Structure and compositeness of hadrons

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2014, Feb. 27th

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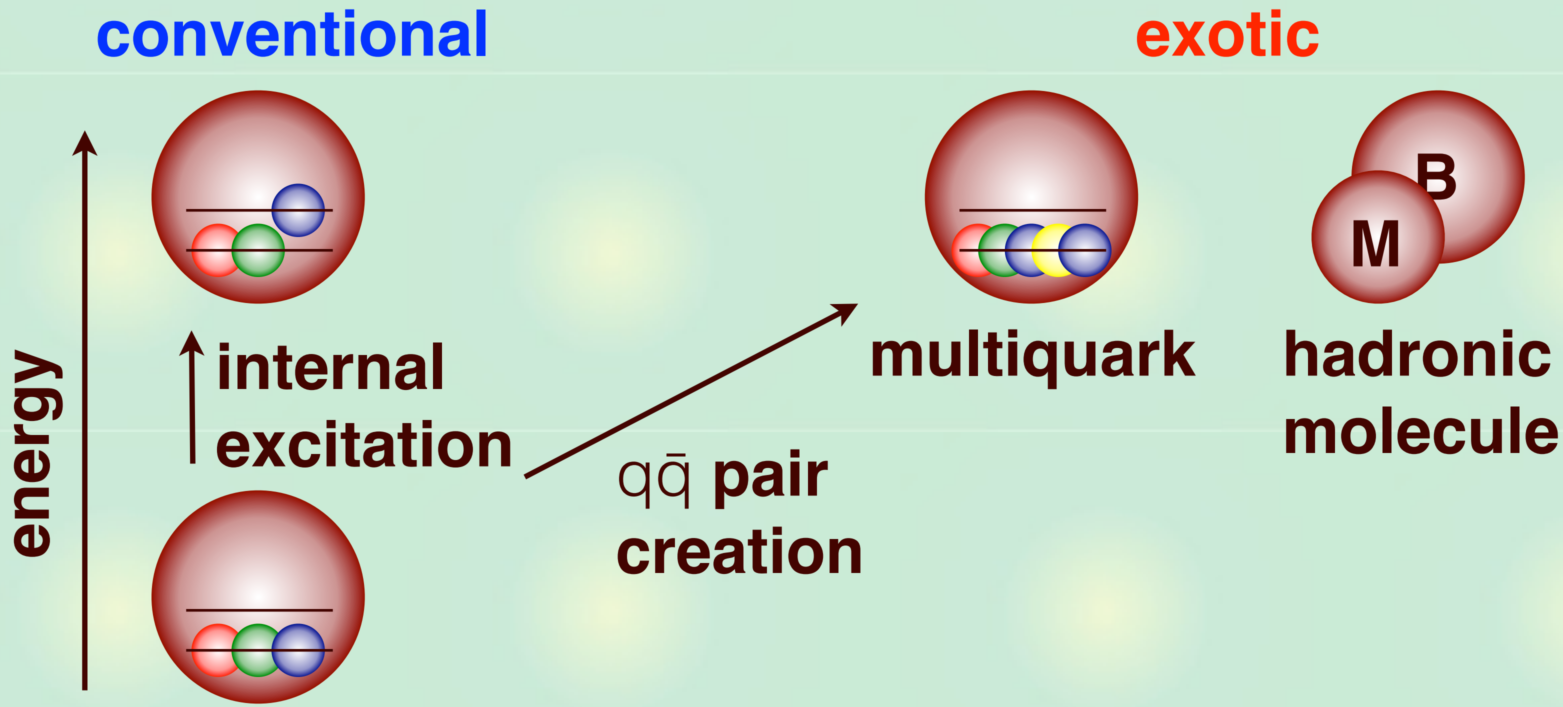
T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

