Hadron Properties in the Nuclear Medium

TECHNISCHE UNIVERSITÄT DARMSTADT

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Outline

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- Motivation
- Vector Mesons and Thermal Dileptons
- In-Medium Spectral Functions from the FRG
  - results for the O(4) model at $T = 0$
  - results for the quark-meson model at $T > 0$ and $\mu > 0$
- Summary and Outlook

Motivation

Strong QCD in the Vacuum

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in the vacuum of QCD chiral symmetry is sponaneously broken:

- quarks condense

$$\left[Q^j_{V,A}, H^0_{QCD}\right] = 0 \quad \text{but} \quad Q^j_A|0\rangle \neq 0 = |\pi^j\rangle$$

$$\rightarrow \quad \langle\bar{q}q\rangle \neq 0$$

- appearence of Goldstone bosons

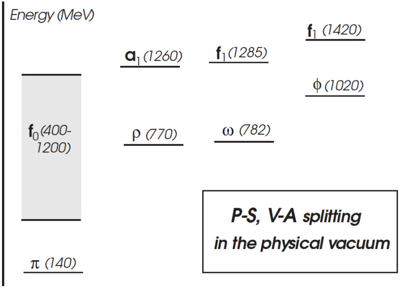
$$\langle 0|j^{\mu,j}_A|\pi_j(p)\rangle = -i\delta_{ik} f_\pi p^\mu e^{-ipx}$$

$$f^2_\pi m^2_\pi = -2\bar{m}\,\langle\bar{q}q\rangle$$

- no parity doublets in the hadron spectrum

$$M(J^P) \neq M(J^{-P})$$

light-meson spectrum



Motivation

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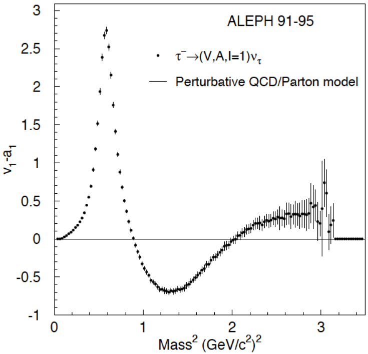
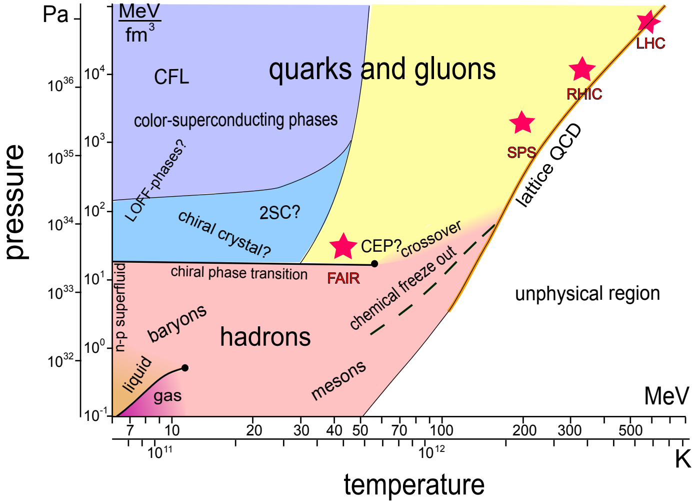
V-A data



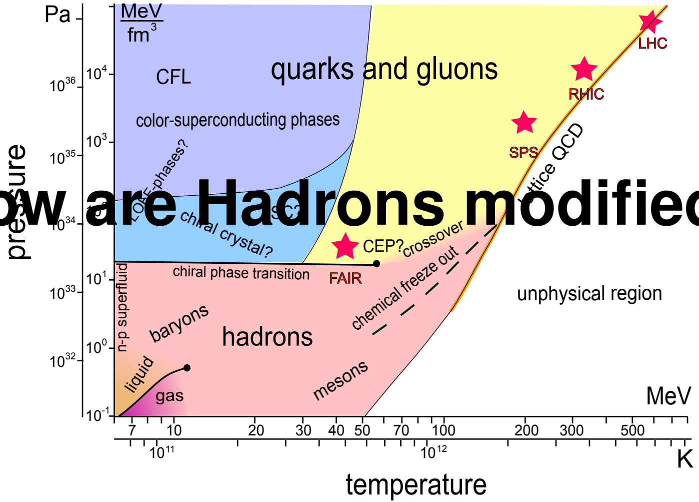
Motivation

Hot and Dense Hadronic Matter



Motivation

Hot and Dense Hadronic Matter



How are Hadrons modified?

