Heavy hadrons by lattice QCD

Yusuke Namekawa(Univ. of Tsukuba)
for the PACS-CS collaboration

S.Aoki, K-I.Ishikawa, N.Ishizuka, K.Kanaya, Y.Kuramashi, Y.Nakamura,
Y.Namekawa, M.Okawa, Y.Taniguchi, A.Ukawa, N.Ukita, T.Yamazaki, T.Yoshie

Contents

1 Introduction

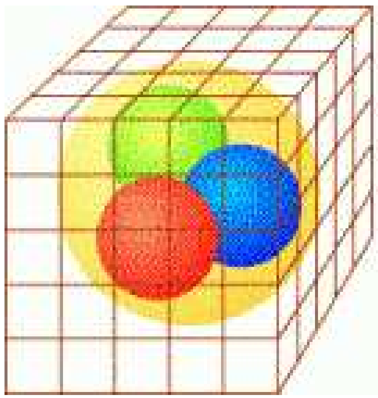
1.1 Basics of lattice field theory

Lattice field theory is a field theory on the discretized spacetime, formulated by K.Wilson(Nobel prize in 1982, 1936-2013).

- Lattice as a regularization: Infrared and ultraviolet regularizations are naturally introduced by finite lattice size L and lattice spacing a.

  ◇ Continuum theory is recovered by taking $L \to \infty, a \to 0$.

- Numerical simulation: Thanks to finite degrees of freedom on the lattice, nonperturbative study by numerical simulations is possible. It is useful, especially for QuantumChromoDynamics(QCD), where nonperturbative effects are crucial, such as quark confinement.



[Expectation value by lattice QCD]

Expectation value of an observable O is defined by

$$\begin{aligned}
\langle O \rangle &:= \int dU O e^{-S[U]} \\
U(x) &:= \exp(ig_0 A_\mu(x)), \\
S[U] &: \text{action} \\
g_0 &: \text{bare coupling constant} \\
A_\mu(x) &: \text{gauge field}
\end{aligned}$$

In lattice QCD, the expectation value is evaluated as

$$\langle O \rangle = \frac{1}{N_{\rm data}} \sum_{i=1}^{N_{\rm data}} O[U^i]$$

where $\{U_i\}$ is a set of gauge configurations generated by Monte Carlo simulation with a probability $P \sim e^{-S}$,

$$\{U^1\} \to \{U^2\} \to \{U^3\} \to \ldots$$

[Static quark-antiquark potential $V(r)$]

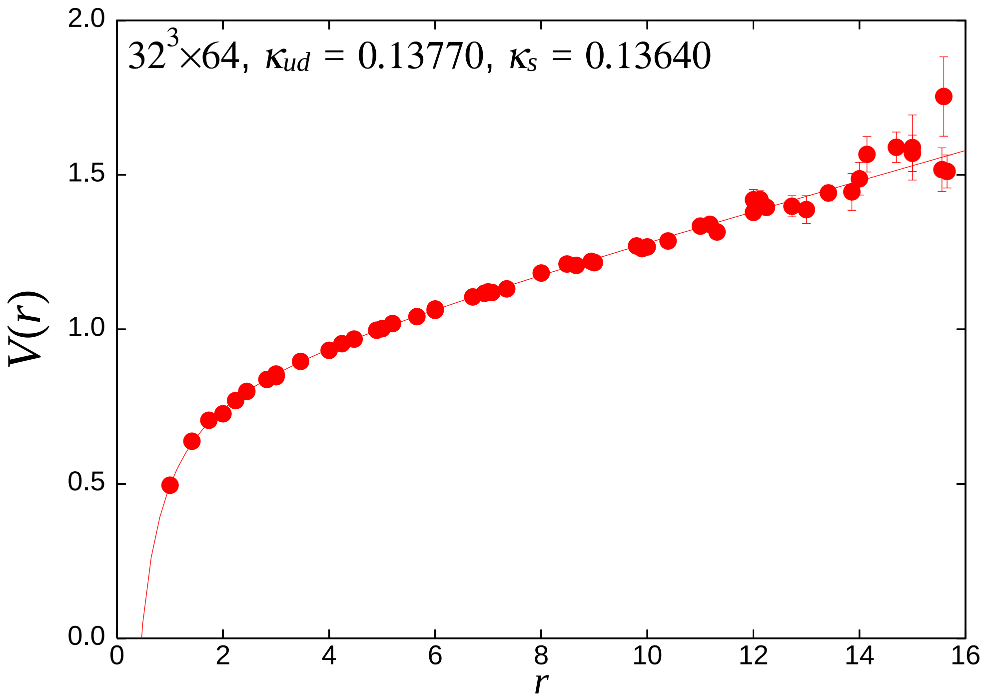
Quark-antiquark potential $V(r)$ can be calculated by lattice QCD.

- At a large distance, $V(r)$ grows linearly in r i.e. quark can not be separated independently.

$$V(r) \sim \sigma r$$

r : distance between quark and antiquark

σ : string tension



PACS-CS, 2009

[Advantages of lattice simulation]

Lattice simulation gives a nonperturbative and model independent result.
cf1. for perturbative studies, see lectures by Yasui-san and Harada-san.
cf2. for model studies, see talks by Yoshida-san, Hyodo-san ...

- Lattice simulation can predict undiscovered hadrons in a nonperturbative and model independent way.
- Lattice simulation can be a guide for uncertain experiments.
- Lattice simulation can determine quark masses, which can not be obtained in experiments due to quark confinement.
- Lattice simulation can extract Cabbibo-Kobayashi-Maskawa matrix element from the experimental data.

[Disadvantage of lattice simulation]

- Lattice simulation needs large computer resources.
  ex. the fastest computer in Japan(K-computer) costs $O(10^{11})$ yen.

[Comparison of model with lattice QCD]

So far, many models have been used for a study of QCD phenomena.

- Correctness of a model must be always checked, because the result is model-dependent.
  $\leftarrow$ Experiment gives a check. In addition, precise lattice QCD calculations can help judging a model.

	Model	Lattice
Result	Model-dependent	Model-independent
Input	Many parameters	α_s, m_{quark} (or hadron masses)
α_s, m_{quark} running	Artificial	QCD running
Heavy quark	$1/M$ expansion	Full order
Cost	Low	High

Recent development of computers and algorithms reduce the cost of lattice simulations significantly $\rightarrow$ Next page.

1.2 Recent development of lattice QCD

- Thanks to recent development of computers and algorithms, realistic simulations on the physical point can be performed i.e. pion mass in lattice QCD simulations reaches the physical value.

Year	Machine	Speed [TFlops]	m_π[MeV]
1996-2005	CP-PACS	0.6	700
2006-2011	PACS-CS	14	160
2008-2014	T2K(Tokyo,Tsukuba,Kyoto)	235	135
	Experiment		135



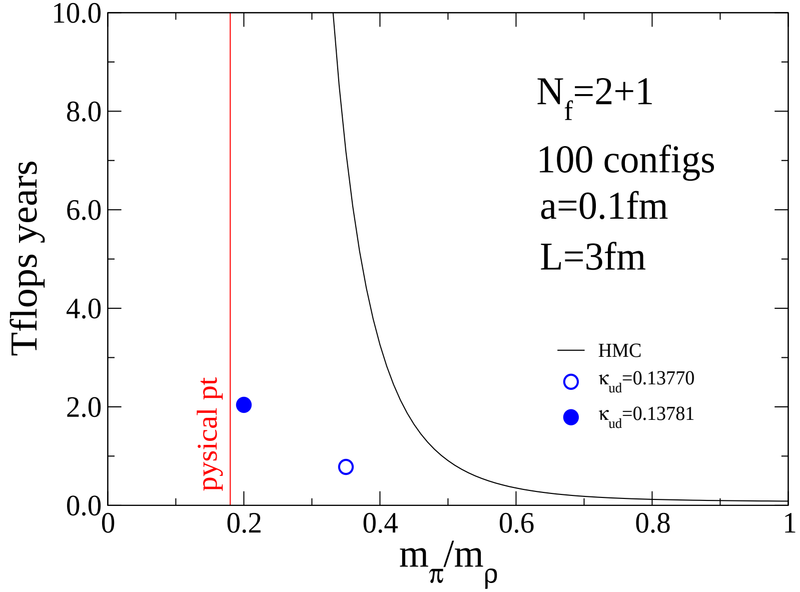
[Recent development of consumer game machine]

- PlayStation 4 is on sale.
- PlayStation 4 is as fast as CP-PACS and PACS-CS.
  - ◇ You can enjoy not only video games but also lattice QCD on PlayStation 4.
- Q. Is PlayStation 4 useful for a study of lattice QCD?
  A. Yes for small scale simulations, but No for large scale simulations, because of limited amount of memories and poor network speeds.

