ヘビーハドロン有効理論

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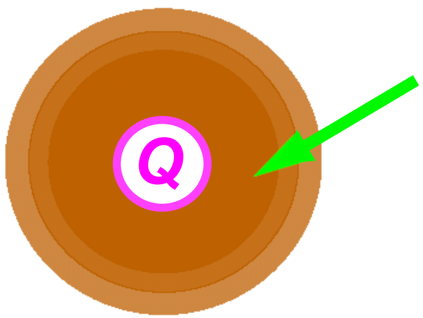
at「ヘビークォークハドロンと原子核のスペクトルと構造」研究会
(KEK, February 27, 2014)

Based on

- M.H., H.Hoshino and Y.L.Ma, Phys. Rev. D85, 114027 (2012)
- M.H. and Y.L.Ma, Phys. Rev. D 87, 056007 (2013)

1. Introduction

☆ Heavy-Light Mesons ($Q\bar{q}$ type) Baryons (Qqq)



"Light-quark cloud" (Brown Muck)

··· made of light quarks and gluons

typical energy scale ~ Λ_{QCD}

◎ **Heavy mesons ··· 3 or 3^{bar}, … of SU(3)$_l$**

◎ **Heavy baryons ··· 6, … of SU(3)$_l$**

Flavor representations, which do not exist in the light quark sector, give a new clue to understand the hadron structure.

Chiral partner structure

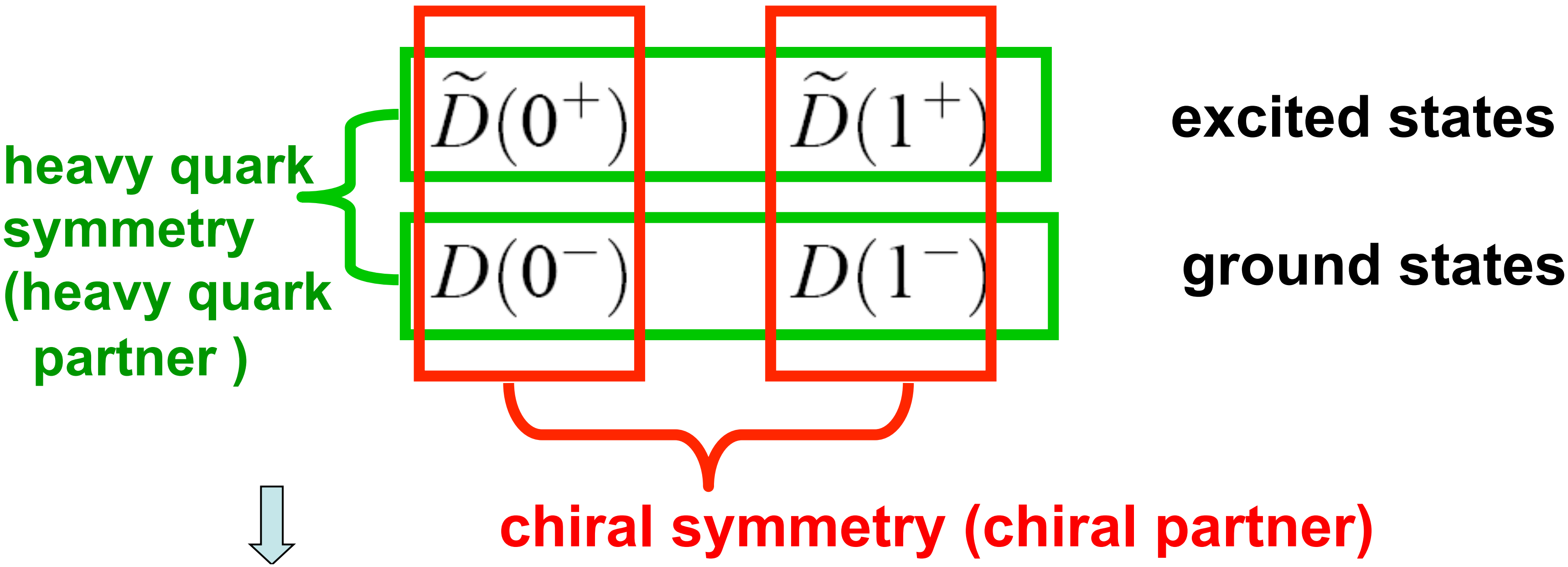
- Chiral symmetry breaking generates the mass splitting between chiral partners.
- examples :
  - N(940) left ⇔ N(940) right

    [N(940) ⇔ N*(1535)]
  - pi(130) ⇔ sigma(600)

    [pi(130) ⇔ rho(770)]
  - (D, D*) ⇔ (D_0*, D_1)

"chiral doubling"

M.A.Nowak, M.Rho and I.Zahed, PRD**48**, 4370 (1993)



$$M(0^+) - M(0^-) \simeq M(1^+) - M(1^-) \sim M_{\text{constituent}}$$

$$M_{D^*_{sJ}(2317)} - M_{D_s^\pm} = 349.3 \pm 0.5\,\text{MeV}$$

$$\mathbf{M_{D(0+,1+)} - M_{D(0-,1-)} \sim 0.43\ GeV}$$

Chiral doubling seems to work.

Outline

1. Introduction
2. Heavy Quark Symmetry
3. Heavy Meson Effective Theory (HMET)
4. HMET for Chiral doubling
5. Study of sigma meson structure using a model based on the chiral doubling
6. Chiral doubling in heavy baryons
7. Summary

2. Heavy Quark Symmetry

☆ Heavy Quark Symmetry

··· Symmetry existing in QCD at $M_Q \to \infty$ limit

◎ velocity super-selection rule

- One Heavy quark interacting with light quarks and gluons

$$P_{\rm light} \simeq \Lambda_{QCD} \qquad\qquad P_{\rm heavy} \simeq M_Q \cdot V$$

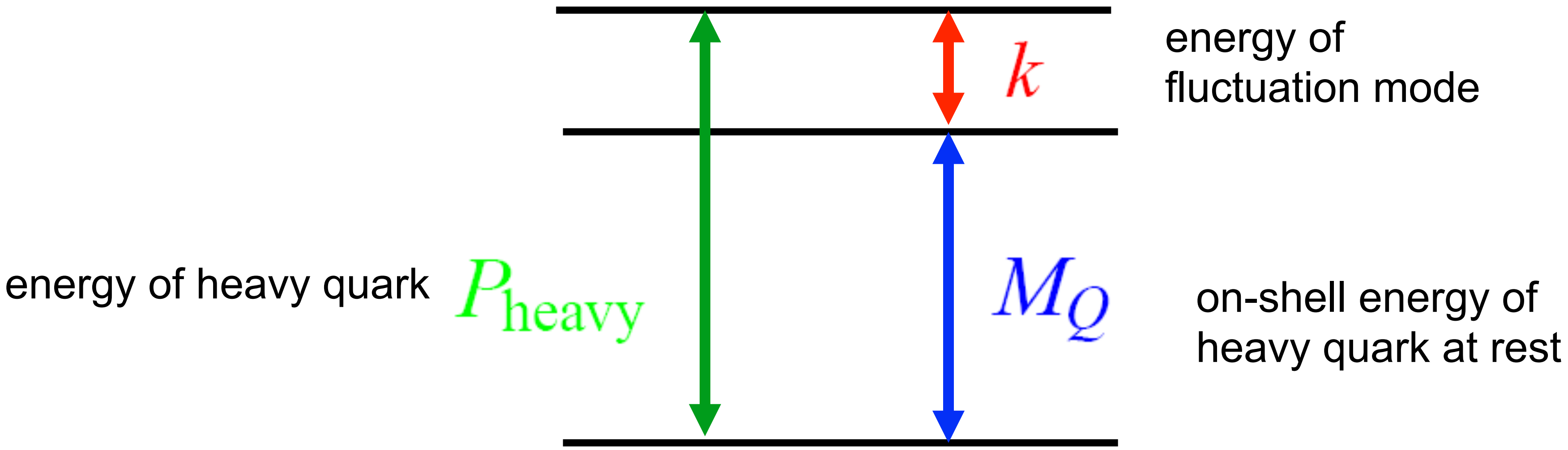
↑ Velocity of Heavy quark
: $V_\mu=(1,0,0,0)$ at rest frame

$\delta P_{\rm light}$ is transferred to the Heavy quark

$$\delta V \simeq \frac{\delta P_{\rm light}}{M_Q} \xrightarrow[M_Q \to \infty]{} 0$$

- Velocity of the heavy quark is not changed by the QCD interaction

- **Fluctuation mode** around the On-shell Heavy Quark



$$P^{\mu}_{\text{heavy}} = M_Q \cdot V^{\mu} + k^{\mu}$$

$$\frac{k^{\mu}}{M_Q} \simeq \frac{\Lambda_{\text{QCD}}}{M_Q} \ll 1$$

··· **expansion parameter**

☆ QCD Lagrangian (heavy quark sector)

$$\mathcal{L}_{\text{heavy}} = \bar{Q}(i\gamma^\mu D_\mu - M_Q)Q$$

$$D_\mu Q = (\partial_\mu - ig_s G_\mu)Q$$

◎ introducing the fluctuation mode

$$Q(x) = \sum_{V_\mu} e^{-iM_Q V\cdot x} Q_V(x)$$

$$Q_V^{(h)} = P_+ Q_V \; ; \quad Q_V^{(\chi)} = P_- Q_V \qquad \left(P_\pm = \frac{1 \pm V^\mu \gamma_\mu}{2} \;:\; \text{Projection} \right)$$

$$V^\mu \gamma_\mu Q_V^{(h)} = Q_V^{(h)} \; ; \quad V^\mu \gamma_\mu Q_V^{(\chi)} = Q_V^{(\chi)}$$

- $(i\gamma^\mu \partial_\mu - M_Q)\, Q$

$$\Rightarrow \; e^{-M_Q V\cdot x}\left[(M_Q \gamma^\mu V_\mu - M_Q)\, Q_V + (i\gamma^\mu \partial_\mu)\, Q_V\right]$$

$$\Rightarrow \; M_Q\,(\gamma^\mu V_\mu - 1)(Q_V^{(h)} + Q_V^{(\chi)}) + i\gamma^\mu \partial_\mu (Q_V^{(h)} + Q_V^{(\chi)})$$

$$= \; i\gamma^\mu \partial_\mu Q_V^{(h)} + (i\gamma^\mu \partial_\mu - 2M_Q)\, Q_V^{(\chi)}$$

$$\mathcal{L}_{\text{heavy}} \Rightarrow \left[\bar{Q}_V^{(h)} + \bar{Q}_V^{(\chi)}\right] \left[i\gamma^\mu D_\mu Q_V^{(h)} + (i\gamma^\mu \partial_\mu - 2M_Q)\, Q_V^{(\chi)}\right]$$

$$= \bar{Q}_V^{(h)} (i\gamma^\mu D_\mu)\, Q_V^{(h)} + \bar{Q}_V^{(\chi)} (i\gamma^\mu \partial_\mu - 2M_Q)\, Q_V^{(\chi)}$$

$$+ \bar{Q}_V^{(\chi)} (i\gamma^\mu D_\mu)\, Q_V^{(h)} + \bar{Q}_V^{(h)} (i\gamma^\mu \partial_\mu - 2M_Q)\, Q_V^{(\chi)}$$

$Q_V^{(h)}$ ··· **massless (effectively)**

$Q_V^{(\chi)}$ ··· **mass of 2M_Q (effectively)**

$\Downarrow$

eliminate $Q_V^{(\chi)}$ **using the EoM**

$$Q_V^{(\chi)} = \frac{1}{2M_Q} (i\gamma^\mu D_\mu)\, Q_V^{(h)} + O\left(\frac{1}{M_Q^2}\right)$$

$$\mathcal{L}_V = \bar{Q}_V^{(h)} (i\gamma^\mu D_\mu)\, Q_V^{(h)} + O\left(\frac{1}{M_Q}\right)$$

$$\bar{Q}_V^{(h)} \gamma^\mu Q_V^{(h)} = \bar{Q}_V^{(h)} \frac{1}{2} \{V_\nu \gamma^\nu, \gamma^\mu\}\, Q_V^{(h)} = \bar{Q}_V^{(h)} V^\mu Q_V^{(h)}$$

$$V^\mu \gamma_\mu Q_V^{(h)} = Q_V^{(h)} \; ; \quad \bar{Q}_V^{(h)} V^\mu \gamma_\mu \bar{Q}_V^{(h)}$$

$$\mathcal{L}_V = \bar{Q}_V^{(h)} (iV^\mu D_\mu)\, Q_V^{(h)} + \cdots$$

$$\mathcal{L}_V = \bar{Q}_V^{(h)} \left(iV^\mu D_\mu\right) Q_V^{(h)}$$

- $Q_V^{(h)}$: anihilate the heavy quark with velocity V_μ

  (not create anti-heavy quark)