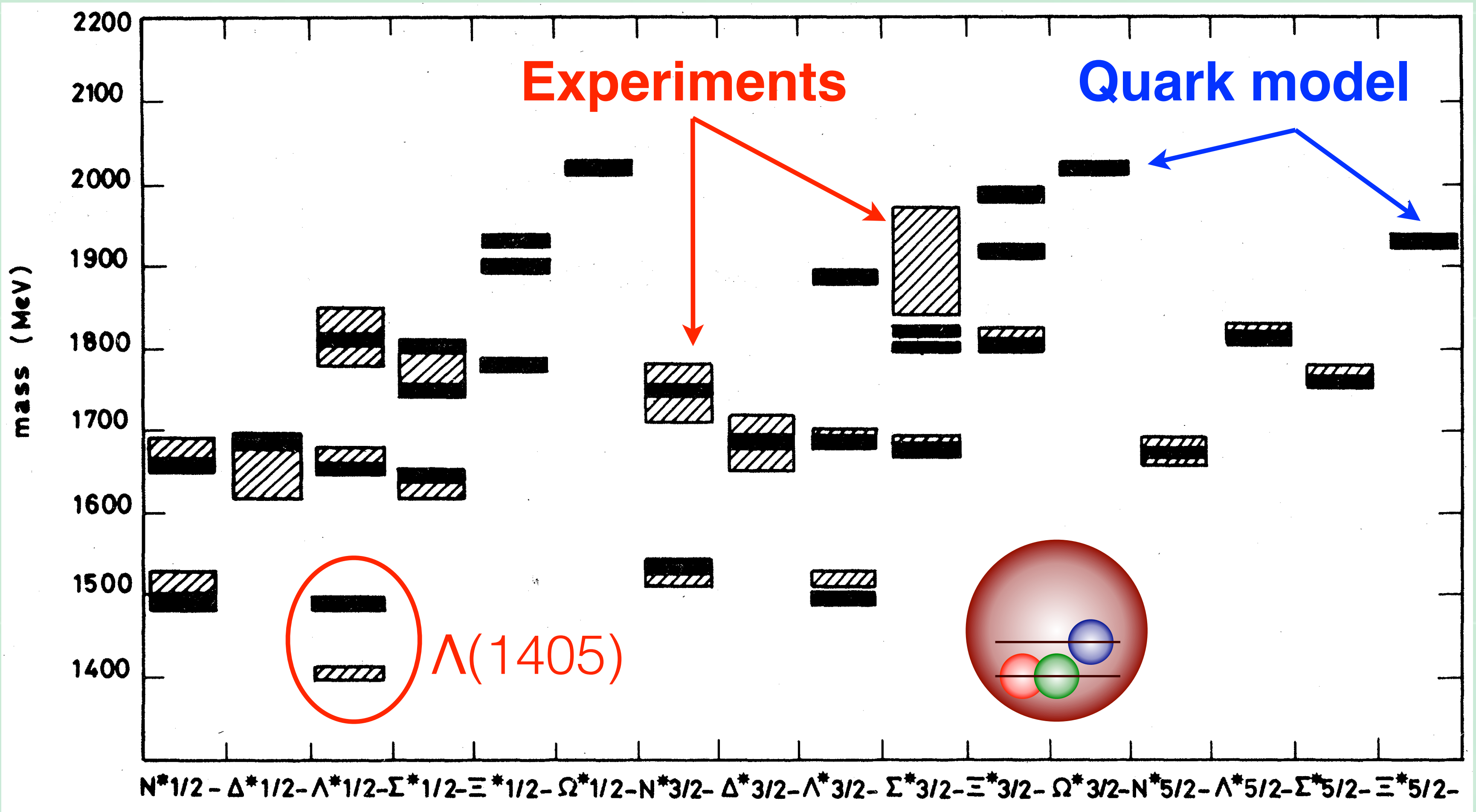
$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\;q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

How can we identify the structure of hadrons?

Λ(1405) in quark model

Baryon excited states in a constituent quark model

N. Isgur, G. Karl, Phys. Rev. D18, 4187 (1978)



Prediction does not fit experimental data of Λ(1405)

Λ(1405) in hadron molecule model

Dynamical coupled-channel scattering model

R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. 153, 1617 (1967)

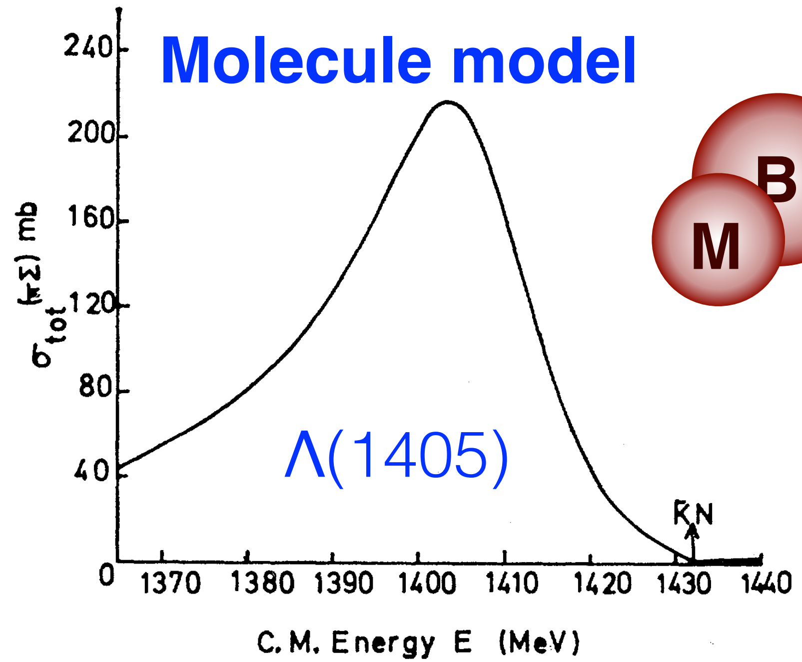
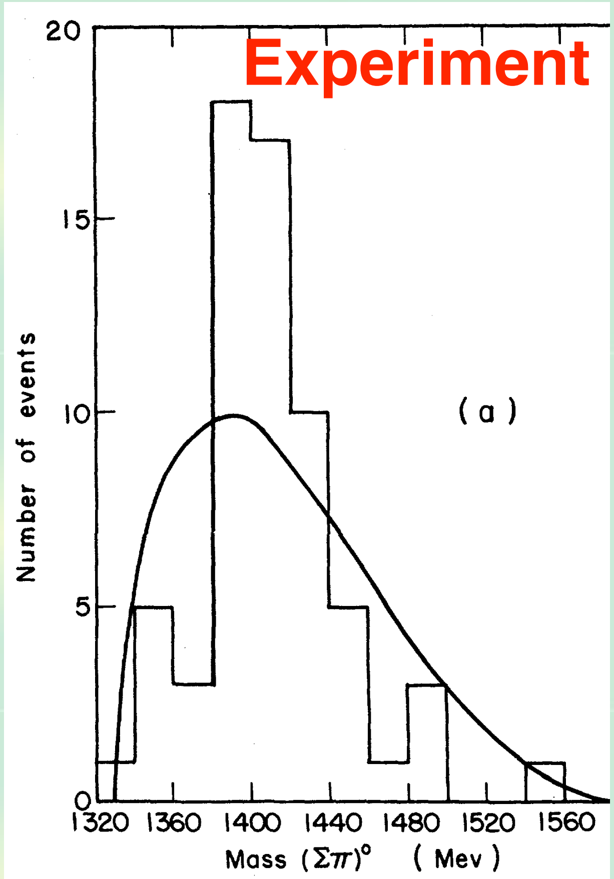