Year	Machine	Speed [TFlops]	m_π[MeV]
1996-2005	CP-PACS	0.6	700
2006-2011	PACS-CS	14	160
2008-2014	T2K(Tokyo,Tsukuba,Kyoto)	235	135
2014	PlayStation 4	2	–
	Experiment		135

[Computational cost]

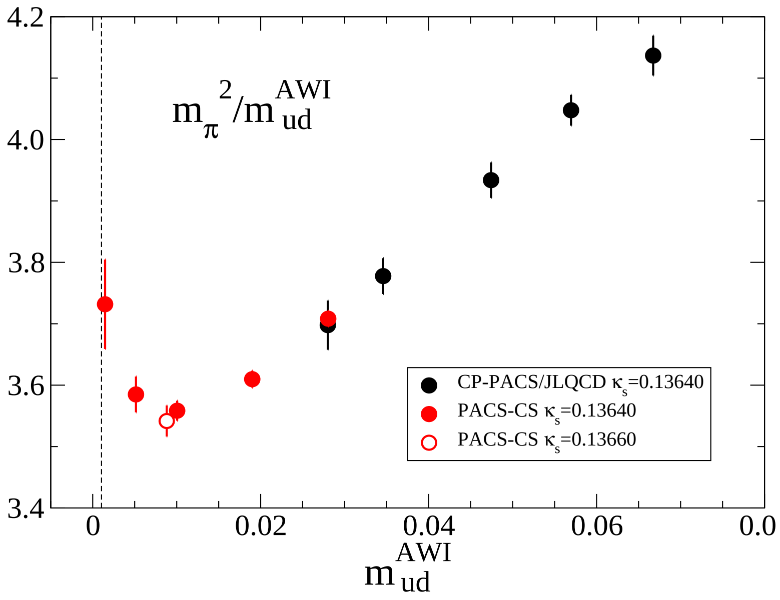
- Rapid increase of the computational cost toward the physical point kept us from performing realistic lattice QCD simulations.
- Rapid increase of the computer power and algorithm improvement help us realistic lattice QCD simulations on the physical point.
  - ◇ Computer powers grow up by $O(1000)$ in the last decade.
  - ◇ Algorithm improvement reduces the cost by $O(10)$.

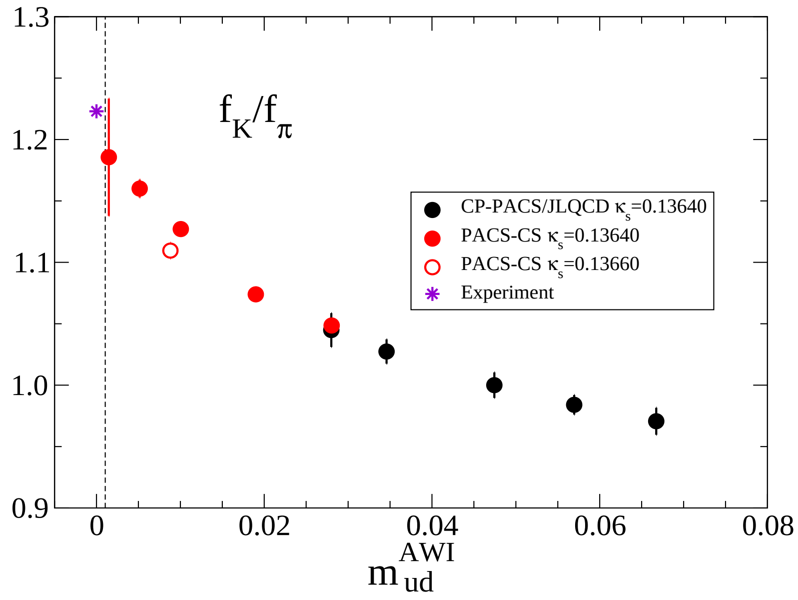


PACS-CS, 2009

[Extrapolation to the physical point]
Realistic simulation is needed for a correct result.

- If we employ artificially heavy quark masses in the lattice simulation, the computational cost is reduced significantly.
- However, an extrapolation from too heavy quark mass data fails to reproduce the experimental value.
  → Simulation near the physical point is required.





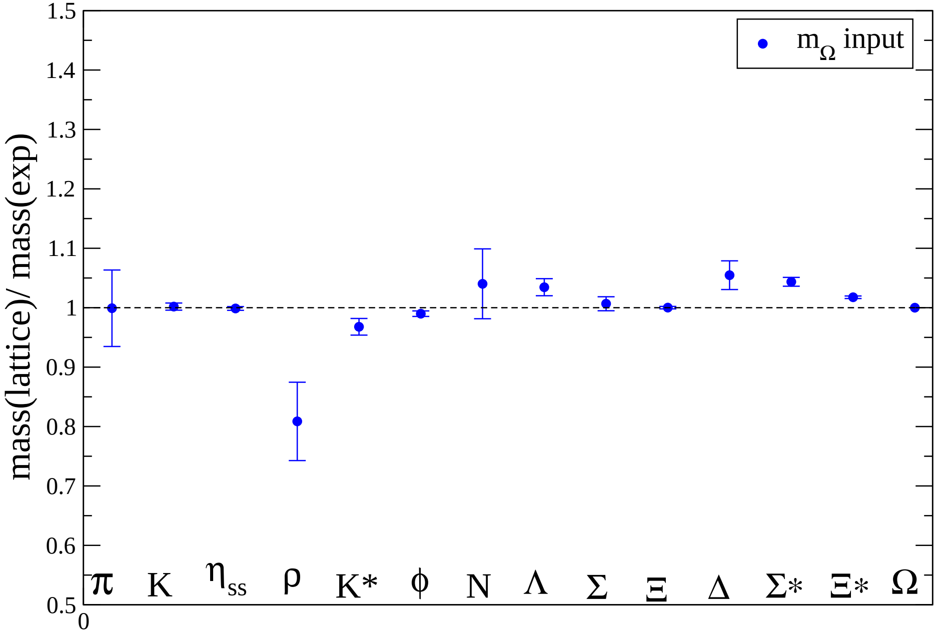
PACS-CS,2009

[Recent result 1 : Light hadron spectrum on the physical point]

Three lattice groups have reached the physical point with dynamical *ud*, *s* quarks. Stable hadron spectrum is reproduced in 5% accuracy.

PACS-CS,2009; BMW,2010; MILC,2012

- Lattice result in ρ channel corresponds to $E_{\pi\pi}$, not m_ρ.
- For unstable hadrons such as ρ, more detailed analysis is needed → Next page.