Photons and Dileptons

penetrating probes of hot and dense matter

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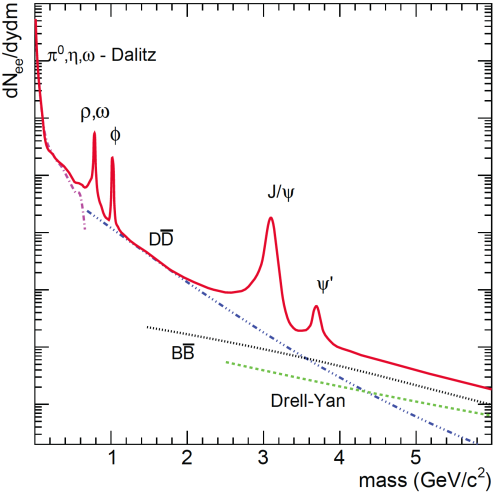

Low-mass Dileptons

theory

dilepton rate : (local thermal equilibrium)

$$\frac{dN_{ll}}{d^4x d^4q} = \underbrace{-\frac{\alpha}{\pi^3 M^2}\frac{1}{3}g_{\mu\nu}}_{L_{\mu\nu}} \mathrm{Im}\Pi^{\mu\nu}_{em}(M,q;\mu,T); \quad M^2 = \omega^2 - \vec{q}^2$$

electromagnetic correlator:

$$\mathrm{Im}\Pi^{\mu\nu}_{em}(M,q;\mu,T) = -i\int d^4x\ e^{i(\omega t - \vec{x}\cdot\vec{q})}\Theta(t)\left\langle\left[j^{\mu}_{em}(x), j^{\nu}_{em}(0)\right]\right\rangle$$

electromagnetic current: (VDM)

$$j^{\mu}_{em}(M \leq 1\mathrm{GeV}) = \frac{m^2_\rho}{g_\rho}\rho^\mu + \frac{m^2_\omega}{g_\omega}\omega^\mu + \frac{m^2_\phi}{g_\phi}\phi^\mu$$

from quark counting:

$$\mathrm{Im}\Pi^{\mu\nu}_{em} \sim \left[\mathrm{Im}D^{\mu\nu}_\rho + \frac{1}{9}\mathrm{Im}D^{\mu\nu}_\omega + \frac{2}{9}\mathrm{Im}D^{\mu\nu}_\phi\right]$$

ρ-Meson

medium modification

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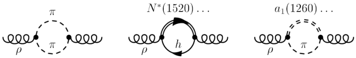
L/T decomposition:

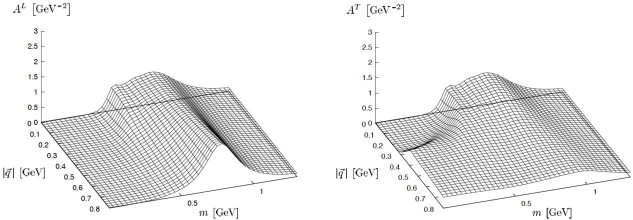
$$D_V^{\mu\nu} = \left(M^2 - m_V^2 - \Sigma_V^L(\omega, q)\right)^{-1} P_L^{\mu\nu} + \left(M^2 - m_V^2 - \Sigma_V^T(\omega, q)\right)^{-1} P_T^{\mu\nu}$$

ρ-meson selfenergy:

$$\Sigma_\rho^{L/T} = \Sigma_{\rho\pi\pi}^{L/T} + \Sigma_{\rho M}^{L/T} + \Sigma_{\rho B}^{L/T}$$



ρ-spectral functions: (low temperature)



ρ-Meson

photo-absorption as a test

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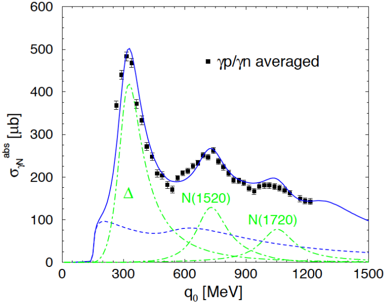
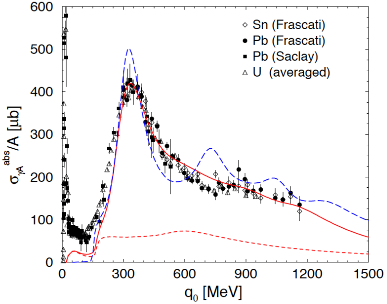
photo-absorption cross section:

$$\frac{\sigma_\gamma}{A} = -\frac{4\pi\alpha}{\omega}\frac{m_\rho^4}{g_\rho^2}\,\mathrm{Im}D_\rho^T(\omega, q=\omega)$$

nucleon



nucleus



Spectral Functions

under HIC conditions

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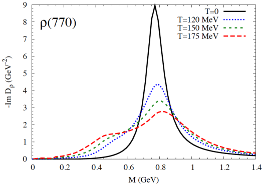
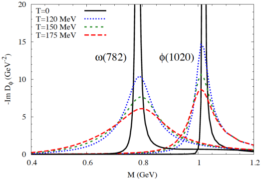
isentropic fireball expansion