FIG. 1. The total s-wave $\pi\Sigma$ scattering cross section calculated for the multichannel potential model for Y_0*(1405) is plotted as a function of the total c.m. energy. The cross section becomes very small at the $\bar{K}N$ threshold, where only the term $\gamma(E)$ contributes to the $\pi\Sigma$ scattering.

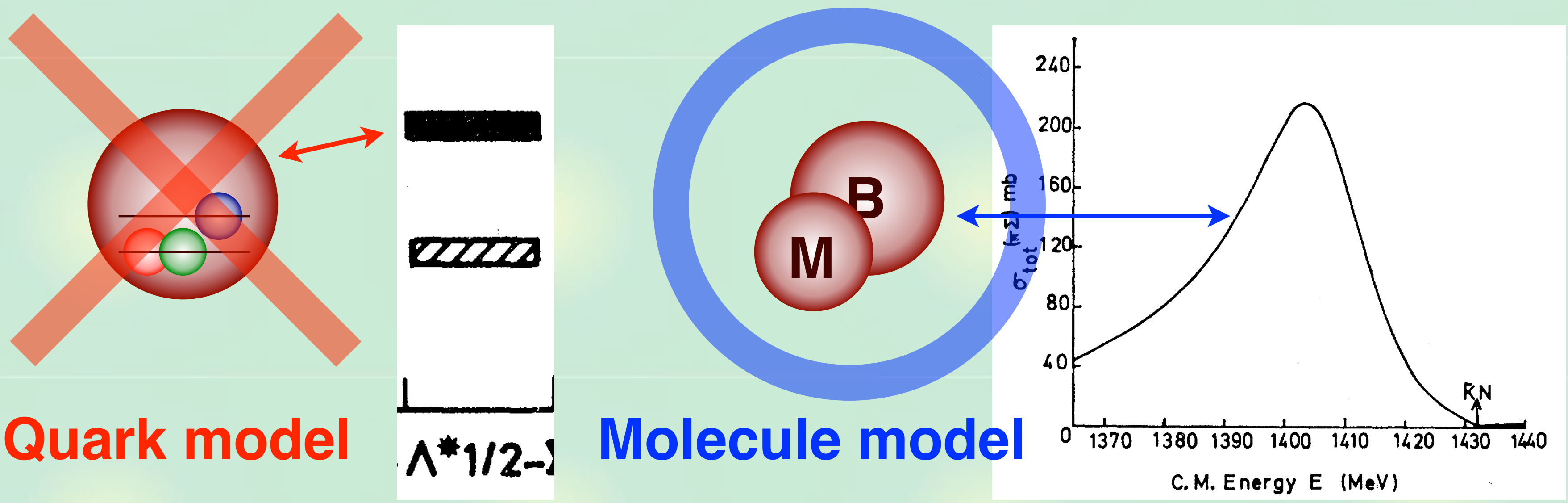


M.H. Alston *et al.*, Phys. Rev. Lett. 6, 698-702 (1961)

Good description of the spectrum (mass and width)

qqq v.s. molecule

Comparison with experimental data



The model prediction contradicts/agrees with data.