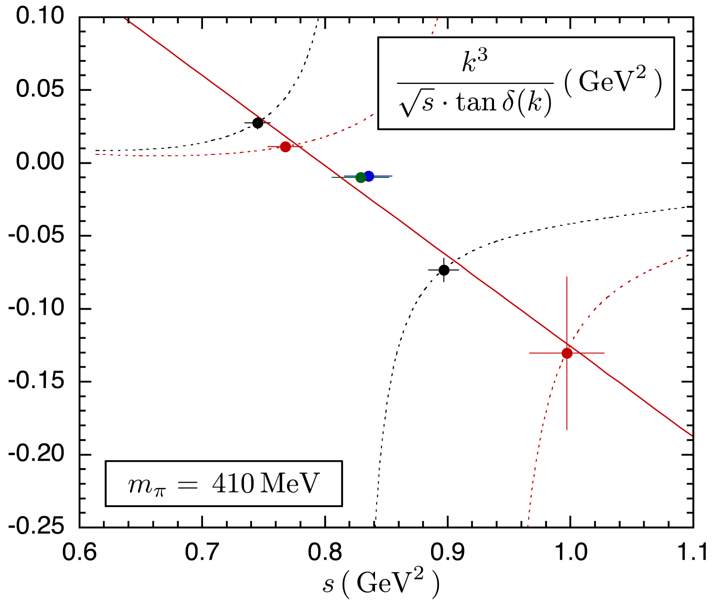
PACS-CS,2009

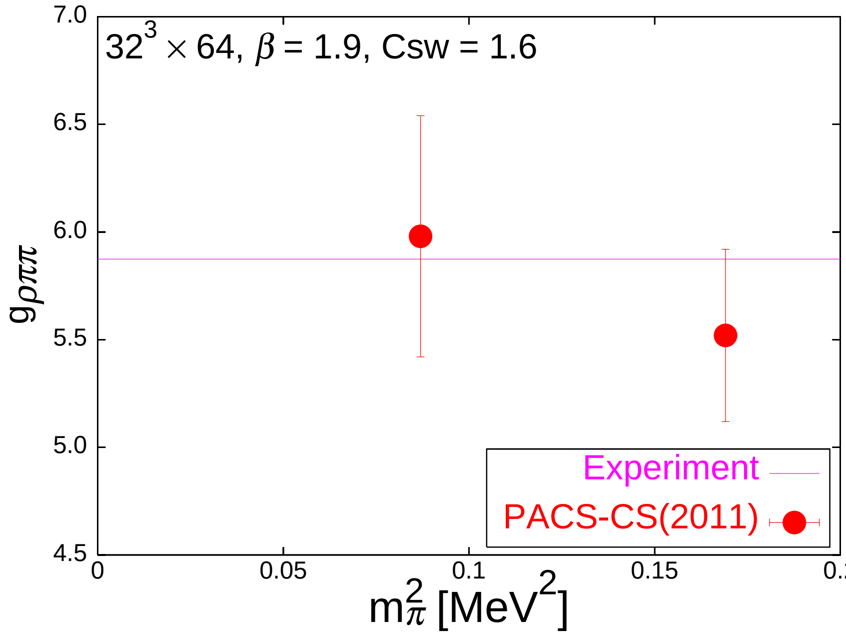
[Recent result 2 : rho meson decay]

- Rho meson resonance mass m_ρ and decay width Γ_ρ are obtained by the scattering phase shift $\delta(k)$ calculated using Lüscher's formula M.Lüscher,1986

  $\rightarrow$ See Appendix for details.

- Although measurements of ρ meson decay on the physical point has not been performed, lattice results agree with the experiment value.



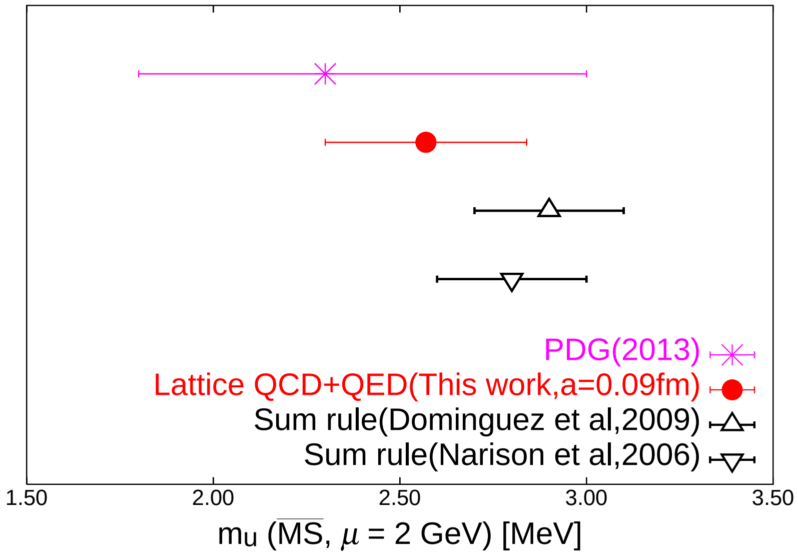


PACS-CS,2011

[Recent result 3 : Quark masses in QCD+QED]

We can perform not only pure QCD but also QCD+QED ($N_f = 1+1+1$) simulation on the physical point with dynamical u, d, s quarks.

$$\langle O(e) \rangle_{QCD+QED} = \left\langle O \frac{\det D_{quark}[U_{QCD} \otimes U_{QED}(e)]}{\det D_{quark}[U_{QCD}(e=0)]} \right\rangle_{QCD+quenched\ QED}$$



PACS-CS,2012

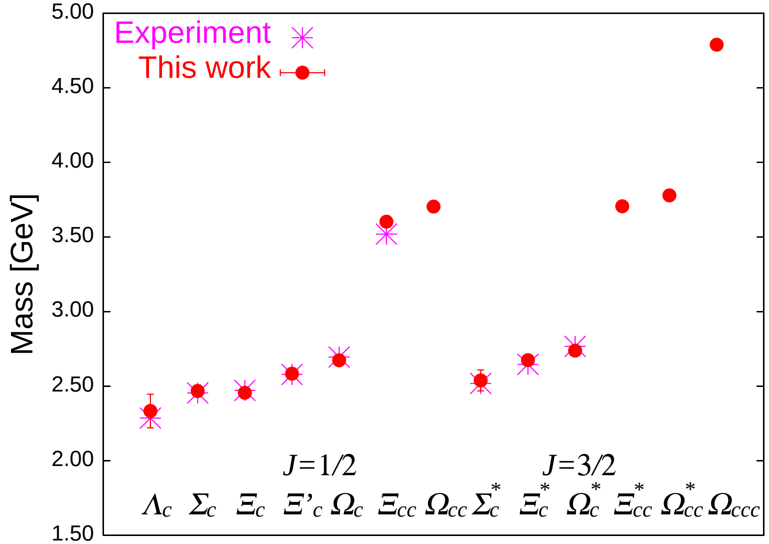
2 Lattice QCD results for heavy hadrons

[Advantage of lattice QCD simulation for heavy hadrons]

- Lattice simulation can predict undiscovered hadron properties in a nonperturbative and model independent way.
  $\rightarrow$ Doubly and triply charmed baryon spectrum.
- Lattice simulation can be a guide for uncertain experiments.
  $\rightarrow$ Ξ_{cc} spectrum.
- Lattice simulation can determine quark masses, which can not be obtained in experiments due to quark confinement.
  $\rightarrow$ Charm quark mass.
- Lattice simulation can extract Cabbibo-Kobayashi-Maskawa matrix element from the experimental data.
  $\rightarrow$ $|V_{cd}|, |V_{cs}|$.

2.1 Doubly and triply charmed baryon spectrum

- Doubly and triply charmed baryons have not been discovered in experiments, except for Ξ_{cc} → Next page.
- Lattice simulation can predict undiscovered hadrons in a nonperturbative and model independent way.



PACS-CS,2013

2.2 Doubly charmed baryon, Ξ_{cc}

[Experimental status of Ξ_{cc}]
A doubly charmed baryon Ξ_{cc} has not been established experimentally yet.
Only one candidate for Ξ_{cc} has been reported.

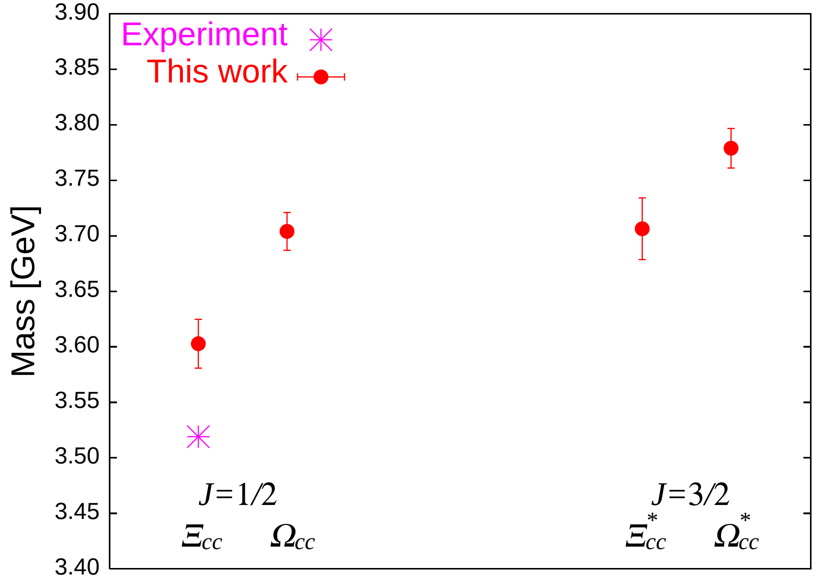
- $\Xi_{cc} = 3519$ [MeV] was found by SELEX(2002,2005).
- BABAR, BELLE and FOCUS found no evidence for Ξ_{cc}.