  ⇒conservation of the heavy quark number

- No γ matrices at $M_Q \to \infty$ limit

  ⇒ conservation of the heavy quark spin

  ・・・ SU(2) spin symmetry

- N_h heavy quarks carrying same velocity of V_μ

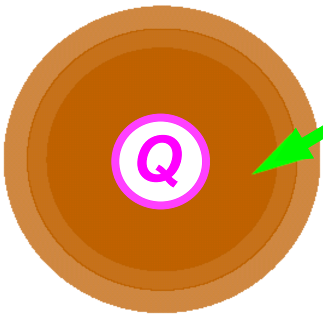
$$\mathcal{L}_V = \sum_{j=1}^{N_h} \bar{Q}_{V(j)}^{(h)} \left(iV^\mu D_\mu\right) Q_{V(j)}^{(h)} + \cdots$$

・・・ conservation of the flavor symmetry

⇒ SU($2N_h$) spin-flavor symmetry

3. Heavy Meson Effective Theory

☆ $Q\bar{q}$ type meson （Qqq type baryon）



"Light-quark cloud" (Brown Muck)
･･･ made from light quarks and gluons
typical energy ～ Λ_{QCD}

◎ total angular momentum
= spin of hadrons

angular moementum carrie
light degrees of freedom

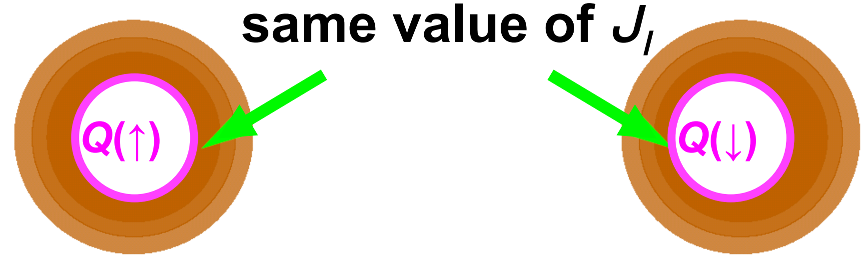
$$\vec{J} = \vec{S}_Q + \vec{J}_l$$

spin of heavy quark

・ $M_Q \to \infty$ limit

$$\left.\begin{array}{l} \left[\vec{S}_Q, H\right] = 0 \\ \left[\vec{J}, H\right] = 0 \end{array}\right\} \Rightarrow \left[\vec{J}_l, H\right] = 0$$

same value of J_l



$J = J_l + \frac{1}{2}$ $J = J_l - \frac{1}{2}$

Heavy Meson Multiplet
･･･ same mass (degenerate)

conservation of J_l
⇒ classify hadrons by J_l

◎ Parity of Mesons

・ Quark model ･･･ Brown muck ⇔ $\bar{q}$

$$\vec{J}_l = \vec{S}_l + \vec{L}$$

angular momentum ($\vec{L}$)

spin of light quark ($\vec{S}_l$)

$$\Rightarrow \quad P_{\text{light}} = -(-1)^L$$

Parity of meson ･･･ $P = (-1)^{L+1}$

◎ classification of mesons

A heavy meson multiplet includes $J_\pm = J_l \pm \frac{1}{2}$ states

・ $J_l = \frac{1}{2}$ ⇒ $J^P =$ $(0^-, 1^-)$ ･･･ grand state
$(0^+, 1^+)$ ･･･ excited state

・ $J_l = \frac{3}{2}$ ⇒ $J^P =$ $(1^+, 2^+)$ ･･･ excited state
････

☆ **multiplet at grandstate** ··· J_l=1/2 ; J^P = (0^- , 1^-)

Pseudoscalar meson *P* ; vector meson *P**

$$P = \begin{pmatrix} D^0 & D^+ & D_s \\ \\ B^- & B^0 & B_s \end{pmatrix} , \qquad P^* = \begin{pmatrix} D^{*0} & D^{*+} & D_s^* \\ \\ B^{*-} & B^{*0} & B_s^* \end{pmatrix}$$

- **fluctuation fields**

$$P = e^{-iM_Q V\cdot x} P' \ , \qquad P_\mu^* = e^{-iM_Q V\cdot x} P_\mu^{*\prime} \ ; \qquad V^\mu P_\mu^{*\prime} = 0$$

- **Bi-spinor field**

$$H_{al} \sim Q_a^{(h)} \bar{\psi}_l$$; $$\psi = \xi_L q_L + \xi_R q_R$$ ··· **constiuent quark field**

a ··· index of heavy flavor SU(2)

l ··· index of light flavor SU(3)

$$H_{al} = \left(\frac{1 + \not{V}}{2} \right) \left[i\gamma_5 P'_{al} + \gamma^\mu \left(P_\mu^{*\prime} \right)_{al} \right]$$

anihilate heavy meson

☆ transformation properties

$$H_{al} = \left(\frac{1+\not{V}}{2}\right)\left[i\gamma_5 P'_{al} + \gamma^\mu \left(P^{*\prime}_\mu\right)_{al}\right] \quad \sim \quad Q_a^{(h)}\bar{\psi}_l$$

$$\psi \to h(\pi, g_L, g_R)\psi$$

$$Q^{(h)} \to z_H Q^{(h)} \;, \quad z_H \in \mathrm{SU}(2)_H$$

$$Q^{(h)} \to S Q^{(h)} \;, \quad S \in \mathrm{SU}(2)_{\mathrm{spin}}$$

$$H_{al} \quad \underset{\text{heavy spin}}{\longrightarrow} \quad S \cdot H_{al}$$

$$H_{al} \quad \underset{\text{heavy flavor}}{\longrightarrow} \quad [z_H]_{aa'} \cdot H_{a'l}$$

$$H \quad \underset{\text{chiral}}{\longrightarrow} \quad H_{al'} \cdot \left[h^\dagger(\pi, g_L, g_R)\right]_{l'l}$$

☆ Effective Lagrangian for Heavy Pseudoscalar and Vector Mesons

- **covariant derivative**

$$D_\mu H \equiv \partial_\mu H + iH\alpha_{\mu\parallel}(\pi) \quad ; \quad \alpha_\parallel^\mu = \frac{1}{2i}\left(\partial^\mu\xi_R\cdot\xi_R^\dagger + \partial^\mu\xi_L\cdot\xi_L^\dagger\right)$$

$$\partial_\mu H \xrightarrow[\text{chiral}]{} (\partial_\mu H)\, h^\dagger(\pi, g_L, g_R) + H\left(\partial_\mu h^\dagger(\pi, g_L, g_R)\right)$$

$$\alpha_{\mu\parallel}(\pi) \xrightarrow[\text{chiral}]{} h(\pi, g_L, g_R)\alpha_{\mu\parallel}(\pi)h^\dagger(\pi, g_L, g_R) + \frac{1}{i}\partial_\mu h(\pi, g_L, g_R)h^\dagger(\pi, g_L, g_R)$$

- **Lagrangian (leading order ; *O*(*p*))**

$$\mathcal{L}_{(1)}/M_Q = \mathrm{tr}\left[H\left(iV^\mu D_\mu\right)\overline{H}\right] + k\,\mathrm{tr}\left[H\gamma_\mu\gamma_5\alpha_\perp^\mu\overline{H}\right]$$

$\overline{H} = \gamma_0 H^\dagger \gamma_0$ ･･･ **create heavy meson**

$\alpha_\perp^\mu = \frac{1}{2i}\left(\partial^\mu\xi_R\cdot\xi_R^\dagger - \partial^\mu\xi_L\cdot\xi_L^\dagger\right)$; ***k*** ･･･ **a constant with 0-dim**

☆ $D^* \to D\pi$ decay

$$k\,\mathrm{tr}\left[H\gamma_\mu\gamma_5\alpha_\perp^\mu \overline{H}\right] \Rightarrow 2i\frac{k}{F_\pi}\left\{\left[P_\mu^{*\prime}\right]_{al}\left[\partial^\mu\pi\right]_{ll'}\left[\overline{P}'\right]_{l'a} - \left[P'\right]_{al}\left[\partial^\mu\pi\right]_{ll'}\left[\overline{P}_\mu^{*\prime}\right]_{l'a}\right\} + \cdots$$

exp (old date, sorry)

$$\Gamma(D^{*0} \to D^0 + \pi^0) = \frac{p_\pi^3}{24\pi M_{D^{*0}}^2}\left(M_Q\frac{k}{F_\pi}\right)^2 \quad \cdots \quad < 1.3\ \text{MeV}$$

$$\Gamma(D^{*\pm} \to D^\pm + \pi^0) = \frac{p_\pi^3}{24\pi M_{D^{*\pm}}^2}\left(M_Q\frac{k}{F_\pi}\right)^2 \quad \cdots \quad 29.5 \pm 6.8\ \text{keV}$$

$$\Gamma(D^{*\pm} \to D^0 + \pi^\pm) = \frac{p_\pi^3}{12\pi M_{D^{*\pm}}^2}\left(M_Q\frac{k}{F_\pi}\right)^2 \quad \cdots \quad 65 \pm 15\ \text{keV}$$

$$p_\pi = \frac{\sqrt{\left(M_{D^*}^2 - (M_D + m_\pi)^2\right)\left(M_{D^*}^2 - (M_D - m_\pi)^2\right)}}{2M_{D^*}}$$

☆ determination of k ··· from $D^{*\pm} \to D^\pm \pi^0$

$$M_{D^{*\pm}} = 2010.1\ \text{MeV}\ ; \quad M_{D^\pm} = 1869.4\ \text{MeV}$$

$$M_{\pi^0} = 134.9766\ \text{MeV}\ ; \quad F_\pi = 92.42 \pm 0.26\ \text{MeV}$$

$$M_Q \equiv \frac{1}{4}\left(M_D + 3M_{D^*}\right) = 1974\ \text{MeV}$$

$$k = 0.59 \pm 0.07$$

4. Heavy Meson Effective Theory for Chiral doubling

Heavy meson multiplets

- **Ground states** ･･･ $J_l = 1/2$; $J^P = (0^-, 1^-)$
  - **Pseudoscalar meson** D ; **Vector meson** D^*
  - $D = (D^0, D^+, D_s)$ $D^* = (D^{*0}, D^{*+}, D_s^*)$

chiral partner

- **Excited states** ･･･ $J_l = 1/2$; $J^P = (0^+, 1^+)$
  - **Scalar meson** D_0^* ; **Axial-vector meson** D_1
  - $D_0^* = (D_0^{*0}, D_0^{*+}, D_{s0}^*)$ $D_1 = (D_1^0, D_1^+, D_{s1})$

Heavy meson effective field

☆ **Ground states** ･･･ $J_l = 1/2$; $J^P = (0^-, 1^-)$

Pseudoscalar meson *D* ; Vector meson *D**

$D = (D^0, D^+)$ $D^* = (D^{*0}, D^{*+})$

・ **Bi-spinor field** $H \sim Q\bar{\Psi}$; Ψ ･･･ light constiuent quark field

$$H = \frac{1+\not{v}}{2}\left[D^{*\mu}\gamma_\mu + iD\gamma_5\right]$$

annihilates heavy mesons (not generate)

☆ **Excited states** ･･･ $J_l = 1/2$; $J^P = (0^+, 1^+)$

Scalar meson D_0^* ; Axial-vector meson D_1

$D_0^* = (D_0^{*0}, D_0^{*+})$ $D_1 = (D_1^0, D_1^+)$

$$G = \frac{1+\not{v}}{2}\left[-iD_1^\mu\gamma_\mu\gamma_5 + D_0^*\right]$$

Heavy meson Lagrangian

◎ chiral doubler fields for heavy mesons

$$\mathcal{H}_L = \frac{1}{\sqrt{2}}[G + iH\gamma_5], \qquad \mathcal{H}_R = \frac{1}{\sqrt{2}}[G - iH\gamma_5]$$

$$\mathcal{H}_L \to \mathcal{H}_L g_L^\dagger, \qquad \mathcal{H}_R \to \mathcal{H}_R g_R^\dagger \qquad g_{L,R} \in SU(2)_{L,R}$$

◎ chiral field for pion

$$U = e^{2i\pi/f_\pi} = \xi^2 \to g_L\, U\, g_R^\dagger$$

Δ term generates mass difference between (D, D*) and (D_0*, D_1).

