Nucleon Spin from Lattice QCD

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χQCD Collaboration

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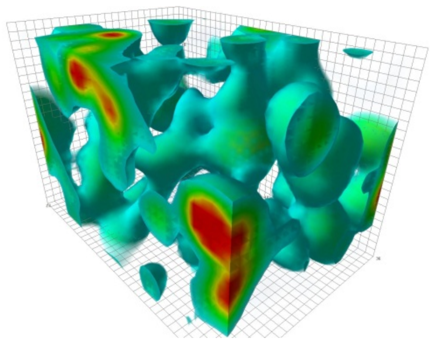
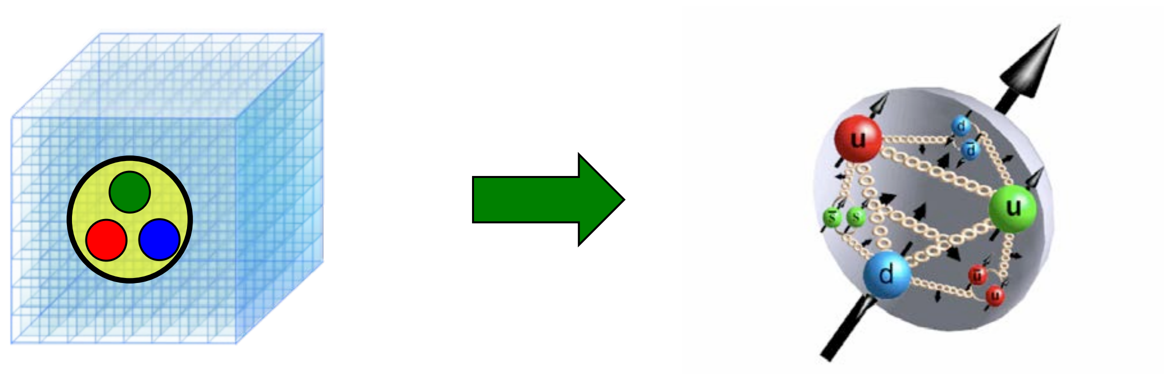
arXiv:1312.4816

2014/03/08

Workshop on High-energy QCD
and nucleon structure @ J-PARC





- <u>Outline</u>
  - Introduction
  - Lattice QCD framework
    - Challenges: Disconnected Insertion and Glue
  - Lattice QCD results
  - Summary & Prospects

(Figs by D.B.Leinweber, K.Murano and STAR Coll.)

Nucleon structure from QCD

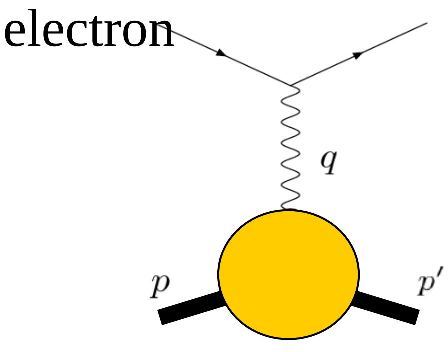
- Nucleon: the only hadron which is stable
  - the structure is crucial to understand nucleon itself, QCD, (& beyond SM)
  - ***Electric/Magnetic structure***

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

    - G_E: electric form factor
    - G_M: magnetic form factor
  - ***Deep Inelastic Scattering (DIS)***

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[W_2 + 2W_1 \tan^2\frac{\theta}{2}\right]$$

    - W_1, W_2 ➔ F_1, F_2 structure functions



Puzzles in Nucleon structure

- Do we know precisely ? Flavor/Glue DoF ?
- Vector form factor
  - One of the most well-determined quantities, but… ➔ “proton size crisis”
  - Strangeness element ←→ constrain G_A^s

(2010/07)

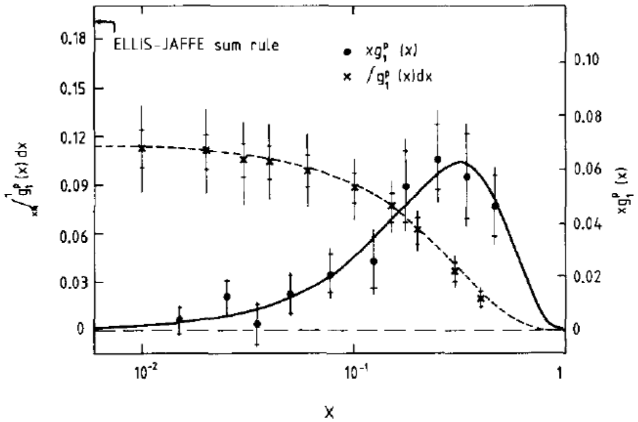
- Scalar form factor
  - Origin of the mass
  - pi-N-Sigma term ←→ pi-N int., rho mass shift in medium
  - Strangeness element ←→ Dark Matter Search
- More for Beyond SM
  - EDM form factor ←→ (strong) CP problem
  - Tensor form factor ←→ Non V-A Int in beta-decay