(hidden) assumption:

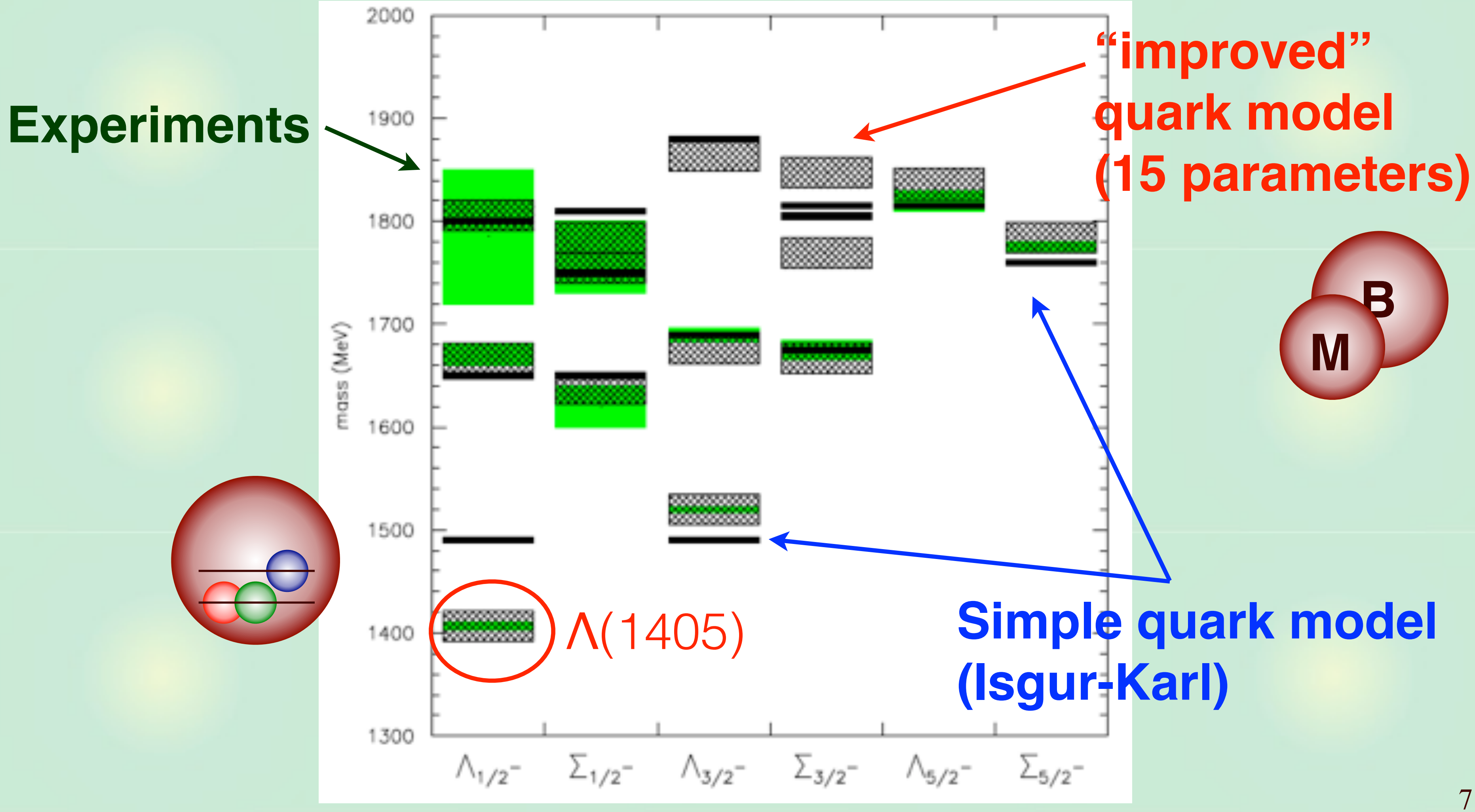
- Model space <--> structure of the predicted state

Is this so simple?

