Electromagnetic properties of charmed baryons from lattice QCD

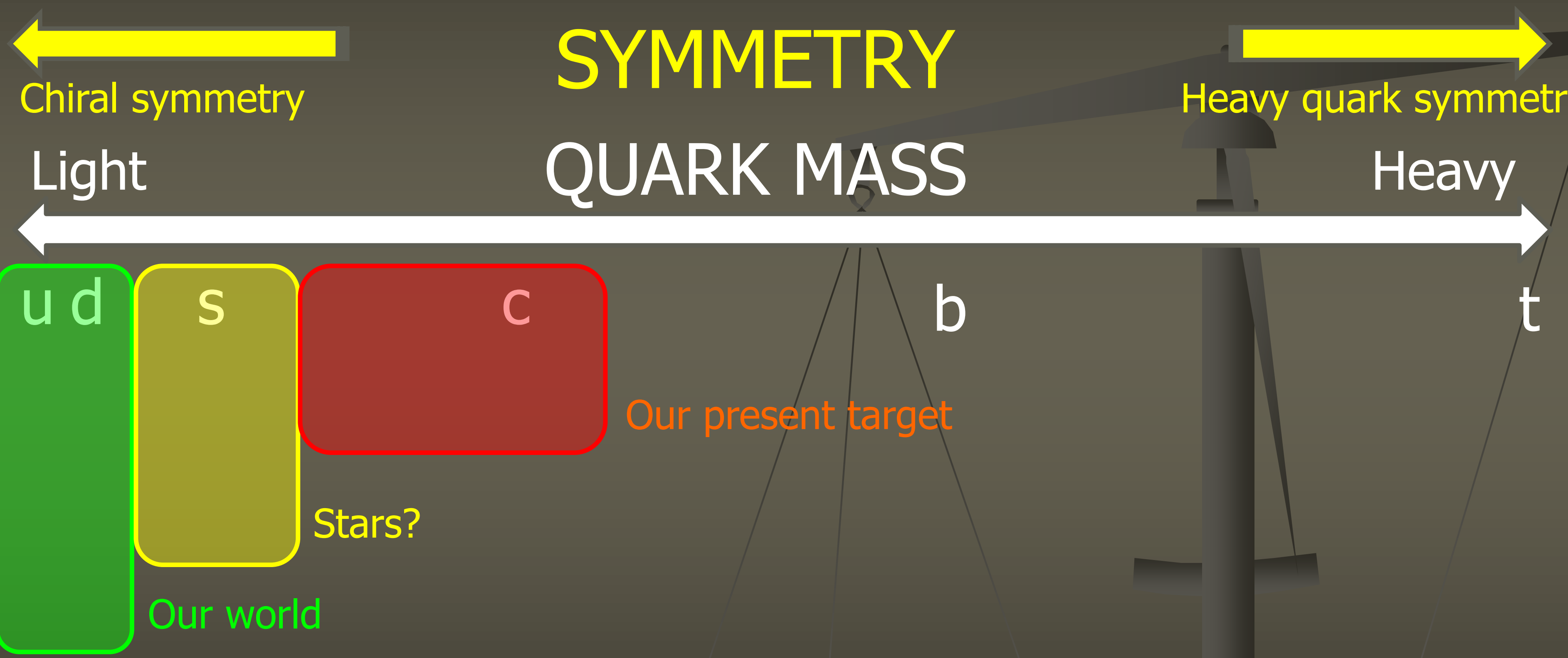
Toru T. Takahashi (Gunma College of Technology)

with
Makoto Oka (TITech)
Guray Erkol, Can Utku (Turkey)

Introduction

Heavy hadrons

Charmed hadrons are attracting much interest.



軽いクォークのみの世界では見られなかった対称性(HQ symmetry)などが現れる。
テトラクォーク, メソン分子, ハイブリッドハドロンなど、新しい複合粒子形態の可能性。

Heavy hadrons

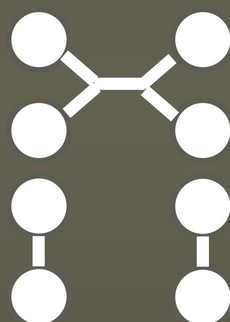
実際、Babar等の実験で、新たなチャームを含む粒子(X, Y, Z) が発見されつつある。

それらの正体？

Tetraquark states $[Qq][\overline{Qq}]$

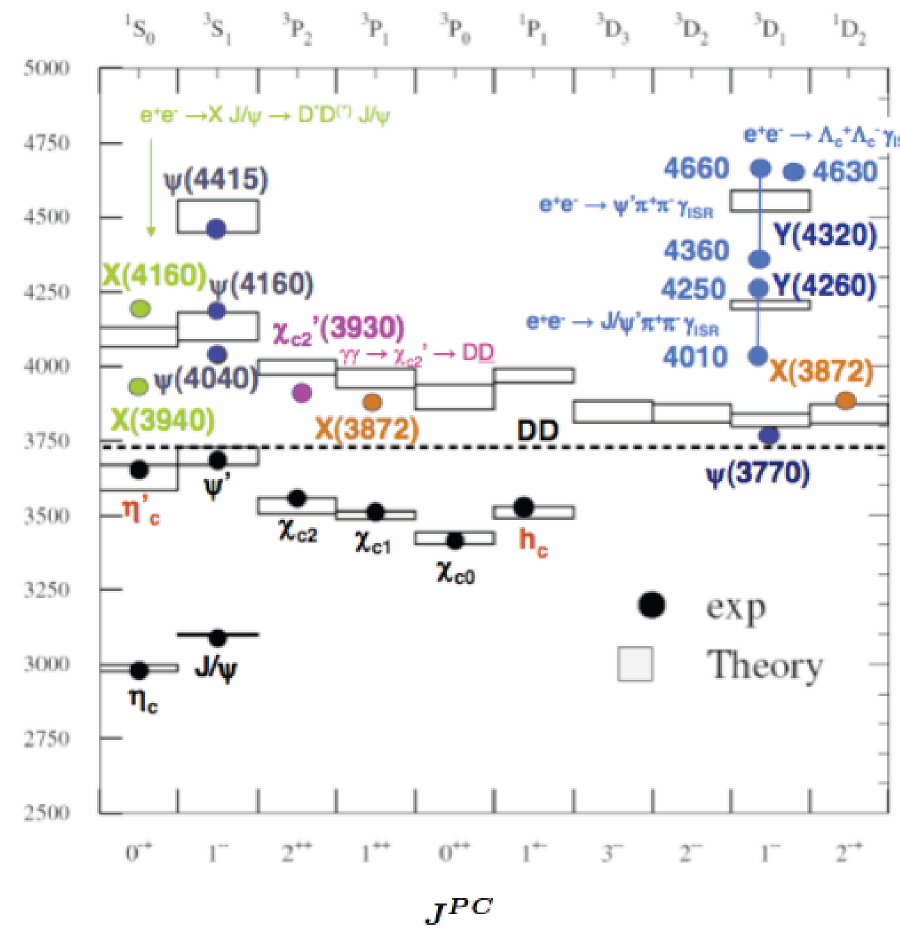
Hadron molecules $(Qq)(\overline{Qq})$

Hybrid hadrons $Q\bar{Q}g$, ...



単純なQQbarを超えた構造
理論的にも興味深い

M. Nielsel (Charm2010)



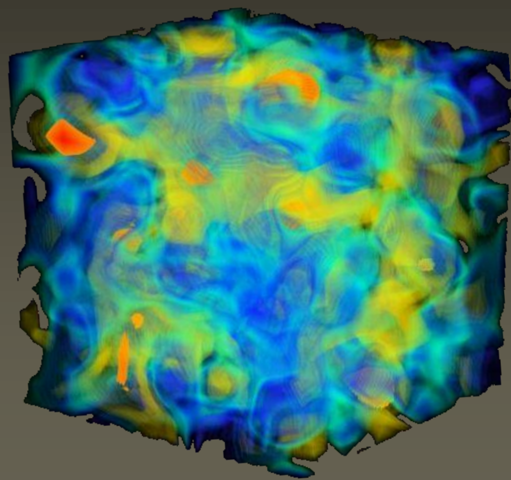
QCDに基づいた理論的考察、予言が望まれる。

Lattice QCD

Lattice QCD as one possible solution for QCD

Continuum QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}(i\gamma^\mu D_\mu - m_f)\psi$$



