L_J$

- "di-neutron" channel $^1S_0$ ➔ central force
- "deuteron" channel $^3S_1 - ^3D_1$ ➔ central & tensor force





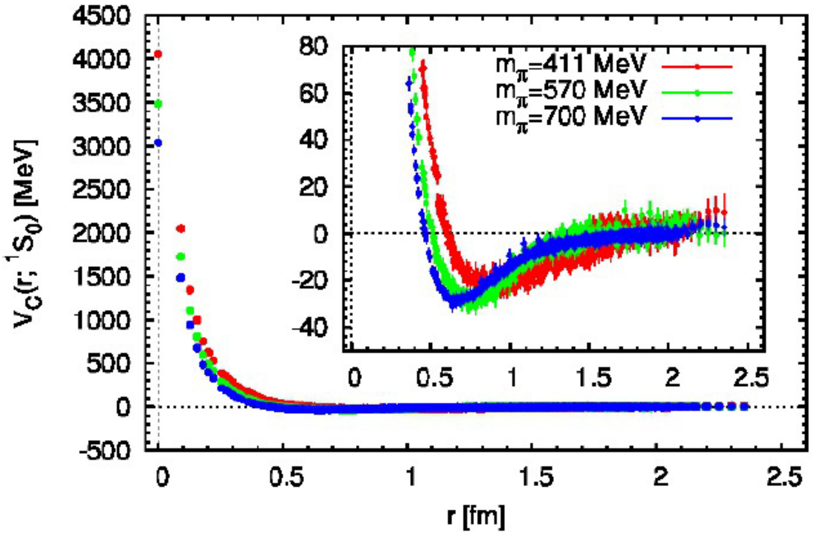
Not Bound $a(^1S_0) = 1.6(1.1)$ fm

Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

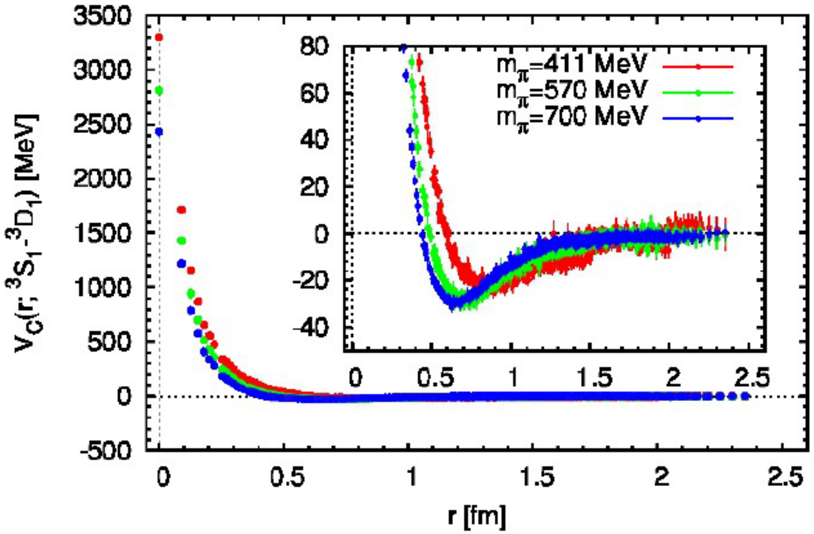
N.Ishii et al. (HAL QCD Coll.)
PLB712(2012)437

Quark mass dependence

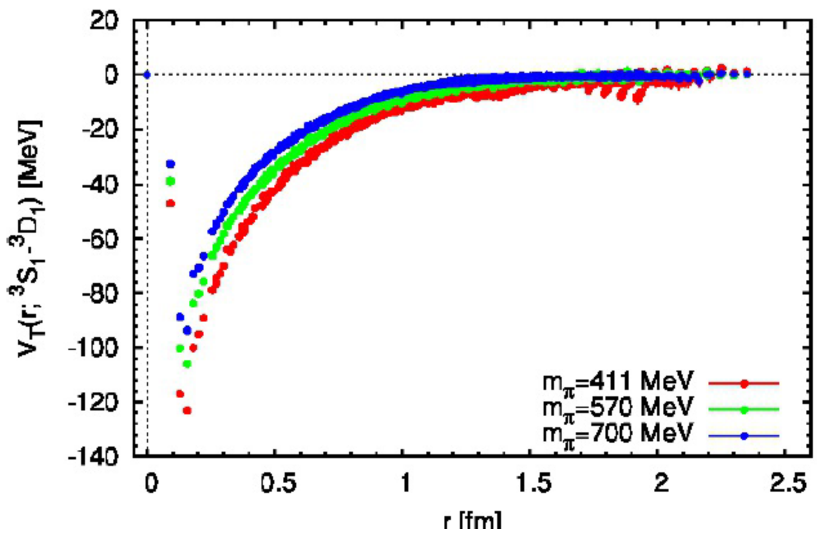
Central in $^1S_0$



$^3S_1$-$^3D_1$ channel



Central



Tensor

Lighter mass corresponds to…

- **Longer interaction range**
- **Larger Repulsive Core**
- **Stronger Tensor Force**