Improvement of models

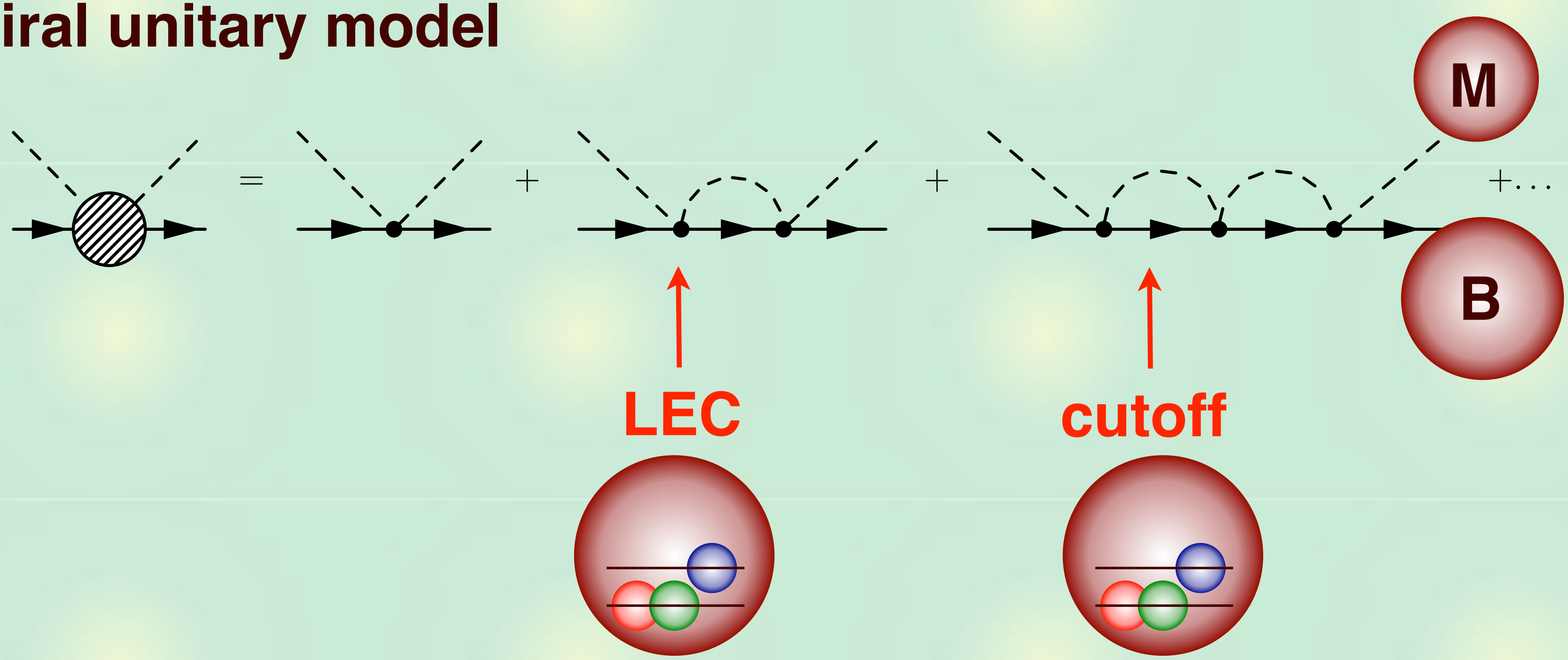
Quark model with more interactions (large N_c expansion)

C.L. Schat, J.L. Goity, N.N. Scoccola, Phys. Rev. Lett. 88, 102002 (2002);
J.L. Goity, C.L. Schat, N.N. Scoccola, Phys. Rev. D 66, 114014 (2002)



Ambiguity in the molecule model

Chiral unitary model



- Resonance saturation in low energy constants (LEC)

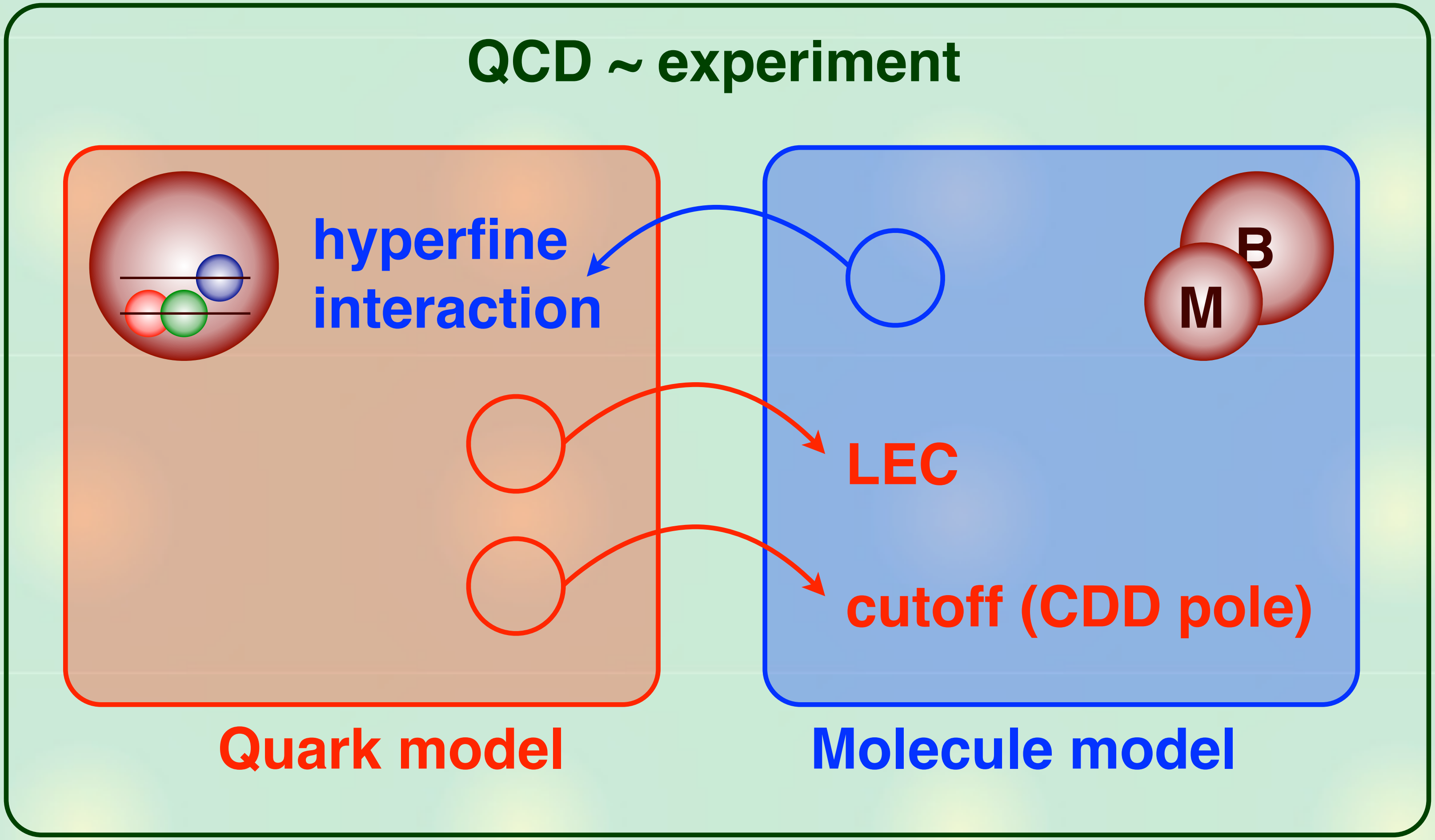
G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