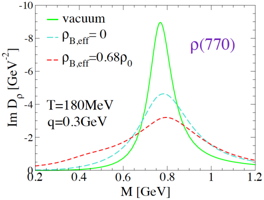
ρ-meson | ω and ϕ-meson

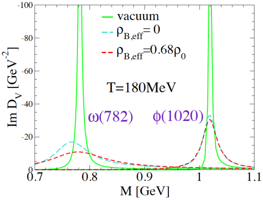
SPS
$\sqrt{s_{NN}}$=17 GeV





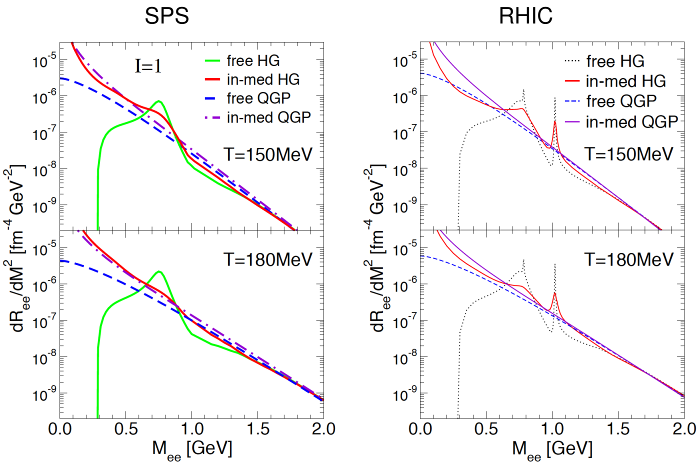
RHIC
$\sqrt{s_{NN}}$=200 GeV





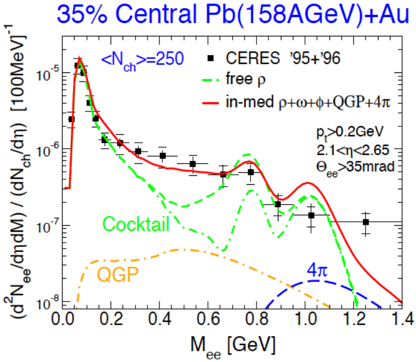
Dilepton Rates

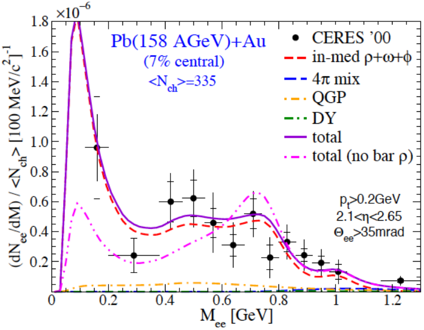
SPS and RHIC conditions



Dilepton Data

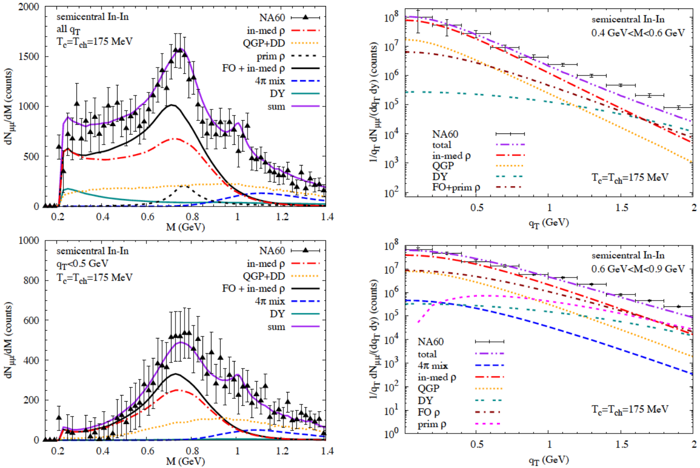
CERES





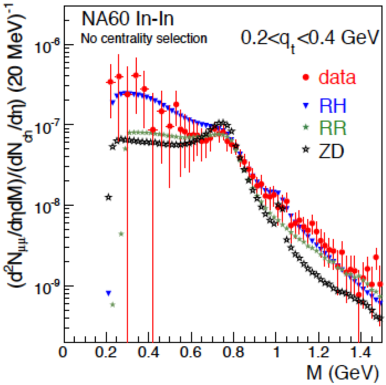
Dilepton Data

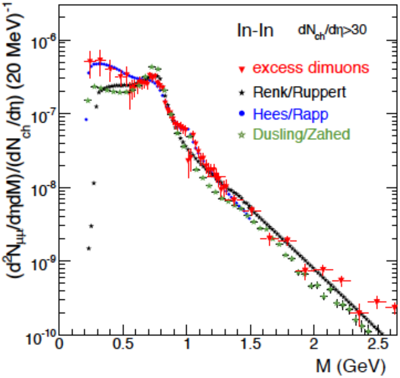
NA60



Dilepton Data

NA60





Dilepton Data

STAR

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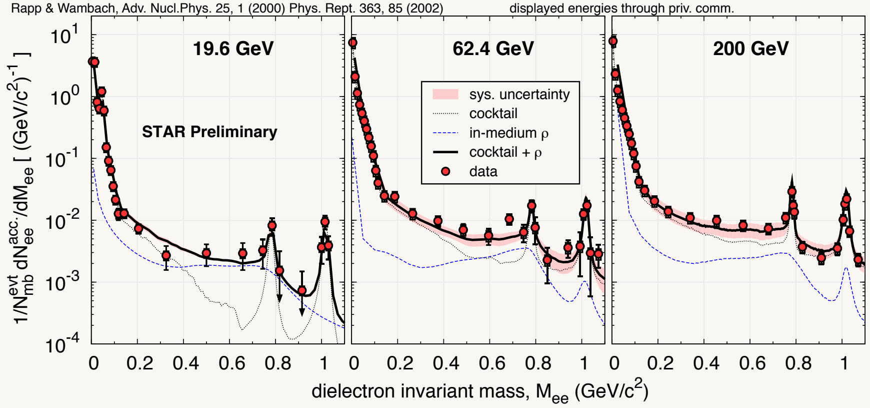

Spectral Functions from the Functional Renormalization Group

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- as chiral symmetry gets restored chiral partners become degenerate

  $\rightarrow$ identical spectral functions of parity partners

  Weinberg sum rule(s)

$$\int_0^\infty \frac{d\omega^2}{\omega^2 - \vec{q}^2} \left(\rho_V^L(\omega, q) - (\rho_A^L(\omega, q) \right) = \int_0^\infty \frac{d\omega^2}{\omega^2 - \vec{q}^2} \rho_\pi(\omega, q) \quad \propto \text{"} f_\pi^{*2} \text{"}$$

- in the presence of phase transitions a method beyond the loop expansion is required!
- the FRG provides such a method

K. Kamikado, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1302.6199 [hep-ph]

R.-A. Tripolt, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1311.0630 [hep-ph], PRD 89, 034010 (2014)

Functional Renormalization Group

a primer

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partition function: (scalar field $\phi(x)$)