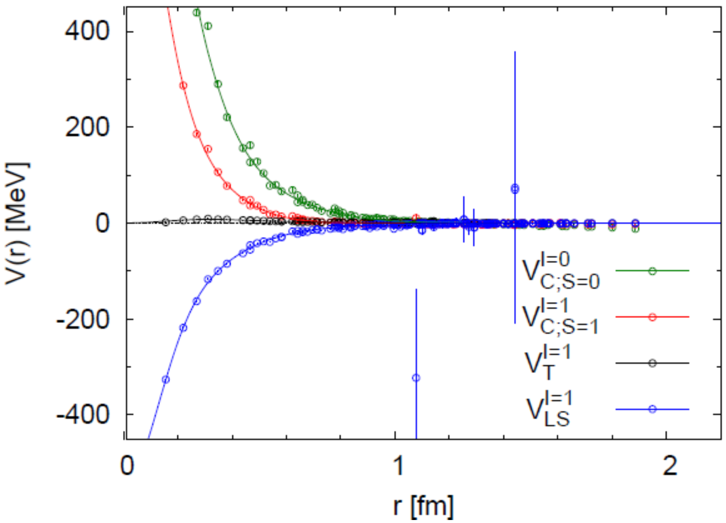
N.Ishii @ Lat2012

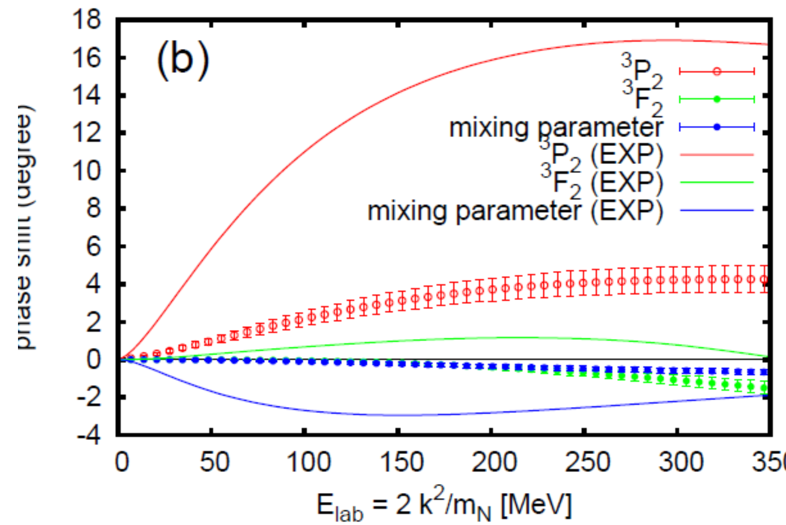
Nuclear Forces (negative parity)

$$^{2S+1}L_J$$

- Central, tensor & LS forces

$$^1P_1,\ ^3P_0,\ ^3P_1,\ ^3P_2{-}^3F_2 \qquad A_1^-, T_1^-, T_2^- \oplus E^-\text{-rep}$$