- CDD pole contributions

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)
G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C 78, 025203 (2008)

Ambiguities in the model analysis

Schematic picture:



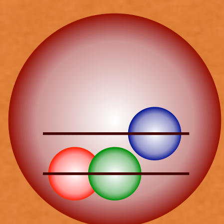
=> model space ≠ structure of the predicted state

Summary of introduction

- **Model space ≠ structure of hadron**

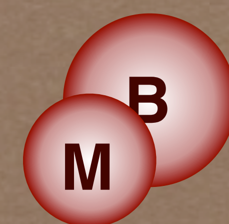
- **What we need is a model-independent measure for the hadron structure.**

QCD ~ experiment



Z ~ 1

Z ~ 0



Compositeness of bound states

Compositeness approach for a bound state |B>

S. Weinberg, Phys. Rev. 137, B672 (1965); T. Hyodo, IJMPA 28, 1330045 (2013)

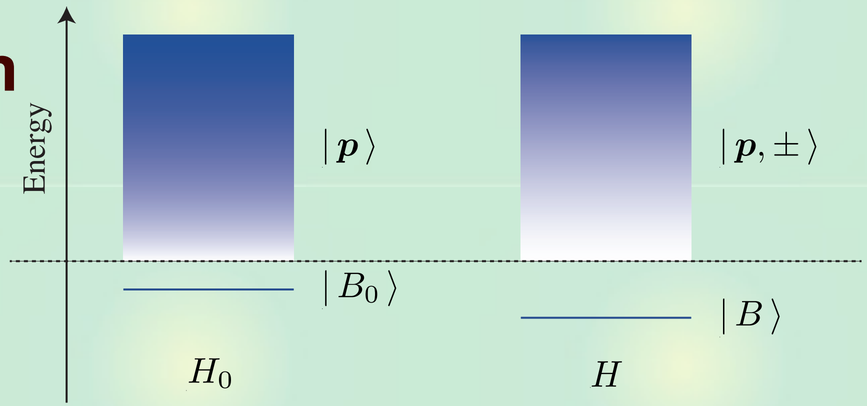
$$H = H_0 + V \qquad H|B\rangle = -B|B\rangle, \quad \langle B|B\rangle = 1$$

Decompose H into free part + interaction

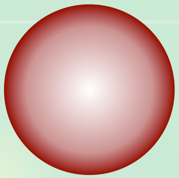
Complete set for free Hamiltonian : bare $|B_0>$ + continuum

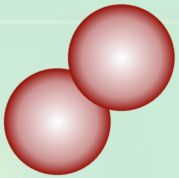
$$1 = |B_0\rangle\langle B_0| + \int d\boldsymbol{p}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|$$

$$1 = \underline{\langle B|B_0\rangle\langle B_0|B\rangle} + \int \underline{d\boldsymbol{p}\langle B|\boldsymbol{p}\rangle\langle\boldsymbol{p}|B\rangle}$$