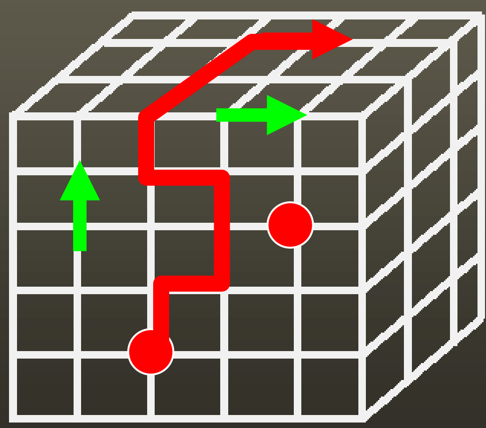
Gluon field : $A_\mu(x)$

Quark field : $q(x)$

Field strength : $F_{\mu\nu}(x)$

Continuum limit ↑ ↓ Discretization

Lattice QCD



Gluon field : $U_\mu(n)$ → lives on links

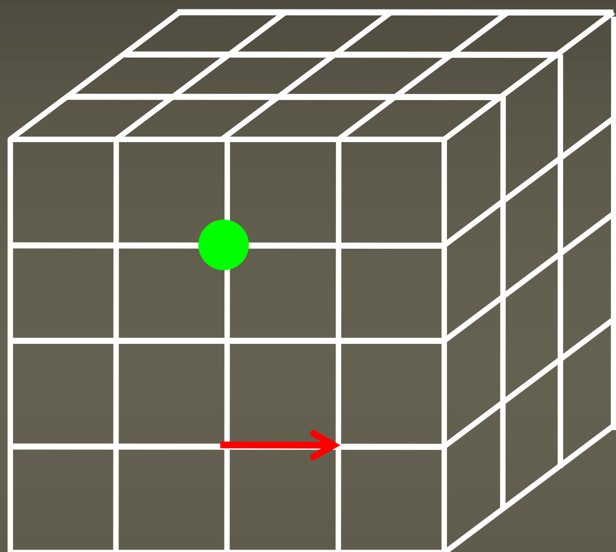
Quark field : $q(n)$ → lives on sites

Field strength : Plaquette (loop)

Lattice QCD as one possible solution for QCD

Compact formalism of QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}(i\gamma^\mu D_\mu - m_f)\psi$$



Quark field $\psi(s)$

Gauge field $U\mu(s) = e^{ig\int A_\mu dx_\mu}$

$U\mu(s) \in \mathrm{SU}(3)$

Lattice + Euclid space

Nonperturbative evaluation of PATH-INTEGRAL

by computers

by hand

Lattice QCD as one possible solution for QCD

格子QCDにおいては、基本的に演算子の期待値を計算する
クォーク演算子は求めたプロパゲータで置き換える

例 中間子の質量を測るなら2点関数を計算する　$M(x) = u(x)\bar{d}(x)$

D_{T0} d-quark propagator で置き換える

$$\langle M(T)\bar{M}(0)\rangle = \langle u(T)\bar{d}(T)\bar{u}(0)d(0)\rangle = D_{T0}U_{T0}$$

U_{T0} u-quark propagator で置き換える

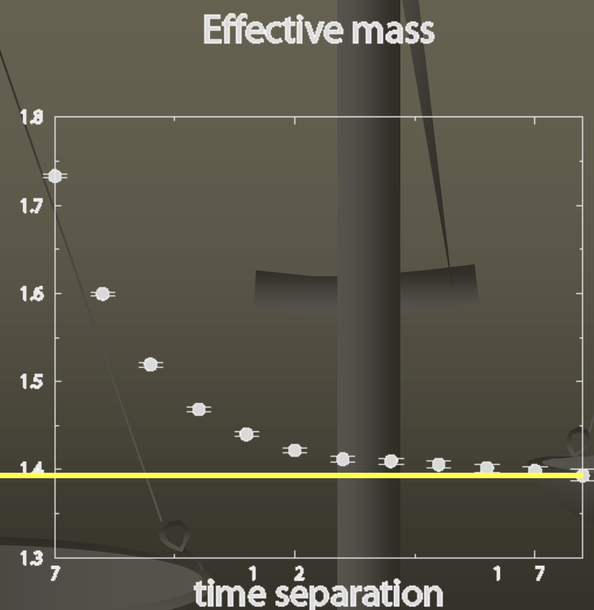
時刻0から時刻Tへの中間子の伝搬

Euclidean time evolution by exp(-Ht)

励起状態は先にdecay

Creation at t=0

Annihilation at t=T



Energy of the ground state

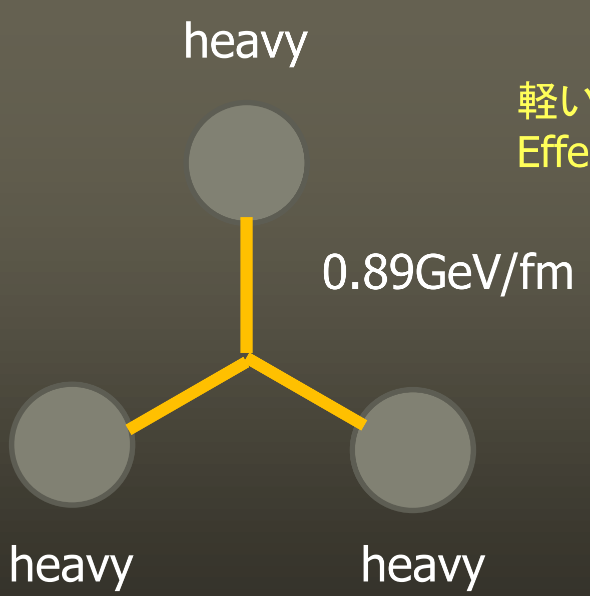
Target

Target

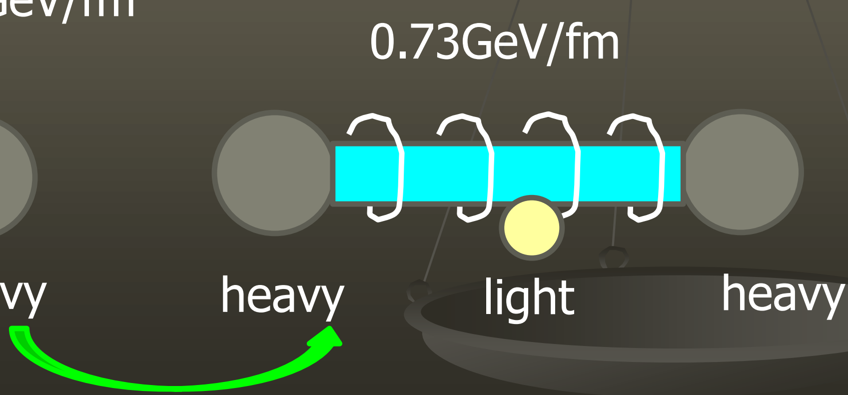
Electromagnetic form factors of singly- or doubly-charmed baryons

→ ヘビークォークを含むハドロンの大きさや内部構造の情報

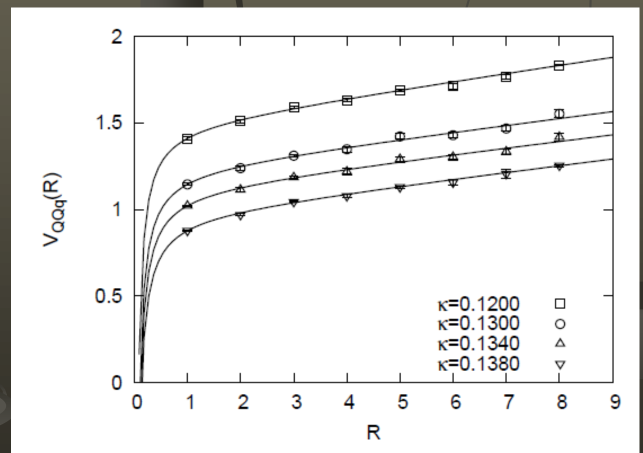
クォークのダイナミクス
HQ間の“effective string tension”の変化



軽いクォークの動きが
Effectiveなstring tensionを変化させる



A. Yamamoto et al.
Phys.Lett. B664 (2008) 129-13



Target

Electromagnetic form factors of singly- or doubly-charmed baryons

→ ヘビークォークを含むハドロンの大きさや内部構造の情報

Large isospin splitting (SELEX data) の起源？

Mass difference in isospin doublet

$$M_n - M_p = 1.29$$
$$M_{D^+} - M_{D^0} = 4.77$$
$$M_{\Xi^-} - M_{\Xi^0} = 6.85$$
$$M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}} = 9$$

