Lattice QCD analysis of partial restoration of chiral symmetry in flux-tube structure between quarks

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ヘビークォークハドロンと原子核のスペクトルと構造
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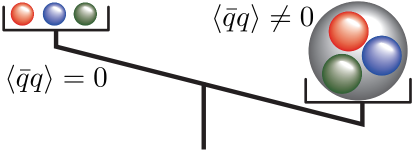
Refs. TI, G. Cossu, and S. Hashimoto,
PoS (LATTICE 2013) 376, arXiv:1311.0218 [hep-lat];
PoS (Hadron 2013) 159, arXiv:1401.4293 [hep-lat].

Non-perturbative QCD Phenomena

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2}\text{Tr}\; G_{\mu\nu}G^{\mu\nu} + \bar{q}\,(i\not{D} - m)\,q$$

Chiral Symmetry Breaking

$\text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V$

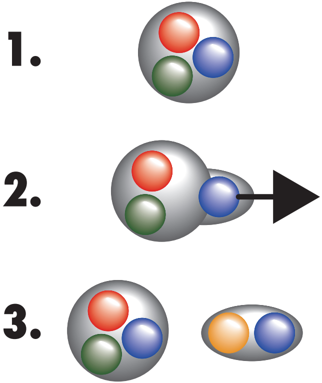


Chiral symmetry is restored

- High temperature
  - Quark Gluon Plasma
- Finite density
  - neutron star
  - nuclear matter (partially ?)

Confinement

There are no isolated quarks.



Quarkonia Spectroscopy and Interquark Potential

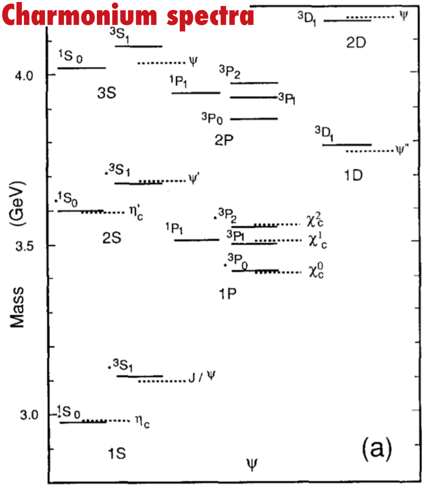
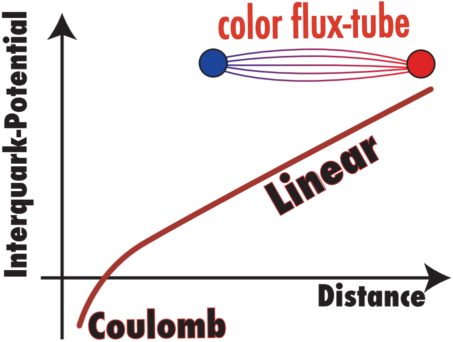


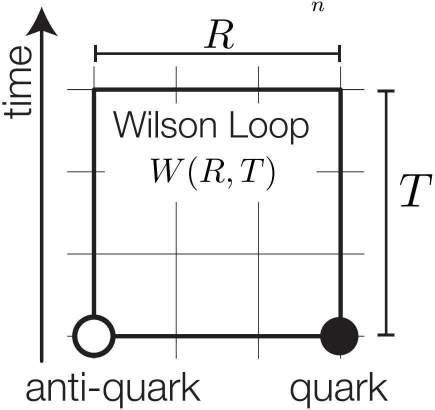
Fig. Mukherjee, et al., '93

■ Coulomb + linear potential

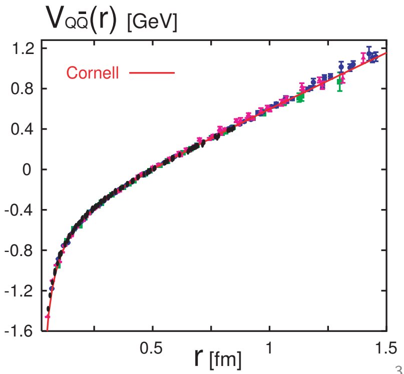


Wilson Loop and Interquark Potential in Lattice QCD

$$W(R,T) \equiv \mathrm{Tr} \prod_{\mathrm{loop}} U_\mu$$
$$= \langle Q\bar{Q}| e^{-HT} |Q\bar{Q}\rangle$$
$$= \sum_n C_n e^{-E_n(R)T} \sim \exp\left(-V_{\mathrm{Q\bar{Q}}}(R)T\right)$$



Coulomb + Linear potential



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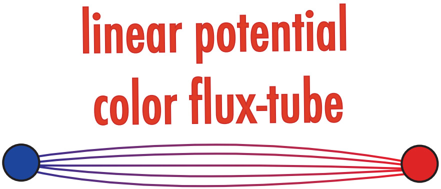

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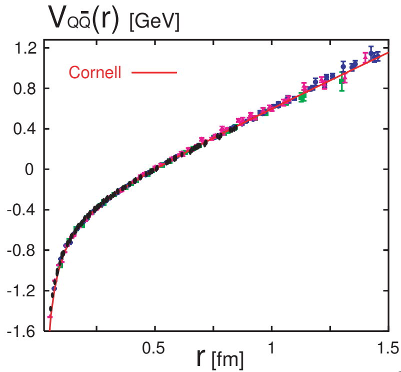
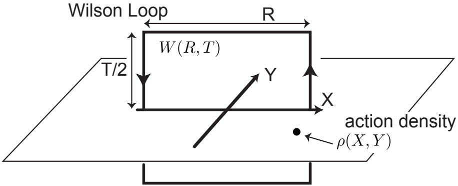


Flux-tube Measurement in Lattice QCD

■ action density $\rho(x)$ around Wilson loop, i.e., $m_Q \to \infty$ (static) Quark-Antiquark pair



$$\langle \rho(x) \rangle_W \equiv \frac{\langle \rho(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle$$