Puzzles in Nucleon structure

- Spin (axial vector)
  - **“Spin crisis”**
    - quark spin is small !

$$\Delta\Sigma = \sum_q [\Delta q + \Delta\bar{q}] = 0.2\text{–}0.3$$

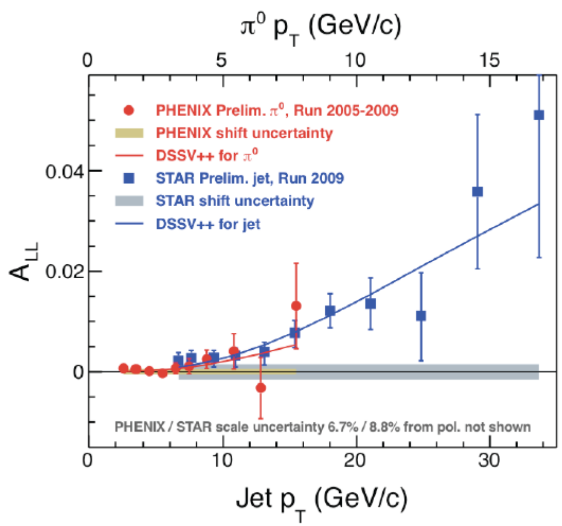
EMC(1988)

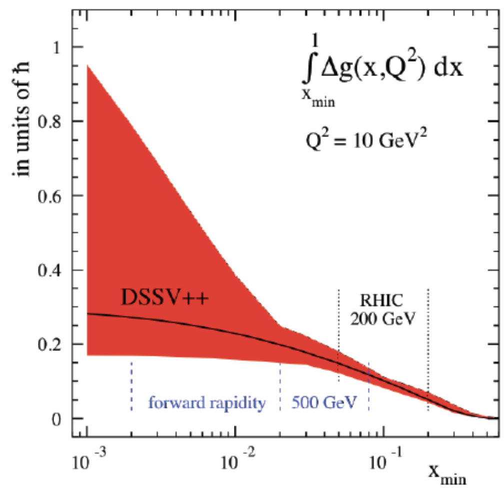


$$g_1(x) \simeq \frac{1}{2}\sum_q e_q^2[\Delta q(x) + \Delta\bar{q}(x)]$$

  - Glue ?

$$\int_{0.05}^{0.2} \Delta g(x)dx = 0.1 \pm^{0.06}_{0.07}$$





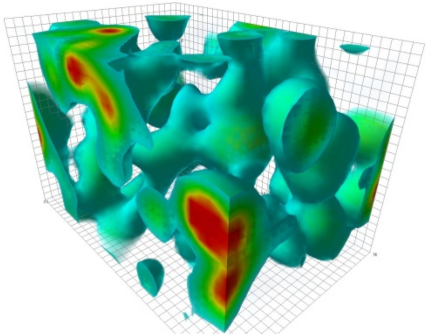
2014/03/08

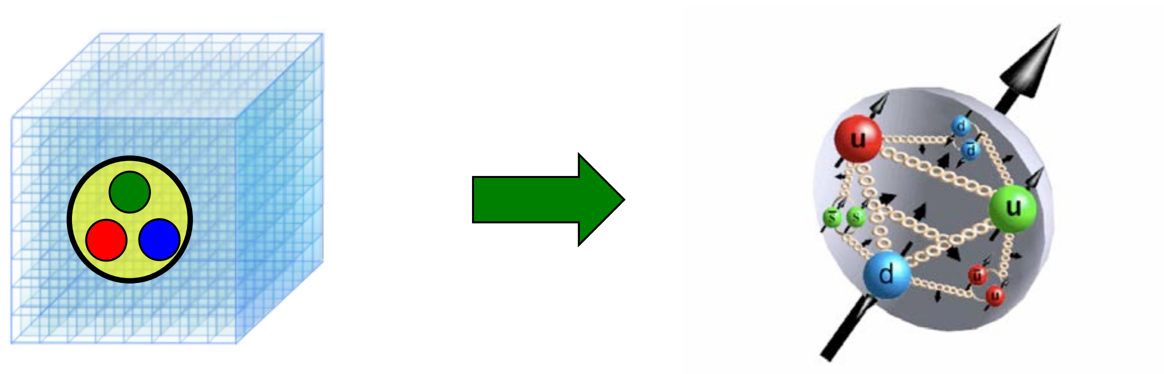
RHIC Spin: arXiv:1304.0079

Where does the proton spin come from ?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_G$$