非常に大きい → Q-q-Q のような配位を取り、非常にコンパクトな内部構造を示唆

Brodsky et al. Phys.Lett. B698 (2011) 251-255

$\sqrt{\langle r^2 \rangle} < 0.26$ fm となることが必要

Target

Electromagnetic form factors of singly- or doubly-charmed baryons

→ ヘビークォークを含むハドロンの大きさや内部構造の情報

第一原理計算である格子QCDから、これらの物理量を評価したい

問題点？

数GeV　　　　　～1.3GeV

・通常用いられる格子cut off に比べて、charm quarkは重く
cut off effectが見えてしまう。（格子が荒すぎる）

→ そのようなエラーを軽減するrelativistic heavy quark action を用いる

・計算量は見るチャネルの数に比例して増える

→ wall operatorを使うことにより、計算量を削減

Problems and Improvements

3 point functions on the lattice

Form factorは3点関数から求める

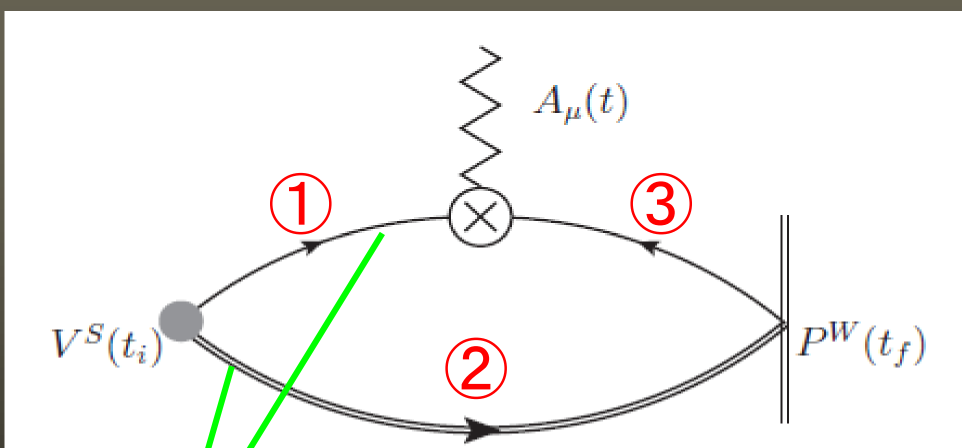
EM current $V_\mu = \sum_q e_q \, \bar{q}(x)\gamma_\mu q(x)$

3pt func. $\langle B(y) V_\mu \bar{B}(x) \rangle \longrightarrow \bar{u}(p)\left[\gamma_\mu F_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2(q^2)\right]u(p)$

B : hadronic operator

→ EM form factor

ダイアグラムで描けば・・・



Quark propagator

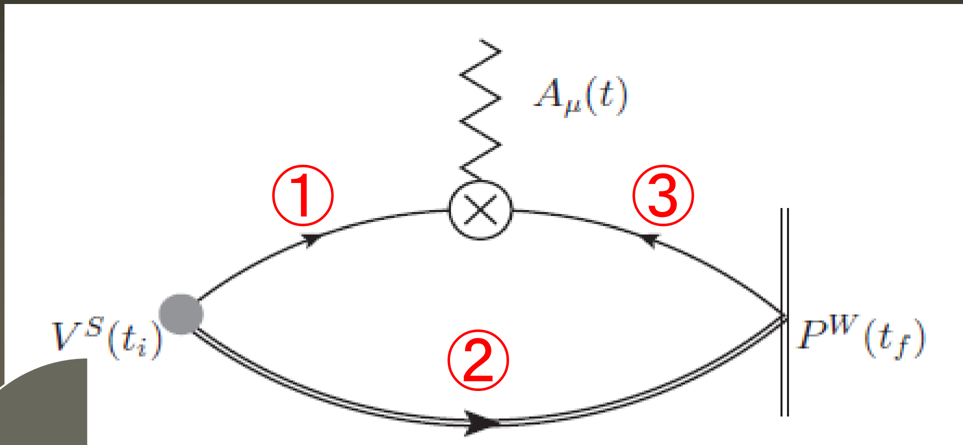
格子QCD計算でやることは・・・

①～③は格子上のquark propagatorであり、原理的には先にpropagatorを求めておいて、後で組み合わせればよい。

現実的にはpropagatorを完全に求めることは困難で、backward propagator ③ をチャネルごとに解く必要がある。

3 point functions on the lattice

通常のsequential source methodを使った場合



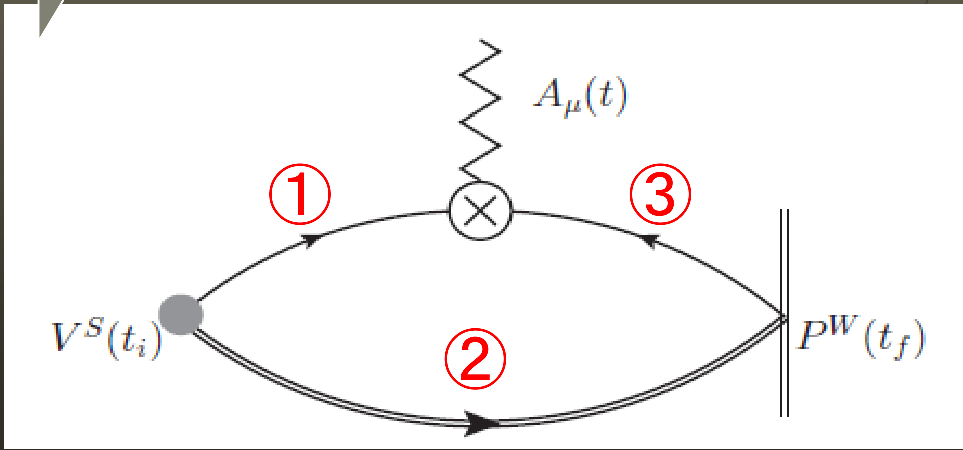
①, ② → 1回求めておけば良い

③ → チャネルごとに計算しなおし
(so-called sequential source method)

Sink operatorを変更して、ハドロン内部のクォーク場を位置に関して全て独立にする

$$\Sigma_x \ \bar{q}_1(x)\Gamma q_2(x) \longrightarrow \Sigma_{x,x'} \ \bar{q}_1(x)\Gamma q_2(x')$$