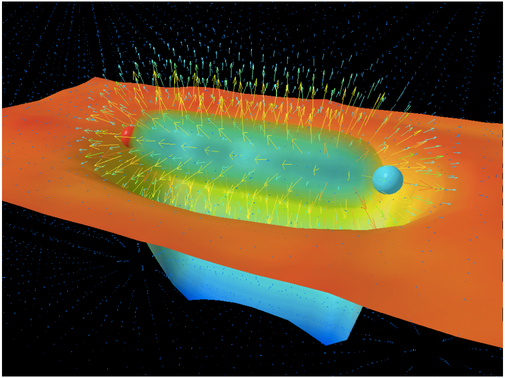
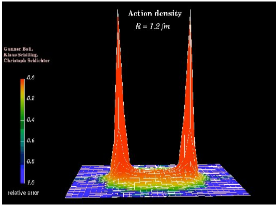


Fig. Leinweber et al. '03

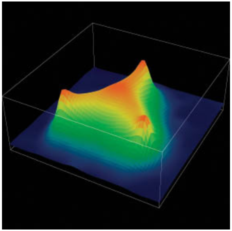
Flux-tube Structures for Multi-Quark Systems

interquark potential is proportional to the flux-tube length

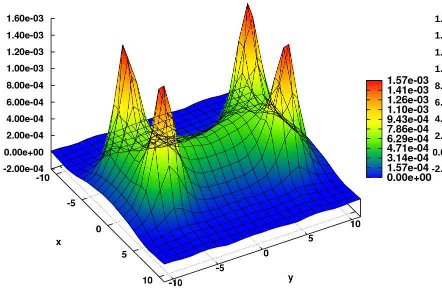
QQ-bar : Bali-Schlichter-Schilling '95

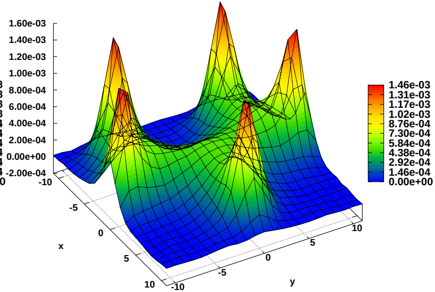


3Q : Ichie, et al. '03



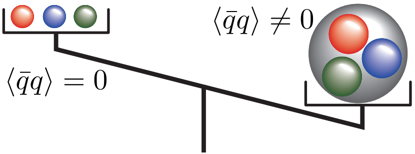
4Q and 5Q : Bicudo-Cardoso-Cardoso '11

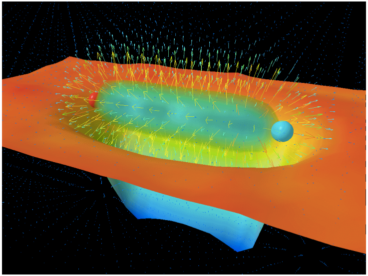




Question

Chiral Symmetry Breaking $\times$ Quark Confinement $=$?

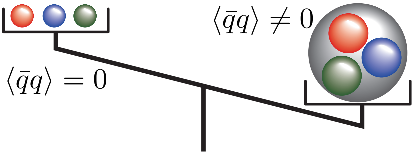


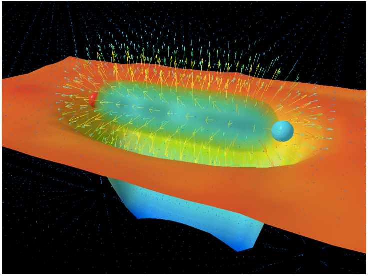


Question

Chiral Symmetry Breaking × Quark Confinement = ?

- Quarks are confined in hadrons by color flux-tube.
- How about "chiral symmetry" in color flux-tube, i.e., "hadron" ?
- From lattice QCD, we analyze chiral condensate inside color flux-tube.





Chiral Condensate and Dirac Eigenmode

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}\ G_{\mu\nu}G_{\mu\nu} + \bar{q}(\not{D} + m)q$$

- chiral condensate $\langle \bar{q}q \rangle \Leftarrow$ *eigenvalue of Dirac operator* $\not{D}\psi_\lambda = \lambda\psi_\lambda$

$$-\langle \bar{q}q \rangle = \langle {\rm Tr} S_q \rangle = \lim_{m\to 0}\lim_{V\to\infty}\langle {\rm Tr}\frac{1}{\not{D}+m}\rangle = \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\sum_\lambda \frac{1}{\lambda + m}$$

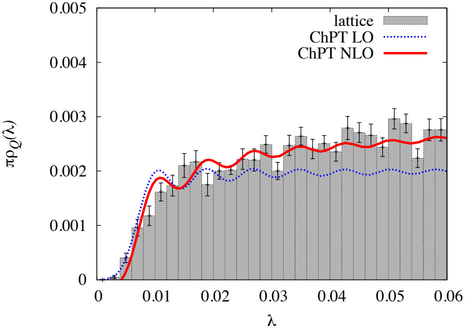
Banks–Casher Relation

$$\langle \bar{q}q \rangle = -\pi\langle \rho(0) \rangle$$

near-zero Dirac mode

$\Longrightarrow$ chiral symmetry breaking

$\rho(\lambda)$: eigenmode density



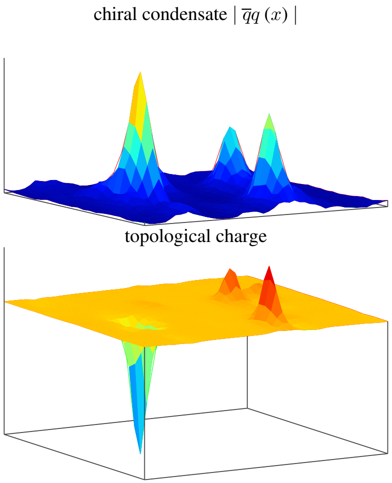
Fukaya et al. (JLQCD Coll.) '10

Local Structure of Chiral Condensate in QCD Vacuum

a snapshot of QCD vacuum

$$\langle \bar{q}q \rangle = -\mathrm{Tr}\frac{1}{\not{D}+m} = -\frac{1}{V}\sum_{\lambda}\frac{1}{\lambda+m}$$
$$= -\frac{1}{V}\sum_{\lambda}\sum_{x}\frac{\psi_{\lambda}^{\dagger}(x)\psi_{\lambda}(x)}{\lambda+m}$$
$$= \frac{1}{V}\sum_{x}\bar{q}q(x)$$

with Dirac eigenfunction $\not{D}\psi_{\lambda} = \lambda\psi_{\lambda}$



■ a local chiral condensate $\bar{q}q(x)$ forms clusters $\Longrightarrow$ instantons