- **Quark spin**: 20-30%
  - – DIS, Lattice
- **Glue spin**: ~20% ?
- **Quark orbital angular momentum:**
  - – Small in Lattice ? (for a part of diagrams)
- **Glue orbital angular momentum ?**
  - – ➔ **Dark-Spin ?**





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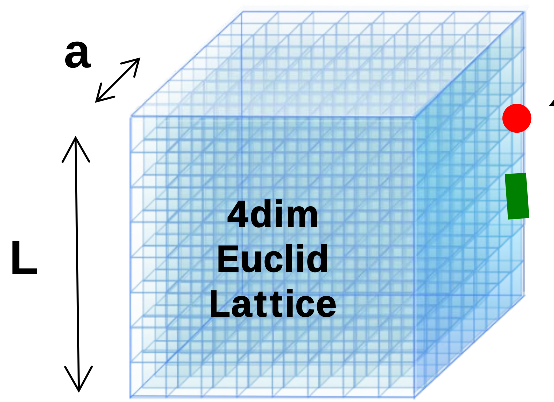
Lattice QCD
First-principles calculation of QCD

$$Z = \int dU\, dq\, d\bar{q}\;\, e^{-S_E}$$



quarks on sites

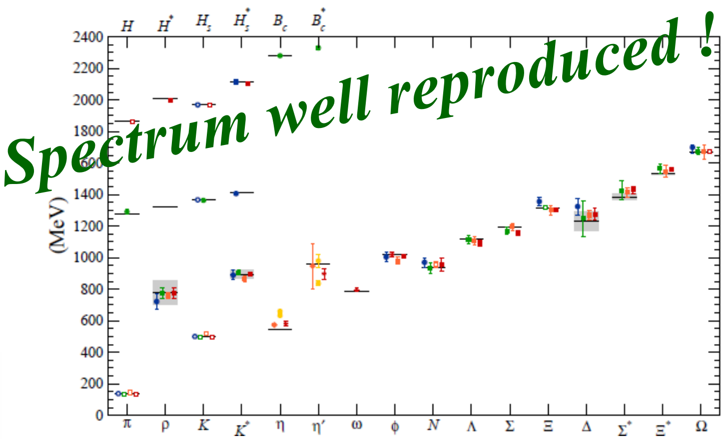
$$q(x)$$

gluons on links

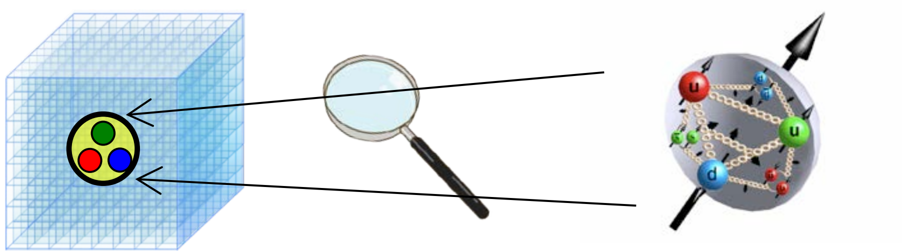
$$U_\mu(x, x+\mu) = \exp[-iaA_\mu]$$

K.G. Wilson

- **Well-defined reguralized system**
- **Gauge-invariance manifest**
- **Fully-Nonperturbative**
- **DoF ~ 10^9 ➔ Monte-Carlo w/ Euclid time**



Summary by Kronfeld, arXiv:1203.1204



Isovector matrix elements

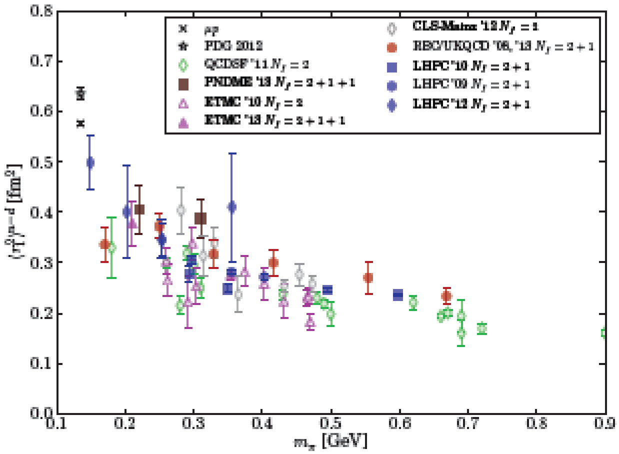
review talk by S.Syritsyn @ Lat13

Gold-plated observables

Nucleon Dirac Radius

$< r_1^2 >$

$$F_1^{u-d}(Q^2) \approx F(0)\left[1 - \frac{1}{6}Q^2 \langle r_1^2 \rangle^{u-d} + \mathcal{O}(Q^4)\right]$$