☆ model Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{heavy}} = &\ \frac{1}{2}\mathrm{Tr}\left[\bar{\mathcal{H}}_L i(v\cdot\partial)\mathcal{H}_L\right] + \frac{1}{2}\mathrm{Tr}\left[\bar{\mathcal{H}}_R i(v\cdot\partial)\mathcal{H}_R\right] \\ & -\frac{\Delta}{2}\mathrm{Tr}\left[\bar{\mathcal{H}}_L\mathcal{H}_L + \bar{\mathcal{H}}_R\mathcal{H}_R\right] - \frac{g_\pi F_\pi}{4}\mathrm{Tr}\left[U^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R + U\bar{\mathcal{H}}_R\mathcal{H}_L\right] \\ & + i\frac{g_A}{2}\mathrm{Tr}\left[\gamma^5\gamma^\mu\partial_\mu U^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R - \gamma^5\gamma^\mu\partial_\mu U\bar{\mathcal{H}}_R\mathcal{H}_L\right]\end{aligned}$$

5. Study of sigma meson structure using a model based on the chiral doubling

Based on

- M.H., H.Hoshino and Y.L.Ma, Phys. Rev. D85, 114027 (2012)

☆ Access to the quark condensate

"σ" particle ･･･ Quantum fluctuation of the condensate

A candidate

･･･ $f_0(500)$: lightest I=0 scalar meson

However, $f_0(500)$ may not be a qq^{bar} meson !

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(500)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $a_0(1450)$	$1^-(0^{++})$	• $\phi(2170)$	$0^-(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++}$
• $f_0(1500)$	$0^+(0^{++})$		or $4^{++})$
$f_1(1510)$	$0^+(1^{++})$	$\eta(2225)$	$0^+(0^{-+})$
• $f_2'(1525)$	$0^+(2^{++})$	[illegible]	$1^+(3^{--})$

PDG 2012

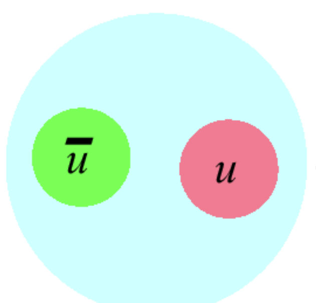
◎ Standard qq^{bar} quark model assignment

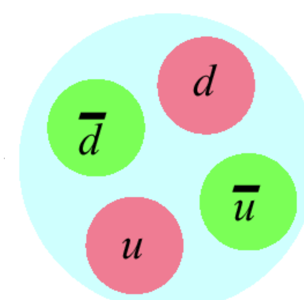
$n\,^{2s+1}\ell_J$	J^{PC}	$I = 1$ $u\bar{d},\ \bar{u}d,\ \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$I = \frac{1}{2}$ $u\bar{s},\ d\bar{s};\ \bar{d}s,\ -\bar{u}s$	$I = 0$ f'	$I = 0$ f
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^{\dagger}$	$h_1(1380)$	$h_1(1170)$
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^{\dagger}$	$f_1(1420)$	$f_1(1285)$

What is $f_0(500)$?

2 quark (qq^{bar}) state

4 quark (qqq^{bar}q^{bar}) state





“σ” particle

Exotic hadron

Outline of this section

5. Study of sigma meson structure using a model based on the chiral doubling
   A) Quark Structure of Scalar Mesons
   B) Linear sigma model for light quark sector including σ meson
   C) An effective model for σ meson coupling to heavy mesons
   D) σ meson in $D_1 \to D\pi\pi$ decay

A) Quark Structure of Scalar Mesons

2-quark picture of scalar mesons

☆ **2-quark picture**

f_0	$I=0$	$\cdots\ \bar{s}s$
κ	$I=1/2$	$\cdots\ \bar{s}u\ ,\quad \bar{s}d\ ,\quad \bar{u}s\ ,\quad \bar{d}s$
a_0	$I=1$	$\cdots\ \bar{u}u\ ,\quad \bar{d}d\ ,\quad \bar{u}d\ ,\quad \bar{d}u$
σ	$I=0$	(same as a_0)

⇔ φ

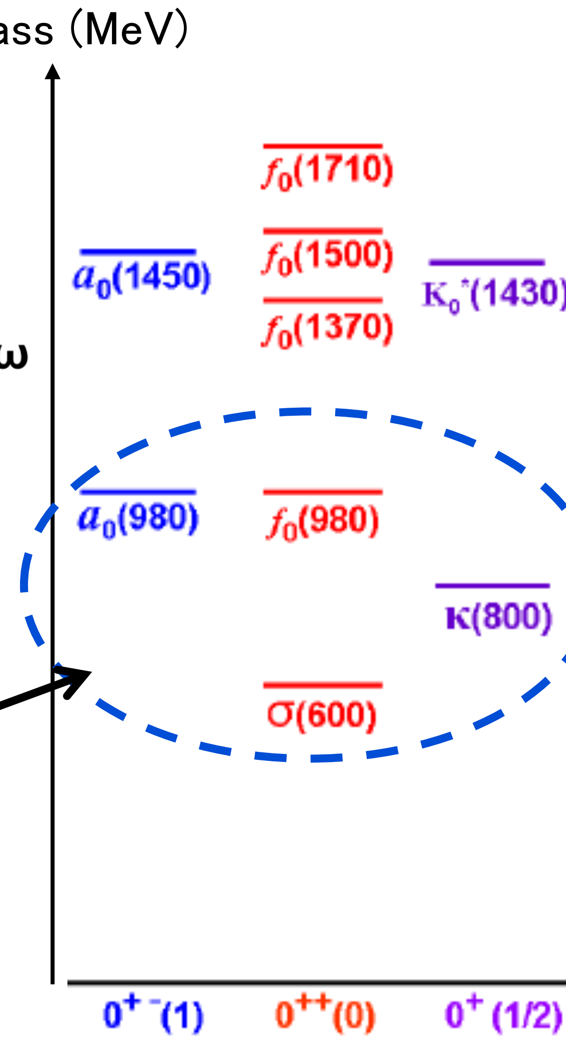
⇔ K^*

⇔ ρ, ω

⇩

⊙ naive expectation of mass hierarchy in $\bar{q}q$ picture

$$M_{f_0} > M_{\kappa} > M_{a_0} = M_{\sigma}$$



Contradiction ?

4-quark picture of scalar mesons

☆ **diquark**

one-gluon exchange gives an attraction in

$$3_c \times 3_c \to \bar{3}_c$$

◎ scalar diquarks in flavor SU(3)

$$\bar{S} = ud \ , \quad \bar{D} = su \ , \quad \bar{U} = ds$$

◎ scalar anti-diquarks in flavor SU(3)

$$S = \bar{u}\bar{d} \ , \quad D = \bar{s}\bar{u} \ , \quad U = \bar{d}\bar{s}$$

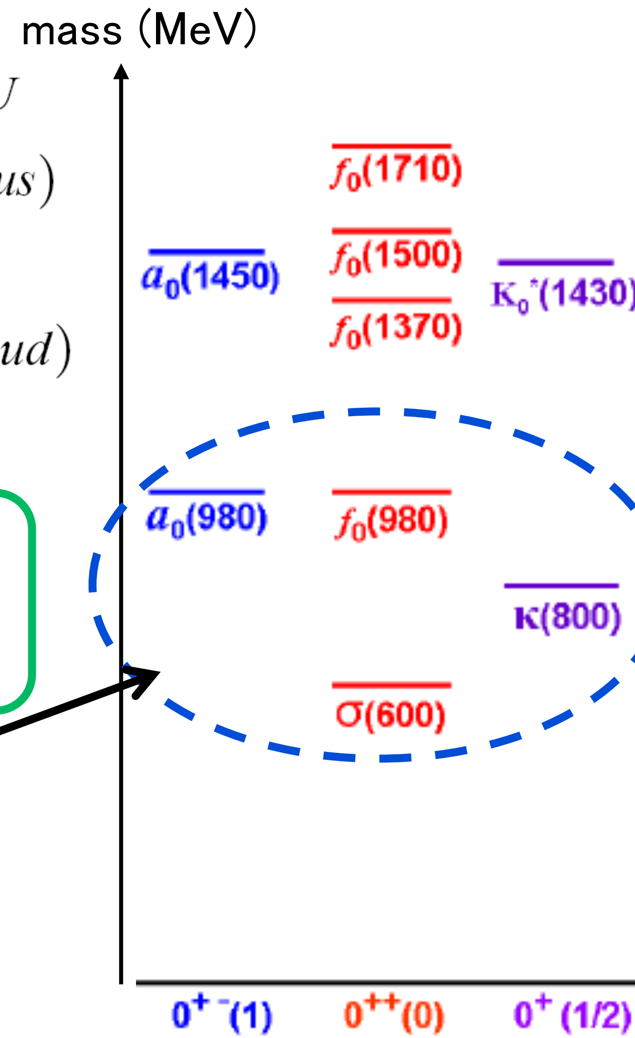
4-qurark picture of scalar mesons

f_0	$I=0$	$\Big\}\cdots$	$\bar{U}U\,,\quad \bar{D}D\,,\quad \bar{U}D\,,\quad \bar{D}U$
a_0	$I=1$		$(\bar{s}\bar{d}ds\,,\quad \bar{s}\bar{u}us\,,\quad \bar{s}\bar{u}ds\,,\quad \bar{s}\bar{d}us)$
κ	$I=1/2$	$\cdots$	$\bar{S}U\,,\quad \bar{S}D\,,\quad \bar{U}S\,,\quad \bar{D}S$
			$(\bar{u}\bar{d}ds\,,\quad \bar{u}\bar{u}us\,,\quad \bar{s}\bar{d}ud\,,\quad \bar{s}\bar{u}ud)$
σ	$I=0$	$\cdots$	$\bar{S}S \quad (\bar{u}\bar{d}ud)$

⊙ naive expectation of mass hierarchy in $\overline{q}\overline{q}qq$ picture

$$M_{f_0} = M_{a_0} > M_\kappa > M_\sigma$$

Consistent ?

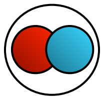


B) Linear sigma model for light quark sector including σ meson

2 and 4 quark states in linear sigma model

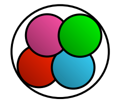
Amir H. Fariborz, Renata Jora, and Joseph Schechter, PRD 72, 034001 (2005)

3×3 matrix fields M & M' (Linear Sigma Model):

 2 quark field ~ $q_L \bar{q}_R$

$$M = S + i\phi$$

Scalar Pseudo scalar

 4 quark field ~ $q_R q_R \bar{q}_L \bar{q}_L$

$$M' = S' + i\phi'$$

Scalar Pseudo scalar

Scalar mesons in 3x3 matrix

☆ **scalar fields in 2-quark picture**

$$
S = \begin{pmatrix} (\sigma + a_0^0)/\sqrt{2} & a_0^+ & \kappa^+ \\ a_0^- & (\sigma - a_0^0)/\sqrt{2} & \kappa_0 \\ \kappa^- & \bar{\kappa}^0 & f_0 \end{pmatrix} \sim \begin{pmatrix} \bar{u}u & \bar{d}u & \bar{s}u \\ \bar{u}d & \bar{d}d & \bar{s}d \\ \bar{u}s & \bar{d}s & \bar{s}s \end{pmatrix}
$$

☆ **scalar fields in 4-quark picture**

$$
S' = \begin{pmatrix} (f_0 + a_0^0)/\sqrt{2} & a_0^+ & \kappa^+ \\ a_0^- & (f_0 - a_0^0)/\sqrt{2} & \kappa_0 \\ \kappa^- & \bar{\kappa}^0 & \sigma \end{pmatrix} \sim \begin{pmatrix} \bar{s}\bar{d}ds & \bar{s}\bar{d}us & \bar{s}\bar{d}ud \\ \bar{s}\bar{u}ds & \bar{s}\bar{u}us & \bar{s}\bar{u}ud \\ \bar{u}\bar{d}ds & \bar{u}\bar{d}us & \bar{u}\bar{d}ud \end{pmatrix}
$$

2 and 4 quark states in linear sigma model

Amir H. Fariborz, Renata Jora, and Joseph Schechter, PRD 72, 034001 (2005)

3×3 matrix fields M & M' (Linear Sigma Model):

2 quark field ~ $q_L \bar{q}_R$ 4 quark field ~ $q_R q_R \bar{q}_L \bar{q}_L$

$M = S + i\phi$ (Scalar, Pseudo scalar) $M' = S' + i\phi'$ (Scalar, Pseudo scalar)

These transform in the same way under SU(3)L×SU(3)R :

$$M \to U_L M U_R{}^\dagger \qquad M' \to U_L M' U_R{}^\dagger$$

[SU(3)R× SU(3)L: $q_L \to U_L q_L, q_R \to U_R q_R$]

Different transformations under U(1)A :

$$M \to M e^{+2i\nu} \qquad M' \to M' e^{-4i\nu}$$

[U(1)A: $q_L \to \exp(+i\nu) q_L, q_R \to \exp(-i\nu) q_R$]

$U(1)_A$ Symmetry ?