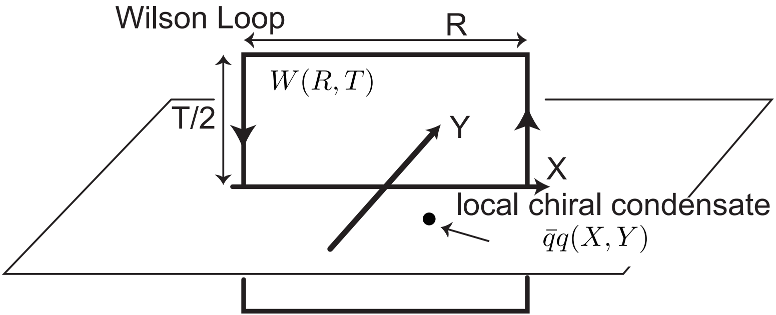
Chiral Condensate inside Color Flux-tube

local chiral condensate is given by

$$\bar{q}q(x) = -\sum_{\lambda} \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{\lambda + m} \quad \text{with } \textit{Dirac eigenmode } \not{D}\psi_\lambda = \lambda\psi_\lambda$$

change of the chiral condensate around the Wilson loop

$$\langle \bar{q}q(x)\rangle_W \equiv \frac{\langle \bar{q}q(x)W(R,T)\rangle}{\langle W(R,T)\rangle} - \langle \bar{q}q\rangle$$



About Lattice QCD Setup

In this study, we mainly use

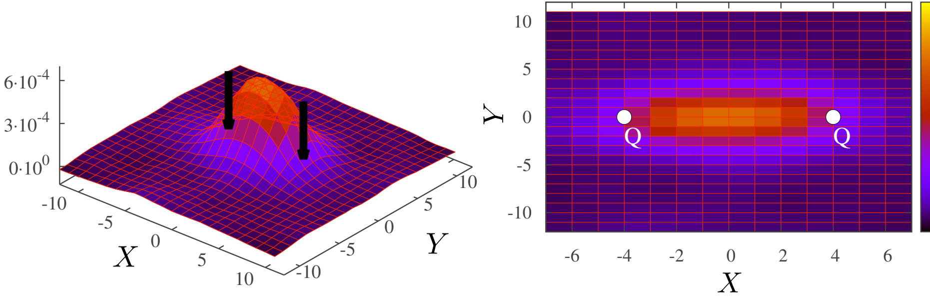
- 2+1 flavor dynamical overlap-fermion configuration by JLQCD Coll.
  $\Longrightarrow$ overlap-fermion keeps "exact chiral symmetry" on lattice
  $\Longrightarrow$ overlap-fermion Dirac eigenmodes are ideal probe
  to analyze chiral properties in lattice QCD
- pion mass $m_\pi \sim 300$ MeV, kaon mass $m_K \sim 500$ MeV
- lattice volume $24^3 \times 48$
- lattice spacing $a^{-1} = 1.759(10)$ GeV $\Longrightarrow$ $a \sim 0.112$ fm

Reduction of Chiral Condensate in the Flux-tube

difference of (truncated) chiral condensate with quark-antiquark

$$\langle \bar{q}q(x)\rangle_W^{(N)} \equiv \frac{\langle \bar{q}q^{(N)}(x)W(R,T)\rangle}{\langle W(R,T)\rangle} - \langle \bar{q}q\rangle^{(N)} \quad (N : \text{eigenmode number})$$

- $\langle \bar{q}q(x)\rangle_W > 0 \Rightarrow |\langle \bar{q}q(x)\rangle|_{\text{flux}} < |\langle \bar{q}q\rangle|$ $\because \langle \bar{q}q\rangle < 0$ in vacuum
- partial restoration of chiral symmetry in the flux ("Bag-model"-like)



(anti-)quark at $(4,0)$ and $(-4,0)$. lattice unit $a \sim 0.11$ fm, pion mass ~ 300 MeV

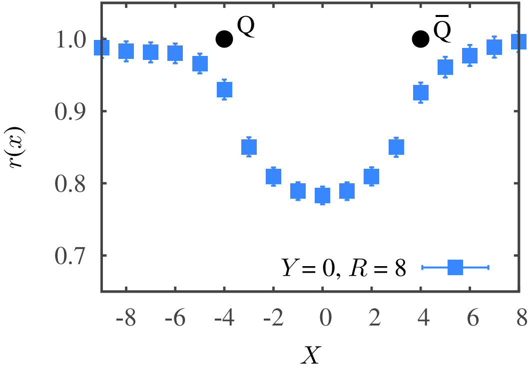
Ratio of Chiral Condensate

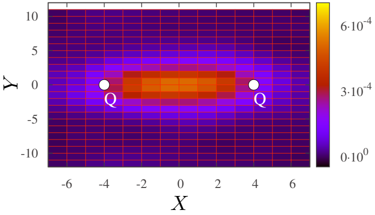
$$r(x) \equiv \frac{\langle \bar{q}q^{(\mathrm{subt})}(x) W(R,T) \rangle}{\langle \bar{q}q^{(\mathrm{subt})} \rangle \langle W(R,T) \rangle} < 1$$