ChPT predicts divergence $\sim \log m_\pi^2$

Larger L_s, smaller $Q^2_{\min}$ are desirable

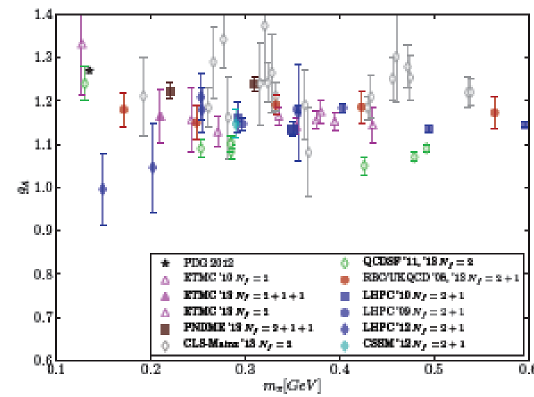
Gold-plated observables

Drama of the Axial Charge

g_A

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = g_A\, \bar{u}_p \gamma^\mu \gamma^5 u_p\,,$$

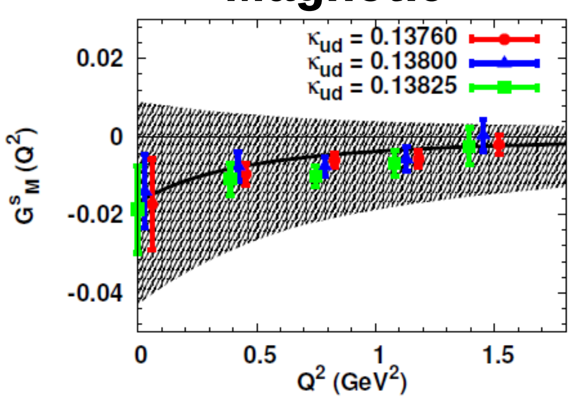
Experiment (W.A.) [PDG'12] $g_A^{\text{ave}} = 1.2701(25)$



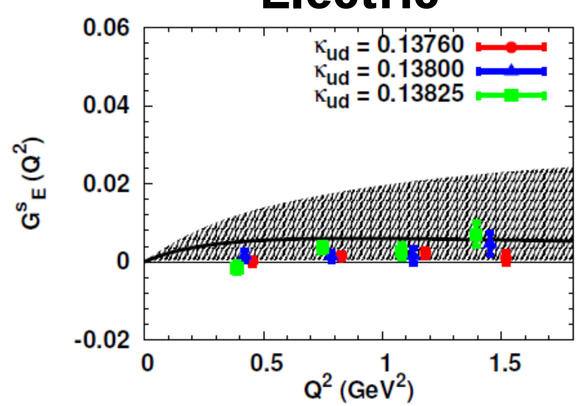
Many lattice calculations underestimated g_A by 10-15%