Wall operator を用いた場合



①, ② → 1回求めておけば良い

③ → 1回求めておけば良い

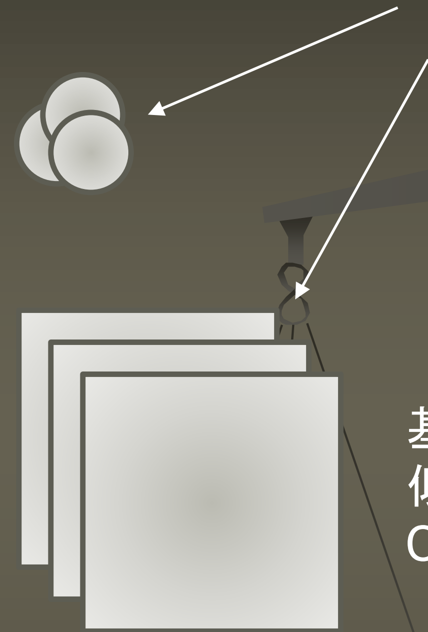
3 point functions on the lattice

Wall operatorを使うデメリット

Quark operatorの分布

通常のlocal baryon operator

$$q_1(x)q_2(x)q_3(x)$$



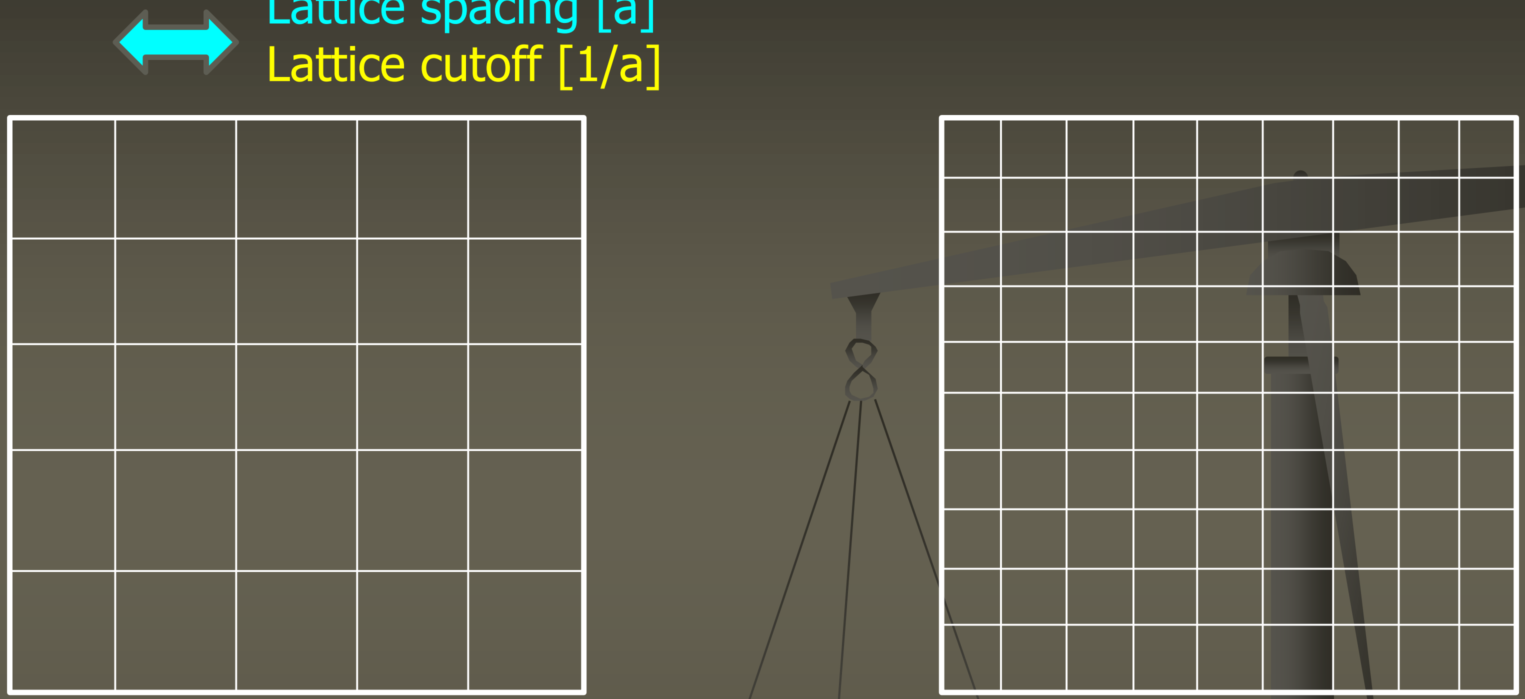
用いるwall operator

$$\sum_{xyz} q_1(x)q_2(y)q_3(z)$$

基底状態ハドロンの波動関数とは
似ても似つかず、
Operatorの結合が小さくなってしまう。

結果、基底状態ハドロンのシグナルは相対的に出にくくなり、
励起状態シグナルによる、systematic errorが増大する可能性がある。

Cut off artifact



Too coarse.
Small numerical cost.
Cutoff is insufficient
for charm quarks (∼1.3 GeV).

Fine.
Cutoff is sufficient.
Huge numerical cost.

Cut off artifact

Charm quark is too heavy and too light

We can do
ordinary calc.
(Wilson action works.)

We can use
1/m expansion

up, down   strange   charm   bottom, top

Quark mass   light ⟷ heavy

Cut off artifact

Wilson (Clover) action, contains no O(a) error but finite O((mQa)^n) errors

$$D_{xy} = \delta_{xy} - \kappa \sum_{\mu} [(1-\gamma_\mu) U_{x,\mu} \delta_{x+\mu,y} + (1+\gamma_\mu) U^{+}_{x,\mu} \delta_{x,y+\mu}]$$
$$-\kappa [c_{SW} \sum_{\mu\nu} F_{\mu\nu}(x) \sigma_{\mu\nu}] \delta_{xy}$$

IMPROVEMENT

Heavy quark action, designed to reduce O((mQa)^n) errors

$$D_{xy} = \delta_{xy} - \kappa_Q \sum_{i} [(r_s - \nu\gamma_i) U_{xi} \delta_{x+i,y} + (r_s + \nu\gamma_i) U^{+}_{x,i} \delta_{x,y+i}]$$
$$-\kappa_Q [(1-\gamma_4) U_{x,4} \delta_{x+4,y} + (1+\gamma_4) U^{+}_{x,4} \delta_{x,y+4}]$$
$$-\kappa_Q [c_B \sum_{i,j} F_{ij}(x) \sigma_{ij} + c_E \sum_{i} F_{i4}(x) \sigma_{i4}] \delta_{xy}$$

The form is essentially the same as the standard clover action.
Parameters were tuned in PACS-CS's paper (Phys.Rev. D84 (2011) 074505)

Cut off artifact

$\kappa_{val}^{u,d}$	m_{Σ_c}	m_{Ω_c}	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
	[GeV]	[GeV]	[GeV]	[GeV]
0.13700	2.841(18)	2.959(24)	3.810(12)	3.861(17)
0.13727	2.753(19)	2.834(19)	3.740(13)	3.806(12)
0.13754	2.647(19)	2.815(26)	3.708(16)	3.788(16)
0.13770	2.584(28)	2.781(26)	3.689(18)	3.781(28)
Lin. Fit	2.553(18)	2.740(24)	3.660(14)	3.755(18)
Quad. Fit	2.525(38)	2.740(67)	3.687(24)	3.791(36)
Exp.	2.455	2.695	3.519	-
PACS-CS [14]	2.467(39)(11)	2.673(5)(12)	3.603(15)(16)	3.704(5)(16)

Cut off artifact そのもの

We employed the simplest FermiLab-method based action.
Mohler et al. Phys.Rev.D87, 034501 (2013)

Always Overestimate the charmed hadron masses.
Relativistic heavy quark action can well reproduce the masses.(PACS-CS)

Simulation setups

Simulation setups

Simulation conditions

2+1 flavor gauge configurations (generated by PACS-CS)
Iwasaki gauge action and the Wilson quark action
$32^3 \times 64$, cutoff$\sim$2.2 GeV, a$\sim$0.1 fm (spatial volume is large)
WALL-type sink operators (to avoid many matrix inversions)

heavy ⟷ light

Kappa_ud →	0.13700,	0.13727,	0.13754,	0.13700	
Pion mass →	700,	569,	411,	295	(MeV)

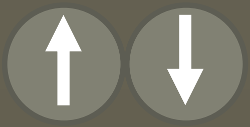
Results (mesons)

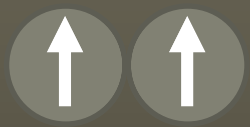
Vector and axial-vector couplings of D and D* mesons in 2+1 flavor Lattice QCD

Phys.Lett. B719 (2013) 103-109   arXiv:1210.0869

Guray Erkol, Utku Can, Makoto Oka , A. Ozpineci and Toru T. Takahashi

D meson form factors from the lattice QCD

D meson J=0 (scalar) 

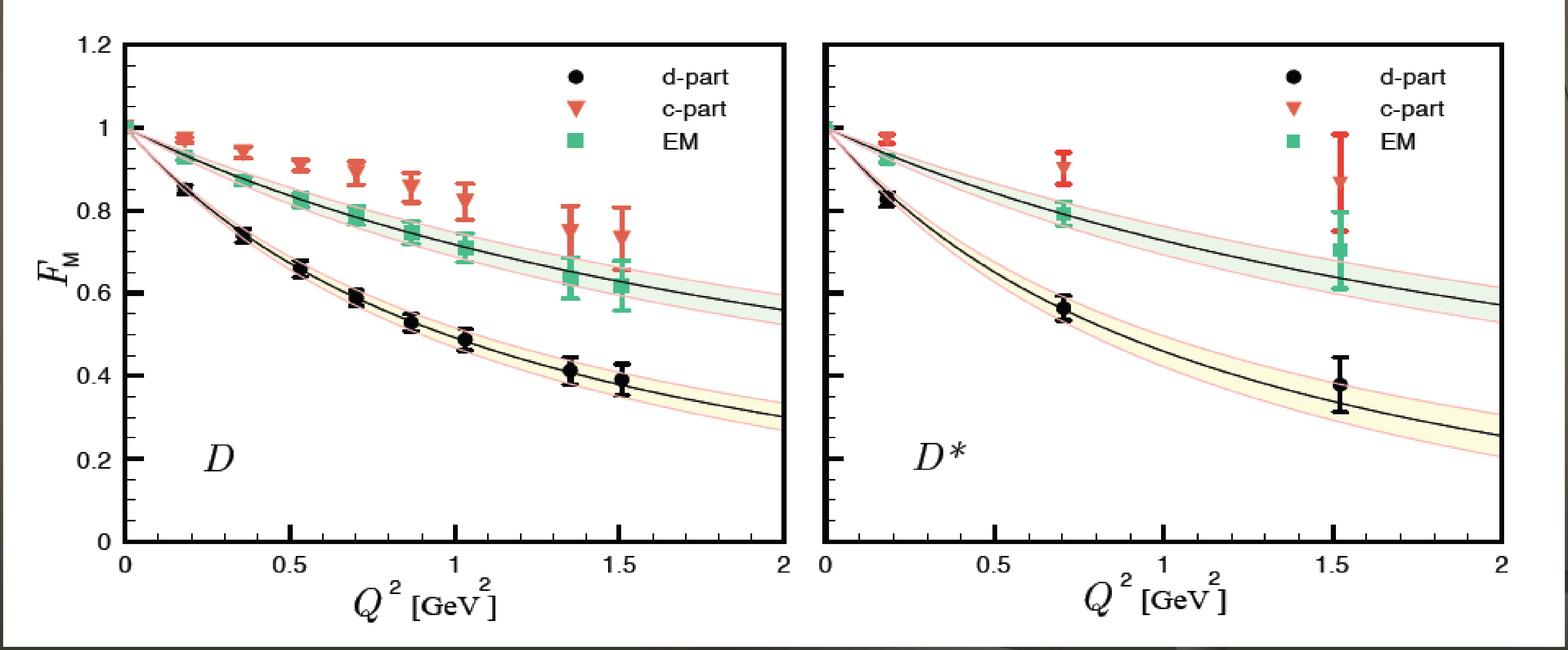
D^* meson J=1 (vector) 

c-quark と d-quarkのスピンの組み方

Lattice QCD results (couplings)