$\Rightarrow$ about 20 % of chiral condensate is reduced at flux core

due to strong color-electric field between quarks

cross section of flux-tube





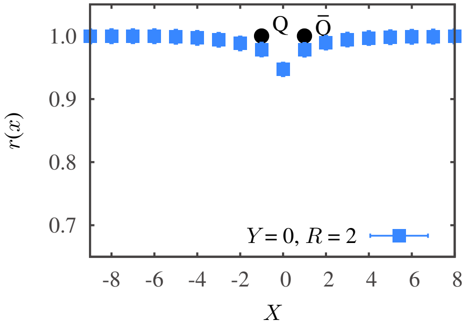
(taking continuum limit $a \to 0$) subtracted condensate

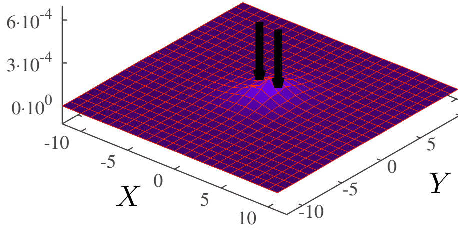
$\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\mathrm{subt})} \rangle + c_1^{(N)} m_q / a^2 + c_2^{(N)} m_q^3$

ref. JLQCD Coll. Noaki '09

Quark Separation Dependence of Restoration

partial restoration increases with the interquark separation R,
until the string breaking occurs around $R \sim 1$ fm





cf. lattice unit $a \sim 0.11$ fm

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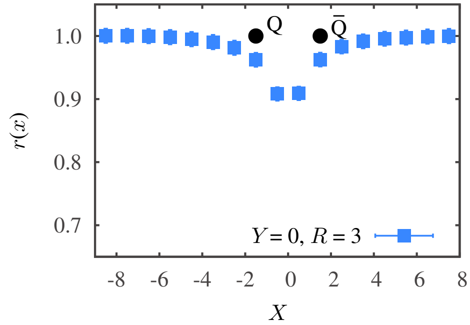

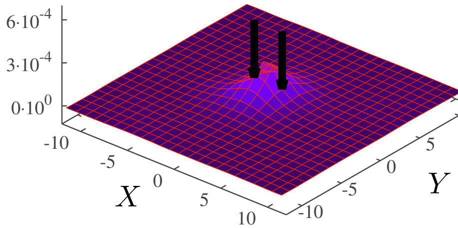

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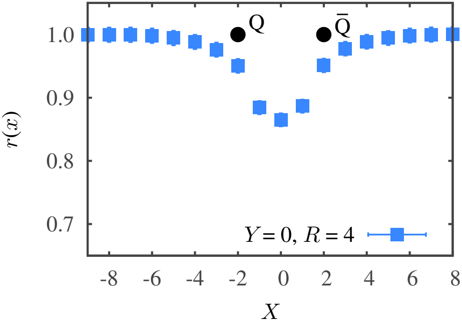

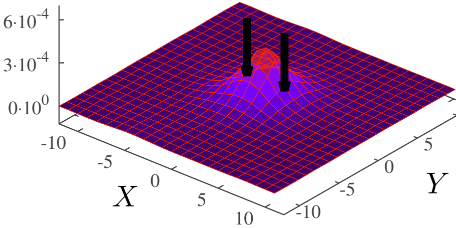

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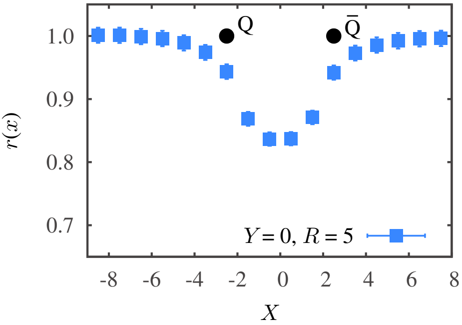

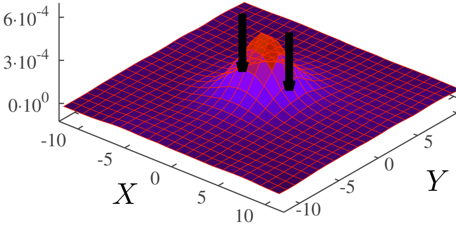

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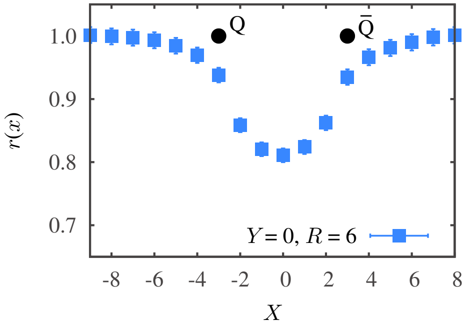

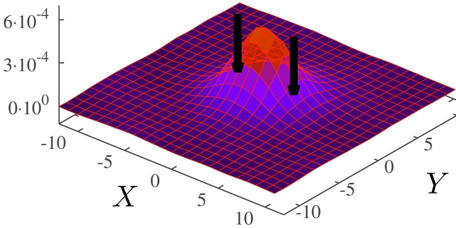

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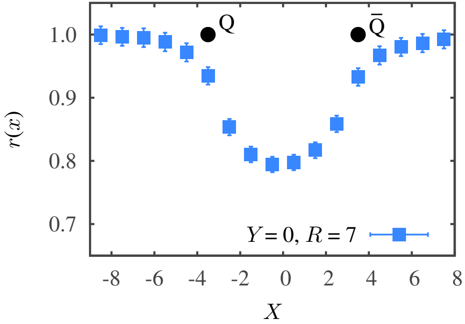

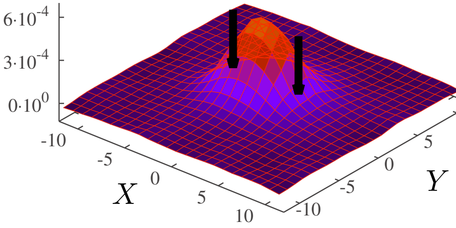

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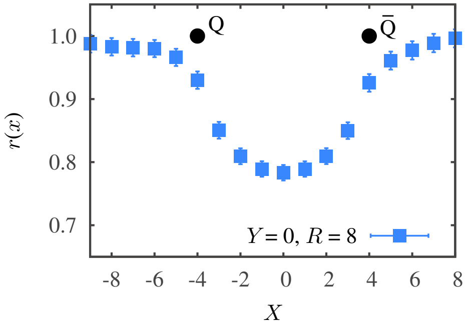