Nf=2 clover (CP-PACS), L=2.5fm, mπ=1.1GeV

We also observe that potentials glow by lighter quark mass

Superfluidity $^3P_2$ in neutron star
←→ neutrino cooling

←→ **Cas A NS: cooling is being measured !**

K.Murano et al., arXiv:1305.2293
K.Murano @ Lat2013

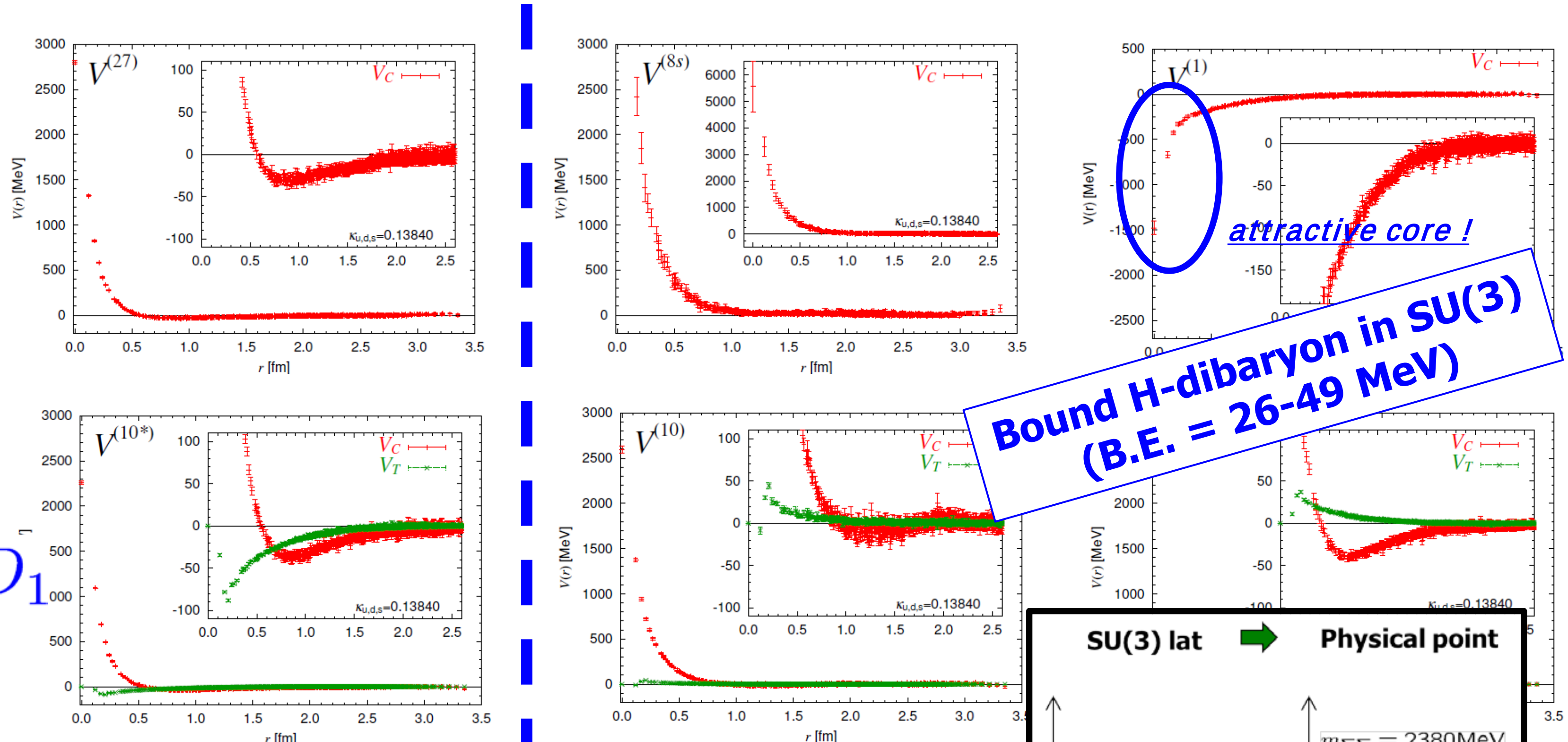
SU(3) study

(2) BB potentials

a=0.12fm, L=3.9fm,
m(PS)= 0.47-1.2GeV

$^1S_0$

$^3S_1 - ^3D_1$

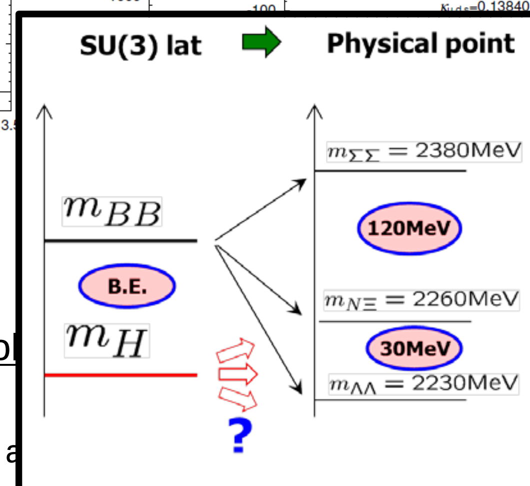


Bound H-dibaryon in SU(3)
(B.E. = 26-49 MeV)

27,10*:
Same as NN

8s,10:
strong repulsive core

ket
e



T.Inoue et al. (HAL QCD Col

Repulsive core
← Pauli principle !

M.Oka et al., NPA464(1987)700

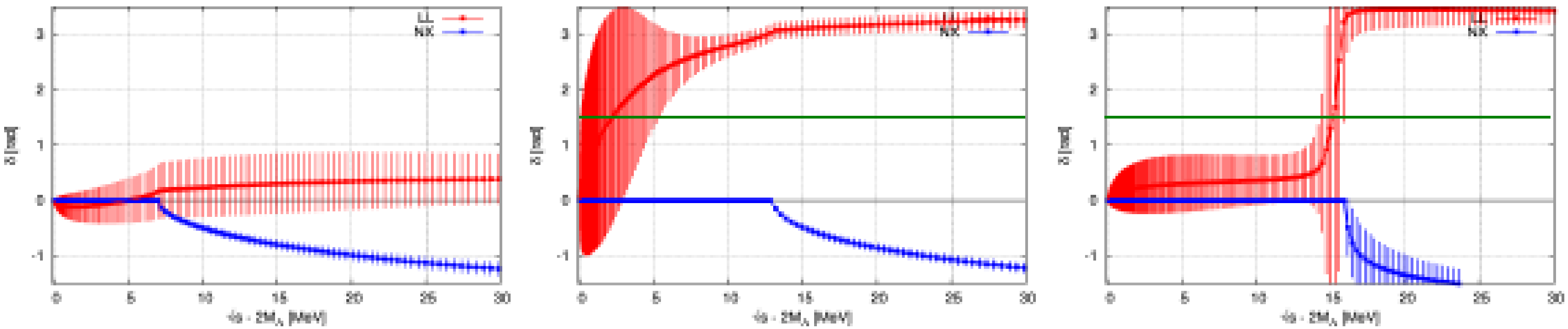
Also seen in SU(2)c , Takahashi et a
Charmonium-N, Kawanai-Sasaki, PRD82(2010)091501
Meson-baryon, Y.Ikeda et al., arXiv:1111.2663

Coupled channel formalism in HAL

$\Lambda\Lambda - N\Xi - \Sigma\Sigma$

ΛΛ and NΞ phase shifts

Esb1 : mπ= 701 MeV **Esb2 : mπ= 570 MeV** **Esb3 : mπ= 411 MeV**



Preliminary!

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter

- Esb1:
  - Bound H-dibaryon
- Esb2:
  - H-dibaryon is near the ΛΛ threshold
- Esb3:
  - The H-dibaryon resonance energy is close to NΞ threshold..

- We can see the clear resonance shape in ΛΛ phase shifts for Esb2 and 3.
- The "binding energy" of H-dibaryon from NΞ threshold becomes smaller as decreasing of quark masses.