Strangeness EM form factor

Magnetic



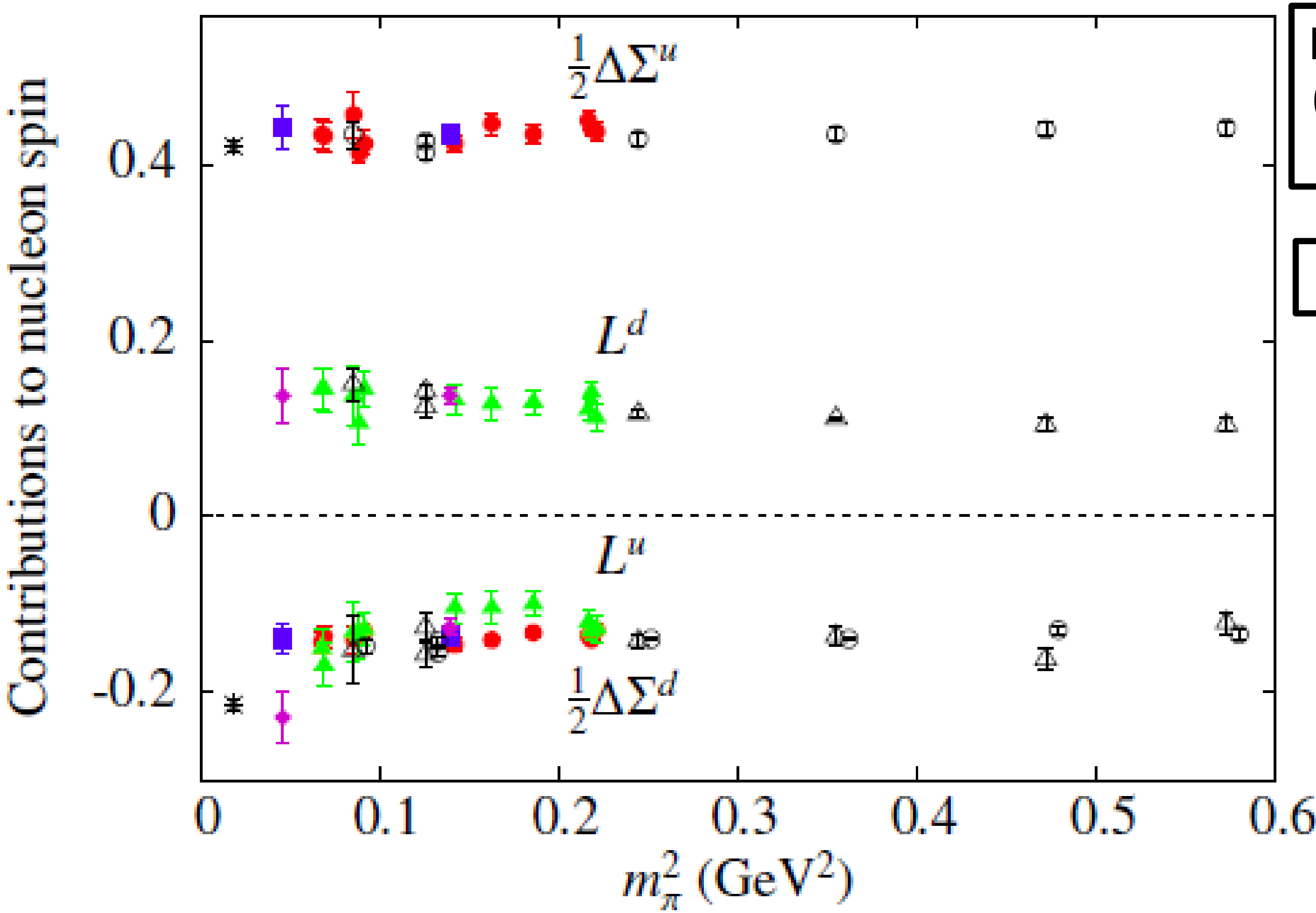
Electric



$$G^s_M(0) = -0.017(25)(07)$$

TD et al., PRD80(2009)094503

How about proton spin ?



Part of u/d quark (Connected Insertion Only)

No Glue calc

Open symbols: Asktad+DWF (LHPC)

Filled symbols: TMF (ETMC)

(Msbar, mu=2GeV)

Fig from C. Alexandrou et al., PRD88(2013)014509

Formulation on the Lattice

- 1st-moment <x> and spin J studied simultaneously
- Matrix elements of energy-momentum tensor

Gauge invariant decomposition

X.Ji (1997)

$$T_q^{\mu\nu} = \frac{i}{4}\left[\bar{q}\gamma^\mu \overrightarrow{D}^\nu q - \bar{q}\gamma^\mu \overleftarrow{D}^\nu q + (\mu \leftrightarrow \nu)\right]$$

$$T_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha}$$

$$T_q^{\mu\nu} \rightarrow \bar{q}\vec{\gamma}\gamma_5 q + \bar{q}[\vec{x} \times (-i\vec{D})]q$$

$$T_g^{\mu\nu} \rightarrow \vec{x} \times (\vec{E} \times \vec{B})$$

Recent developments:
Chen et al., Wakamatsu,
Hatta, Leader & Lorce, ...

- Nucleon matrix elements

$$\langle p, s | T^{\mu\nu} | p', s' \rangle = \bar{u}(p,s)\left[T_1(q^2)\gamma^\mu \bar{p}^\nu + T_2(q^2)\bar{p}^\mu i\sigma^{\nu\alpha}/2m + T_3(q^2)(q^\mu q^\nu - g^{\mu\nu} q^2)/2m + T_4(q^2) g^{\mu\nu} m/2\right] u(p', s')$$

$$\langle x \rangle = T_1(0) \qquad J = \frac{1}{2}[T_1(0) + T_2(0)]$$

(angular) momentum sum rules

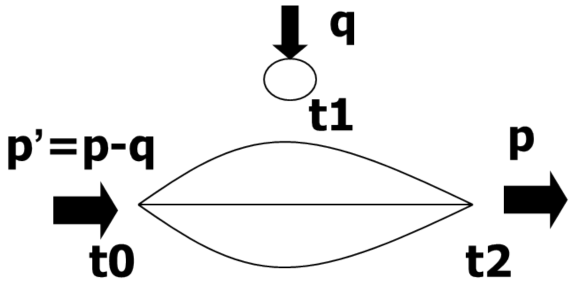
$$\langle x \rangle_q + \langle x \rangle_G = 1 \qquad J_q + J_G = 1/2$$