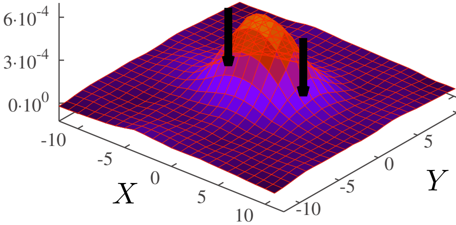

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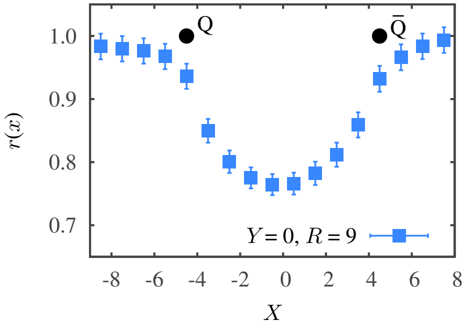

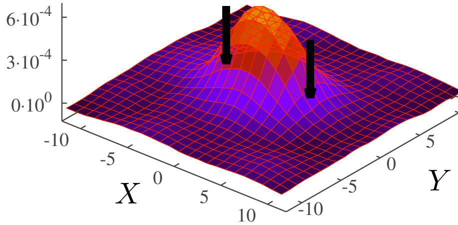

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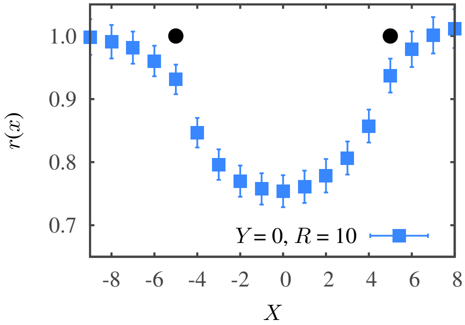

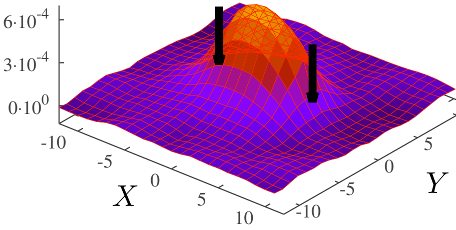

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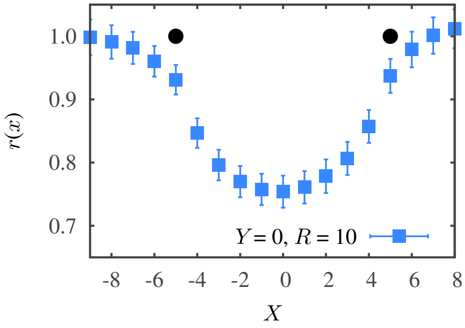
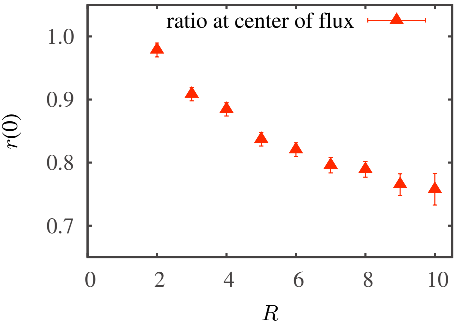


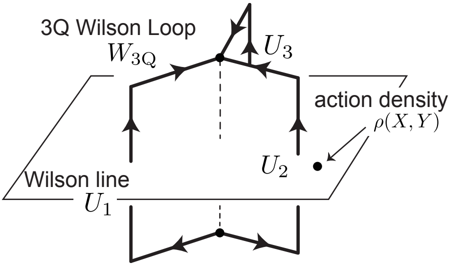


cf. lattice unit $a \sim 0.11$ fm

3-Quark System

3Q-Wilson loop

$$W_{3Q} \equiv \frac{1}{3!}\varepsilon_{abc}\varepsilon_{a'b'c'}U_1^{aa'}U_2^{bb'}U_3^{cc'}$$



with Wilson lines U_k
cf. Takahashi-Suganuma '01

Y-type flux formation in 3Q-system

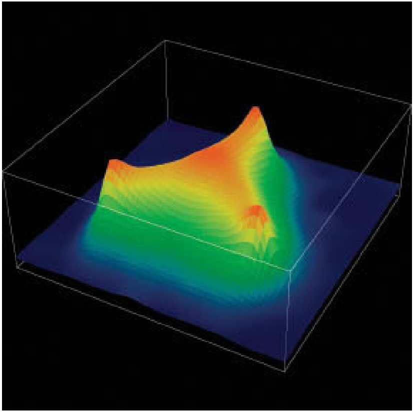


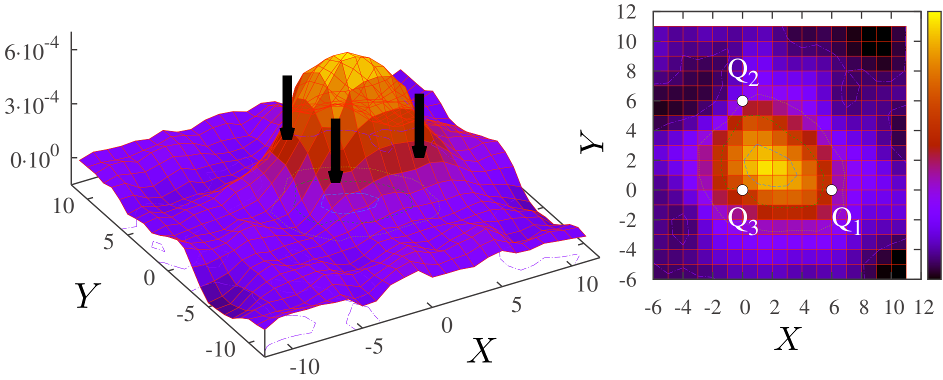
Figure : Ichie et al. '03

Partial Restoration of Chiral Symmetry in 3Q system

■ chiral condensate in 3Q system

$$\langle \bar{q}q(x) \rangle_{W_{3Q}} \equiv \frac{\langle \bar{q}q(x) W_{3Q} \rangle}{\langle W_{3Q} \rangle} - \langle \bar{q}q \rangle > 0$$