Form factors of D-meson

$$\langle \mathcal{D}(p')|V_\mu(q)|\mathcal{D}(p)\rangle = \frac{(p+p')}{2\sqrt{E_D\,E_{D'}}}\,[2/3F_D^c(q^2)+1/3F_D^d(q^2)]$$



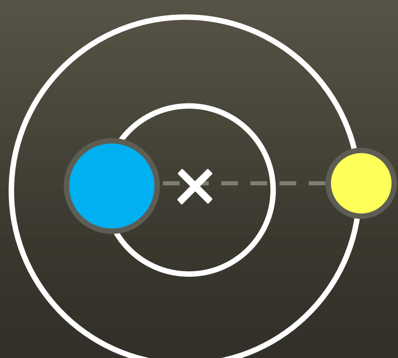
$D - meson$ $D^* - meson$

Lattice QCD results (couplings)

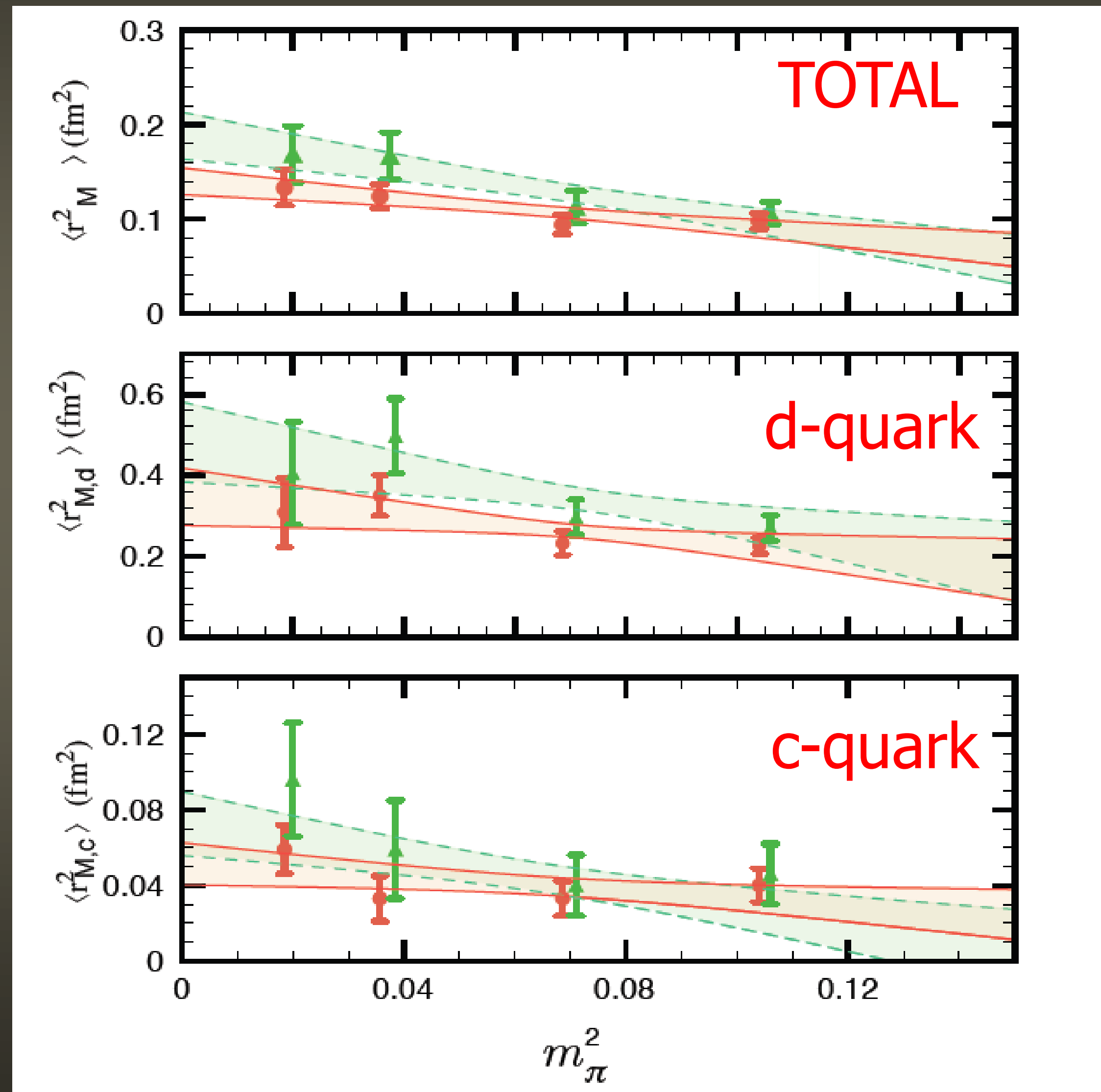
D* (vector) は D (scalar)より
常にサイズが大きい

クォークを重くすると、大きさは縮退
← Color-Magnetic 相互作用が
構造の違いの主原因

C-quark の広がりは d-quark より
かなり小さい
← 重心がc-quark側にシフトしている



D, D* mesonの荷電半径



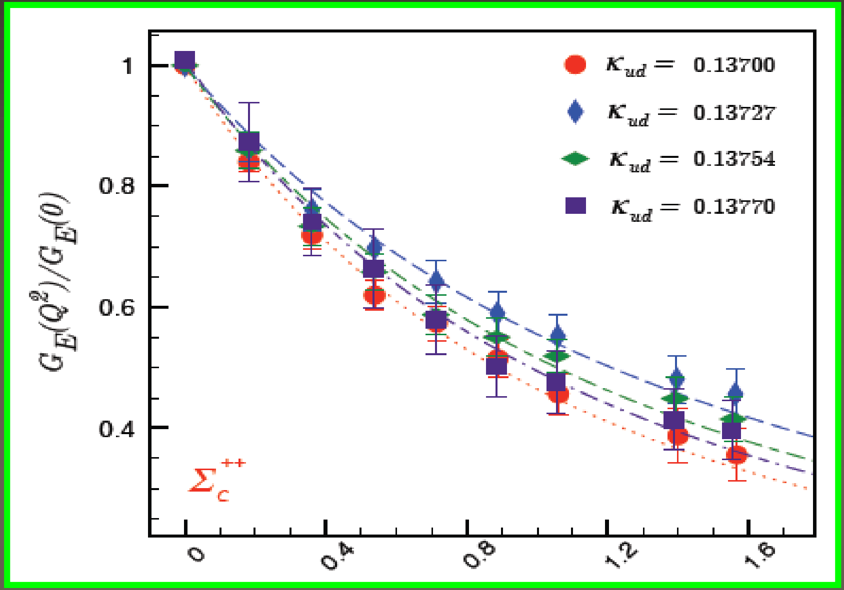
Results (baryons)

Charmed baryon form factors from the lattice QCD

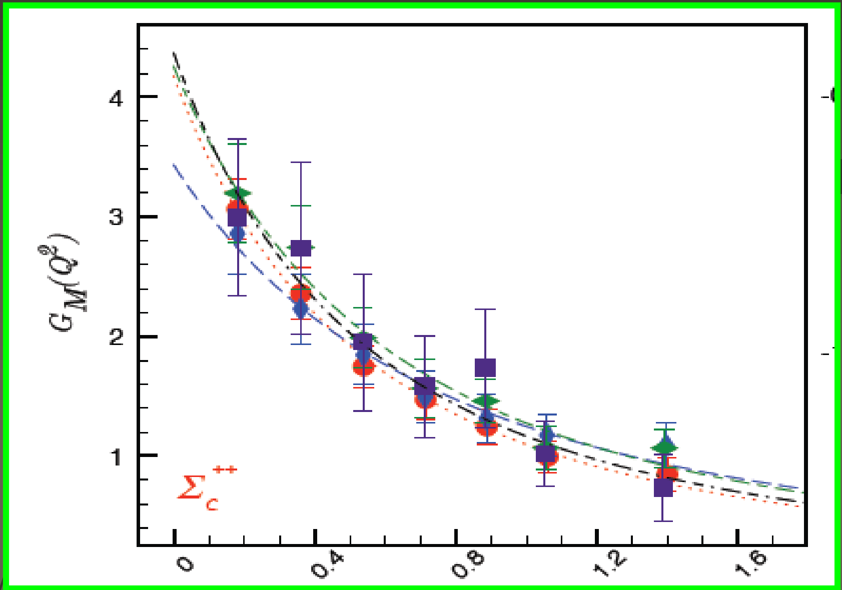
	w light quark	w/o light quark
Singly charmed	Σ_c qqc	Ω_c ssc
Doubly charmed	Ξ_{cc} qcc	Ω_{cc} scc