$$Z[j] = e^{W[j]} = \int [\mathcal{D}\phi] \; e^{-S[\phi] + \int d^4x \; \phi(x) j(x)}$$

generating functional:

$$\left. \frac{\delta W[j]}{\delta j(x)} \right|_{j=0} = \frac{1}{Z[0]} \int [\mathcal{D}\phi] \phi e^{-S[\phi]} = \langle \phi(x) \rangle \equiv \varphi(x)$$

two-point correlation function: (Euclidean)

$$\left. \frac{\delta^2 W[j]}{\delta j(x) \delta j(y)} \right|_{j=0} = \langle \phi(x)\phi(y) \rangle - \langle \phi(x) \rangle \langle \phi(y) \rangle \equiv G(x, y)$$

effective action: ((Legendre transform of W)

$$\Gamma[\varphi] = -W[j] + \int d^4x \; \varphi(x) j(x)$$

stationarity condition: and thermodynamic potential:

$$\left. \frac{\delta \Gamma[\varphi]}{\delta \varphi} \right|_{\varphi=\varphi_0} = 0; \quad \rightarrow \quad \Omega(T) = \frac{T}{V} \Gamma[\varphi_0]$$

Functional Renormalization Group

Wilsonian coarse graining

at given resolution scale k split ϕ into low- and high-frequency modes:

$$\phi(x) = \phi_{q\leq k}(x) + \phi_{q>k}(x)$$

$$\rightarrow \quad Z[j] = \int [\mathcal{D}\phi]_{q\leq k} \underbrace{\int [\mathcal{D}\phi]_{q>k}\; e^{-S[\phi]+\int d^4x\;\phi j}}_{=Z_k[j]} \;; \qquad \lim_{k\to 0} Z_k[j] = Z[j]$$

regulator $R_k(q)$:

$$\lim_{k\to 0} R_k(q) = 0$$

$$\lim_{k\to \Lambda} R_k(q) = \infty$$

$$Z_k[j] = \int [\mathcal{D}\phi]\; e^{-S[\phi]-\Delta S_k[\phi]+\int d^4x\;\phi j}$$

$$\Delta S_k[\phi] = \underbrace{\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\;\phi(-q)R_k(q)\phi(q)}_{\text{acts like a mass term } m_k}$$

effective action:

$$\rightarrow \quad \Gamma_k[\varphi] = -\ln Z_k[j] + \int d^4x\;\varphi(x)j(x) - \Delta S_k[\varphi]$$