◎ **Anomaly is suppressed in the large Nc QCD**

Current is conserved.

$U(1)_A$ is spontaneously broken
by the quark condensate.

◎ **Definition of the spontaneously broken charge**

Light-front axial charge is well-defined.

S. Weinberg, Phys. Rev. 177 (1969) 2604.

mixing

When the $U(1)_A$ symmetry exists, **2-quark state** and **4-quark state** do **not mix** with each other. But, the $U(1)_A$ symmetry is broken
- by **anomaly** explicitly
- by **spontaneous chiral symmetry breaking**

⇒ mixing between 2-quark state and 4-quark state

$$\begin{pmatrix} \phi_\pi \\ \phi'_\pi \end{pmatrix} = \begin{pmatrix} \cos\theta_\pi & -\sin\theta_\pi \\ \sin\theta_\pi & \cos\theta_\pi \end{pmatrix} \begin{pmatrix} \pi(140) \\ \pi(1300) \end{pmatrix}$$

$$\begin{pmatrix} \bar{u}u + \bar{d}d \\ \bar{s}s \\ \bar{u}\bar{s}us + \bar{d}\bar{s}ds \\ \bar{u}\bar{d}ud \end{pmatrix} = \begin{pmatrix} U_{1\sigma} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \sigma \\ f_0 \\ \\ \\ \end{pmatrix} \begin{matrix} \textbf{Lightest} \\ \textbf{2}^{\textbf{nd}} \\ \textbf{3}^{\textbf{rd}} \\ \textbf{Heaviest} \end{matrix}$$

An effective Lagrangian

Linear sigma model including 2-nonet fields

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) - \frac{1}{2}\mathrm{Tr}(\partial_\mu M' \partial_\mu M'^\dagger)$$
$$-V_0(M, M') - V_{SB}$$

$$V_0 = V_{inv} + V_{anom}$$

V_{inv} : **SU(3)L×SU(3)R invariant, U(1)A invariant.**

V_{anom} : **SU(3)L×SU(3)R invariant, U(1)A breaking (anomaly).**

constrained by anomaly matching with QCD

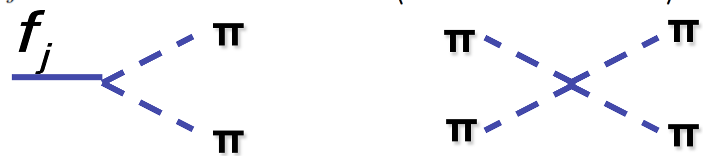
V_{SB} : Explicit SU(3)L×SU(3)R×U(1)A breaking terms.

(effects of current quark masses)

ππ scattering in the linear σ model

Relations among coupling constants due to the chiral symmetry

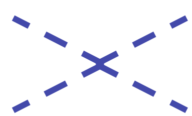
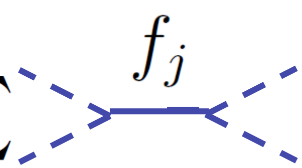
$$\sum_{j=1}^{4} \frac{1}{m_j^2} \left\langle \frac{\partial^3 V}{\partial \pi^0 \partial \pi^0 \partial f_j} \right\rangle^2 = \frac{1}{3} \left\langle \frac{\partial^4 V}{\partial \pi^0 \partial \pi^0 \partial \pi^0 \partial \pi^0} \right\rangle$$

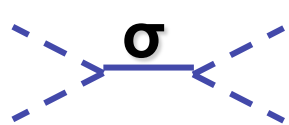


$$\sum_{j=1}^{4} \frac{1}{m_j^4} \left\langle \frac{\partial^3 V}{\partial \pi^0 \partial \pi^0 \partial f_j} \right\rangle^2 = \frac{2}{F_\pi^2}$$

ππ scattering amplitude includes σππ couping and sigma mass

in the low energy region

$$A(s,t,u) =$$  $$+ \sum_{j=1}^{4}$$ 

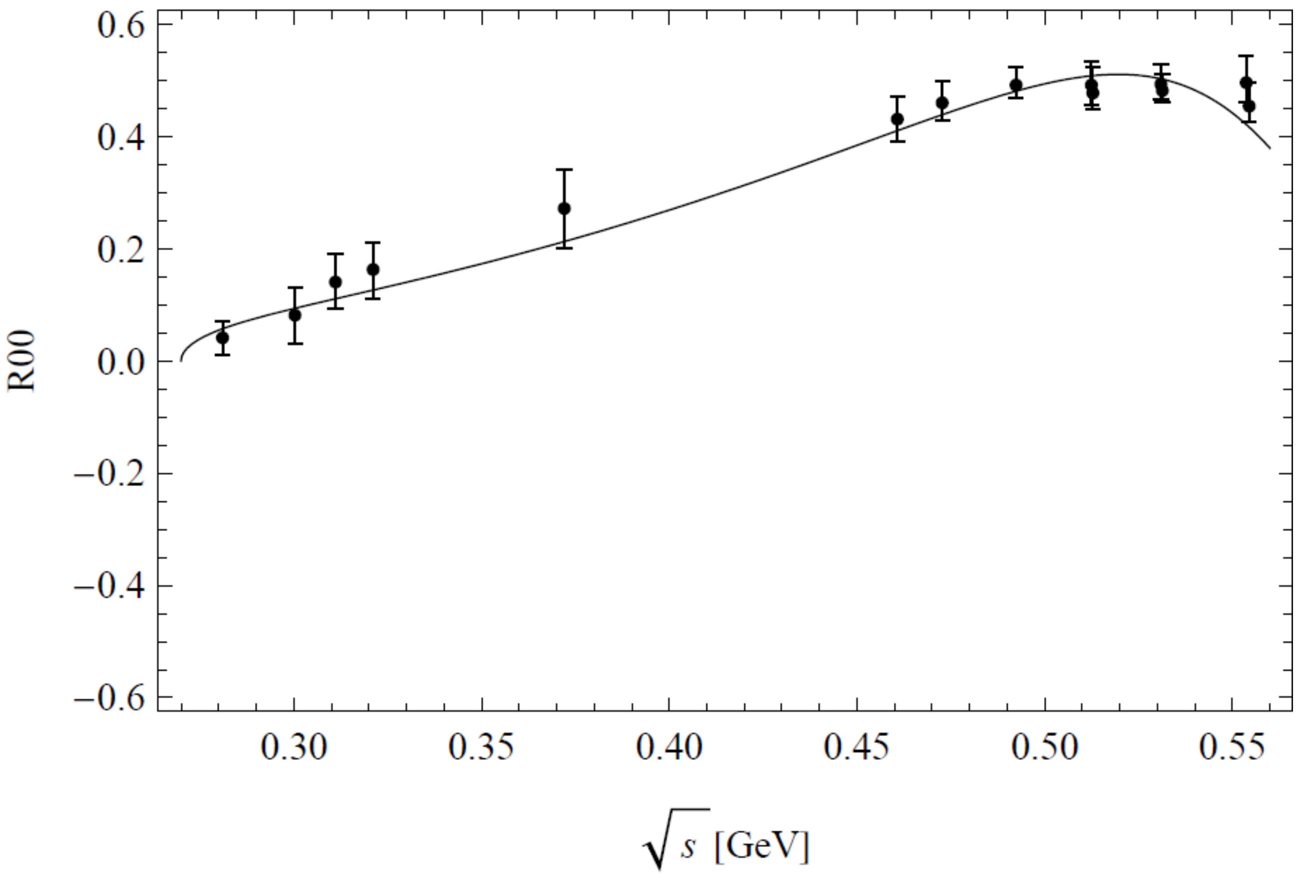
$$\sim$$ 

$$- \frac{g_{\sigma\pi\pi}^2}{m_\sigma^2} + \left(\frac{2}{F_\pi^2} - \frac{g_{\sigma\pi\pi}^2}{(m_\sigma^2)^2} \right) s$$

$$\left(g_{\sigma\pi\pi}^2 \equiv \left\langle \frac{\partial^3 V}{\partial f_1 \partial \pi \partial \pi} \right\rangle^2 \right)$$

expansion of $\frac{s}{m_j^2}$ **and approximation** $\left(\sqrt{s} \le 560\ MeV\right)$

Fit to ππ scattering data



$$T_0^0(s) \sim \rho(s)\left[F_1\left(s;\, g_{\sigma\pi\pi},\, m_\sigma\right) - \frac{g^2_{\sigma\pi\pi}}{m^2_\sigma} + \left(\frac{2}{F^2_\pi} - \frac{g^2_{\sigma\pi\pi}}{(m^2_\sigma)^2}\right) s \right]$$

$$g^2_{\sigma\pi\pi} \sim (2.2\,GeV)^2$$

$$m^2_\sigma \sim 606\,MeV$$

C) An effective model for σ meson coupling to heavy mesons

Heavy meson effective field

☆ **Ground states** ･･･ $J_l = 1/2$; $J^P = (0^-, 1^-)$

Pseudoscalar meson *D* ; **Vector meson *D****

$D = (D^0, D^+, D_s)$ $D^* = (D^{*0}, D^{*+}, D_s^*)$

・ **Bi-spinor field** $H \sim Q\bar{\Psi}$; Ψ ･･･ **light constiuent quark field**

$$H = \frac{1+\not{v}}{2}[D^{*\,\mu}\gamma_\mu + iD\gamma_5]$$

annihilates heavy mesons (not generate)

☆ **Excited states** ･･･ $J_l = 1/2$; $J^P = (0^+, 1^+)$

Scalar meson D_0^* ; **Axial-vector meson D_1**

$D_0^* = (D_0^{*0}, D_0^{*+}, D_{s0}^*)$ $D_1 = (D_1^0, D_1^+, D_{s1})$

$$G = \frac{1+\not{v}}{2}[-iD_1^\mu\gamma_\mu\gamma_5 + D_0^*]$$

Heavy meson effective field

◎ chiral doubler fields

$$\mathcal{H}_L = \frac{1}{\sqrt{2}}[G + iH\gamma_5], \qquad \mathcal{H}_R = \frac{1}{\sqrt{2}}[G - iH\gamma_5]$$

- transformation under the chiral symmetry

$$\mathcal{H}_L \to \mathcal{H}_L g_L^\dagger, \qquad \mathcal{H}_R \to \mathcal{H}_R g_R^\dagger \qquad tr\left[H_L M \overline{H}_R\right]$$

- **transformation under U(1)A**

$$\mathcal{H}_L \to \mathcal{H}_L e^{-i\nu}, \qquad \mathcal{H}_R \to \mathcal{H}_R e^{i\nu}$$

$M \to M e^{+2i\nu}$ 2-quark meson

$M' \to M' e^{-4i\nu}$ 4-quark meson

$tr\left[H_L M \overline{H}_R\right]$ **is allowed, but** $tr\left[H_L M' \overline{H}_R\right]$ **is not**

→ **only 2-quark meson can couple to heavy meson**

Model Lagrangian

$$\mathcal{L}_{\text{heavy}} = \frac{1}{2}\text{Tr}\left[\bar{\mathcal{H}}_L i(v\cdot\partial)\mathcal{H}_L\right] + \frac{1}{2}\text{Tr}\left[\bar{\mathcal{H}}_R i(v\cdot\partial)\mathcal{H}_R\right]$$
$$-\frac{\Delta}{2}\text{Tr}\left[\bar{\mathcal{H}}_L\mathcal{H}_L + \bar{\mathcal{H}}_R\mathcal{H}_R\right]$$
$$-\frac{g_\pi}{4}\text{Tr}\left[M^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R + M\bar{\mathcal{H}}_R\mathcal{H}_L\right]$$
$$+i\frac{g_A}{2F_\pi}\text{Tr}\left[\gamma^5\gamma^\mu\partial_\mu M^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R - \gamma^5\gamma^\mu\partial_\mu M\bar{\mathcal{H}}_R\mathcal{H}_L\right]$$

Δ term generates mass difference between (D, D*) and (D_0*, D_1).
We use physical masses as inputs to determine Δ.