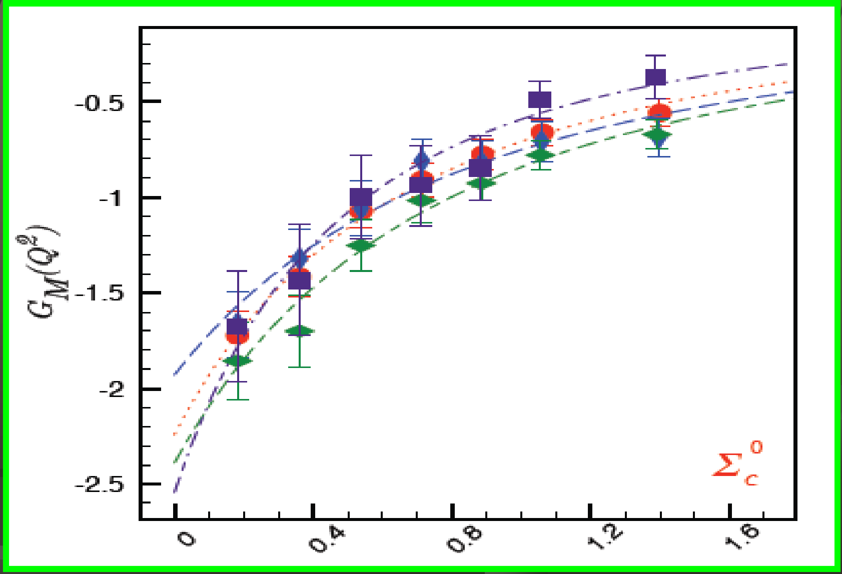
Singly charmed baryons

Electric form factors



Magnetic form factors

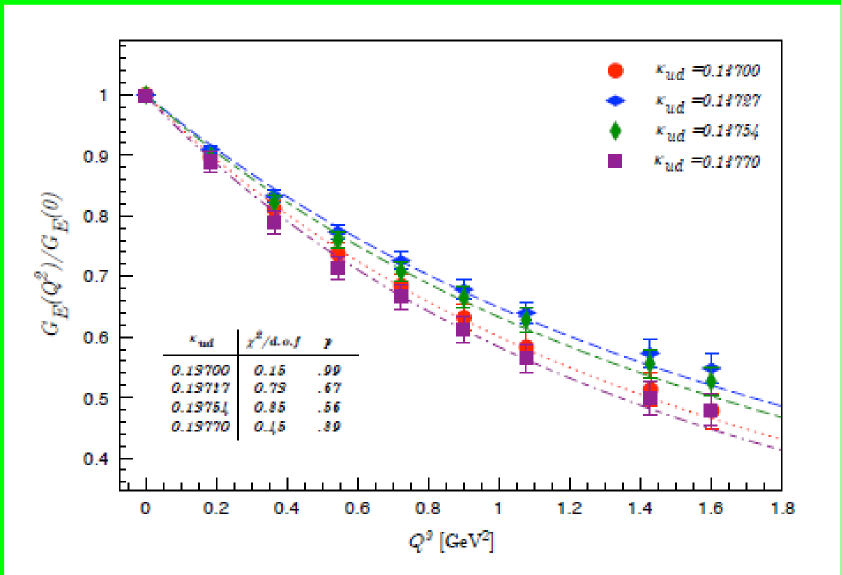


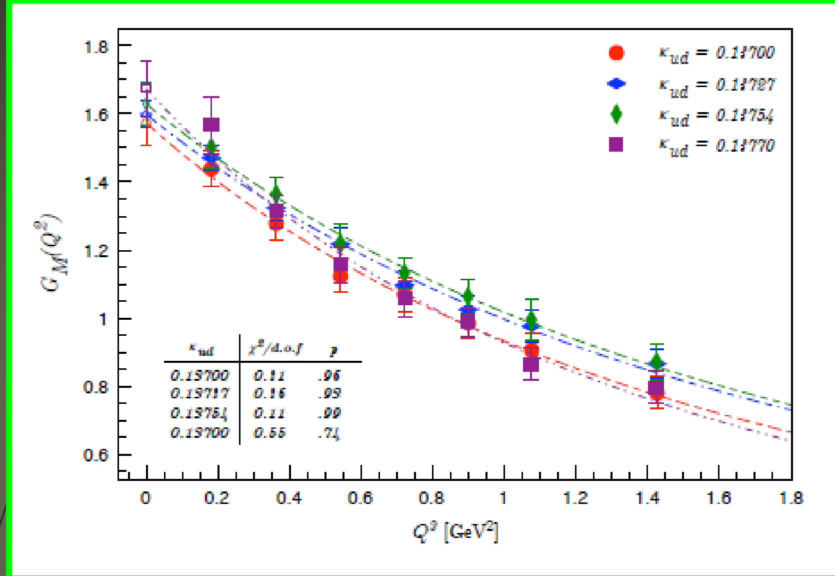


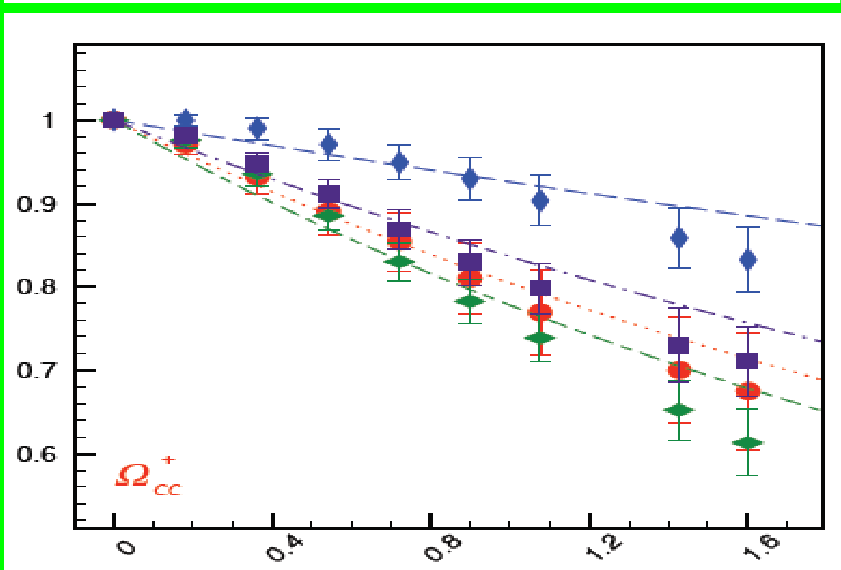
Doubly charmed baryons

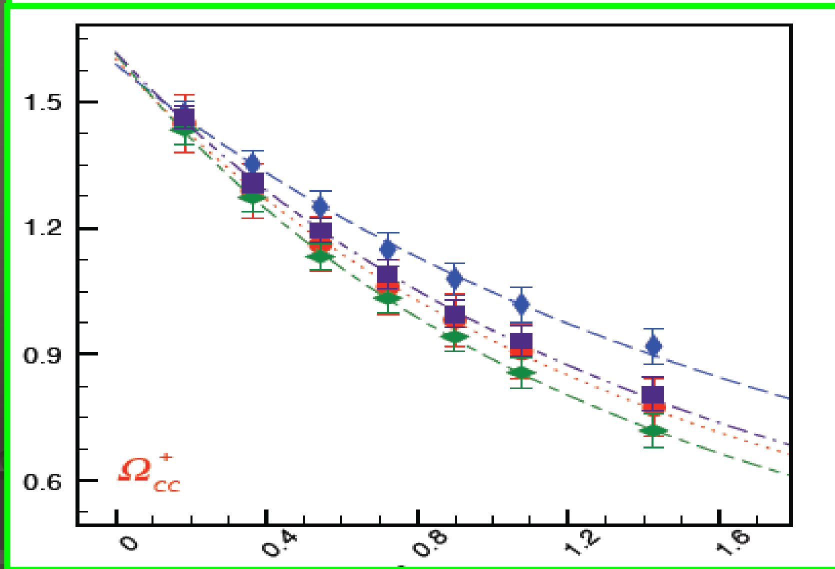
Electric form factors

Magnetic form factors









Electric charge radii

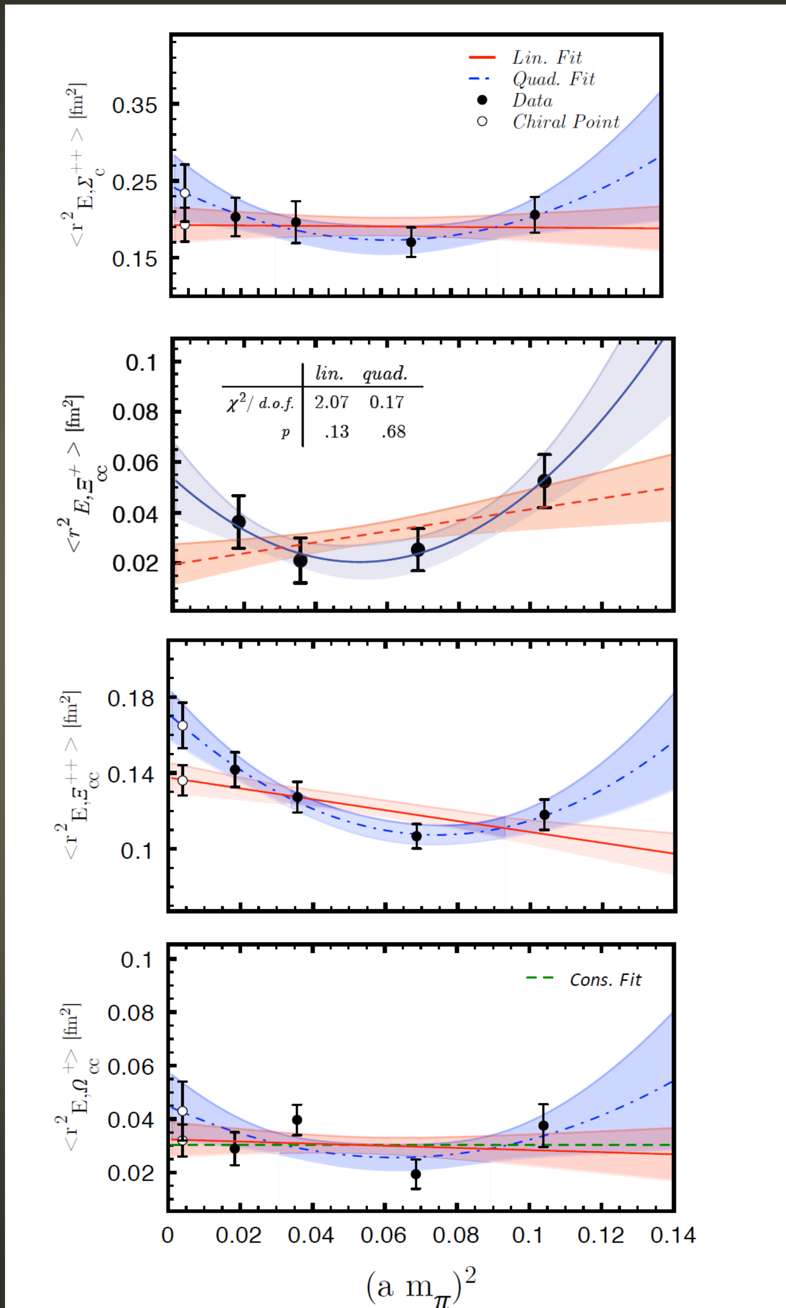
Vertical axis : values
Horizontal axis : valence (sea)quark mass

Σc

Ξcc

Ξcc

Ωcc



Small quarkmass dependence?

Very small
Brodsky et al. の予想に合致

Small (sea)quark mass dependence
Linear and quadratic fits give similar results.

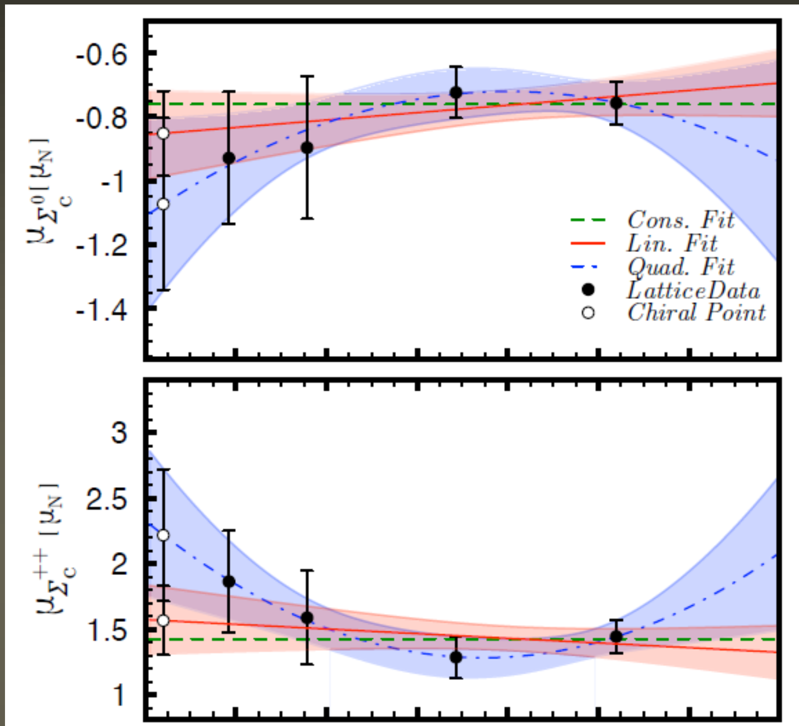
※contains no valence (light) quark

Magnetic moment

Vertical axis : values
Horizontal axis : valence (sea)quark mass

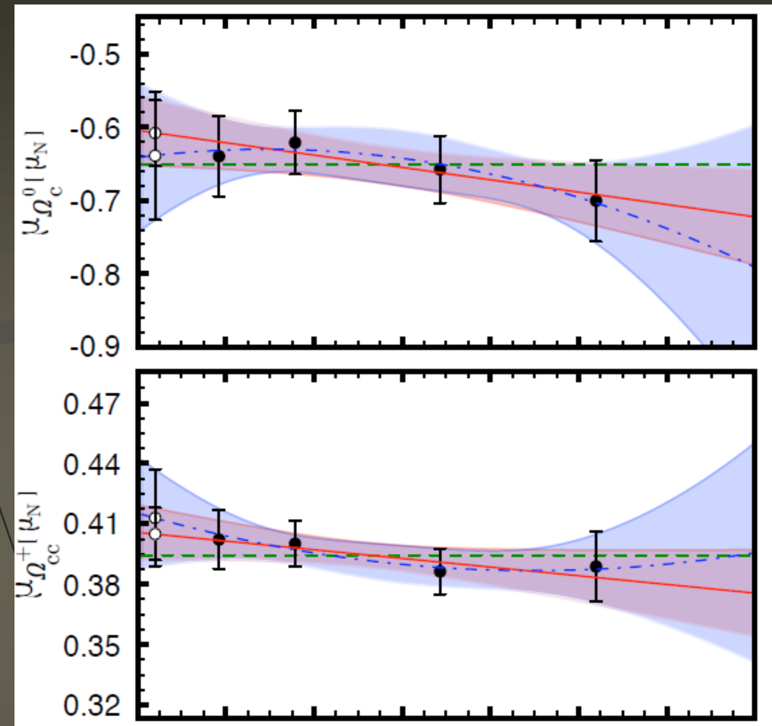
Σc

Σc



Ωc

Ωcc



Comparison with model calculations

esult Quad. fit	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]	[24]
-1.117(198)	-1.78	-1.04	-	-1.043	-1.60	-1.391	-1.17	-1.015	-1.6(2)
2.027(390)	3.07	1.76	-	1.679	2.20	2.44	2.18	2.279	2.1(3)
0.425(29)	0.94	0.72	$0.785^{+0.050}_{-0.030}$	0.722	0.84	0.774	0.77	-	-
-0.639(88)	-0.90	-0.85	-	-0.774	-0.90	-0.85	-0.92	-0.960	-
0.413(24)	0.74	0.67	$0.635^{+0.012}_{-0.015}$	0.668	0.697	0.639	0.70	0.785	-

Almost consistent with each other (but underestimated)

Coupling constants

Heavy hadron form factors

Charmed (or bottomed) hadron couplings or form factors are also important.

HHχPT

describes int. of heavy-light hadrons and NG bosons.
contains three axial couplings at the leading order.
precise knowledge of parameters → B-physics, physics beyond the SM.

Validity check of models

Lattice QCD is not almighty. Model calculations,
which describes hadronic interactions, are still needed in several situations.
Lattice QCD estimation of couplings could be used for the consistency check.

Possible new hadronic state?