Γ_k interpolates between $k = \Lambda$ (no fluct.) and $k = 0$ (full quantum action)

$$\lim_{k\to\Lambda} \Gamma_k[\varphi] = S[\varphi]; \qquad \lim_{k\to 0} \Gamma_k[\varphi] = \Gamma[\varphi]$$

Functional Renormalization Group

flow equation **C. Wetterich (1993)**

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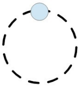
flow equation for the effective action:

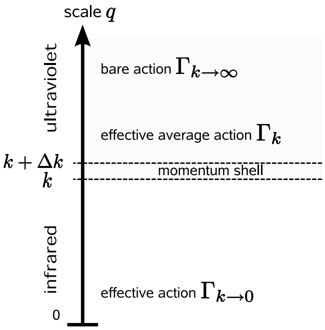
$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

$$\Gamma_k^{(2)}(q) = \frac{\delta^2 \Gamma_k[\varphi]}{\delta\varphi(-q)\delta\varphi(q)}$$

[C. Wetterich, Phys. Lett. B301 (1993) 90]

$$\partial_k \Gamma_k \; = \; \frac{1}{2}$$





O(4) Model

flow equation

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effective action: ('Local Potential Approximation')

$$\varphi = (\varphi_1, \dots, \varphi_4) = (\vec{\pi}, \sigma)$$

$$\Gamma_k[\varphi] = \int d^4x \left\{ \frac{1}{2}(\partial_\mu \varphi)^2 + U_k(\varphi^2) - c\sigma \right\}; \quad \varphi^2 = \varphi_i \varphi^i = \sigma^2 + \vec{\pi}^2$$

$$\Gamma^{(2)}_{k,ij}(q) = \Gamma^{(2)}_{k,\pi}(q) \left\{ \delta_{ij} - \frac{\varphi_i \varphi_j}{\varphi^2} \right\} + \Gamma^{(2)}_{k,\sigma}(q) \frac{\varphi_i \varphi_j}{\varphi^2}$$

flow equation for the effective potential:

$$\partial_k U_K = I_\sigma + 3 I_\pi; \quad I_i = \frac{1}{2} \mathrm{Tr}_q \left(\partial_k R_k(q) \left[\Gamma^{(2)}_{k,i}(q) + R_k(q) \right]^{-1} \right)$$

for

$$R_k(q) = (k^2 - q^2)\Theta(k^2 - q^2)$$

one gets

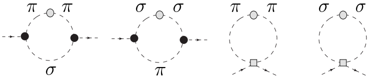
$$I_i = \frac{k^4}{3\pi^2} \frac{1}{2E_i}, \qquad \text{with} \qquad E_\pi = \sqrt{k^2 + 2U'}; \quad E_\sigma = \sqrt{k^2 + 2U' + 4U''\varphi^2}; \quad U' = \left.\frac{\partial U}{\partial \varphi}\right|_{\varphi = \varphi_0} \text{ etc.}$$

O(4) Model

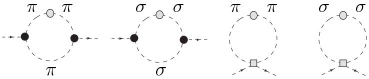
flow equations for 2-point functions

taking two functional derivatives of the flow equation for Γ_k yields

$$\frac{\partial \Gamma^{(2)}_{\pi k}}{\partial k} \sim$$



$$\frac{\partial \Gamma^{(2)}_{\sigma k}}{\partial k} \sim$$



approximation:

$$\Gamma^{(3)}_{ijm} \rightarrow \frac{\delta^3 \Gamma_k}{\delta\varphi^m \delta\varphi^j \delta\varphi^i}; \qquad \Gamma^{(4)}_{ijmn} \rightarrow \frac{\delta^4 \Gamma_k}{\delta\varphi^n \delta\varphi^m \delta\varphi^j \delta\varphi^i}$$

ensures that truncation is consistent with effective action

$$\partial_k \Gamma^{(2)}_{k,\pi}(p=0) = 2\partial_k U'_k$$

$$\partial_k \Gamma^{(2)}_{k,\sigma}(p=0) = 2\partial_k U'_k + 4\partial_k U'' \varphi^2$$

and yields Nambu-Golstone boson in the chiral limit

O(4) Model

analytic continuation

- first solve flow equation for the effective potential, $\partial_k U_k$
- substitute p_0 by continuous real frequency ω

$$\Gamma^{(2),R}_{k,j}(\omega) = \lim_{\epsilon \to 0} \Gamma^{(2),R}_{k,j}(p_0 = -i(\omega + i\epsilon), \vec{p} = 0); \quad \text{for} \quad j = \pi, \sigma$$