Formulation on the Lattice

- Calculate 3pt (& 2pt) ➔ matrix elements

$$\Pi^{3pt}(\vec{p}, t_2; \vec{q}, t_1) = \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{p}\cdot\vec{x}_2} e^{+i\vec{q}\cdot\vec{x}_1} \langle 0 | \mathrm{T}\left[J_N(\vec{x}_2, t_2) T^{\mu\nu}(\vec{x}_1, t_1) \bar{J}_N(0) \right] 0 \rangle$$

$$(T^{\mu\nu} = T^{4i})$$

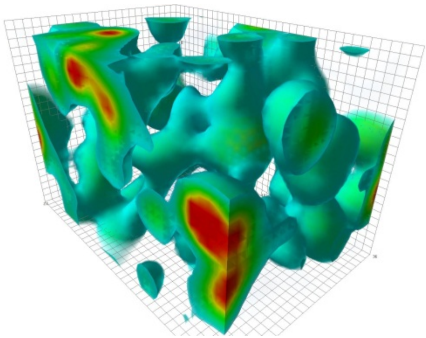


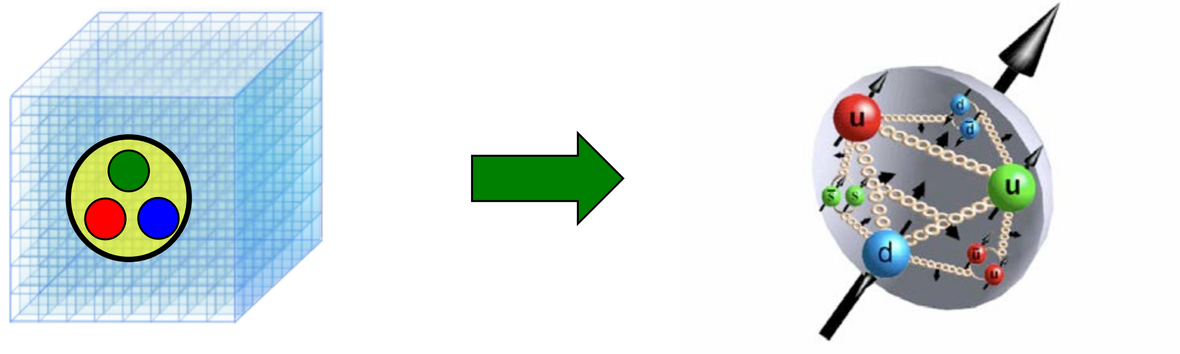
- – Typical examples:

$$\mathrm{Tr}\left[\Gamma_e \Pi^{3pt}_{T_{4i}}(\vec{p}, t_2; \vec{q} = \vec{0}, t_1)\right] = C \cdot e^{-m(t_2 - t_1)} e^{-Et_1} \left[2p_i \cdot T_1(0)\right]$$

$$\mathrm{Tr}\left[\Gamma_m \Pi^{3pt}_{T_{4i}}(\vec{p} = \vec{0}, t_2; \vec{q}, t_1)\right] = C \cdot e^{-m(t_2 - t_1)} e^{-Et_1} \left[-i\epsilon_{ijm} q_j (T_1(-q^2) + T_2(-q^2))\right]$$

- – Other momentum combinations are calculated and T_1, T_2, (T_3) are determined simultaneously





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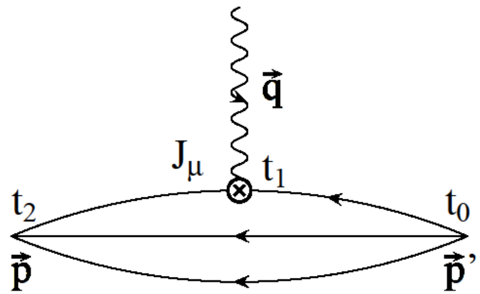
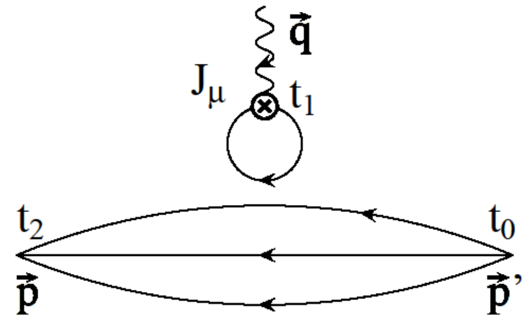
Challenges in Lattice QCD
(1) Disconnected Insertion (DI)