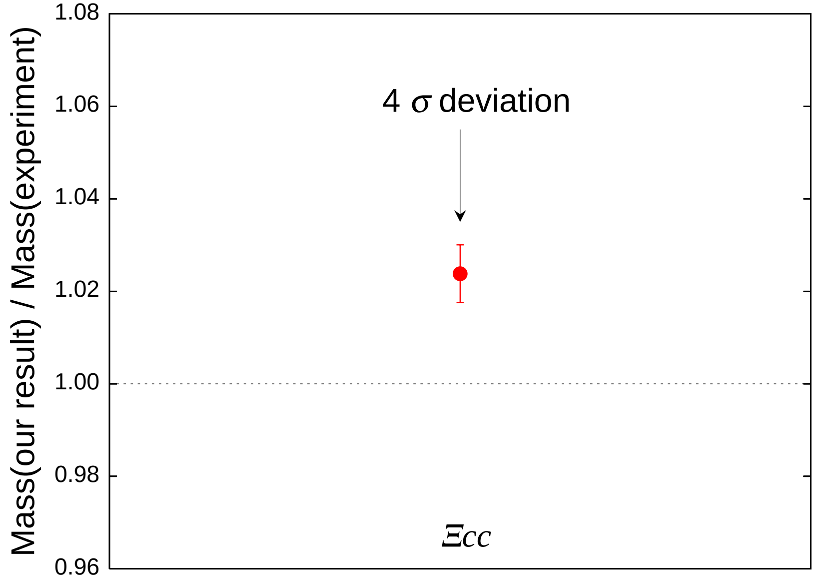
  ♢ Ξ_{cc} is 1-star and has been omitted from PDG.

Lattice QCD result can be a guide for Ξ_{cc}

[Lattice results for doubly charmed baryons]

- Our result does not agree with the experimental value of Ξ_{cc}.
  - ◇ Our result for Ξ_{cc} is higher than SELEX value by 100 MeV (4σ deviation).



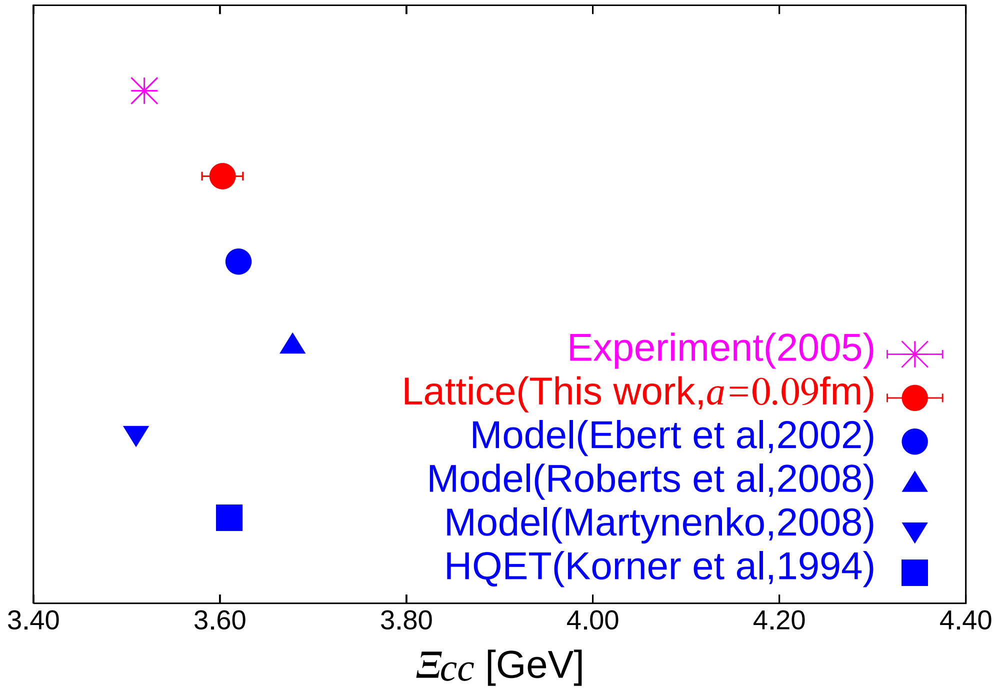


PACS-CS,2013

[Comparison of Ξ_{cc} with models]

Typical model calculations are compared with our lattice result.

- Many models and our result give Ξ_{cc} mass higher than SELEX value by around 100 MeV.
- (There is a model that is consistent with SELEX result.)

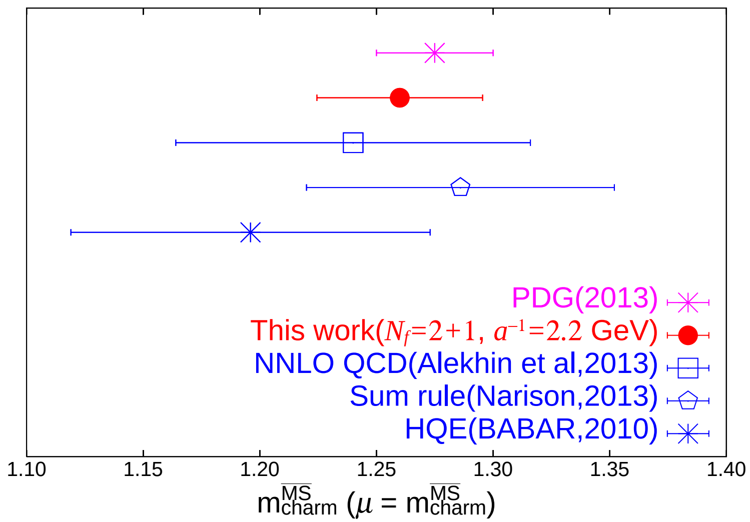


2.3 Charm quark mass

- Charm quark mass is obtained by the axial Ward-Takahashi identity.
- Our result is consistent with other continuum calculations.

$$m_{charm}^{\overline{\mathrm{MS}}} = (Z_{A_4}/Z_P) m_{charm}^{AWI}, \quad m_{charm}^{AWI} = m_{PS} \frac{\langle 0 | A_4^{imp} | PS \rangle}{\langle 0 | PS | PS \rangle},$$

$$A_\mu^{imp} = \sqrt{2\kappa_q}\sqrt{2\kappa_Q} Z_{A_\mu} \{ \bar{q}(x)\gamma_\mu\gamma_5 Q(x) - c_{A_\mu}^{+} \partial_\mu^{+} (\bar{q}(x)\gamma_\mu\gamma_5 Q(x)) - c_{A_\mu}^{-} \partial_\mu^{-} (\bar{q}(x)\gamma_\mu\gamma_5 Q(x)) \},$$



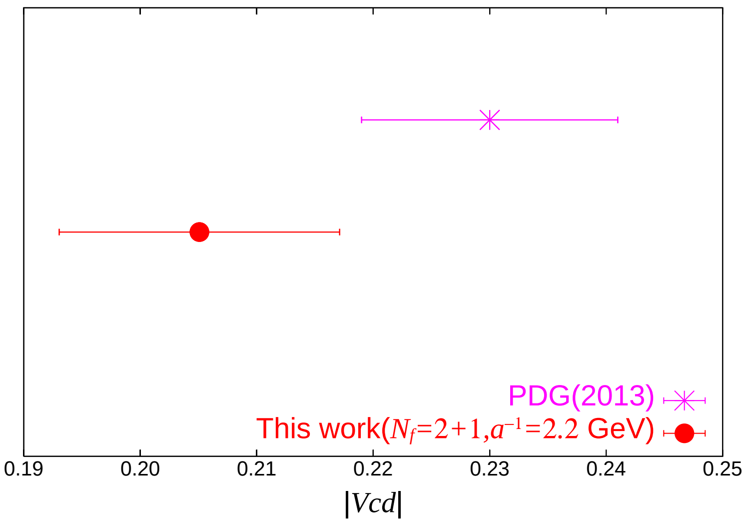
PACS-CS,2011

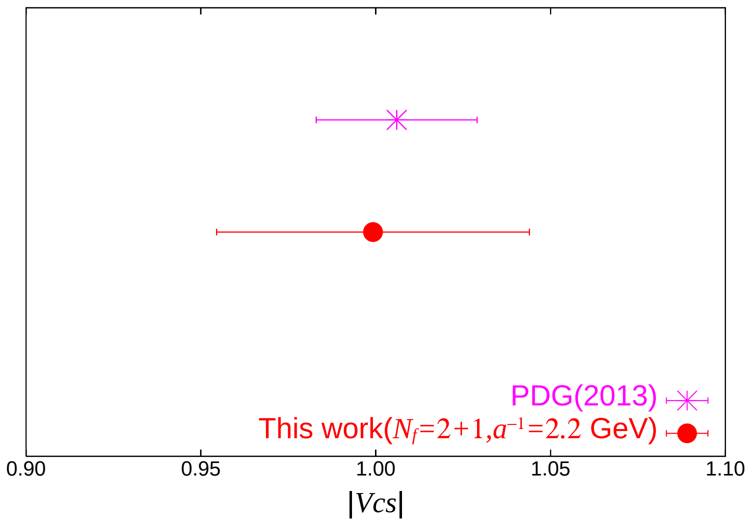
2.4 Cabbibo-Kobayashi-Maskawa matrix element