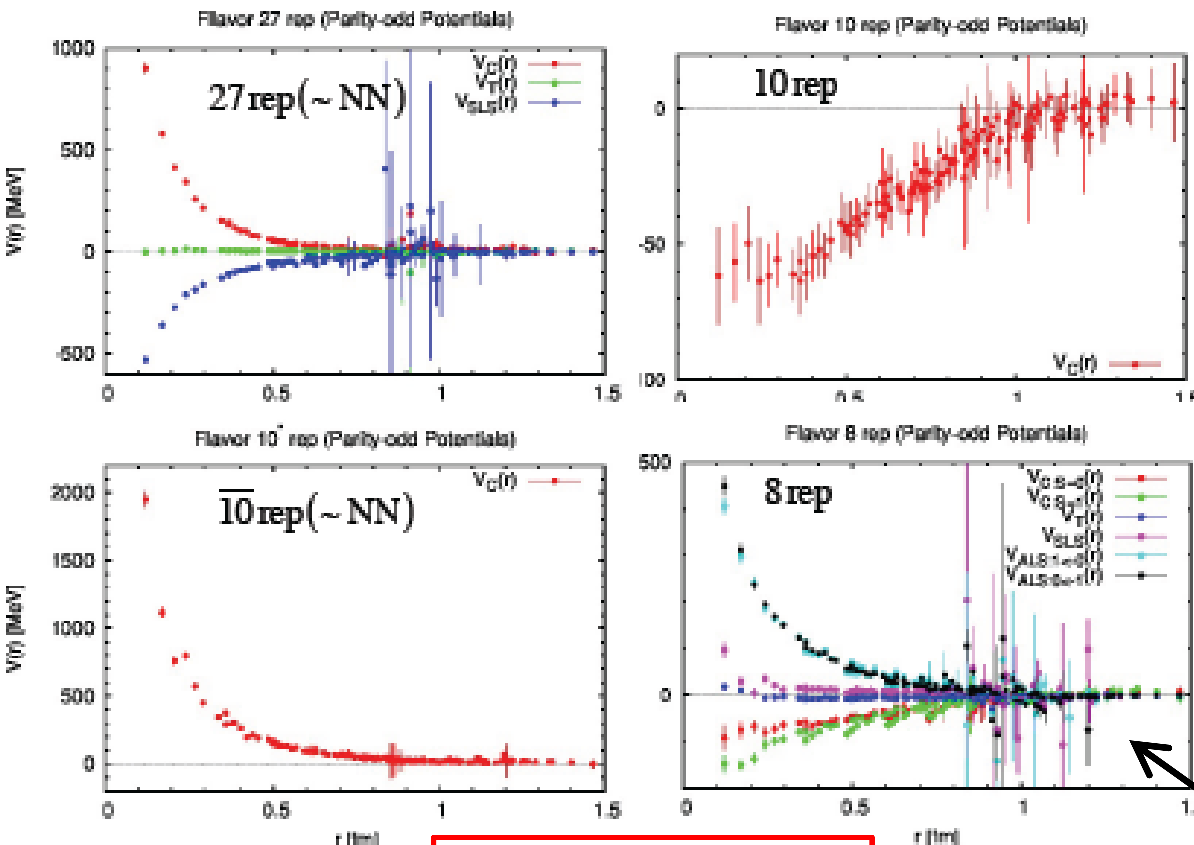
[K. Sasaki]

Symmetric LS and Anti-Symmetric LS (ALS) forces

Parity-odd hyperon potentials in the flavor SU(3) limit. [N.Ishii@Lattice 2013]

$$V_{BB} = V_{C;S=0}(r)\mathbb{P}^{(S=0)} + V_{C;S=1}(r)\mathbb{P}^{(S=1)} + V_T(r)\left(3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)$$
$$+ V_{SLS}(r)\vec{L}\cdot\vec{S}_+ + V_{ALS}(r)\vec{L}\cdot\vec{S}_- + O(\nabla^2)$$

$$\vec{S}_\pm \equiv \frac{1}{2}(\vec{\sigma}_1 \pm \vec{\sigma}_2)$$



- Repulsive core for 27 and 10^* rep.
- No repulsive core for 10 and 8 rep. (consistent with quark model)

- Weak tensor potential (8 rep)
- Weak symmetric LS potential (8 rep)
- Strong anti-symmetric LS potential (8 rep)

SU(3) study

Qualitative behaviors are reasonable

8s – 8a mixes by ALS

(m(PS)= 1.01GeV)

Symmetric LS and Anti-Symmetric LS (ALS) forces

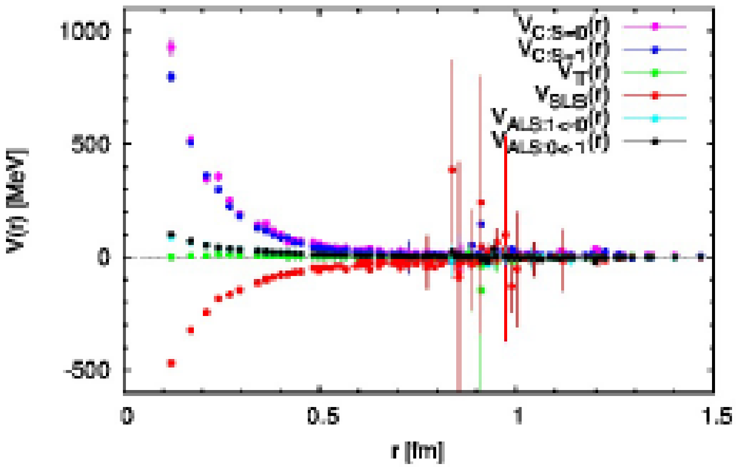
SU(3) study

[N. Ishii @ Lat2013]