- Two kinds of calc in Lattice:



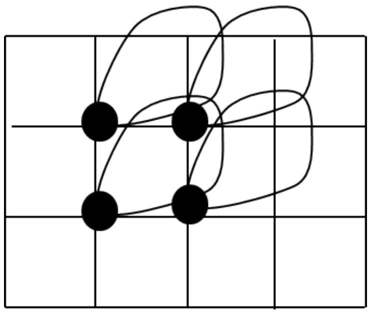
Connected Insertion (CI)



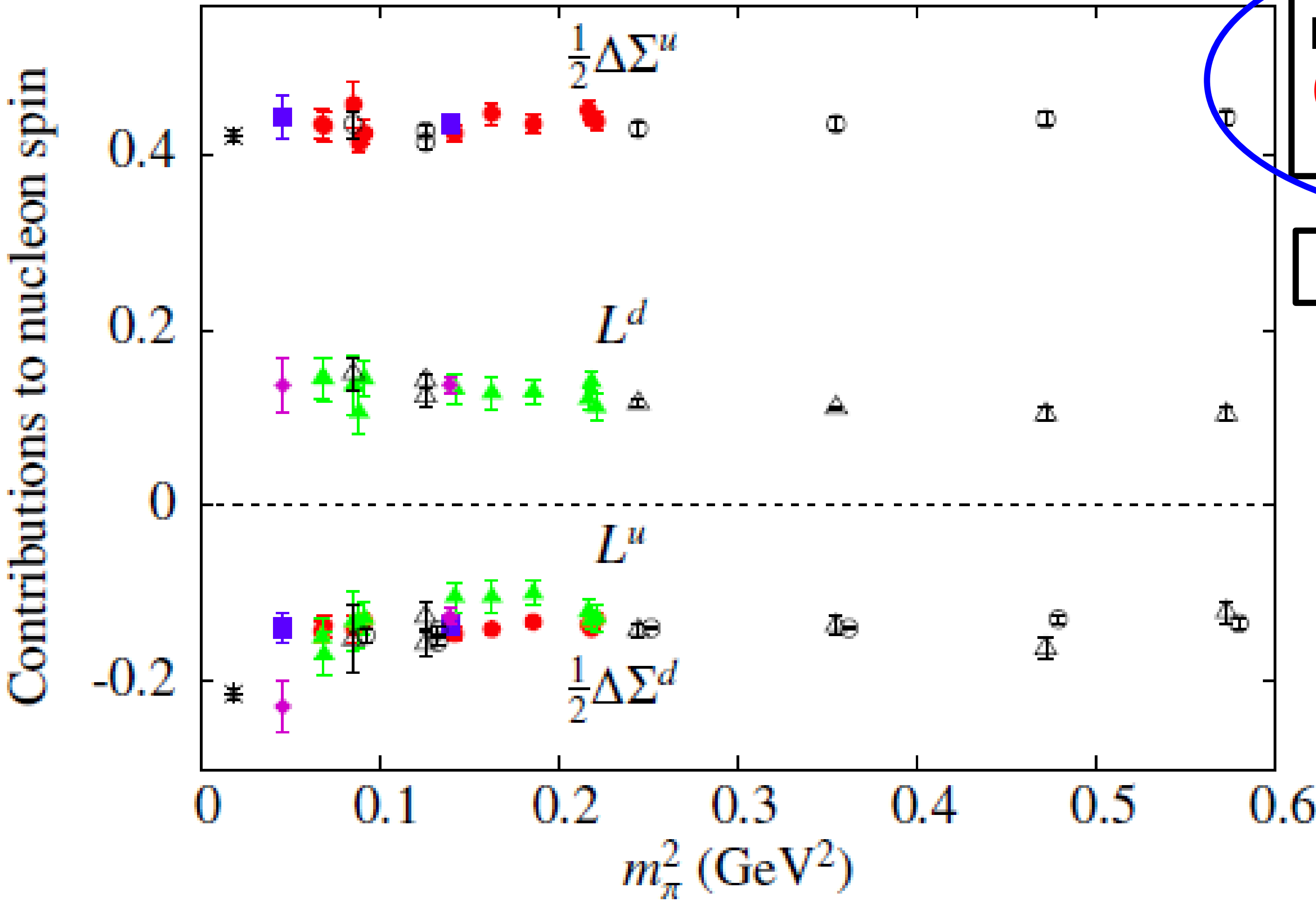
Disconnected Insertion (DI)