- CKM matrix elements are extracted from our mass and pseudoscalar decay constant combined with experimental values for the leptonic decay width.

$$\Gamma(D \to l\nu) = \frac{G_F^2}{8\pi} m_l^2 m_D f_D^2 \left(1 - \frac{m_l^2}{m_D^2}\right)^2 |V_{cd}|^2$$

$$\Gamma(D_s \to l\nu) = \frac{G_F^2}{8\pi} m_l^2 m_{D_s} f_{D_s}^2 \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2 |V_{cs}|^2$$





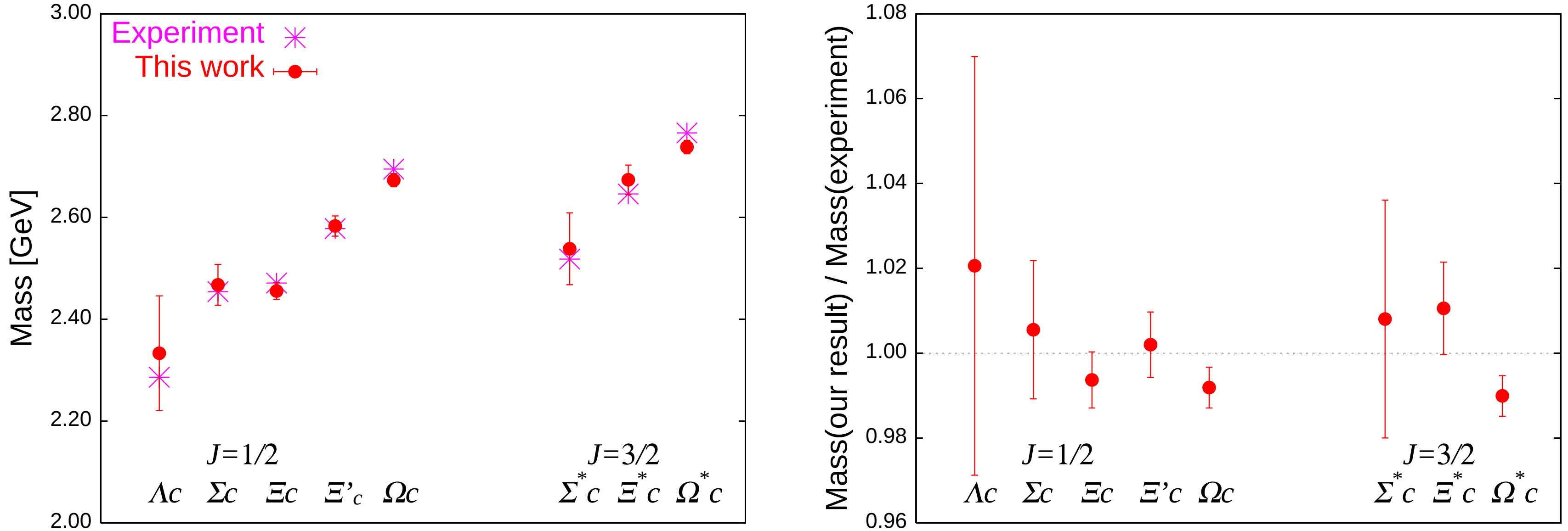
PACS-CS,2011

2.5 Other results by lattice QCD

- Mass spectrum of singly charmed baryons PACS-CS, 2013
- Mass spectrum of charm-strange mesons PACS-CS, 2011
- Mass spectrum of charmonium PACS-CS, 2011
- Works using PACS-CS configurations
  - ◊ Electromagnetic form factors Can et.al., 2013
  - ◊ Search for charmed tetraquark states T_{cc}, T_{cs} HAL, 2013
  - ◊ Search for $Y(4140)$ Ozaki and Sasaki, 2013

[Mass spectrum of singly charmed baryons]

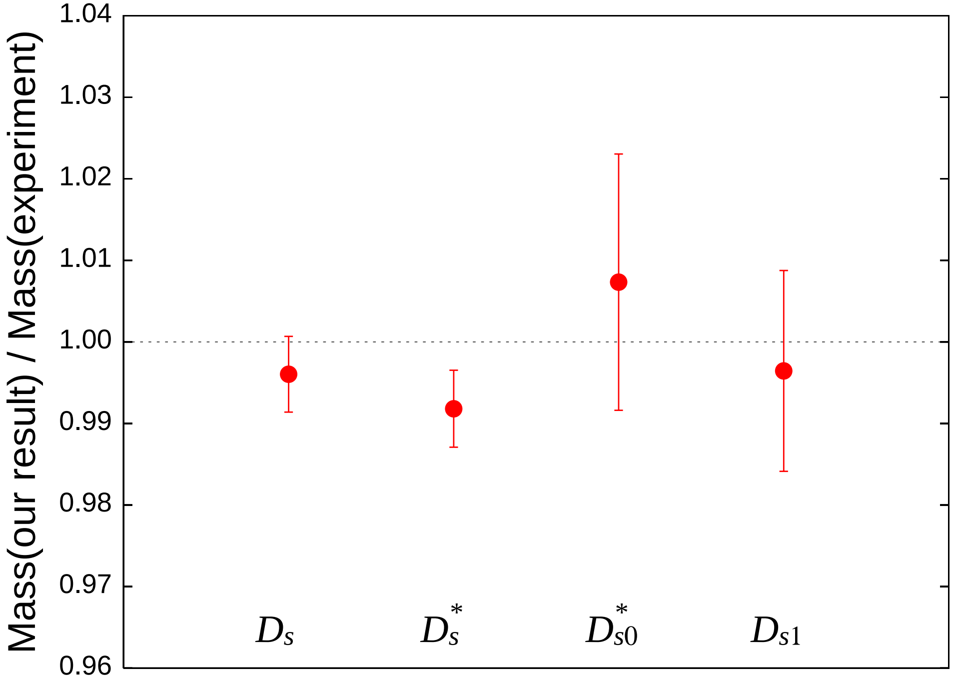
- In contrast to doubly charmed baryons, singly charmed baryons are well explored, 3 and 4-star in PDG.
- Our results agree with experiments in 2σ level.



PACS-CS,2013

[Mass spectrum of charm-strange mesons]

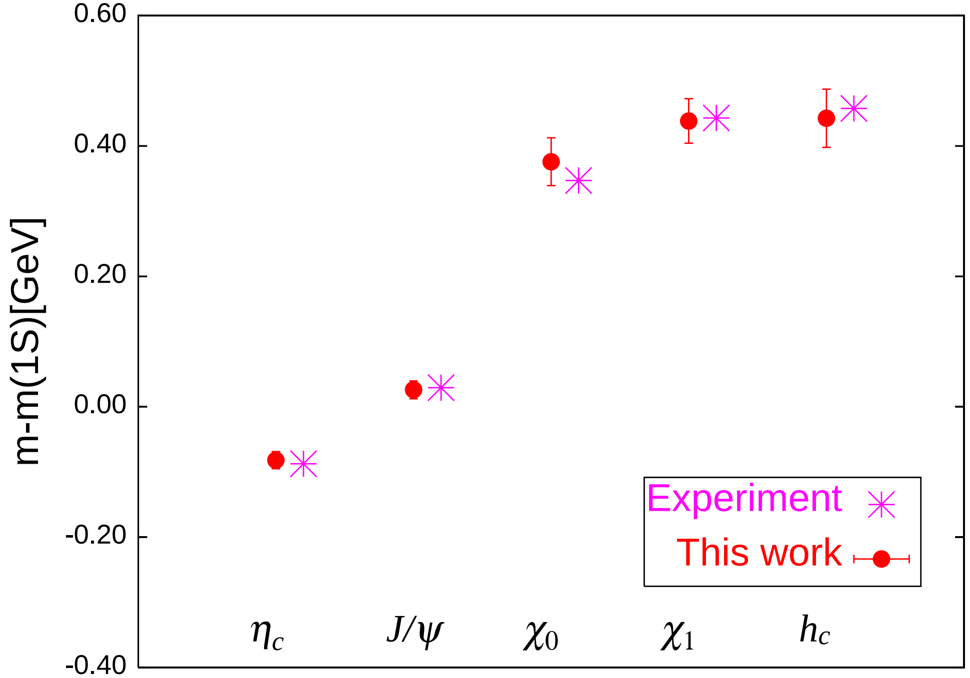
- Our calculation reproduces the charm-strange spectrum in 2σ level.
- (Naive constituent quark model fails to reproduce D_{s0}^* mass.)