Detemination of parameters

- g_A etemined from D* → D π decay

$$\mathcal{L}_{D^* D\pi} = -\frac{i g_A}{F_\pi} D\partial_\mu \Phi D^{*\mu\dagger} + \text{H.c.}$$

$$\begin{pmatrix} \phi_\pi \\ \phi'_\pi \end{pmatrix} = \begin{pmatrix} \cos\theta_\pi & -\sin\theta_\pi \\ \sin\theta_\pi & \cos\theta_\pi \end{pmatrix} \begin{pmatrix} \pi(140) \\ \pi(1300) \end{pmatrix}$$

$$|g_A \cos\theta_\pi| = 0.56$$

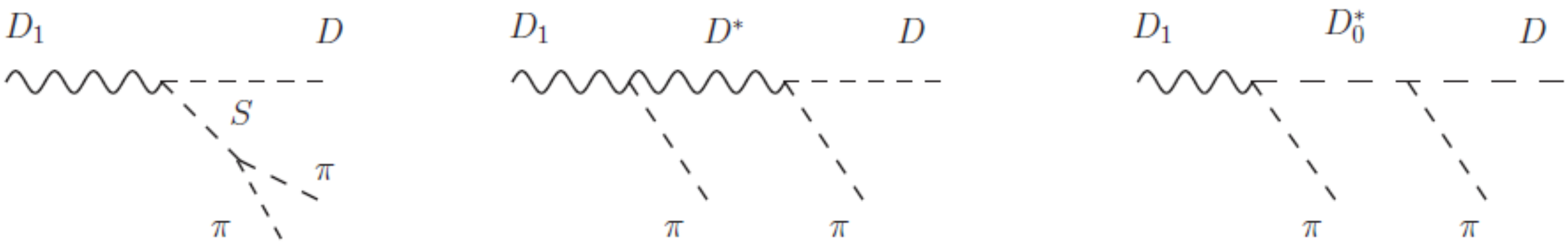
- g_π etemined from D_0* → D π decay

$$\mathcal{L}_{D_0^* D\pi} = \frac{i g_\pi}{2} D\Phi D_0^{*\dagger} + \text{H.c.}$$

$$|g_\pi \cos\theta_\pi| = 3.61$$

D) σ meson in $D_1 \to D\pi\pi$ decay

$D_1 \to D\pi\pi$ decay amplitude



$$\mathcal{M} = -\sqrt{2m_G m_H}\frac{g_A}{F_\pi}\epsilon_\mu(v)\left\{\sum_{i=1}^{4} g_{f_i\pi\pi}\frac{(p_{\pi_1}+p_{\pi_2})^\mu}{s-m_{f_i}^2+im_{f_i}\Gamma_{f_i}(s)}(U_f^{-1})_{ai} + \frac{g_\pi}{2\sqrt{2}}\cos^2\theta_\pi\frac{p_{\pi_2}^\mu - v^\mu v\cdot p_{\pi_2}}{v\cdot k_{D^*}+i\Gamma_{D^*}/2} + \frac{g_\pi}{2\sqrt{2}}\cos^2\theta_\pi\frac{p_{\pi_1}^\mu}{v\cdot k_{D_0^*}+i\Gamma_{D_0^*}/2}\right\}$$

reduced in the low energy region [s << m_{fj} (j=2,3,4)]

$$\mathcal{M} = -\sqrt{2m_G m_H}\frac{g_A\cos\theta_\pi}{F_\pi}\epsilon_\mu(v)\left\{h\, g_{\sigma\pi\pi}\frac{(p_{\pi_1}+p_{\pi_2})^\mu}{s-m_\sigma^2+im_\sigma\Gamma_\sigma(s)} - (p_{\pi_1}+p_{\pi_2})^\mu\left(\frac{\sqrt{2}}{F_\pi}-\frac{h\, g_{\sigma\pi\pi}}{m_\sigma^2}\right) + \frac{g_\pi\cos\theta_\pi}{2\sqrt{2}}\frac{p_{\pi_2}^\mu - v^\mu v\cdot p_{\pi_2}}{v\cdot k_{D^*}+i\Gamma_{D^*}/2} + \frac{g_\pi\cos\theta_\pi}{2\sqrt{2}}\frac{p_{\pi_1}^\mu}{v\cdot k_{D_0^*}+i\Gamma_{D_0^*}/2}\right\}$$

mixing parameter

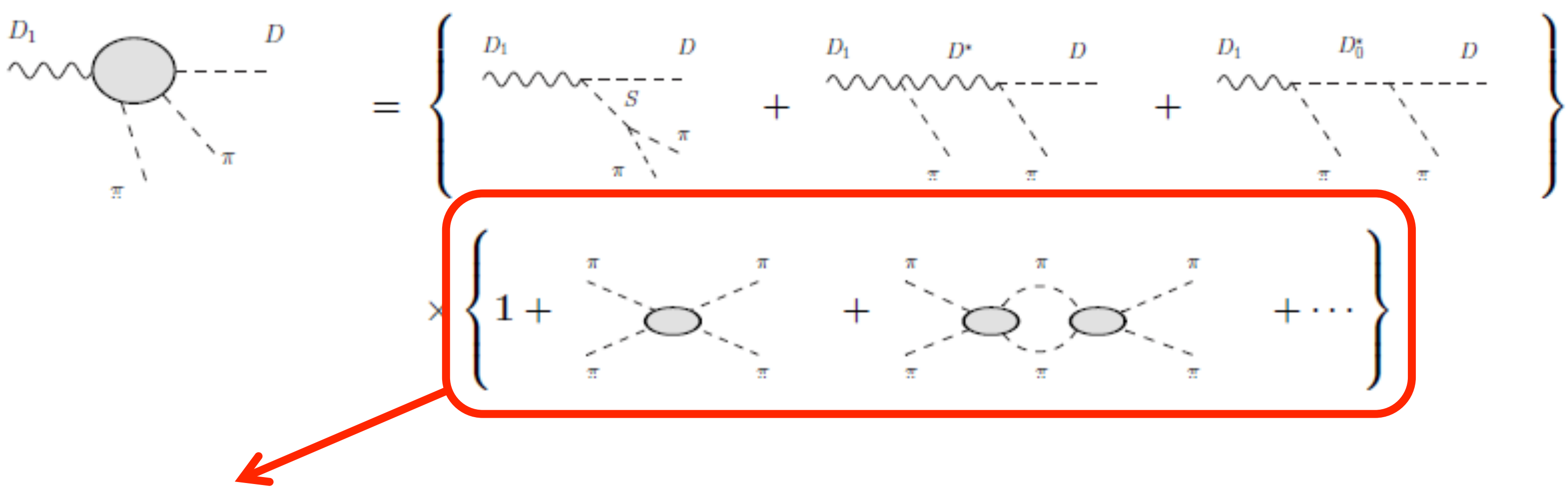
$$h = \frac{(U_f^{-1})_{a1}}{\cos\theta_\pi}$$

$$\begin{pmatrix} u\bar{u}+d\bar{d} \\ s\bar{s} \\ us\overline{us}+ds\overline{ds} \\ ud\overline{ud} \end{pmatrix} = \begin{pmatrix} U_{1\sigma} & \cdots \\ \vdots & 4\times 4\ matrix \end{pmatrix}\begin{pmatrix} \sigma \\ (f_0)_2 \\ (f_0)_3 \\ (f_0)_4 \end{pmatrix}$$

Isospin & partial wave projection

To make the sigma meson contribution clearer, we made the projection of the amplitude onto I = 0, S-wave amplitude.

From this, we can see that the final state interaction do not change the decay width.



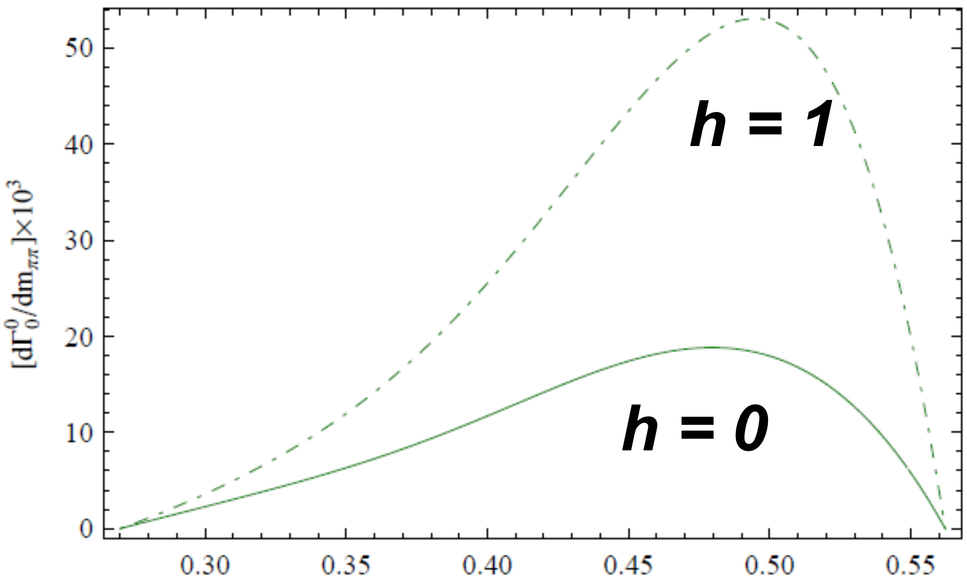
$e^{i\delta}$se shift ofscattering)

$$\Gamma \propto |\mathrm{Amp}|^2 = \left|\mathcal{M} e^{i\delta}\right|^2 = |\mathcal{M}|^2$$

$D_1 \to D\pi\pi$ decay width

Note : There is 4-way ambiguity of signs of $g_{\sigma\pi\pi}$ and g_π .

$g_{\sigma\pi\pi} > 0$
$g_\pi > 0$



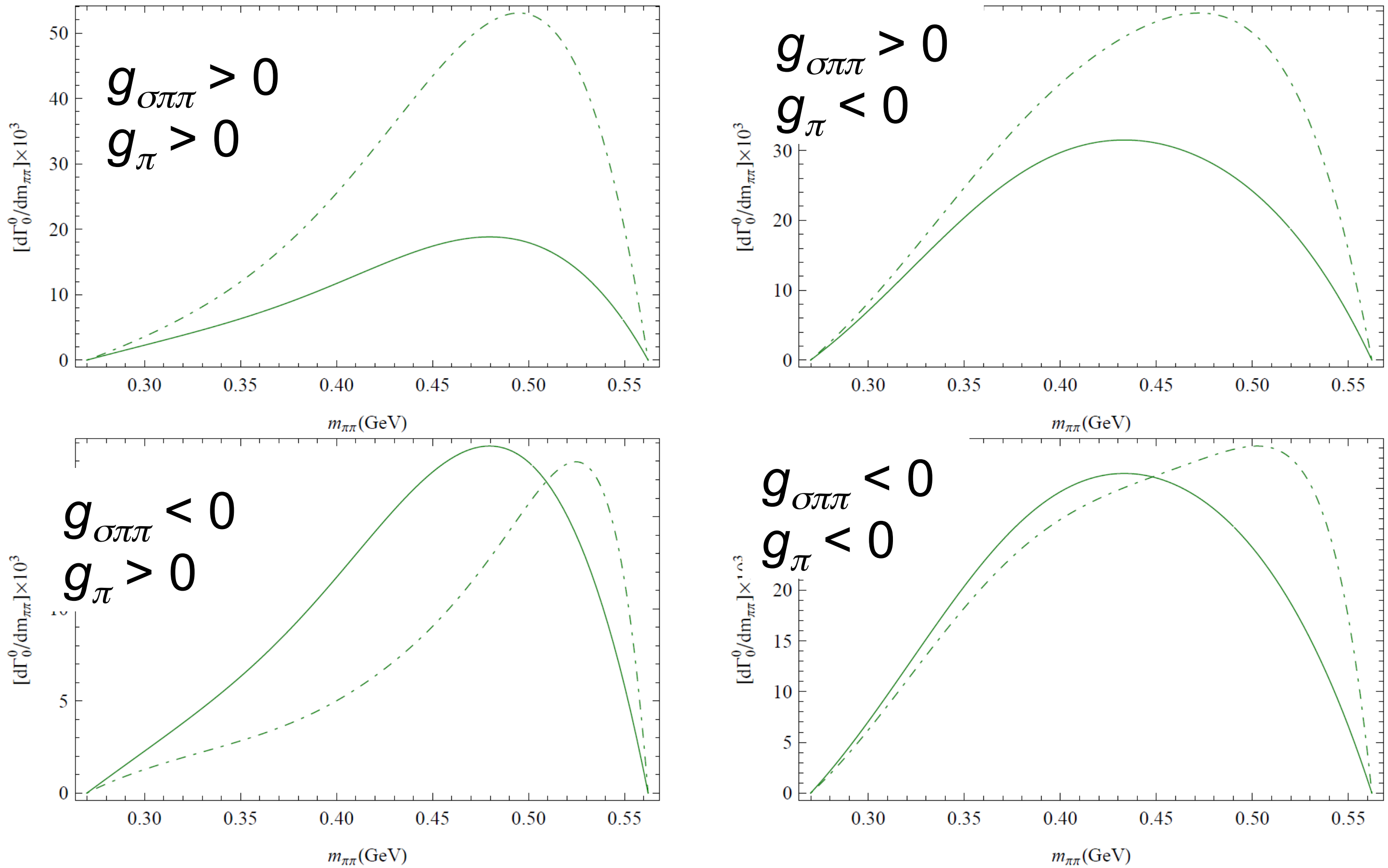
$$h = \frac{(U_f^{-1})_{a1}}{\cos\theta_\pi}$$

We expect that π is dominantly 2-quark state, so $\cos\theta_\pi$ 1.