◦ Y-type flux is smeared due to small quark separation



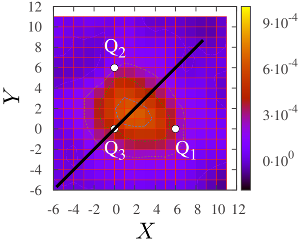
right triangle configuration : $Q_1 = (6, 0), Q_2 = (0, 6), Q_3 = (0, 0)$

lattice unit $a \simeq 0.11$ fm

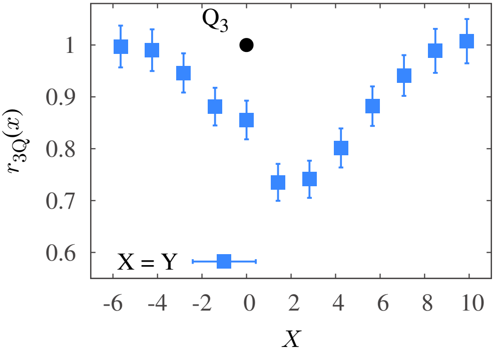
Ratio of Chiral Condensate in 3Q system

$$r_{3Q}(x) \equiv \frac{\langle \bar{q}q^{(\mathrm{subt})}(x) W_{3Q} \rangle}{\langle \bar{q}q^{(\mathrm{subt})} \rangle \langle W_{3Q} \rangle} < 1 \qquad \text{inside flux}$$

- about 20 ~ 30 % of chiral condensate is reduced in 3Q-system
- "partial restoration of chiral symmetry" in "Baryon"

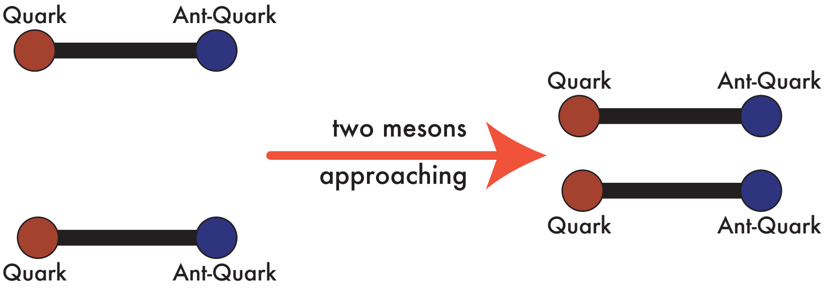


cross-section of 3Q-flux

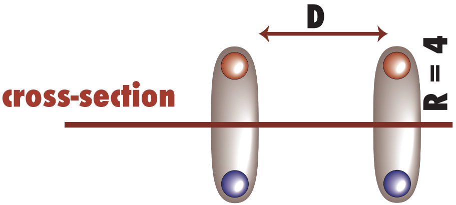


Meson-Meson System

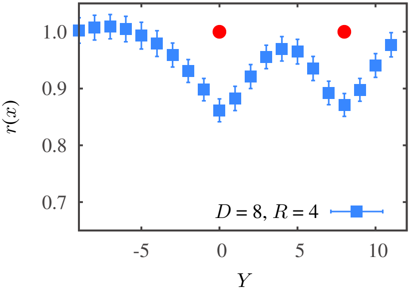
Considering two mesons are approaching,
how about chiral symmetry restoration ?
enhanced or not ?