Lamba-N potential (parity-odd sector)
is obtained as linear combination of irreps. 8, 10^* and 27:

$$V_{\Lambda N} = \left(\frac{1}{2}V_C^{(\overline{10})} + \frac{1}{2}V_{C,S=0}^{(8)}\right)\mathbb{P}^{(S=0)} + \left(\frac{1}{10}V_{C,S=1}^{(8)} + \frac{9}{10}V_C^{(27)}\right)\mathbb{P}^{(S=1)} + \left(\frac{1}{10}V_T^{(8)} + \frac{9}{10}V_T^{(27)}\right)S_{12}(\hat{r}) + \left(\frac{1}{10}V_{SLS}^{(8)} + \frac{9}{10}V_{SLS}^{(27)}\right)\vec{L}\cdot\vec{S}_+ + \frac{1}{2\sqrt{5}}V_{ALS}^{(8)}\cdot\vec{L}\cdot\vec{S}_-$$

ΛN potential (parity-odd)



Experimental suggestion

(from high precision spectroscopy of hypernuclei)

➔

the anti-symmetric LS is strong enough
to cancel the symmetric LS.

[H.Akikawa et al., PRL88(2002)082501.]

- We obtain weak cancellation
  - Symmetric LS is strong. It comes from 27 rep. (90%), i.e., NN LS

$$V_{SLS}^{(\Lambda N)} = \frac{1}{10}V_{SLS}^{(8)} + \frac{9}{10}V_{SLS}^{(27)}$$

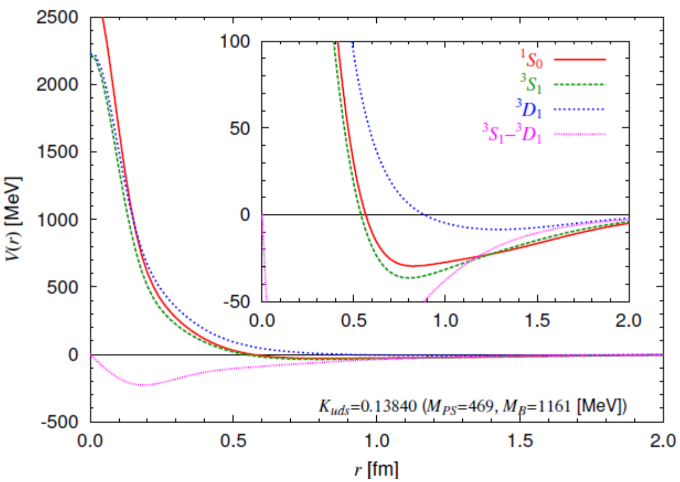
  - Anti-symmetric LS is weak. It is weakened by a numerical factor 1/(2*sqrt(5))

$$V_{ALS}^{(\Lambda N)} = \frac{1}{2\sqrt{5}}V_{ALS}^{(8)}$$

- Quark mass dep. should be studied by breaking the flavor SU(3) symmetry.

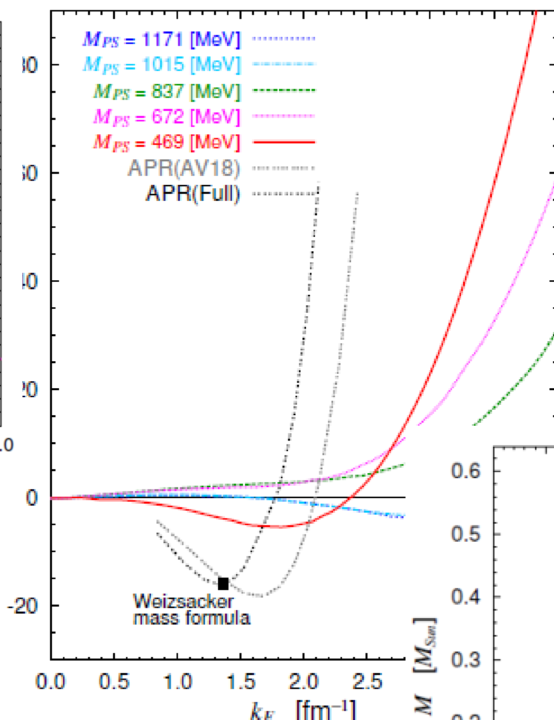
From QCD to nuclear matter

QCD → (**Lat QCD**) → **Lat NN forces** → (**BHF**) → **EoS** → (**TOV eq.**) → **Neutron Star**