$\left(U_f^{-1}\right)_{a1} = 1 \Leftrightarrow$ sigma is pure 2-quark state

$\left(U_f^{-1}\right)_{a1} = 0 \Leftrightarrow$ sigma is pure 4-quark state

$D_1 \rightarrow D\pi\pi$ decay width



Constituent of sigma meson may be detemined by future experiment

6. Chiral doubling in heavy baryons

Based on

- M.H. and Y.L.Ma, Phys. Rev. D 87, 056007 (2013)

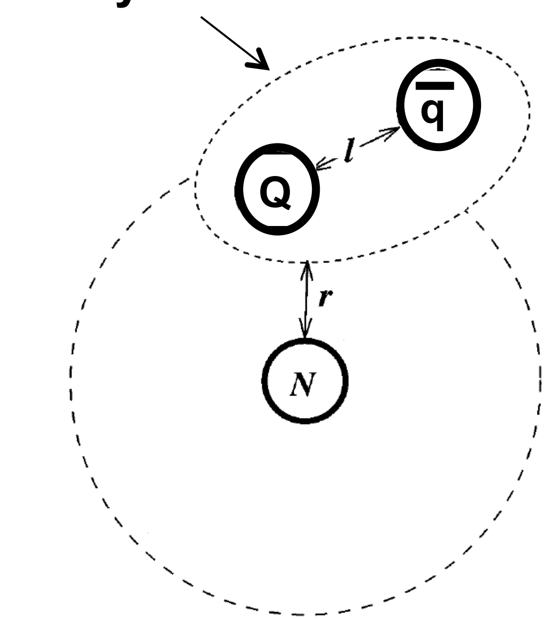
Chiral doubling in heavy baryons

・・・ based on the boundstate approach to heavy baryons

☆ Boundstate approach

heavy baryons (qqQ type)

= heavy meson ($q^{bar}Q$) bound to nculeon (qqq) as a soliton



	r = 0	r = 1
l=0 D(0⁻,1⁻)	$\Lambda_Q(\frac{1}{2}^+)$ $\{\Sigma_Q(\frac{1}{2}^+),\Sigma_Q(\frac{3}{2}^+)\}$	$\{\Lambda_Q(\frac{1}{2}^-),\Lambda_Q(\frac{3}{2}^-)\}$ $\Sigma_Q(\frac{1}{2}^-)$ $\{\Sigma_Q(\frac{1}{2}^-),\Sigma_Q(\frac{3}{2}^-)\}$ $\{\Sigma_Q(\frac{3}{2}^-),\Sigma_Q(\frac{5}{2}^-)\}$
l=1 D(0⁺,1⁺) D(1⁺,2⁺)	$\Lambda_Q(\frac{1}{2}^-)$ $\{\Lambda_Q(\frac{1}{2}^-),\Lambda_Q(\frac{3}{2}^-)\}$ $\{\Lambda_Q(\frac{3}{2}^-),\Lambda_Q(\frac{5}{2}^-)\}$ $\{\Sigma_Q(\frac{1}{2}^-),\Sigma_Q(\frac{3}{2}^-)\}$	...
⋮		

・kinematical structure is same as the constituent quark model

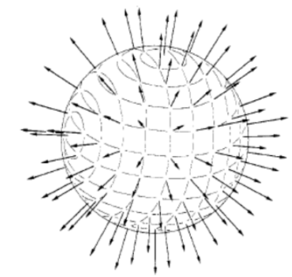
M.H., F.Sannino, J.Schechter and H.Weigel, PRD56, 4098 (1997)

Nucleon as Skyrme soliton

Skyrme model

$$\mathcal{L}_{\rm Skyr} = \frac{F_\pi^2}{4}{\rm Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{\epsilon^2}{4}{\rm Tr}\left\{\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]\left[U^\dagger\partial^\mu U, U^\dagger\partial^\nu U\right]\right\}$$

hedgehog ansatz



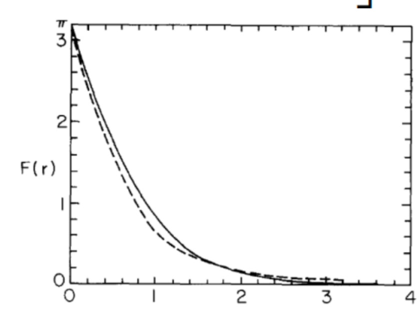
$$U(\mathbf{x}) = \exp\left(i\boldsymbol{\tau}\cdot\hat{\mathbf{x}}F(r)\right) = \cos F(r) + i\boldsymbol{\tau}\cdot\hat{\mathbf{x}}\sin F(r)$$

☆ EoM for F(r)

$$F'' + \frac{2}{r}F' - \frac{1}{r^2}\sin 2F - 8\frac{\varepsilon^2}{F_\pi^2}\left[\frac{\sin 2F\sin^2 F}{r^4} - \frac{F'^2\sin 2F}{r^2} - \frac{2F''\sin^2 F}{r^2}\right] = 0$$

☆ Solution with Baryon number = 1

$$F(r=0) = \pi, \qquad F(r\to\infty) = 0.$$



☆ Soliton mass

$$M_{\rm Skyr} = 4\pi\int_0^\infty r^2 dr\left\{\frac{F_\pi^2}{2}\left[F'^2 + \frac{2\sin^2 F}{r^2}\right] + 4\varepsilon^2\frac{\sin^2 F}{r^2}\left[2F'^2 + \frac{\sin^2 F}{r^2}\right]\right\}$$

Nucleon as Soliton in the HLS

Y. -L. Ma, Y. Oh, G. -S. Yang, M. Harada, H. K. Lee, B. -Y. Park and M. Rho, Phys. Rev. D 86, 074025 (2012).
Y. -L. Ma, G. -S. Yang, Y. Oh and M. Harada, Phys. Rev. D 87, 034023 (2013).

$$\mathcal{L}_{\rm HLS} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \mathcal{L}_{\rm anom},$$

$$\mathcal{L}_{(2)} = f_\pi^2 \,\mathrm{Tr}\left(\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp}^{\mu}\right) + a f_\pi^2 \,\mathrm{Tr}\left(\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel}^{\mu}\right) - \frac{1}{2g^2}\mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu}\right),$$

$$\begin{aligned}\mathcal{L}_{(4)y} = {} & y_1 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp}^{\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_{\perp}^{\nu}\right] + y_2 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_{\perp}^{\mu}\hat{\alpha}_{\perp}^{\nu}\right] + y_3 \mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_{\parallel}^{\nu}\right] + y_4 \mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel}^{\nu}\right] \\ & + y_5 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp}^{\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_{\parallel}^{\nu}\right] + y_6 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel}^{\nu}\right] + y_7 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_{\parallel}^{\nu}\hat{\alpha}_{\parallel}^{\mu}\right] \\ & + y_8 \left\{\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\perp\nu}\hat{\alpha}_{\parallel}^{\nu}\right] + \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_{\perp}^{\nu}\hat{\alpha}_{\parallel}^{\mu}\right]\right\} + y_9 \mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\parallel\nu}\hat{\alpha}_{\perp}^{\mu}\hat{\alpha}_{\parallel}^{\nu}\right],\end{aligned}$$

$$\mathcal{L}_{(4)z} = i z_4 \mathrm{Tr}\left[V_{\mu\nu}\hat{\alpha}_{\perp}^{\mu}\hat{\alpha}_{\perp}^{\nu}\right] + i z_5 \mathrm{Tr}\left[V_{\mu\nu}\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel}^{\nu}\right],$$

$$\mathcal{L}_{\rm anom} = \frac{N_c}{16\pi^2}\sum_{i=1}^{3} c_i \mathcal{L}_i$$

$$\begin{aligned}\mathcal{L}_1 &= i\,\mathrm{Tr}[\hat{\alpha}_{\rm L}^3\hat{\alpha}_{\rm R} - \hat{\alpha}_{\rm R}^3\hat{\alpha}_{\rm L}], \\ \mathcal{L}_2 &= i\,\mathrm{Tr}[\hat{\alpha}_{\rm L}\hat{\alpha}_{\rm R}\hat{\alpha}_{\rm L}\hat{\alpha}_{\rm R}], \\ \mathcal{L}_3 &= \mathrm{Tr}[F_{\rm V}\,(\hat{\alpha}_{\rm L}\hat{\alpha}_{\rm R} - \hat{\alpha}_{\rm R}\hat{\alpha}_{\rm L})],\end{aligned}$$

We determined 17 parameters from holographic QCD model

$M_{\rm sol}$	1184
Δ_M	448
$\sqrt{\langle r^2\rangle_W}$	0.433
$\sqrt{\langle r^2\rangle_E}$	0.608

Heavy meson Lagrangian

◎ chiral doubler fields for heavy mesons

$$\mathcal{H}_L = \frac{1}{\sqrt{2}}[G + iH\gamma_5], \qquad \mathcal{H}_R = \frac{1}{\sqrt{2}}[G - iH\gamma_5]$$

$$\mathcal{H}_L \to \mathcal{H}_L g_L^\dagger, \qquad \mathcal{H}_R \to \mathcal{H}_R g_R^\dagger \qquad g_{L,R} \in SU(2)_{L,R}$$

◎ chiral field for pion

$$U = e^{2i\pi/f_\pi} = \xi^2 \to g_L\, U\, g_R^\dagger$$

Δ term generates mass difference between (D, D*) and (D_0*, D_1).

☆ model Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{heavy}} = {} & \frac{1}{2}\text{Tr}\left[\bar{\mathcal{H}}_L i(v\cdot\partial)\mathcal{H}_L\right] + \frac{1}{2}\text{Tr}\left[\bar{\mathcal{H}}_R i(v\cdot\partial)\mathcal{H}_R\right] \\ & - \frac{\Delta}{2}\text{Tr}\left[\bar{\mathcal{H}}_L\mathcal{H}_L + \bar{\mathcal{H}}_R\mathcal{H}_R\right] - \frac{g_\pi F_\pi}{4}\text{Tr}\left[U^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R + U\bar{\mathcal{H}}_R\mathcal{H}_L\right] \\ & + i\frac{g_A}{2}\text{Tr}\left[\gamma^5\gamma^\mu\partial_\mu U^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R - \gamma^5\gamma^\mu\partial_\mu U\bar{\mathcal{H}}_R\mathcal{H}_L\right]\end{aligned}$$

Inclusion of vector mesons with HLS

$$[SU(2)_L \times SU(2)_R]_{global} \times [U(2)_V]_{local} \to [SU(2)_V]_{global}$$

$$U = e^{2i\pi/F_\pi} = \xi_L^\dagger \xi_R$$

$$\xi_{L,R} = e^{i\sigma/F_\sigma} e^{\pm i\pi/F_\pi} \to h\, \xi_{L,R}\, g_{L,R}^\dagger$$

$$\begin{cases} h \in [U(2)_v]_{local} \\ g_{L,R} \in SU(2)_{L,R} \end{cases}$$

F_π , F_σ ･･･ Decay constants of π and σ

- Maurer-Cartan 1-forms

$$\hat{\alpha}^\mu_{\perp,\parallel} = \left(D^\mu \xi_L \cdot \xi_L^\dagger \mp D^\mu \xi_R \cdot \xi_R^\dagger\right) / (2i)$$

$$D_\mu \xi_L = \partial_\mu \xi_L - i V_\mu \xi_L \qquad D_\mu \xi_R = \partial_\mu \xi_R - i V_\mu \xi_R$$

$$V_\mu = \frac{g}{2}(\omega_\mu + \rho_\mu)$$: HLS gauge field

変換性： $\hat{\alpha}^\mu_{\perp,\parallel} \to h\, \hat{\alpha}^\mu_{\perp,\parallel}\, h^\dagger$