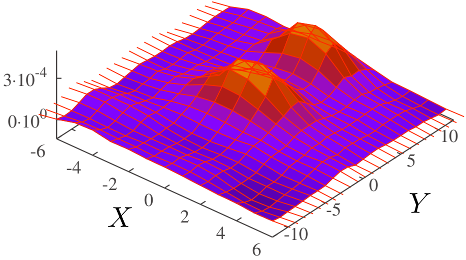


Quark-Antiquark Pair



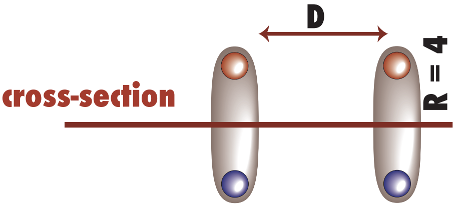
ratio of chiral condensate



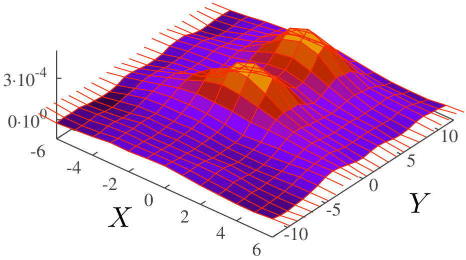


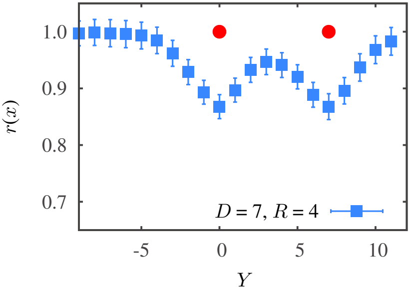
cf. lattice unit $a \sim 0.11$ fm

Quark-Antiquark Pair



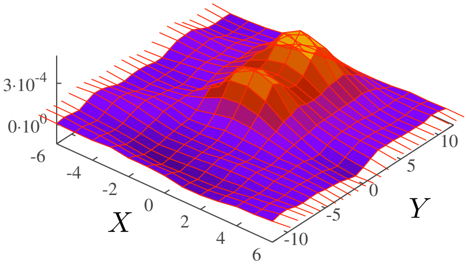
ratio of chiral condensate

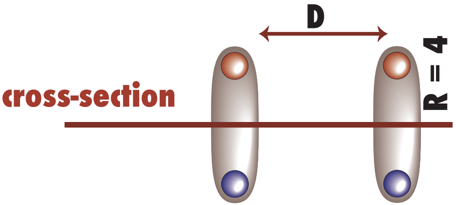




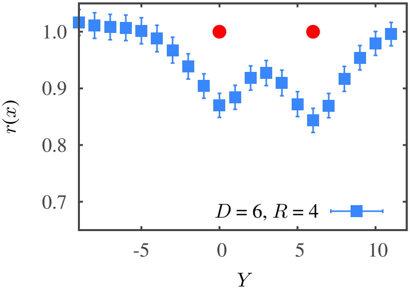
cf. lattice unit $a \sim 0.11$ fm

Quark-Antiquark Pair



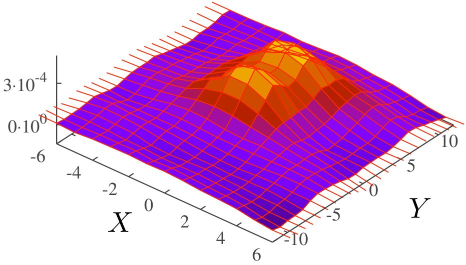


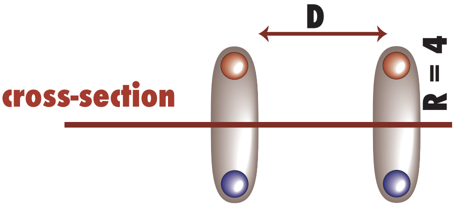
ratio of chiral condensate



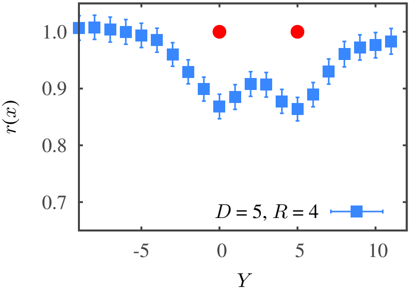
cf. lattice unit $a \sim 0.11$ fm

Quark-Antiquark Pair



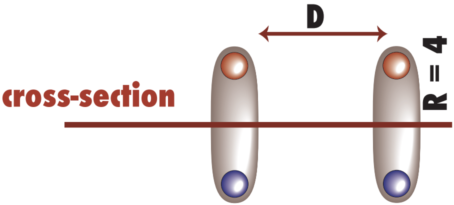


ratio of chiral condensate

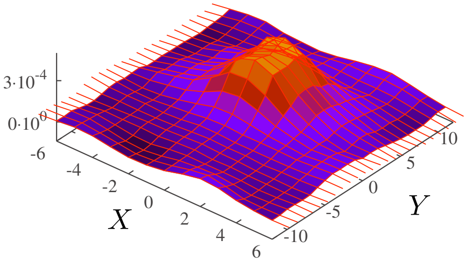


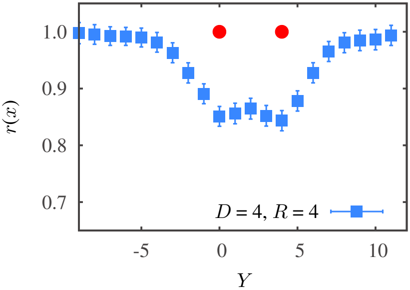
cf. lattice unit $a \sim 0.11$ fm

Quark-Antiquark Pair



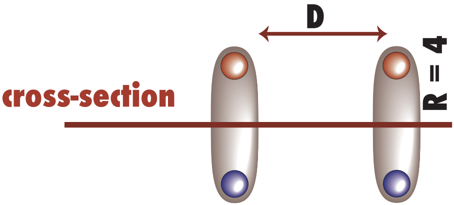
ratio of chiral condensate



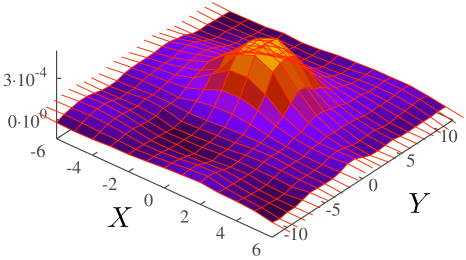


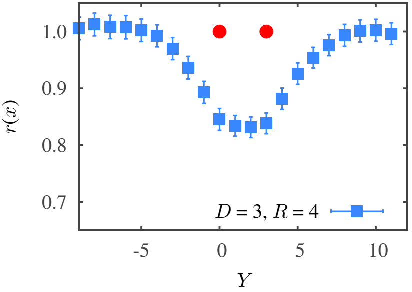
cf. lattice unit $a \sim 0.11$ fm

Quark-Antiquark Pair



ratio of chiral condensate



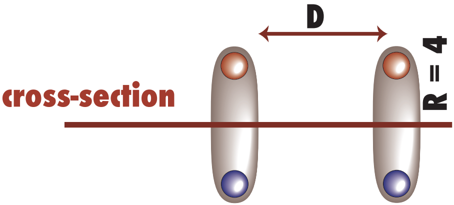


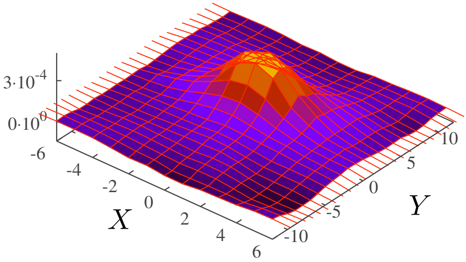
cf. lattice unit $a \sim 0.11$ fm

Quark-Antiquark Pair

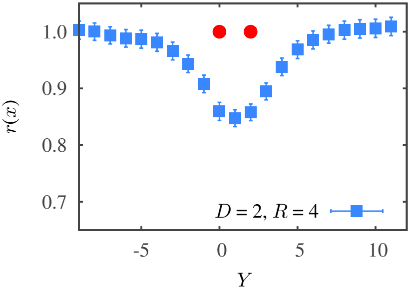
Partial restoration of chiral symmetry is the same as QQ-bar-system.