Z : elementary   X : composite





In QCD,

H_0 **: free hadrons**

V **: hadron interaction**

Z, X **: real and nonnegative --> probabilistic interpretation**

$$\Rightarrow 0 \le Z \le 1, \quad 0 \le X \le 1$$

Weak binding limit

In general, Z **depends on the choice of the potential** V**.**

- Z **: model-(scheme-)dependent quantity**

$$1 - Z = \int d\boldsymbol{p} \frac{|\langle \boldsymbol{p} | V | B \rangle|^2}{(E_p + B)^2}$$ ⟵ **V-dependent**

At the weak binding (R ≫ $R_{\rm typ}$)**,** Z **is related to observables.**

S. Weinberg, Phys. Rev. 137, B672 (1965); T. Hyodo, IJMPA 28, 1330045 (2013)

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\rm typ}), \quad r_e = \frac{-Z}{1-Z} R + \mathcal{O}(R_{\rm typ}),$$

a **: scattering length,** r_e **: effective range**

$R = (2\mu B)^{-1/2}$ **: radius (binding energy)**

$R_{\rm typ}$ **: typical length scale of the interaction**

Criterion for the structure:

$$\begin{cases} a \sim R_{\rm typ} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\rm typ} & \text{(composite dominance)}. \end{cases}$$

Z ~ 1

Z ~ 0 **(deuteron)**

Interpretation of negative effective range

For $Z > 0$, effective range is always negative.

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\rm typ}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\rm typ}),$$

$$\begin{cases} a \sim R_{\rm typ} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\rm typ} & \text{(composite dominance)}. \end{cases}$$

Simple attraction (no barrier, energy-indep., ...) : $r_e > 0$

--> only "composite dominance" is possible.

$r_e < 0$: energy- (momentum-)dependence of the potential

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

<-- pole term/Feshbach projection of coupled-channel effect

Negative r_e --> Something other than $|p\rangle$: CDD pole

Generalization to resonances

Compositeness approach of bound states

$$1 - Z = \int d\boldsymbol{p} \frac{|\langle \boldsymbol{p} | V | B \rangle|^2}{(E_p + B)^2}$$

Extension to general resonances in chiral models

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012)

$$1 - Z = -g^2 \frac{dG(W)}{dW}\bigg|_{W \to M_B} \quad \to \quad 1 - Z = -g_{II}^2 \frac{dG_{II}(W)}{dW}\bigg|_{W \to z_R}$$

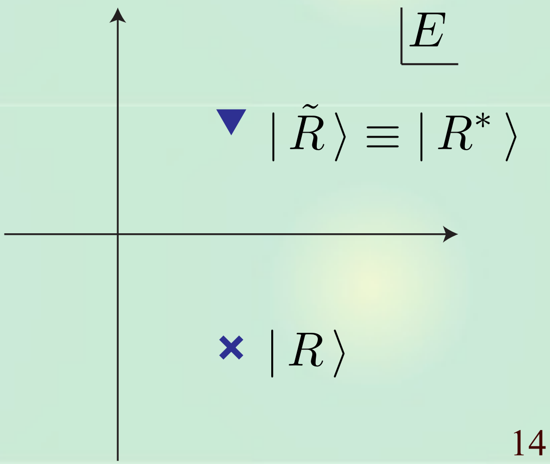
- Z is in general complex. Interpretation?

$$\langle R | R \rangle \to \infty, \quad \langle \tilde{R} | R \rangle = 1$$

$$1 = \langle \tilde{R} | B_0 \rangle \langle B_0 | R \rangle + \int d\boldsymbol{p} \langle \tilde{R} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | R \rangle$$

complex

$$\langle \tilde{R} | B_0 \rangle = \langle B_0 | R \rangle \neq \langle B_0 | R \rangle^*$$



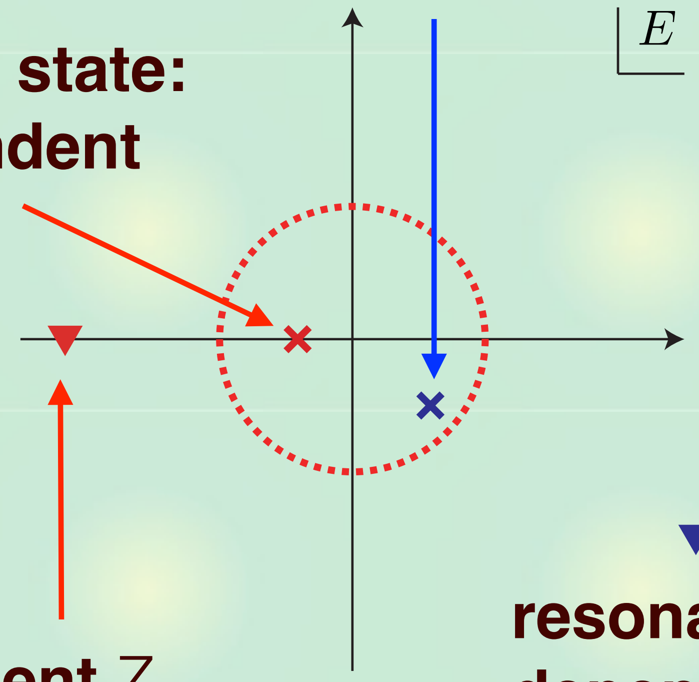
Generalization to resonances

Compositeness approach at the weak binding:

- **Model-independent (no potential, wavefunction, ...)**
- **Related to experimental observables**

What about near-threshold resonances (~ small binding) ?

shallow bound state: model-independent structure



bound state: model-dependent Z

resonance: model-dependent complex Z

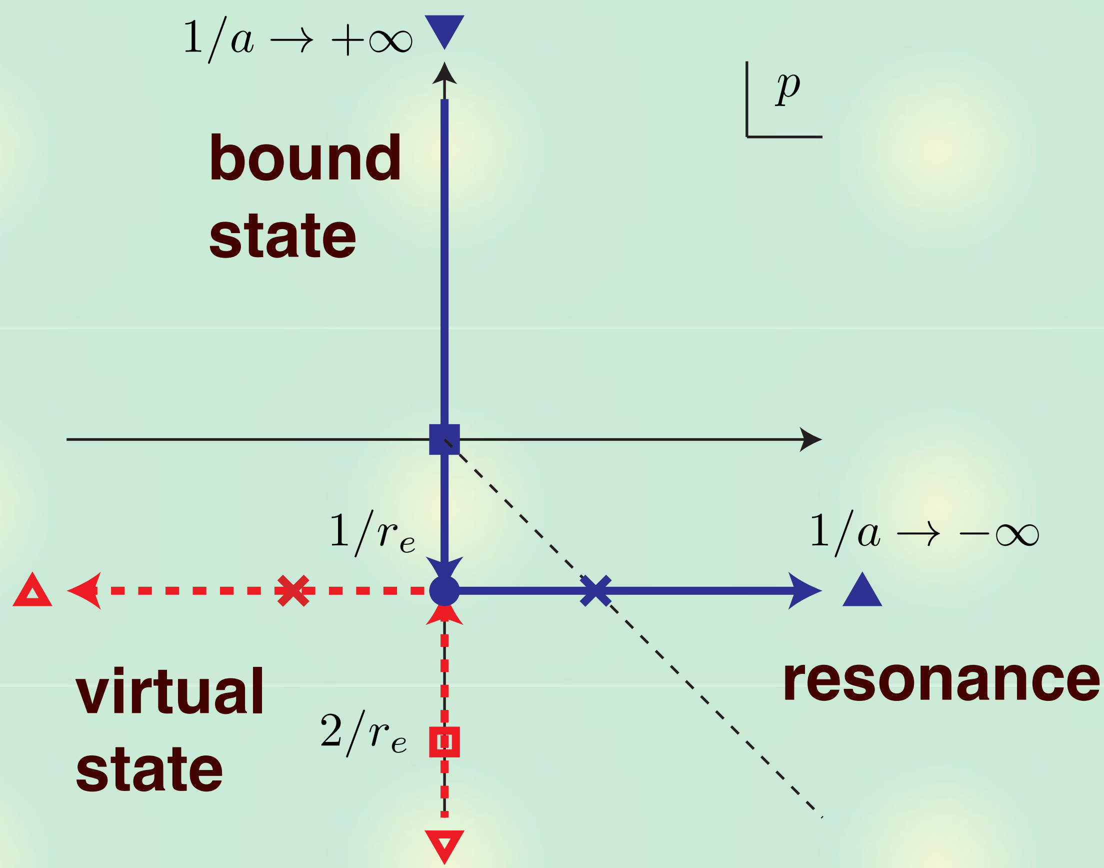
Poles of the amplitude

Near-threshold phenomena: effective range expansion

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013) with opposite sign of scattering length

$$f(p) = \left(-\frac{1}{a} + \frac{r_e^2}{2}p^2 - ip\right)^{-1}$$

$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a} - 1}$$



Pole trajectories with a fixed $r_e < 0$

Resonance pole position <--> (a, r_e)

Example of resonance: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering

- central values in PDG

$$E = 0.67 \text{ MeV}, \quad \Gamma = 2.59 \text{ MeV} \qquad\qquad p^{\pm} = \sqrt{2\mu(E \mp i\Gamma/2)}$$

- deduced threshold parameters of $\pi\Sigma_c$ scattering

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5 \text{ fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5 \text{ fm}$$

- field renormalization constant: complex

$$Z = 1 - 0.608i$$

Large negative effective range

<-- substantial elementary contribution other than $\pi\Sigma_c$
(three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ molecule**

Summary

Composite/elementary nature of resonances

- Renormalization constant Z measures elementariness of a stable bound state.
- In general, Z of a resonance is complex.
- Negative effective range r_e : CDD pole
- Near-threshold resonance : pole position is related to r_e --> elementariness

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013)
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)