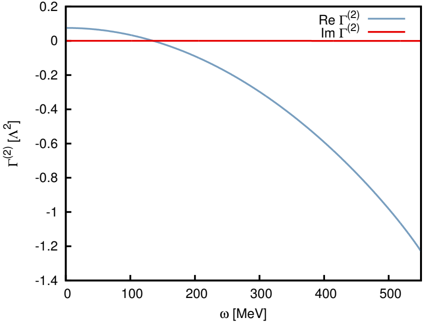
- then solve flow equations $\mathrm{Re}\,\partial_k \Gamma^{(2),R}_k$, $\mathrm{Im}\,\partial_k \Gamma^{(2),R}_k$ at global minimum of $U_{k\to 0}$
- finally, spectral functions are given by discontinuity of the propagators, i.e.

$$\rho_j(\omega) = -\frac{1}{\pi} \frac{\mathrm{Im}\,\Gamma^{(2),R}_j(\omega)}{\left(\mathrm{Re}\,\Gamma^{(2),R}_j(\omega)\right)^2 + \left(\mathrm{Im}\,\Gamma^{(2),R}_j(\omega)\right)^2}; \qquad \Gamma^{(2),R}_j(\omega) = \lim_{k\to 0} \Gamma^{(2),R}_{k,j}(\omega)$$

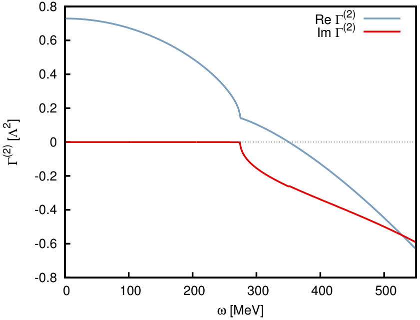
Results for O(4) Model in vacuum

Re $\Gamma^{(2),R}(\omega)$ and Im $\Gamma^{(2),R}(\omega)$

pion:



sigma:



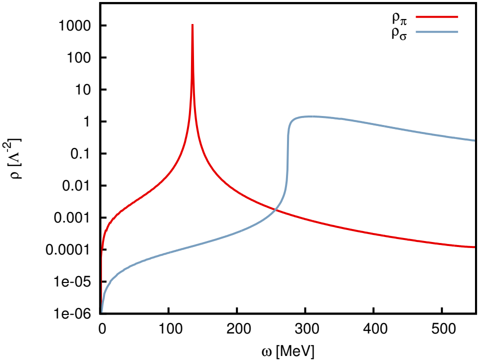
[K. Kamikado, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1302.6199 [hep-ph]]

Results for O(4) Model in vacuum

spectral functions

sigma and pion spectral functions

physical pion mass



[K. Kamikado, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1302.6199 [hep-ph]]

Spectral Functions in a Thermal Medium

Quark-Meson Model

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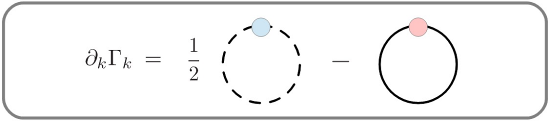
effective action:

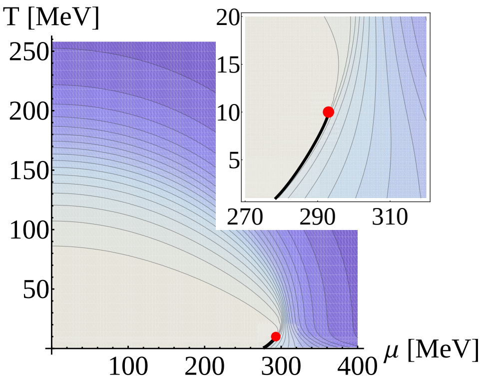
$$\Gamma_k[\bar{\psi}, \psi, \varphi] = \int d^4x \left\{ \bar{\psi} \left(\partial\!\!\!/ + h(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5) - \mu\gamma_0 \right) \psi + \frac{1}{2}(\partial_\mu \varphi)^2 + U_k(\varphi^2) - c\sigma \right\}$$

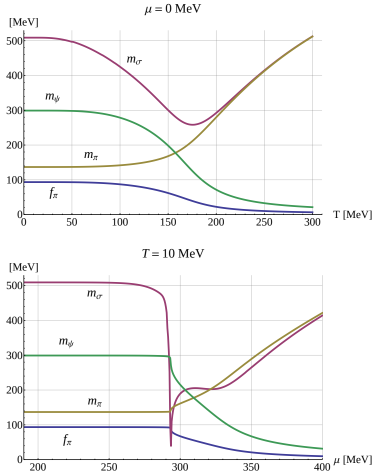
- effective low-energy model for QCD with two flavors
- describes spontaneous and explicit chiral symmetry breaking
- flow equation for the effective action:



Spectral Functions in the Medium

phase diagram, masses and order parameter

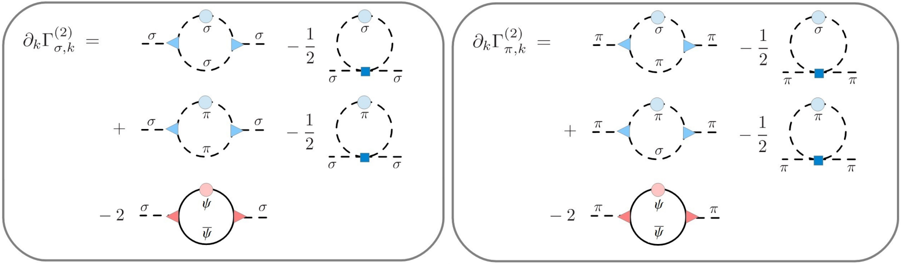




[R.-A. Tripolt, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1311.0630 [hep-ph]]

Spectral Functions in the Medium

flow of the two-point functions



- quark-meson vertices given by $\Gamma^{(2,1)}_{\bar{\psi}\psi\sigma} = h$, $\Gamma^{(2,1)}_{\bar{\psi}\psi\vec{\pi}} = ih\gamma^5\vec{\tau}$
- meson vertices from scale-dependent effective potential: $\Gamma^{(0,3)}_{\phi_i\phi_j\phi_m}$, $\Gamma^{(0,4)}_{\phi_i\phi_j\phi_m\phi_n}$

Results for Quark-Meson Model

spectral functions at μ = 0

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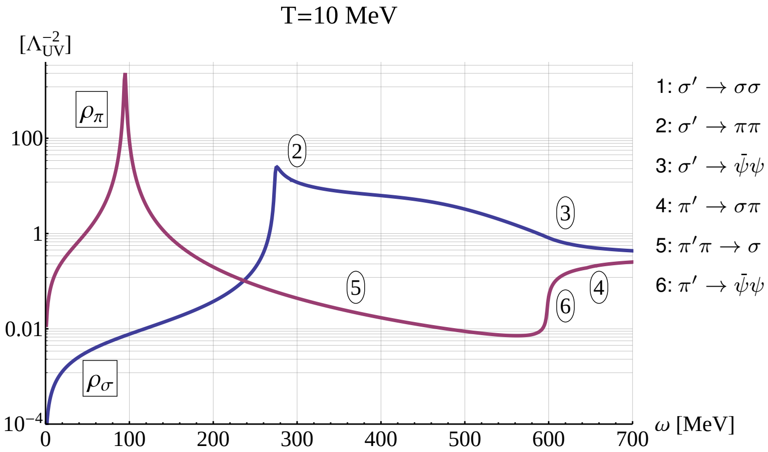
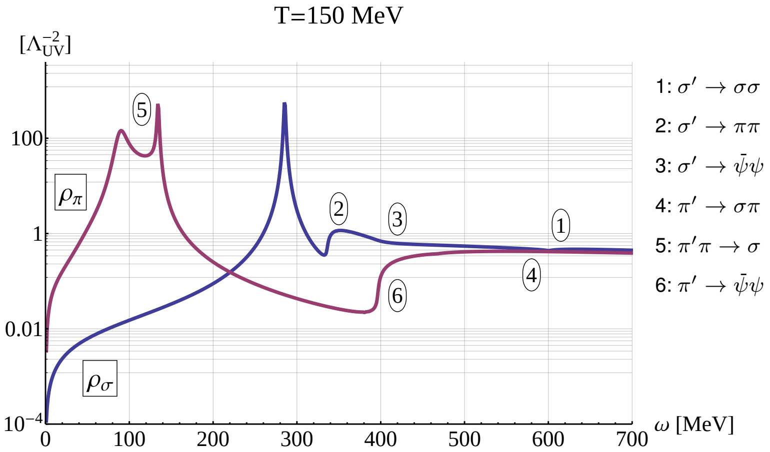


[R.-A. Tripolt, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1311.0630 [hep-ph]]

Results for Quark-Meson Model

spectral functions at μ = 0



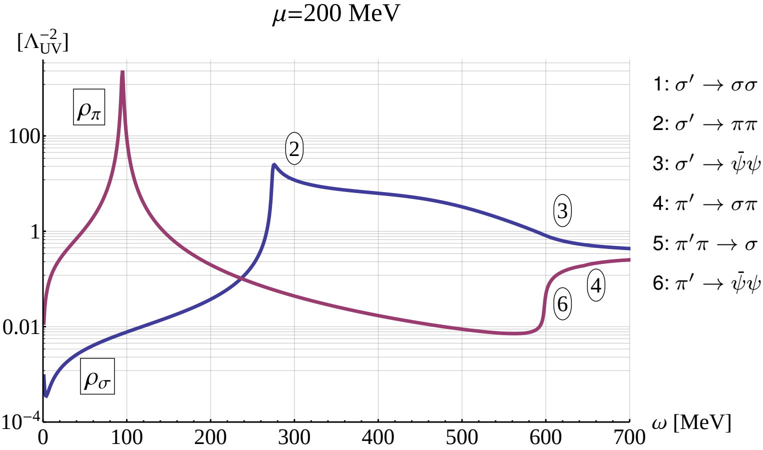
[R.-A. Tripolt, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1311.0630 [hep-ph]]