PACS-CS,2011

[Mass spectrum of charmonium]

- Our results are consistent with experiments at a percent accuracy.

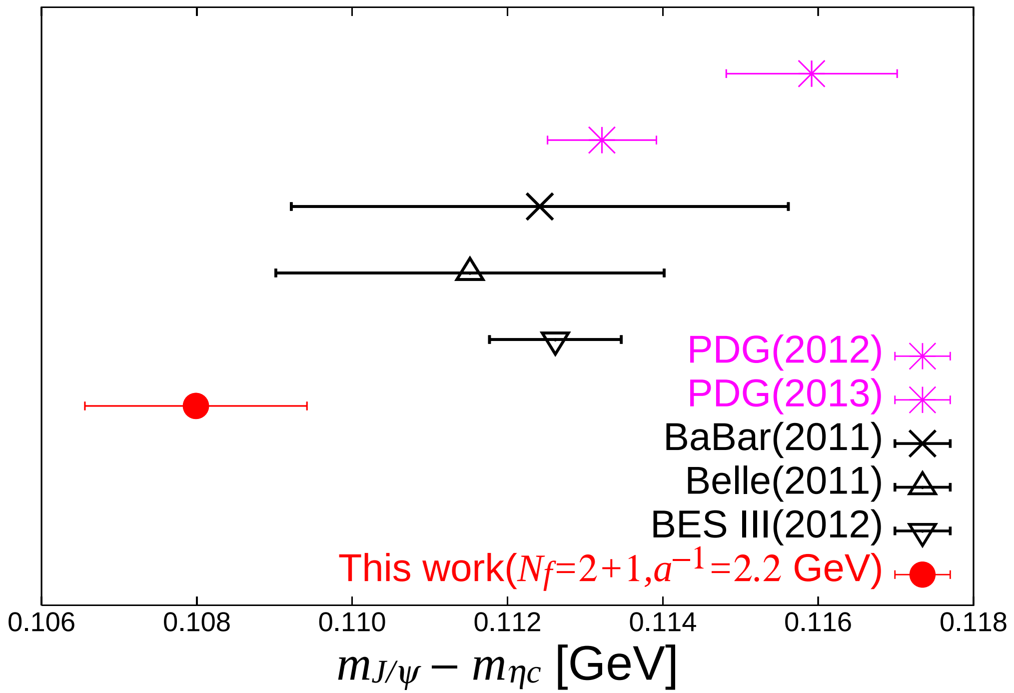


PACS-CS,2011

[Mass spectrum of charmonium(continued)]

- Our charmonium hyperfine splitting is consistent with the recent Belle and BaBar experimental values in 2σ level.
- Our charmonium hyperfine splitting deviates from BES III experiment(2012) by 4% (3σ).

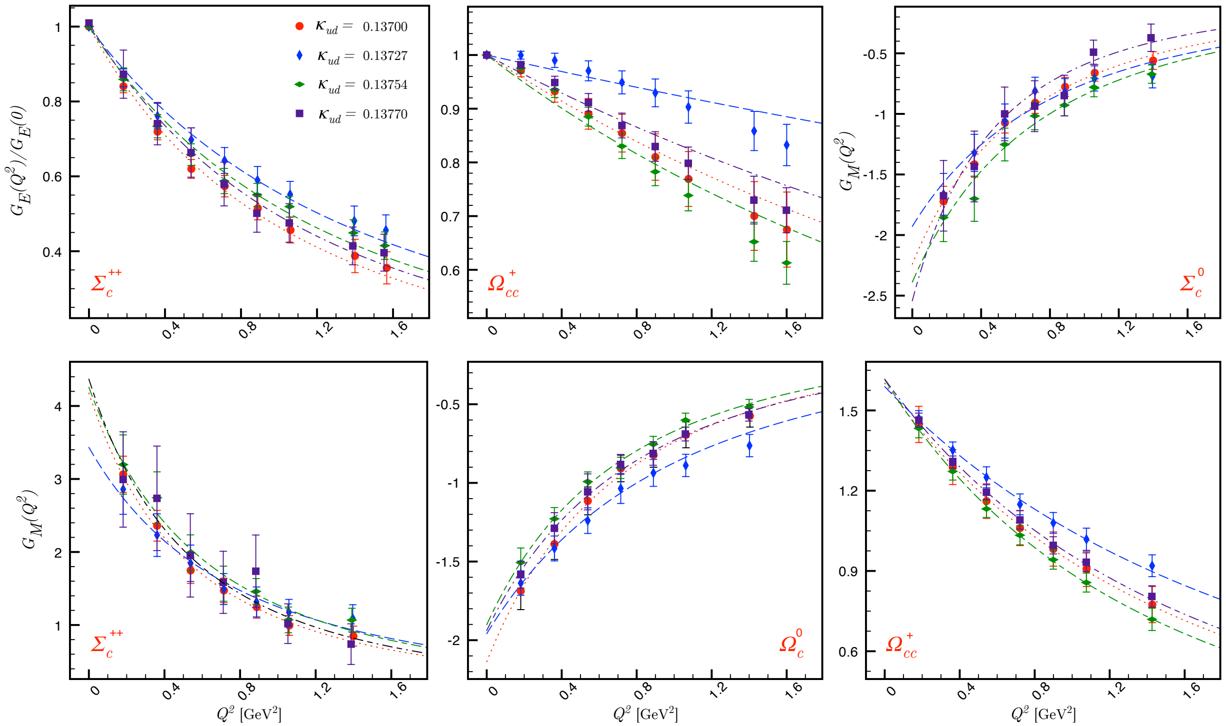
$\longrightarrow$ We have not evaluated the following systematic errors: scaling violations, dynamical charm quark effects, disconnected loop contributions, EM effects.



PACS-CS,2011

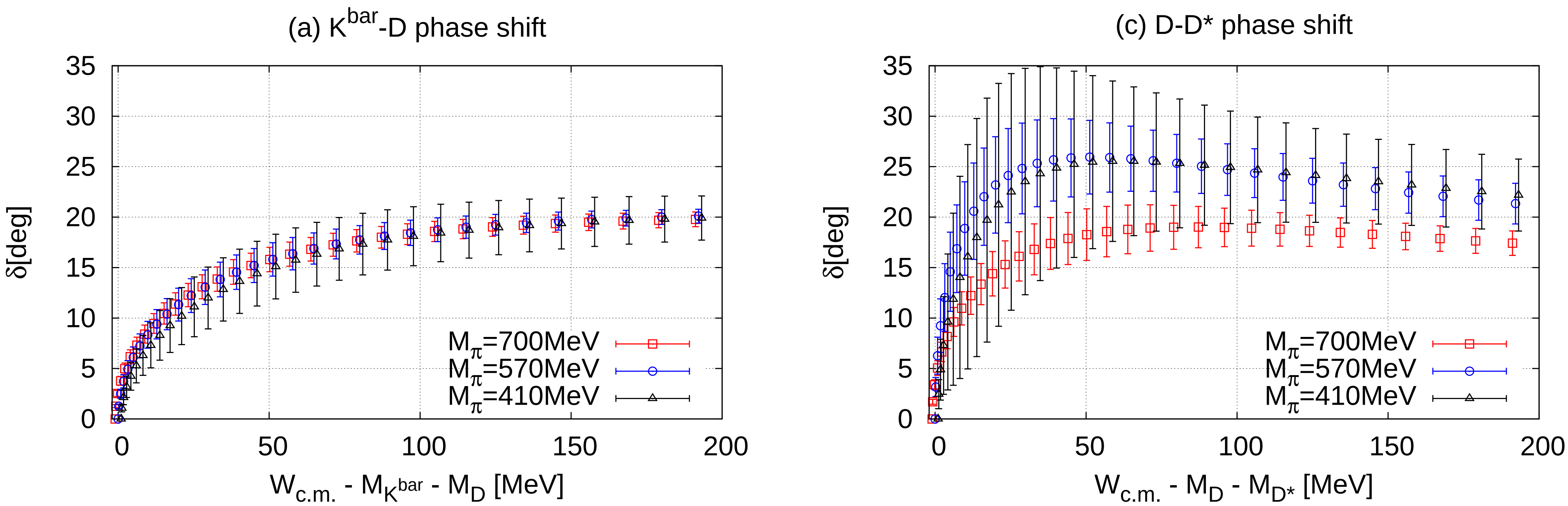
[Electromagnetic form factors] Can et.al., 2013