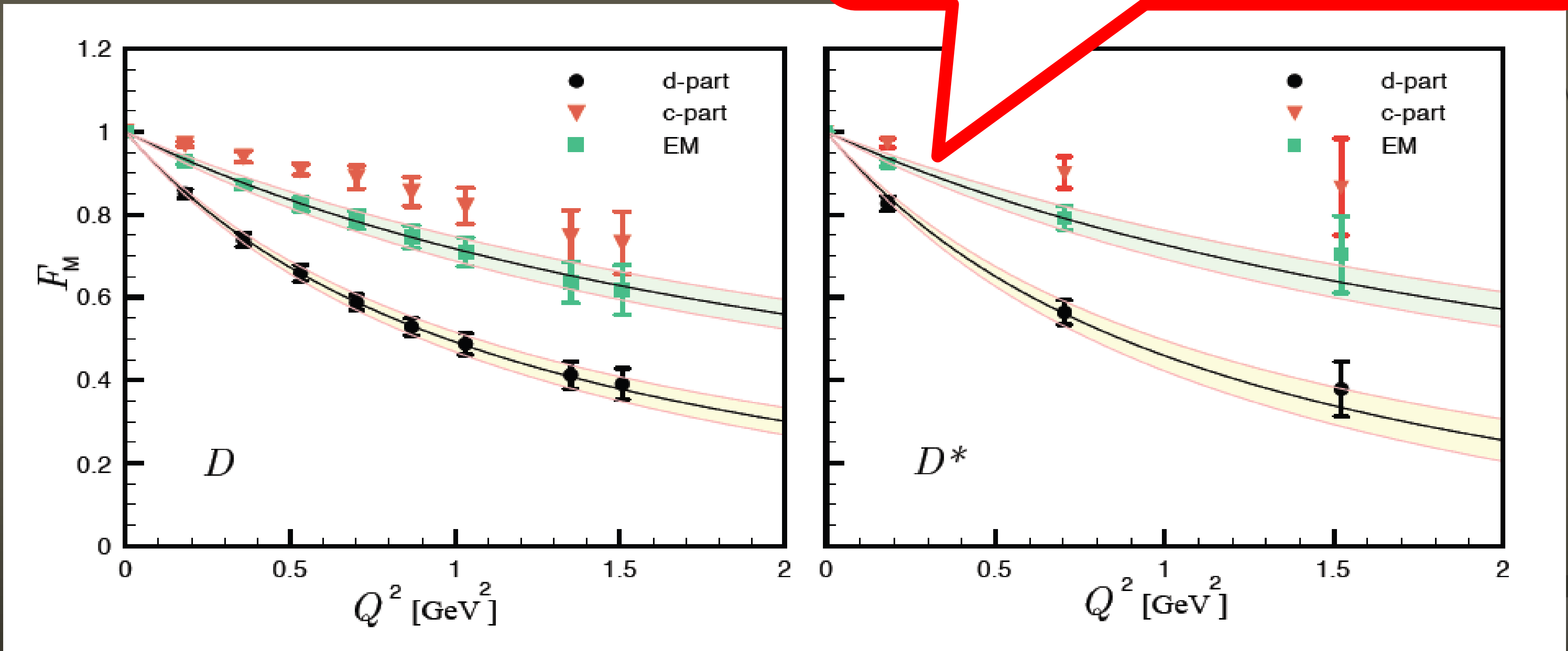
Charmed hadron-pion coupling is responsible for new hadronic states.

Lattice QCD results (couplings)

Form factors of D-meson

VMD-ansatz reproduces data well.
ここから結合定数を見積る。

$$F_V(Q^2) = \left[1 - \frac{Q^2}{m_\rho^2 + Q^2}\frac{g_{DD\rho}}{g_\rho}\right].$$

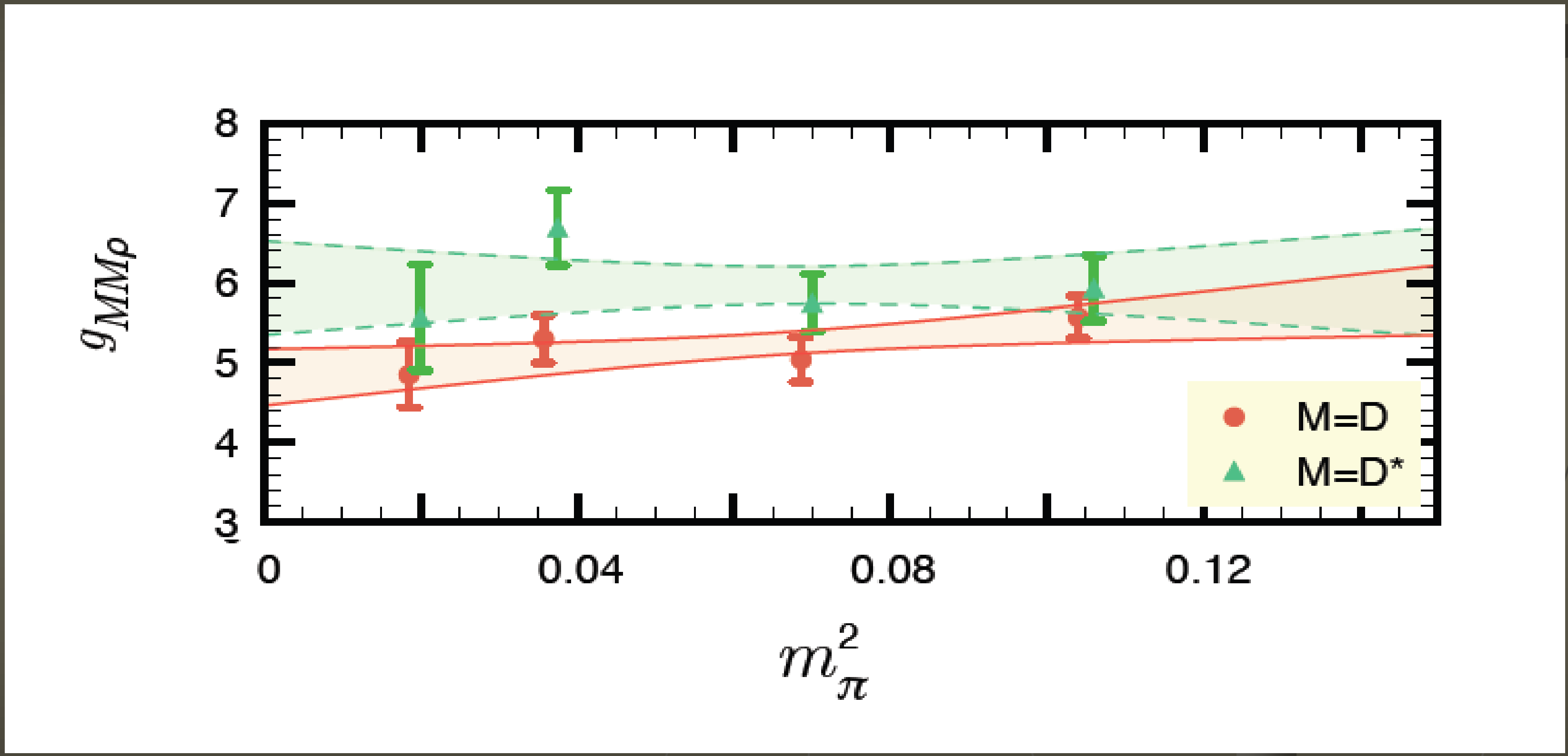


$D - meson$ $D^* - meson$

Lattice QCD results (couplings)

Quark-mass dependences of $DD\rho$ and $D^*D^*\rho$ coupings

$g_{\{D^*D^*\rho\}}$ is systematically smaller than $g_{DD\rho}$



Our chiral-fit -> 4.48±0.34 , 5.94±0.56

Lattice QCD results (couplings)

		Other models	
	Our chiral-fit	QCD Sum Rule	DS equation
D D ρ →	4.48±0.34	2.9 ± 0.4	
D*D*ρ→	5.94±0.56	5.2 ± 0.3	5.05

D*D*ρ coupling is consistent with other models
D D ρ coupling is still controversial
VMD-ansatz well reproduces form factors

BBρ coupling constants

We extract BBρ coupling constants by VMD-ansatz for form factors.

VMD-ansatz

$$F_V(Q^2) = \frac{m_\rho^2}{m_\rho^2 + Q^2} \frac{g_{\mathcal{BB}\rho}}{g_\rho},$$

Coupling is obtained as a fitted parameter

※rho-meson masses are determined at each quark mass on the lattice.

ρ meson coupling constant

$g_{\Xi_{cc}\Xi_{cc}\rho}$
6.555(428)
5.651(211)
5.721(263)
5.536(230)
5.700(245)
6.098(380)

$g_{\Sigma_c\Sigma_c\rho}$
13.392(883)
12.485(920)
11.073(676)
11.228(694)
10.554(512)
10.947 (1.056)

700 MeV (heavier)

Valence quark mass

300 MeV (heavier)

Chiral limit

Summary

We have been investigating charmed-hadron coupling constants by means of lattice QCD.

- 2+1 flavor gauge configurations generated by PACS-CS.
- Cutoff → 2.17 GeV, latice spacing → 0.09 fm
- Pion mass → 700∼300 MeV
- WALL sink method instead of so-called sequential source method

We successfully extracted some of charmed-meson, charmed-baryon coupling constants and form factors.

- Clarification of other channels and interpretation into physics are in order.
- Technical improvement is essentially needed.

There are not many lattice studies of charmed hadrons.

- Still, charmed hadron spectra are to be clarified in a more sophisticated way.
- Wavefunction measurement of scattering states may be interesting.
- Comparison of couplings with flavor SU(3) sectors.