Temperature Evolution

animation

Results for Quark-Meson Model

spectral functions at finite μ



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Results for Quark-Meson Model

spectral functions at finite μ

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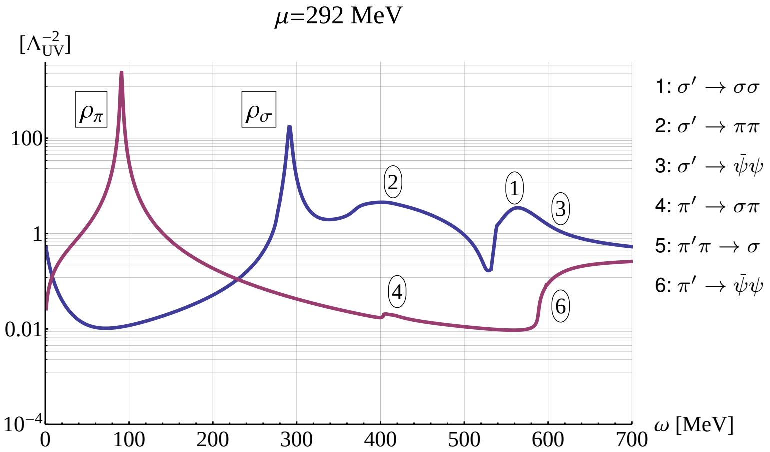
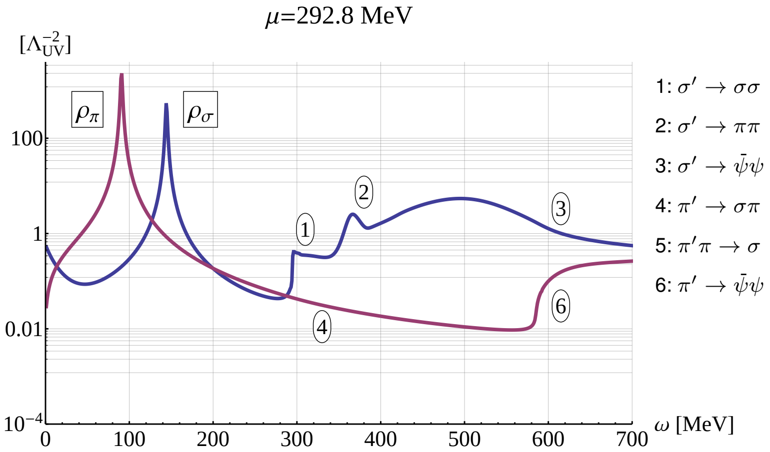


[R.-A. Tripolt, N. Strodthoff, L. von Smekal and J. Wambach, arXiv:1311.0630 [hep-ph]]

Results for Quark-Meson Model

spectral functions at finite μ



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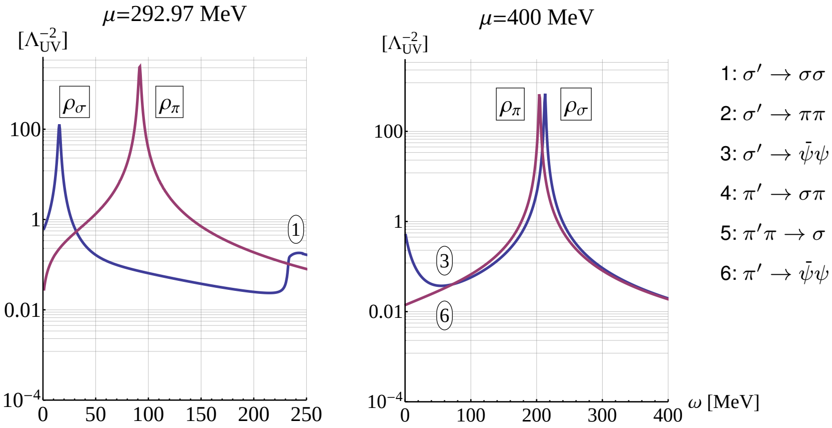

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Summary and Outlook

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- Part 1: in-medium vector mesons
  - hadronic (phen.) description of vector mesons in a hot and dense medium
  - application to low-mass dilepton spectra in HIC's
  - good account of the measurements (vector mesons acquire a large width)

- Part 2: spectral functions from the FRG
  - presented a tractable method to obtain hadronic spectral functions at finite T and μ from the FRG
  - results reveal complicated structure for in-medium spectral functions
  - inclusion of finite external spatial momenta will allow for calculation of transport coefficients like shear viscosity