- Electromagnetic form factors are calculated using PACS-CS configurations.
  → See Takahashi-san's talk



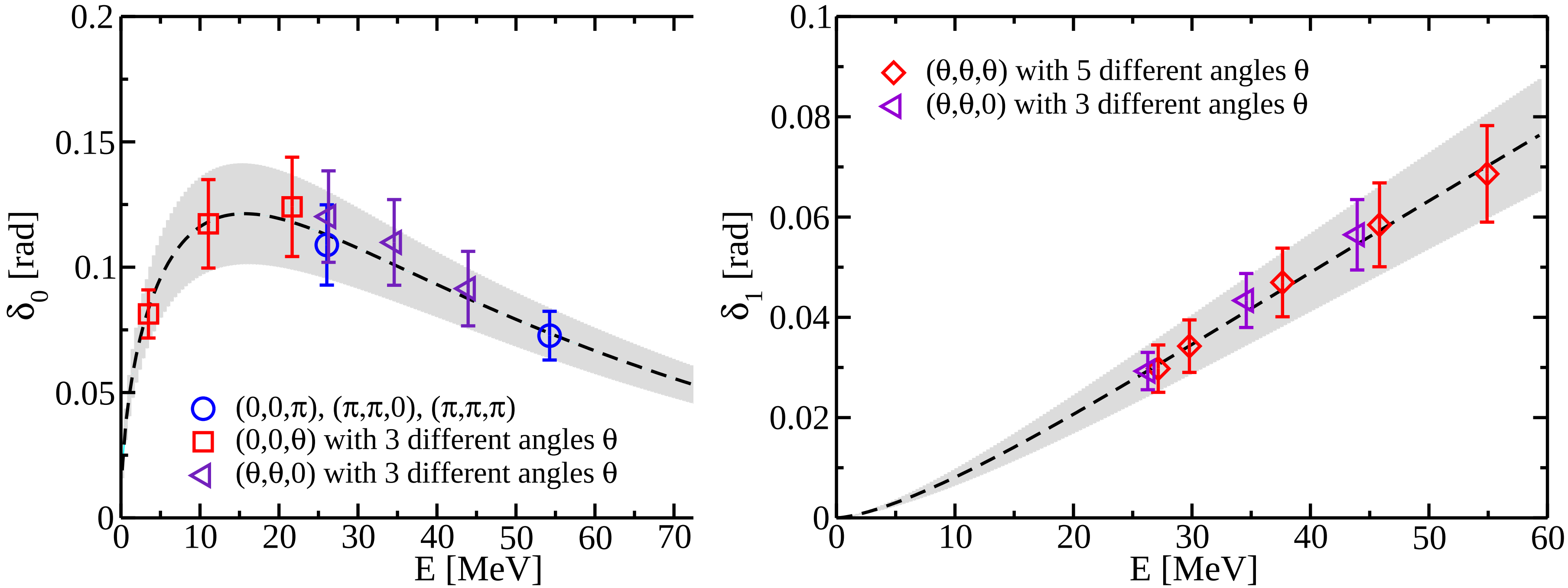
[Search for charmed tetraquark states T_{cc}, T_{cs}] HAL, 2013

- Phase shifts for charmed tetraquark states T_{cc}, T_{cs} are calculated using PACS-CS configurations.
  → See Ikeda-san's talk
- No bound or resonance states are observed.



[Search for $Y(4140)$] Ozaki and Sasaki, 2013

- Phase shifts for $Y(4140)$ are calculated using PACS-CS configurations.
- No bound or resonance states are observed.



3 Technical details

[Hardness of heavy quarks on the lattice]

- Heavy quarks are hard to be treated on the lattice due to mass corrections $O(ma)$.
  - ◇ For light quark masses, $O(m_{ud}a)$ corrections are negligible.
  - ◇ $m_{charm}a \sim 0.5$ for a typical lattice spacing, $a^{-1} = 2$ [GeV].
    ← We need improvement of calculation method to eliminate the corrections (or very small a is required).

$$
\begin{aligned}
A_{lattice}(a) \quad &= \quad A_{continuum} \\
&\quad +O(a\Lambda_{QCD}) + O((a\Lambda_{QCD})^2) + ... \\
&\quad +O(am) + O((am)(a\Lambda_{QCD})) + O((am)(a\Lambda_{QCD})^2) + ...
\end{aligned}
$$

[Hardness of heavy quarks on the lattice]

- Heavy quarks are hard to be treated on the lattice due to mass corrections $O(ma)$.
  - ♢ For light quark masses, $O(m_{ud}a)$ corrections are negligible.
  - ♢ $m_{charm}a \sim 0.5$ for a typical lattice spacing, $a^{-1} = 2$ [GeV].
    ← We need improvement of calculation method to eliminate the corrections (or very small a is required).
- Improved action removes a part of the corrections.

$$
\begin{aligned}
A_{lattice}(a) \quad = \quad & A_{continuum} \\
& + \cancel{O(a\Lambda_{QCD})} + O((a\Lambda_{QCD})^2) + ... \\
& + \cancel{O(am) + O((am)(a\Lambda_{QCD}))} + O((am)(a\Lambda_{QCD})^2) + ...
\end{aligned}
$$

[Improved actions for the charm quark]
A lot of improved actions are proposed to reduce mass corrections $O(ma)$.

- Fermilab action El-Khadra et.al.(1997); Oktay and Kronfeld(2008)
  - ◇ Improved Wilson quark with non-relativistic power counting.
- Relativistic heavy quark action Aoki et.al.(2003); Christ et.al.(2007)
  - ◇ Improved Wilson quark with relativistic power counting.
- Highly improved staggered quark action Hasenfratz and Knechtli(2001)
- Brillouin quark action Creutz et.al.(2010); Cho et.al.(2013)
- (Quark action on the anisotropic lattice($a_s \neq a_t$)) Klassen,(1999)
  - ◇ Anisotropic lattice suppresses $O(ma_t)$ correction by taking $a_s \ll a_t$ at classical level.
  - ◇ Anisotropic lattice is not efficient at quantum level, without non-perturbative determination of the action parameters. Aoki et.al.(2003); Hashimoto and Okamoto (2003)

[Improved action for the charm quark(continued)]

We employ the relativistic heavy quark action(Tsukuba-type). S.Aoki et al, 2003

- Since the charm quark is not too heavy, relativistic approach is needed.
- This action is designed to control heavy quark mass corrections.

$\leftarrow$ $O(m_Q a)$ and $O((m_Q a)(a\Lambda_{QCD}))$ terms are removed, once all of the parameters in the heavy quark action are determined nonperturbatively.

◇ For r_s, $C_{SW}^{s,t}$, tadpole improved 1-loop values are used. $C_{SW}^{s,t}$ are non-perturbatively improved at the massless point,
$C_{SW}^{s,t} = C_{SW}(NP, m=0) - C_{SW}^{s,t}(PT, m=0) + C_{SW}^{s,t}(PT, m \neq 0)$.