$r_{Q\bar{Q}+Q\bar{Q}} = r_{Q\bar{Q}}$





ratio of chiral condensate



cf. lattice unit $a \sim 0.11$ fm

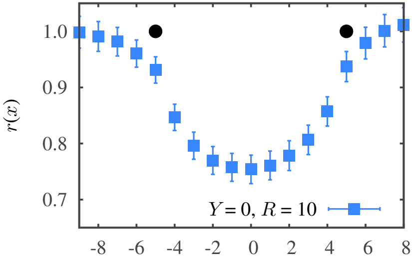
Summary

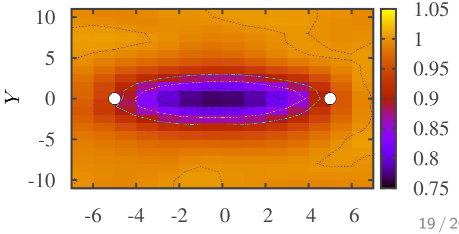
- A color flux-tube appears between quarks, which produces linear confining potential.
- By measuring the chiral condensate $\bar{q}q(x)$ inside flux-tube, we show partial restoration of chiral symmetry

$$\frac{\langle\bar{q}q\rangle_{\text{in flux}}}{\langle\bar{q}q\rangle_{\text{vacuum}}} = 0.7 \sim 0.8$$

$\Rightarrow$ effect of strong *color-electric fields* between quarks

- Partial restoration inside the flux-tube may change
  - ○ light quark properties in QQq, QQqq ?
  - ○ quark pair production rate ?

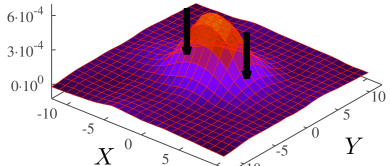


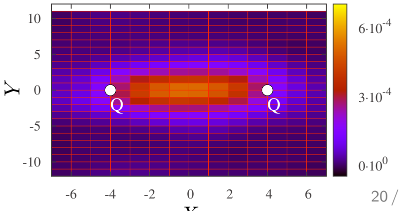


Summary and Outlooks

In this work, we probe local chiral condensate $\bar{q}q(x)$ inside the flux-tube.

- We can use various kinds of operators such as
  - quark number density, energy density
    $\Longrightarrow$ color charge screening, string breaking, hadronization
  - topological charge density
    $\Longrightarrow$ $\mathrm{U_A}(1)$ anomaly inside hadron
  - energy-momentum tensor
    $\Longrightarrow$ energy of flux-tube, pressure of bag
  - *and so on*
- It is also possible to apply tetra-quark, meson-meson systems.
- In addition to hadrons, color flux also appears after heavy ion collision.
- We can also generalize this method in QGP phase at finite temperature.





4 Appendix

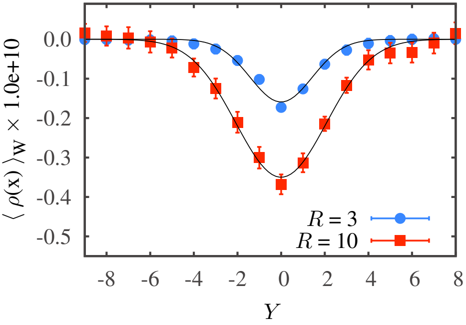
Thickness of Flux-tube and Chiral Symmetry Restoration

- By increasing the length of quantum flux-tube R,
  the thickness σ grows as

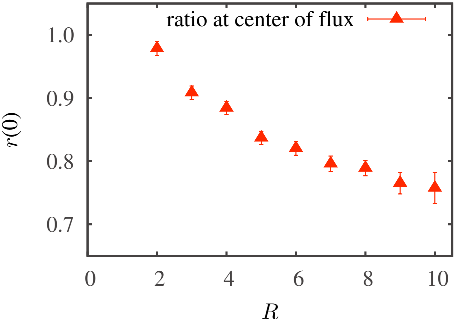
$$\sigma \sim \ln R$$

  which is called as "Roughening." [1]
- Partial restoration of chiral symmetry relates its thickness ?

Cross-section of flux-tube



Ratio of chiral condensate



[1] Hasenfratz-Hasenfratz-Hasenfratz '80, Luscher-Munster-Weisz '81

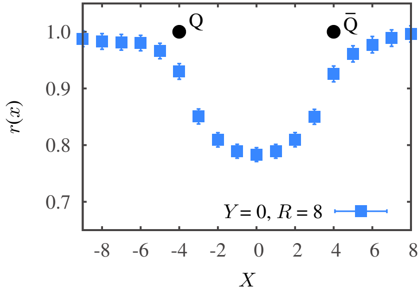
Flux-tube and Chiral Condensate Ratio

flux-tube — action density distribution around static color sources

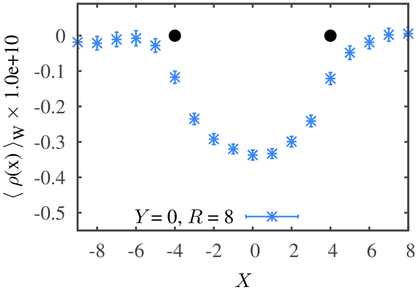
$$\langle \rho(x) \rangle_W \equiv \frac{\langle \rho(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle$$

$\Rightarrow$ chiral condensate is reduced inside "flux-tube"

ratio of chiral condensate



action density distribution



Quark Mass Dependence of Chiral Condensate Reduction

$16^3 \times 48$ lattice with low-lying 120 eigenmodes

- $m_{\rm ud} = 0.015$: $m_\pi \sim 0.30$ GeV
- $m_{\rm ud} = 0.050$: $m_\pi \sim 0.53$ GeV