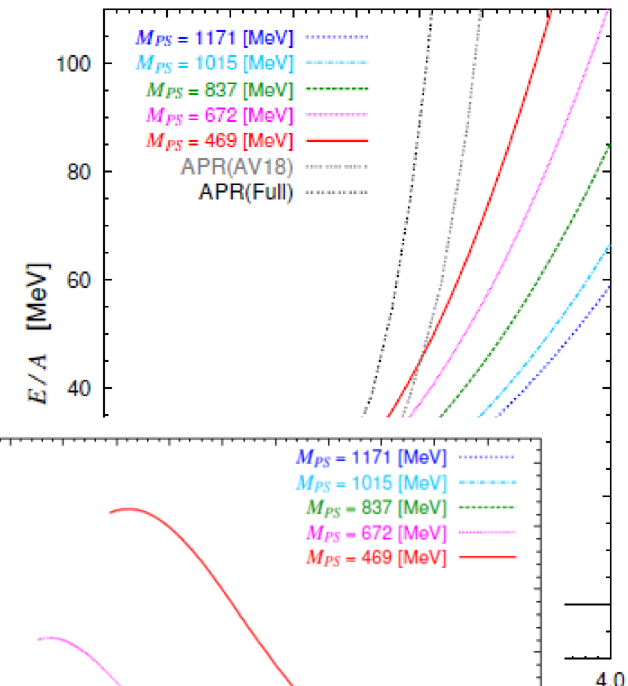


SU(3) study
m(PS)=0.47GeV

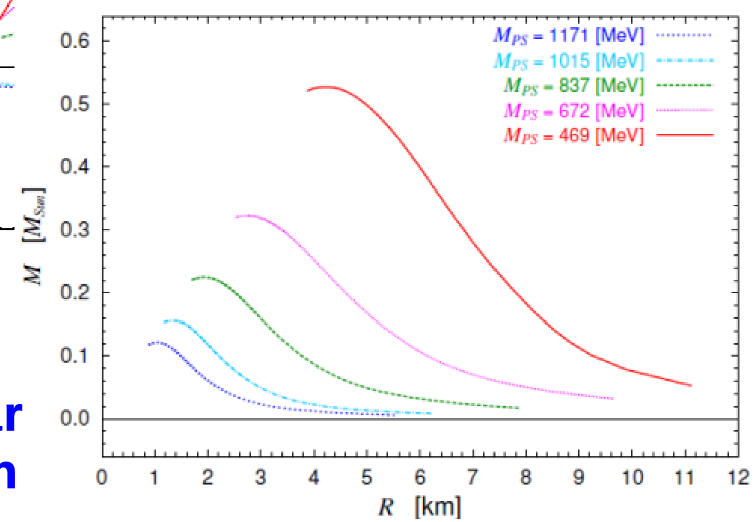
Sym. Nuclear matter



Neutron matter



Neutron Star
M-R relation

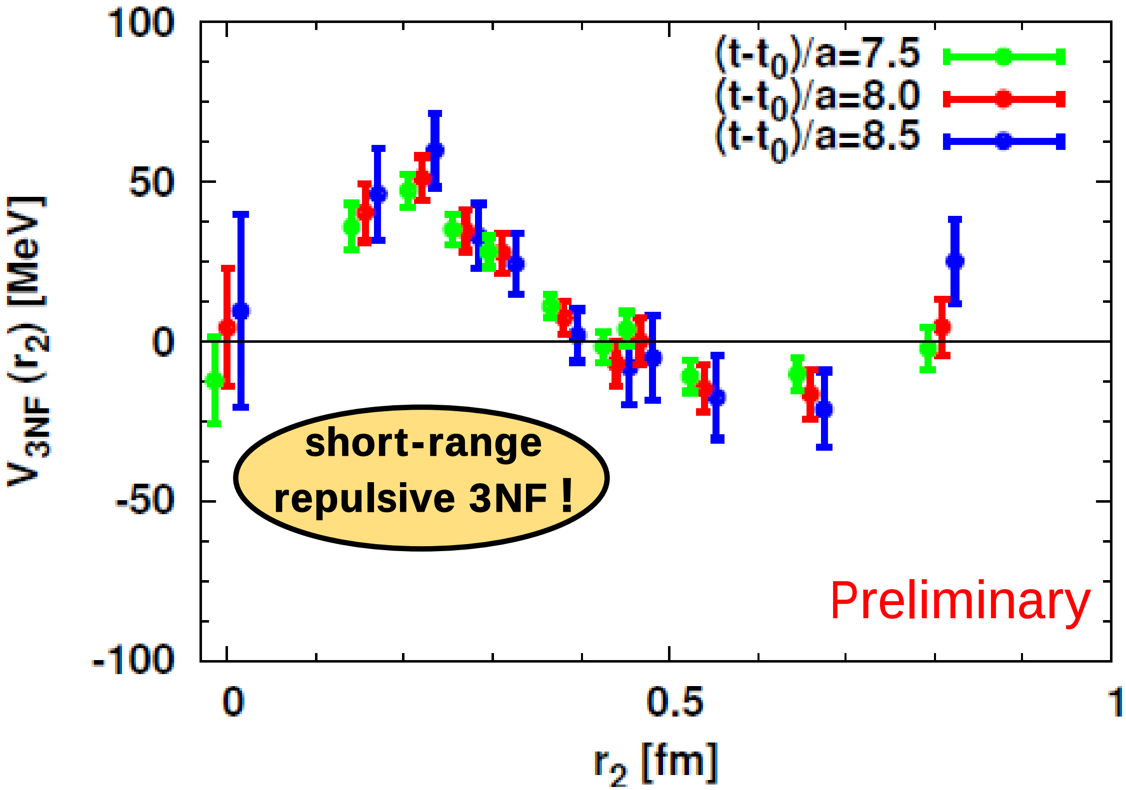


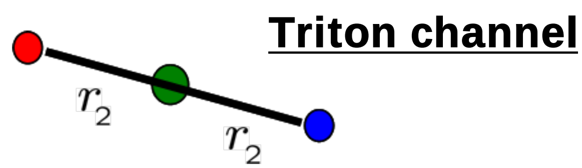
T.Inoue et al., PRL111(2013)112503

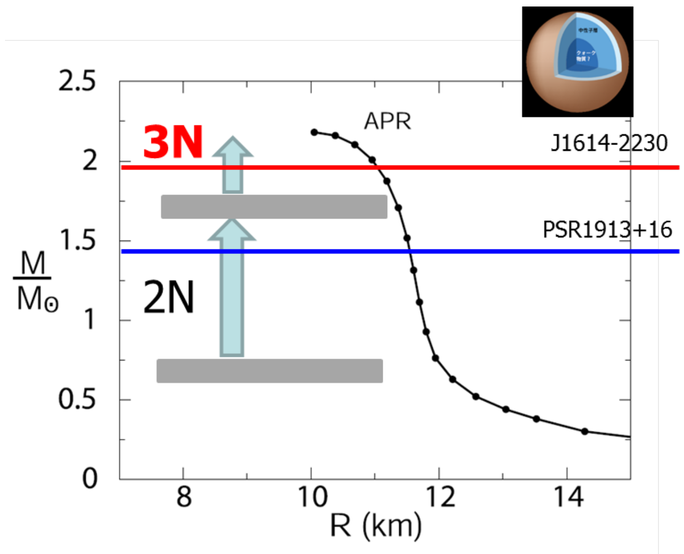
(3) 3N-forces (3NF) on the lattice

T.D. et al. (HAL QCD Coll.) PTP127(2012)723

+ t-dep method updates etc.







Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=1.1GeV, m_N=2.1GeV

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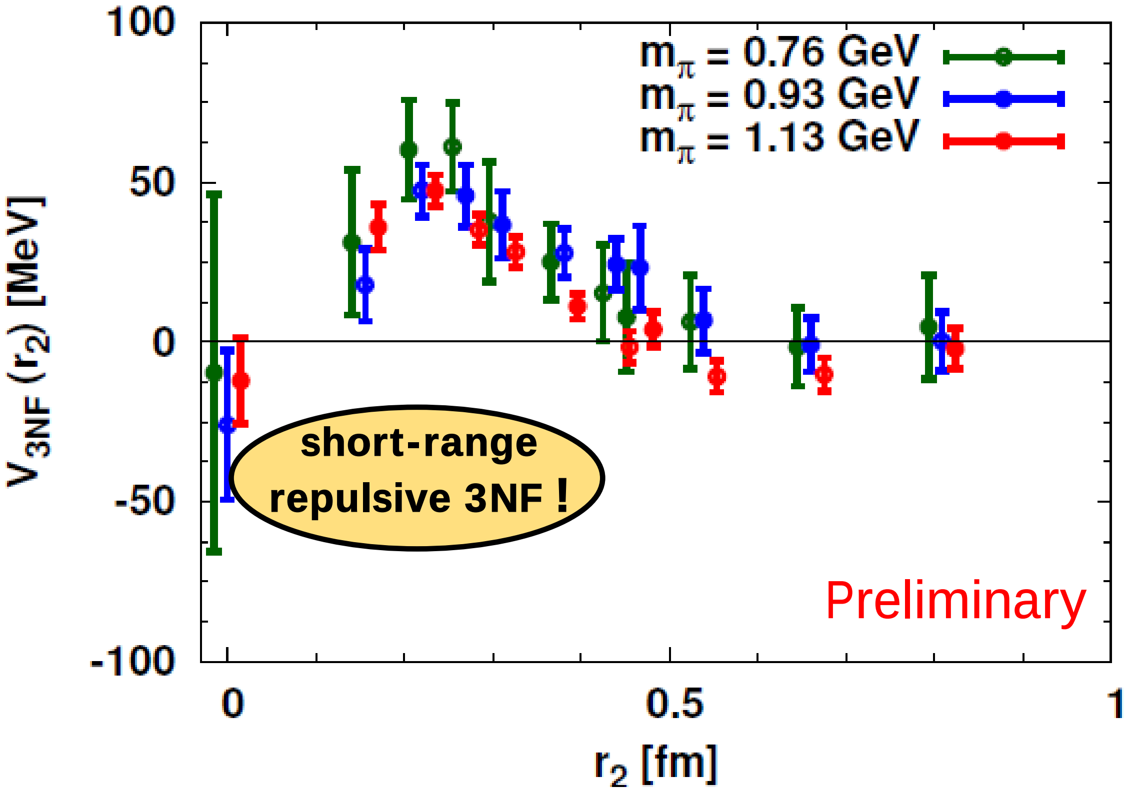
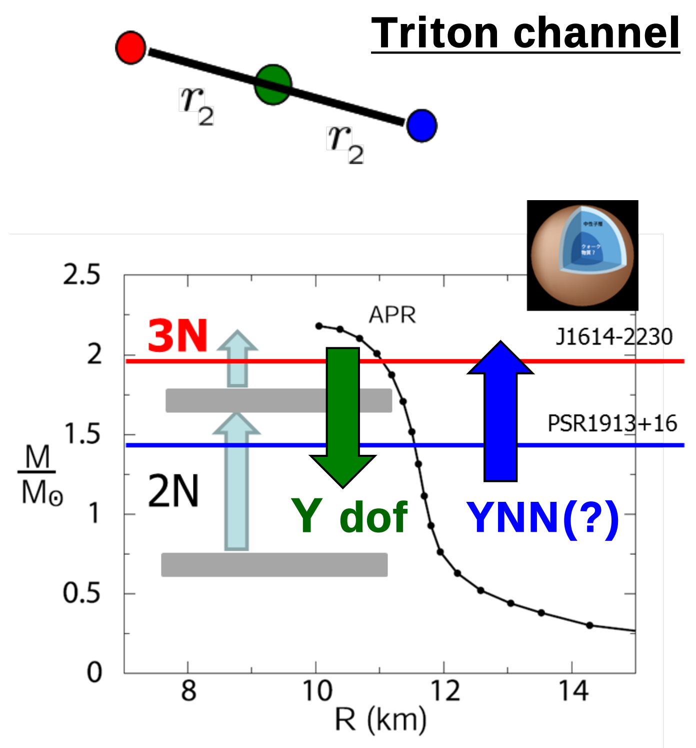
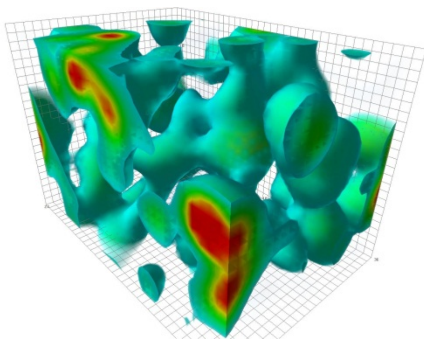