◇ ν is non-perturbatively determined to reproduce the relativistic dispersion relation (the effective speed of light is tuned to be unity, $c_{eff} = 1.002(4)$).

$$\begin{aligned}
S_{RHQ} &= \sum_{x,y} \bar{q}(x) D(x,y) q(y), \\
D(x,y) &\equiv \delta_{x,y} - \kappa_{\text{heavy}} \Big\{ (1-\gamma_4)U_4(x)\delta_{x+4,y} + (1+\gamma_4)U_4^\dagger(x)\delta_{x,y+4} \\
&\quad + \sum_i \Big((r_s - \nu\gamma_i)U_i(x)\delta_{x+i,y} + (r_s + \nu\gamma_i)U_i^\dagger(x)\delta_{x,y+i} \Big) \Big\} \\
&\quad - \delta_{x,y} \kappa_{\text{heavy}} \Big\{ C_{SW}^s \sum_{i<j} \sigma_{ij} F_{ij} + C_{SW}^t \sum_i \sigma_{4i} F_{4i} \Big\}.
\end{aligned}$$

[Simulation setup]

We perform $N_f = 2+1$ full QCD simulation (including dynamical up,down and strange quarks) for charmed baryons on the physical point.

- Action : Iwasaki gauge + $O(a)$ improved Wilson fermion for light sea quarks + relativistic heavy fermion for valence charm quark
- Lattice size : $32^3 \times 64$ ($L = 3$ fm, $a^{-1} = 2.2$ GeV ($\beta = 1.90$))
- Sea and valence quark masses : on the physical point (i.e. $m_\pi = 135$ MeV)
- Inputs : m_π,m_K,m_Ω for m_{ud},m_s,a; $m(1S) \equiv \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$ for m_{charm}

$m_{ud}^{\overline{\rm MS}}(\mu = 2{\rm GeV})$[MeV]	$m_s^{\overline{\rm MS}}(\mu = 2{\rm GeV})$[MeV]	N_{conf} (MD time)
3	93	80 (2000)

[Operators for mesons and baryons]

- We use the following relativistic operators, because the relativistic heavy quark is employed in our simulation.

$$M_{\Gamma}^{fg}(x) = \bar{q}_f(x)\Gamma q_g(x), \quad \Gamma = I, \gamma_5, \gamma_\mu, i\gamma_\mu\gamma_5, i[\gamma_\mu, \gamma_\nu]/2.$$

$$(J = 1/2) \quad O_{\alpha}^{fgh}(x) = \epsilon^{abc}((q_f^a(x))^T C\gamma_5 q_g^b(x))q_{h\alpha}^c(x),$$
$$C = \gamma_4\gamma_2, \alpha = 1, 2.$$

$$\begin{aligned}
(J = 3/2) \quad D_{3/2}^{fgh}(x) &= \epsilon^{abc}((q_f^a(x))^T C\Gamma_+ q_g^b(x))q_{h1}^c(x), \\
D_{1/2}^{fgh}(x) &= \epsilon^{abc}[((q_f^a(x))^T C\Gamma_0 q_g^b(x))q_{h1}^c(x) \\
&\quad -((q_f^a(x))^T C\Gamma_+ q_g^b(x))q_{h2}^c(x)]/3, \\
D_{-1/2}^{fgh}(x) &= \epsilon^{abc}[((q_f^a(x))^T C\Gamma_0 q_g^b(x))q_{h2}^c(x) \\
&\quad -((q_f^a(x))^T C\Gamma_- q_g^b(x))q_{h1}^c(x)]/3, \\
D_{-3/2}^{fgh}(x) &= \epsilon^{abc}((q_f^a(x))^T C\Gamma_- q_g^b(x))q_{h2}^c(x), \\
&\quad \Gamma_\pm = (\gamma_1 \mp i\gamma_2)/2, \Gamma_0 = \gamma_3.
\end{aligned}$$

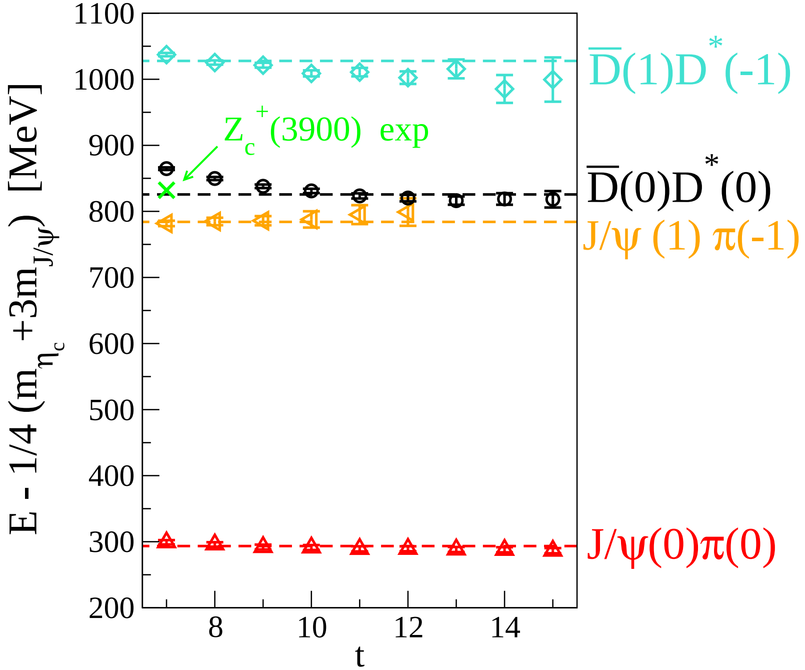
4 Summary

Lattice QCD results for heavy hadrons are presented.

- Lattice simulation can predict undiscovered hadrons in a nonperturbative and model independent way.
  $\rightarrow$ Doubly and triply charmed baryon spectrum are predicted.
  cf. Takahashi-san and Ikeda-san talks for other observables.
- Lattice simulation can be a guide for uncertain experiments.
  $\rightarrow$ Ξ_{cc} spectrum is obtained, which is higher than SELEX experimental value by 100 MeV.
- Lattice simulation can determine quark masses, which can not be obtained in experiments due to quark confinement.
  $\rightarrow$ Charm quark mass is determined.
- Lattice simulation can extract Cabbibo-Kobayashi-Maskawa matrix element from the experimental data.
  $\rightarrow$ $|V_{cd}|, |V_{cs}|$ are obtained.

[Future works]

- Explore excited states, including scattering and resonance states, such as X, Y, Z...
  - ◇ cf. Prelovseka and Leskovec investigated $Z(3900)$ in $J^{PC} = 1^{+-}$, $I = 1$ by lattice QCD, though their simulation is still unphysical, $m_\pi = 266(4)$ [MeV], $a^{-1} = 1$ [GeV].
    $\rightarrow$ No signal for $Z(3900)$.



Prelovseka and Leskovec(2013)

[New computers]

- New computers are very fast. They help us calculate our future targets.

Year	Machine	Speed [TFlops]
1996-2005	CP-PACS	0.6
2006-2011	PACS-CS	14
2008-2014	T2K(Tokyo,Tsukuba,Kyoto)	235
2014	PlayStation 4	2
2012-	HA-PACS	802
2012-	BlueGene/Q	1258
2012-	K-computer	8162

[Open source code for lattice QCD]
We have developed an open source code of lattice QCD "Bridge++" for not only experts but also beginners.

- Bridge++ has a great deal of readability, keeping a sufficient performance for frontier works.

    ◇ Core project members:
    S.Aoki, T.Aoyama, K.Kanaya, H.Matsufuru, S.Motoki, Y.Namekawa, H.Nemura, Y.Taniguchi, S.Ueda, N.Ukita

Appendix

[Recent result : rho meson decay]

Rho meson resonance mass m_ρ and decay width Γ_ρ are obtained by the scattering phase shift $\delta(p)$ calculated using Lüscher's formula M.Lüscher,1986,

$$\tan\delta(p_n) = \frac{\pi^{3/2} n}{Z_{00}(1,n)},$$

$$Z_{00}(1,n) := \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{m}\in Z^3} \frac{1}{m^2 - n^2},$$

$$p_n = \frac{2\pi}{L} n,\ n \notin Z, \quad \text{NB. } n \in Z \text{ for non-interacting case.}$$

Once we obtain $\delta(p)$ in lattice simulations, we can extract m_ρ and Γ_ρ by

$$\tan\delta(p) = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{p^3}{\sqrt{s}(m^2_\rho - s)},\ \sqrt{s} = 2\sqrt{m^2_\pi + p^2},$$

$$\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{(m^2_\rho/4 - m^2_\pi)^{3/2}}{m_\rho}$$