Heavy meson Lagrangian with HLS

◎ Introduce new fields for heavy mesons

$$\hat{\mathcal{H}}_L = \mathcal{H}_L \xi_L^\dagger, \quad \hat{\mathcal{H}}_R = \mathcal{H}_R \xi_R^\dagger,$$

$$\hat{\mathcal{H}}_L \to \hat{\mathcal{H}}_L h^\dagger(x), \quad \hat{\mathcal{H}}_R \to \hat{\mathcal{H}}_R h^\dagger(x).$$

◎ Heavy meson Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{heavy}} = \; & \frac{1}{2}\mathrm{Tr}\left[\hat{\mathcal{H}}_L (iv\cdot\widetilde{D})\bar{\hat{\mathcal{H}}}_L\right] + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathcal{H}}_R (iv\cdot\widetilde{D})\bar{\hat{\mathcal{H}}}_R\right] - \frac{\Delta}{2}\mathrm{Tr}\left[\bar{\hat{\mathcal{H}}}_L\hat{\mathcal{H}}_L + \bar{\hat{\mathcal{H}}}_R\hat{\mathcal{H}}_R\right] \\ & - \frac{g_\pi F_\pi}{4}\mathrm{Tr}\left[\bar{\hat{\mathcal{H}}}_L\hat{\mathcal{H}}_R + \bar{\hat{\mathcal{H}}}_R\hat{\mathcal{H}}_L\right] - g_A \mathrm{Tr}\left[\gamma^5\gamma^\mu \hat{\alpha}_{\perp\mu}\left(\bar{\hat{\mathcal{H}}}_L\hat{\mathcal{H}}_R + \bar{\hat{\mathcal{H}}}_R\hat{\mathcal{H}}_L\right)\right]\end{aligned}$$

$$\widetilde{D}_\mu = \partial_\mu - iV_\mu - i\kappa\alpha_{\parallel\mu}$$

Heavy baryon as bound state

- We solve the equation of motion for the heavy meson field with the background nucleon as soliton.

$$
\mathcal{L}_{\text{heavy}} =
$$

$$
\frac{1}{2}\mathrm{Tr}\left[\hat{\mathcal{H}}_L(iv\cdot\widetilde{D})\bar{\hat{\mathcal{H}}}_L\right]+\frac{1}{2}\mathrm{Tr}\left[\hat{\mathcal{H}}_R(iv\cdot\widetilde{D})\bar{\hat{\mathcal{H}}}_R\right]-\frac{\Delta}{2}\mathrm{Tr}\left[\bar{\hat{\mathcal{H}}}_L\hat{\mathcal{H}}_L+\bar{\hat{\mathcal{H}}}_R\hat{\mathcal{H}}_R\right]
$$

$$
-\frac{g_\pi F_\pi}{4}\mathrm{Tr}\left[\bar{\hat{\mathcal{H}}}_L\hat{\mathcal{H}}_R+\bar{\hat{\mathcal{H}}}_R\hat{\mathcal{H}}_L\right]-g_A\mathrm{Tr}\left[\gamma^5\gamma^\mu\hat{\alpha}_{\perp\mu}\left(\bar{\hat{\mathcal{H}}}_L\hat{\mathcal{H}}_R+\bar{\hat{\mathcal{H}}}_R\hat{\mathcal{H}}_L\right)\right]
$$

- Ansaz for classical solution

$$
\hat{H}=\begin{pmatrix}0 & \mathbb{H}\\ 0 & 0\end{pmatrix},\ \hat{G}=\begin{pmatrix}\mathbb{G} & 0\\ 0 & 0\end{pmatrix}
$$

$$
\mathbf{H}^{\dagger a}_{lh}=u(\mathbf{x})(\boldsymbol{\tau}\cdot\hat{\mathbf{x}})_{ad}\psi_{dl}\chi_h
$$

a ⋯ isospin of heavy light meson

l ⋯ spin of the light degree of freedom

h ⋯ heavy quark spin

Heavy meson Lagrangian

☆ Integrating out scalar mesons and keeping pion only $(M \to F_\pi U)$

$$\mathcal{L}_{\rm heavy} = \frac{1}{2}{\rm Tr}\left[\bar{\mathcal{H}}_L i(v\cdot\partial)\mathcal{H}_L\right] + \frac{1}{2}{\rm Tr}\left[\bar{\mathcal{H}}_R i(v\cdot\partial)\mathcal{H}_R\right]$$
$$-\frac{\Delta}{2}{\rm Tr}\left[\bar{\mathcal{H}}_L\mathcal{H}_L + \bar{\mathcal{H}}_R\mathcal{H}_R\right] - \frac{g_\pi F_\pi}{4}{\rm Tr}\left[U^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R + U\bar{\mathcal{H}}_R\mathcal{H}_L\right]$$
$$+i\frac{g_A}{2}{\rm Tr}\left[\gamma^5\gamma^\mu\partial_\mu U^\dagger\bar{\mathcal{H}}_L\mathcal{H}_R - \gamma^5\gamma^\mu\partial_\mu U\bar{\mathcal{H}}_R\mathcal{H}_L\right]$$

- Redefine the fields as $\hat{\mathcal{H}}_L = \mathcal{H}_L\xi_L^\dagger,\quad \hat{\mathcal{H}}_R = \mathcal{H}_R\xi_R^\dagger \qquad U = \xi_L^\dagger\xi_R$

$$\hat{\mathcal{H}}_L = \frac{1}{\sqrt{2}}[\hat{G} - i\hat{H}\gamma_5],\quad \hat{\mathcal{H}}_R = \frac{1}{\sqrt{2}}[\hat{G} + i\hat{H}\gamma_5]$$

- Ansaz for classical solution

$$\hat{H} = \begin{pmatrix} 0 & \mathbb{H} \\ 0 & 0 \end{pmatrix},\ \hat{G} = \begin{pmatrix} \mathbb{G} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathbf{H}^{\dagger a}_{lh} = u(\mathbf{x})(\tau\cdot\hat{\mathbf{x}})_{ad}\psi_{dl}\chi_h$$

a ⋯ isospin of heavy light meson

l ⋯ spin of the light degree of freedom

h ⋯ heavy quark spin

Quantum number & Binding energy

- Spin of heavy baryon (bound state)

$$\vec{J} = \vec{S}_{\text{heavy}} + \vec{r} + \vec{K} \qquad \vec{K} = \vec{J}_{\text{light}} + \vec{I}_{\text{light}}$$

$\vec{S}_{\text{heavy}}$: heavy quark spin

$\vec{r}$: relative angular momentum between heavy meson and nucleon

- Binding energy for H ~ (D, D*) with r = 0 (ground state)

$$V_H = -\int d^3x \mathcal{L}^H_{\text{heavy}} = \frac{1}{2}(1+\kappa)g\,\omega(0) - g_A F'(0)\left[k(k+1) - \frac{3}{2}\right]$$

$$F'(0) = 626.1 \text{ MeV}; \qquad \omega(0) = -74.5 \text{ MeV}$$

Y. -L. Ma, Y. Oh, G. -S. Yang, M. Harada, H. K. Lee, B. -Y. Park and M. Rho, Phys. Rev. D 86, 074025 (2012).
Y. -L. Ma, G. -S. Yang, Y. Oh and M. Harada, Phys. Rev. D 87, 034023 (2013).

- **Assume g_A > 0 and $|\kappa| \leq 1$ to have a bound state in *K*=0**
  ⇒ $\Lambda_c(1/2^+)$ is the ground state

Heavy Baryon Masses

$$m_{I,j}^M = M_{\rm sol} + \bar{M}_M + V_M + H_{\rm coll}$$

- $M_{\rm sol}$: Soliton mass.
- $\bar{M}_M$: weight-averaged heavy meson masse, $\bar{M}_H = (3m_{D^*} + m_D)/4$ and $\bar{M}_G = (3m_{D_1'} + m_{D_0^*})/4$.
- $V_M(M = H, G)$: the binding energy

$$V_{\rm H} = \frac{1}{2}(1+\kappa)g\omega(0) + g_A F'(0)\left[k(k+1) - \frac{3}{2}\right],$$
$$V_{\rm G} = \frac{1}{2}(1+\kappa)g\omega(0) - g_A F'(0)\left[k(k+1) - \frac{3}{2}\right].$$

- The collective rotated Hamiltonian

$$H_{\rm coll} = \frac{1}{2\mathcal{I}_{\rm HLS}}\left[[1-\chi(k)]\mathbf{I}^2 + \chi(k)[\chi(k)-1]\mathbf{K}^2 + \chi(k)(\mathbf{j}-\mathbf{r})^2\right],$$
$$\chi(k) = [k(k+1) + 3/4 - j_l(j_l+1)]\,/\,[2k(k+1)]\,.$$

$\Lambda_c(1/2^+)$

$M(\Lambda(1/2+)) = M_N + M_{D(0-,1-)} + V_H$ ($H_{coll} = 0$)

$$V_H = -0.177(1+\kappa) + 0.626 g_A\left[k(k+1) - \frac{3}{2}\right] [\text{GeV}].$$

$M_{D(0-,1-)} = (M_{D(0-)} + 3\, M_{D(1-)})/4 \sim 1.97$ (GeV)

$M_N = 0.94$ (GeV)

$g_A = 0.56$ from $D(1^-) \to D(0^-) + \pi$ decay

$M(\Lambda(1/2^+))^{exp} = 2.286$ (GeV)
is used to determin $\kappa = -0.83$

Chiral partner to $\Lambda_c(1/2^+)$?

◎ Binding energy for G ~ (D_0, D_1^*) with $r = 0$

$$V_G = -0.177(1+\kappa) - 0.626 g_A \left[k(k+1) - \frac{3}{2} \right] [\mathrm{GeV}]$$

- $g_A = 0.56$, $\kappa = -0.83$ ⇒ **bound state is realized for $K = 1$**

Chiral partner to $\Lambda_c(1/2^+)$ = [$\Lambda_c(1/2^-)$, $\Lambda_c(3/2^-)$]

◎ Mass $\quad$ $M_{D(0+,1+)} = (M_{D(0+)} + 3 M_{D(1+)})/4 \sim 2.4(\mathrm{GeV})$

$$M(\Lambda) = M_N + M_{D(0+,1+)} + V_G + H_{coll} = 3.13\ (\mathrm{GeV})$$

・Λ_c(1/2-;2595) is unlikely the chiral partner to Λ_c(1/2+;2286)

・{Λ_c(1/2-;2595) , Λ_c(3/2-;2625) }

・・・ r = 1 boundstate of D(0-,1-) and nucleon

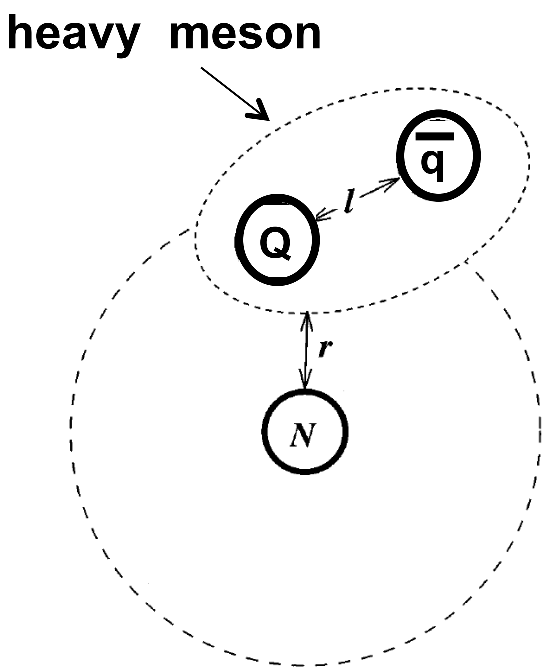
☆ Chiral partner to Λ(1/2+)

◎ excited heavy baryons (qqQ type)

= **heavy meson** + **nculeon** with angular momentum

or

excited heavy meson + **nucleon**



r, *l* : angular momentum

	r = 0	r = 1
l=0 D(0^-,1^-)	$\Lambda_Q(\frac{1}{2}^+)$ $\{\Sigma_Q(\frac{1}{2}^+),\Sigma_Q(\frac{3}{2}^+)\}$	$\{\Lambda_Q(\frac{1}{2}^-),\Lambda_Q(\frac{3}{2}^-)\}$ $\Sigma_Q(\frac{1}{2}^-)$ $\{\Sigma_Q(\frac{1}{2}^-),\Sigma_Q(\frac{3}{2}^-)\}$ $\{\Sigma_Q(\frac{3}{2}^-),\Sigma_Q(\frac{5}{2}^-)\}$
l=1 D(0^+,1^+) D(1^+,2^+)	$\Lambda_Q(\frac{1}{2}^-)$ $\{\Lambda_Q(\frac{1}{2}^-),\Lambda_Q(\frac{3}{2}^-)\}$ $\{\Lambda_Q(\frac{3}{2}^-),\Lambda_Q(\frac{3}{2}^-)\}$ $\{\Sigma_Q(\frac{1}{2}^-),\Sigma_Q(\frac{3}{2}^-)\}$	**3.13 GeV** · · ·

⋮

Prediction for Σ_c and bottomed baryosn

TABLE I. Predicted mass for the charmed baryon for the H doublet.