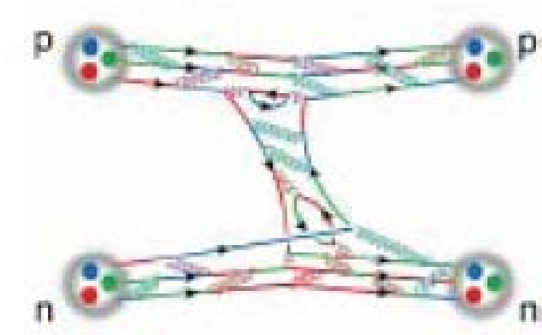


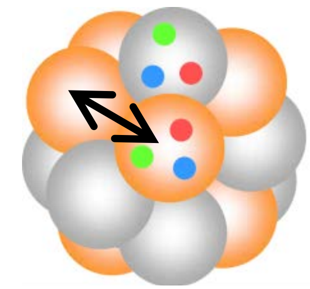
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=0.76-1.1GeV, m_N=1.6-2.1GeV

How about other geometries ?

How about YNN, YYN, YYY ?



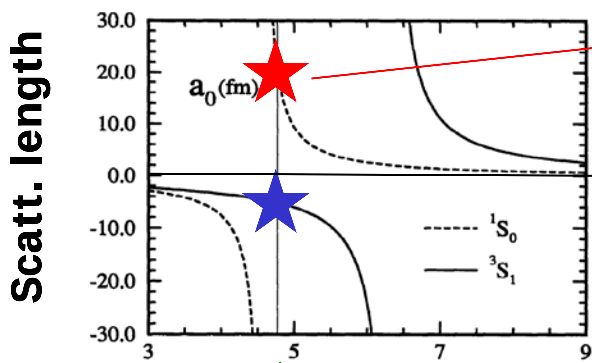




- **<u>Outline</u>**
  - Introduction
  - Theoretical framework: Interactions on the lattice
  - Challenges in multi-baryons & solutions
  - Lattice QCD results at heavier quark masses
  - Prospects toward physical quark mass point
  - Summary

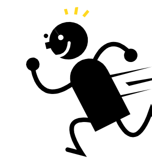
Towards realistic potential

- **Physical mass point**, **Infinite V limit**, **continuum limit**
  – Physical mπ crucial for OPEP, chiral extrapolation won't work



$a_0(^1S_0) \sim 20\text{fm}$

"Unitary Region"



→ m_q

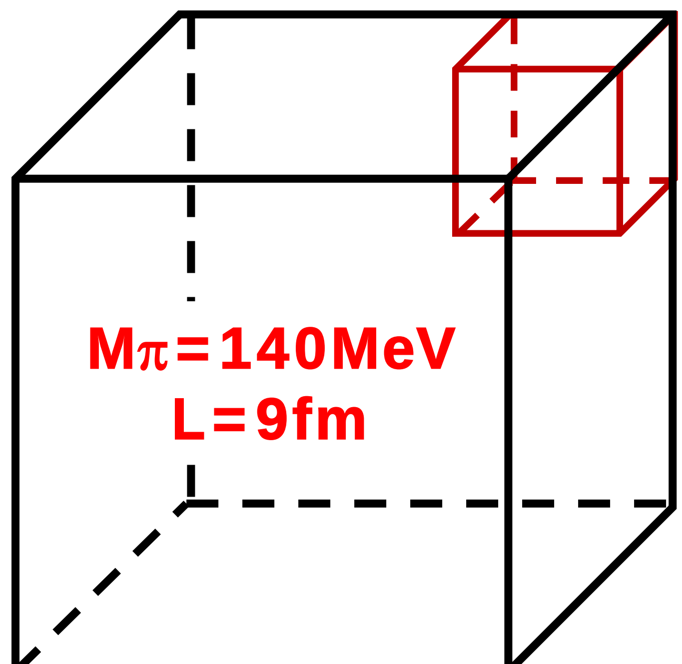
m$_q$

Y.Kuramashi, PTPS122(1996)153

Phys. point

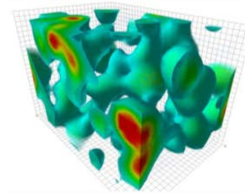
We are here

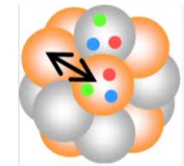
K computer



Mπ>=400MeV
L=3fm

Summary and Prospects

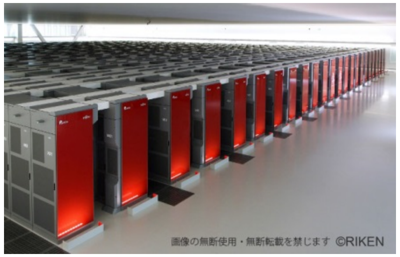




- Hadron Interactions by 1st principle Lat calc
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics

- Lattice QCD results for NN, **YN/YY**, NNN, etc.
  - Intriguing physics even at heavy quark masses

- **Toward** <u>physical quark mass point:</u>
  - Breakthroughs in S/N issue & Comput. cost issue



➔ Realistic hadron interactions
➔ Nuclear Physics on the Lattice !