I	j	States	M^H (MeV)
0	0	$\Lambda_c(\frac{1}{2}^+)$	2286.46 (input)
1	1	$\Sigma_c(\frac{1}{2}^+)$, $\Sigma_c(\frac{3}{2}^+)$	2481.13

TABLE II. Predicted mass for the charmed baryon for the G doublet.

I	j	$I(j_B^P)$	M^G (MeV)
0	1	$\Lambda_c(\frac{1}{2}^-)$, $\Lambda_c(\frac{3}{2}^-)$	3131.66
1	0	$\Sigma_c(\frac{1}{2}^-)$	3131.66
1	1	$\Sigma_c(\frac{1}{2}^-)$, $\Sigma_c(\frac{3}{2}^-)$	3228.99
1	2	$\Sigma_c(\frac{3}{2}^-)$, $\Sigma_c(\frac{5}{2}^-)$	3423.66

TABLE III. Predicted mass for the bottom baryon for the H doublet.

I	j	$I(j_B^P)$	M^H (MeV)
0	0	$\Lambda_b(\frac{1}{2}^+)$	5625.07
1	1	$\Sigma_b(\frac{1}{2}^+)$, $\Sigma_b(\frac{3}{2}^+)$	5819.74

TABLE IV. Predicted mass for the bottom baryon for the G doublet.

I	j	$I(j_B^P)$	M^G (MeV)
0	1	$\Lambda_b(\frac{1}{2}^-)$, $\Lambda_b(\frac{3}{2}^-)$	6470.27
1	0	$\Sigma_b(\frac{1}{2}^-)$	6470.27
1	1	$\Sigma_b(\frac{1}{2}^-)$, $\Sigma_b(\frac{3}{2}^-)$	6567.6
1	2	$\Sigma_b(\frac{3}{2}^-)$, $\Sigma_b(\frac{5}{2}^-)$	6762.27

Application to pentaquark

$$V^5_{\rm H} = 0.177(1+\kappa) - 0.626 g_A\left[k(k+1) - \frac{3}{2}\right][\mathrm{GeV}] = -0.146\,\mathrm{GeV}$$

K = 1 gives a bound state.

M(Θ_c(1/2-, 3/2-)) ~ 2.75 (GeV)

cf : M(Θ_c(1/2-)) ~ 2.7 GeV without ω contribution.

Y.Oh, B.-Y.Park, and D.P.Min, PLB331, 362 (1994)

note : CHORUS exp. did not observe Θ_c(2710). NPB 763 (2007) 268

☆ **chiral partner to pentaquark ?**

$$V^5_{\rm G} = 0.177(1+\kappa) + 0.626 g_A\left[k(k+1) - \frac{3}{2}\right][\mathrm{GeV}] = -0.496\,\mathrm{GeV}$$

***K* = 0**

M(Θ_c(1/2+)) ~ 2.79 (GeV) !

cf : M(Θ_c(1/2+)) = 3052 ± 60 MeV M.A.Nowak et al., PRD70, 031503(2004)

Prediction for Bottomed pentaquarks

TABLE VII. Predicted mass for the bottom pentaquark state for the H doublet.

I	j	Candidates	M^{5,H_b} (MeV)
0	1	$\Theta_b(\frac{1}{2}^+)$	6083.76

TABLE VIII. Predicted mass for the bottom pentaquark state for the G doublet.

I	j	Candidates	M^{5,G_b} (MeV)
0	0	$\Theta_b(\frac{1}{2}^-)$	6130.39

7. Summary

◆ Based on the chiral doubling structure of D mesons I showed the following 2 analyses:

◎ effect of the sigma meson to $D_1 \rightarrow D\pi\pi$ decay

- Our result indicates that we can get some clue to understand the composition of the sigma meson from future experiment.

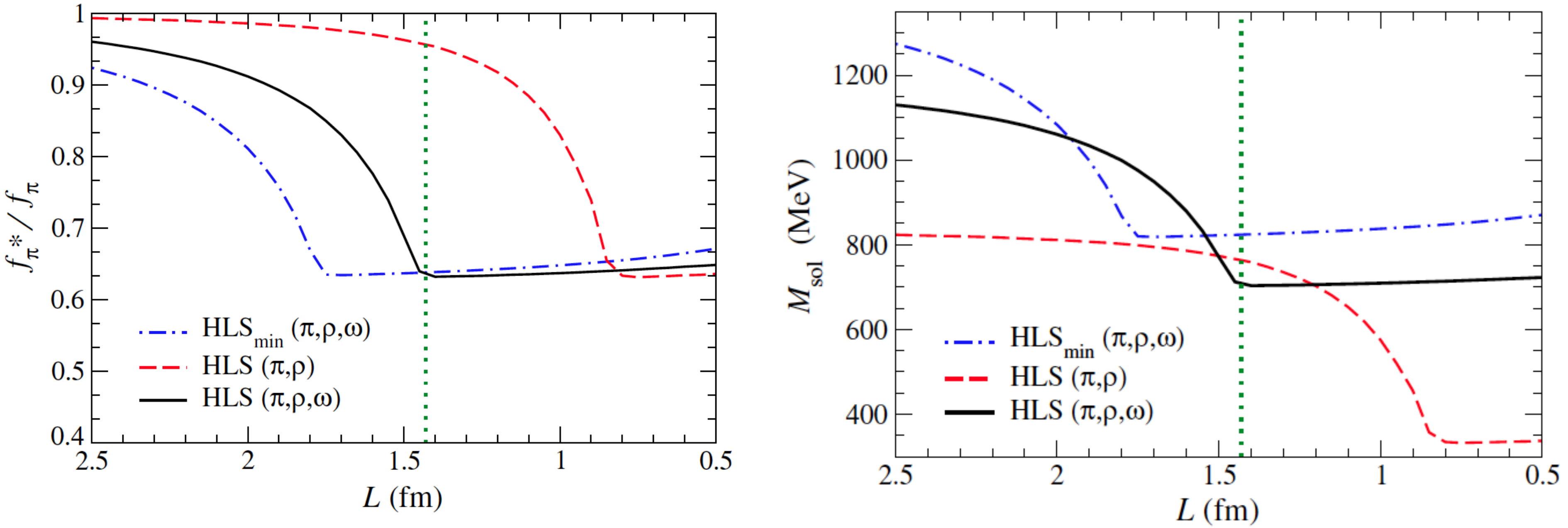
◎ Chiral doubling of heavy baryons

- Our result implies that the chiral partner to $\Lambda_c(1/2^+)$ is [$\Lambda_c(1/2^-)$, $\Lambda_c(3/2^-)$], whose mass is 3.13 GeV.
- Then, $\{\Lambda_c(1/2-;2595)$, $\Lambda_c(3/2-;2625)\}$ is r = 1 boundstates of D(0-,1-) and nucleon
- Two types of pentaquarks exist below *Dp* threshold.
- $M(\Theta_c(1/2-, 3/2-)) = 2.75$ (GeV) ;
- $M(\Theta_c(1/2+)) = 2.79$ (GeV)

Medium modification ?

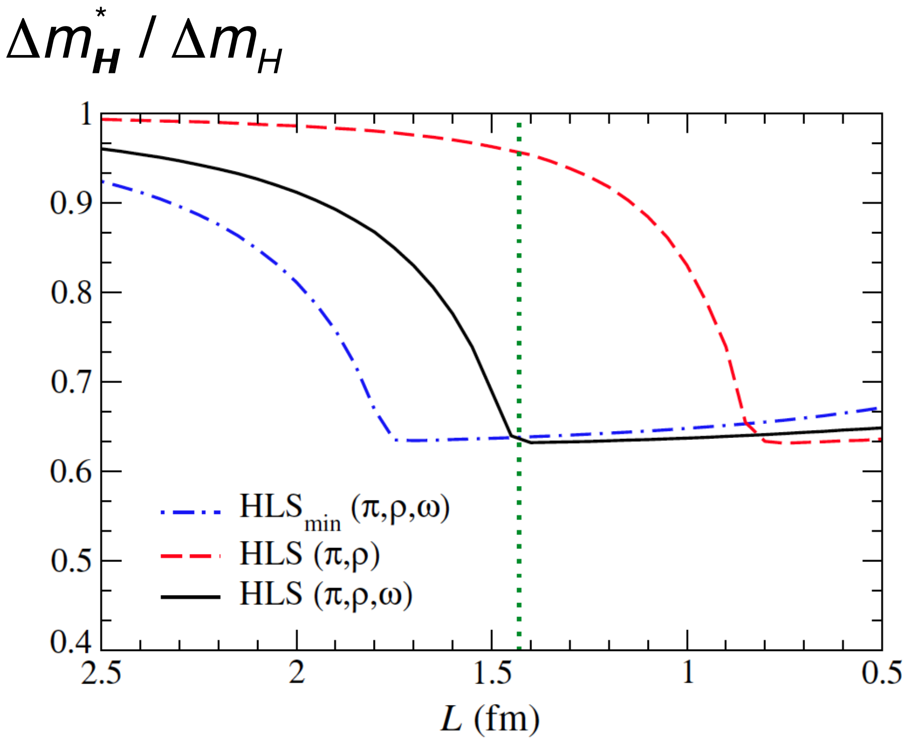
We studied the medium modification of the pion decay constant and soliton mass using the crystal structure.

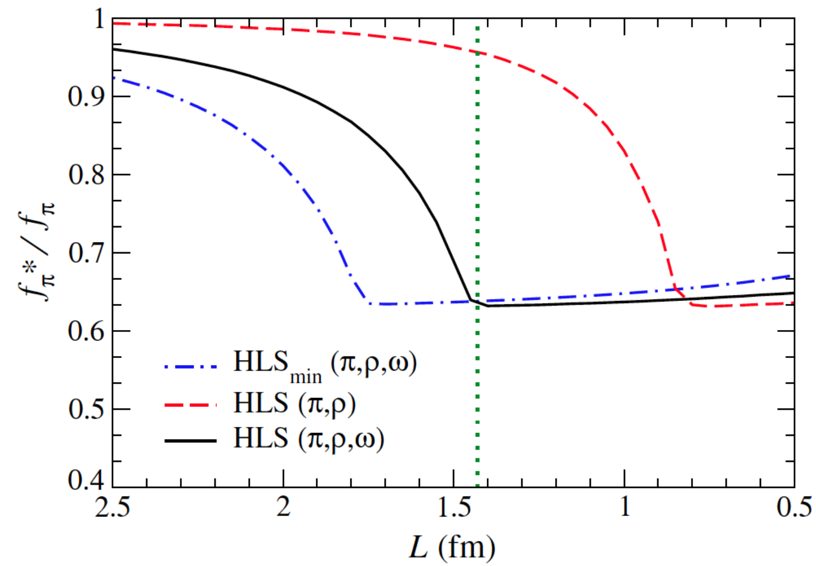
Y.-L.Ma, M. Harada, H. K.Lee, Y.Oh, B.-Y.Park and M.Rho, ``Dense Baryonic Matter in Hidden Local Symmetry Approach: Half-Skyrmions and Nucleon Mass,'' Phys. Rev. D 88, 014016 (2013)



Medium modification of masses of heavy mesons and heavy baryons ?

Medium modification (a naïve expectation ?)





$M_{D(0+,1+)} - M_{D(0-,1-)} \sim 0.43$ GeV

→ $M_{D(0+,1+)} - M_{D(0-,1-)} \sim 0.25$ GeV in medium

Widths of D(0+,1+) will be narrow in medium ?

The End