- DI is inevitable for flavor singlet quantities, but…
  - **All(source)-to-all(sink)** propagator is necessary
  - Straightforward calculation **impossible**
    - **O(10^5) inversions** for O(10^6) x O(10^6) matrix

$$\mathrm{Tr}[\Gamma M^{-1}] = \sum_x \mathrm{Tr}^{\mathrm{spin}}_{\mathrm{color}}[\Gamma M^{-1}(x,x)]$$



How about proton spin ?



Part of u/d quark (Connected Insertion Only)

No Glue calc

Open symbols:
Asktad+DWF (LHPC)

Filled symbols:
TMF (ETMC)

(Msbar, mu=2GeV)

Fig from C. Alexandrou et al., PRD88(2013)014509

The approach for disconnected insertion

- Stochastic Method for DI
  - Use Z(4) (or Z(N)) noises such that

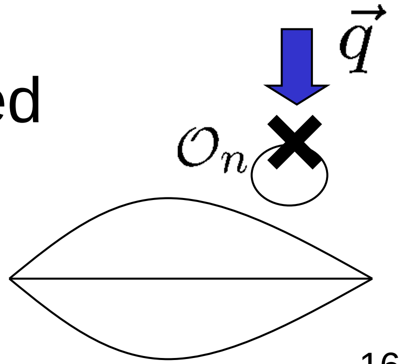
S.-J.Dong, K.-F.Liu, PLB328(1994)130

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta_i^{l\,\dagger}\eta_j^l=\delta_{ij}$$

  - DI loop can be calculated as

$$\mathrm{Tr}[\Gamma M^{-1}]=\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta^{l\,\dagger}(\Gamma M^{-1}\eta^l)$$

- Introduce new source for noises (“off-diagonal” part)
  - ➔ Unbiased subtraction using hopping parameter expansion (HPE)
  - Off-diagonal contaminations are estimated in unbiased way



c.f. other approaches
All-to-all (Foley et al., 2005)
CAA/AMA (Blum et al., 2012)

Stochastic method for DI

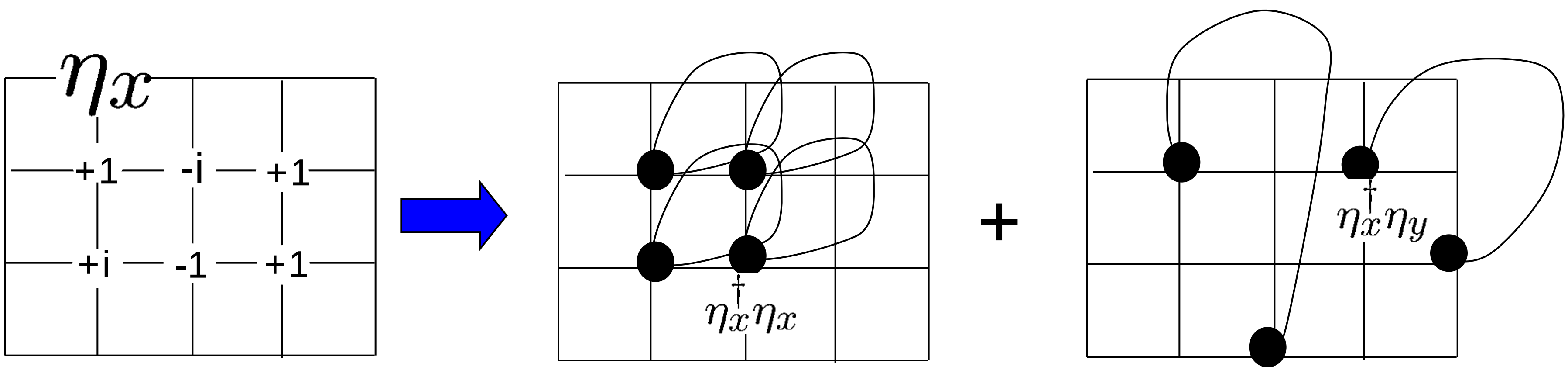
- Stochastic Method for DI
  - Noise

$$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta_i^{l\,\dagger}\eta_j^l=\delta_{ij}$$

  - DI loop

$$\mathrm{Tr}[\Gamma M^{-1}]=\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta^{l\,\dagger}(\Gamma M^{-1}\eta^l)$$

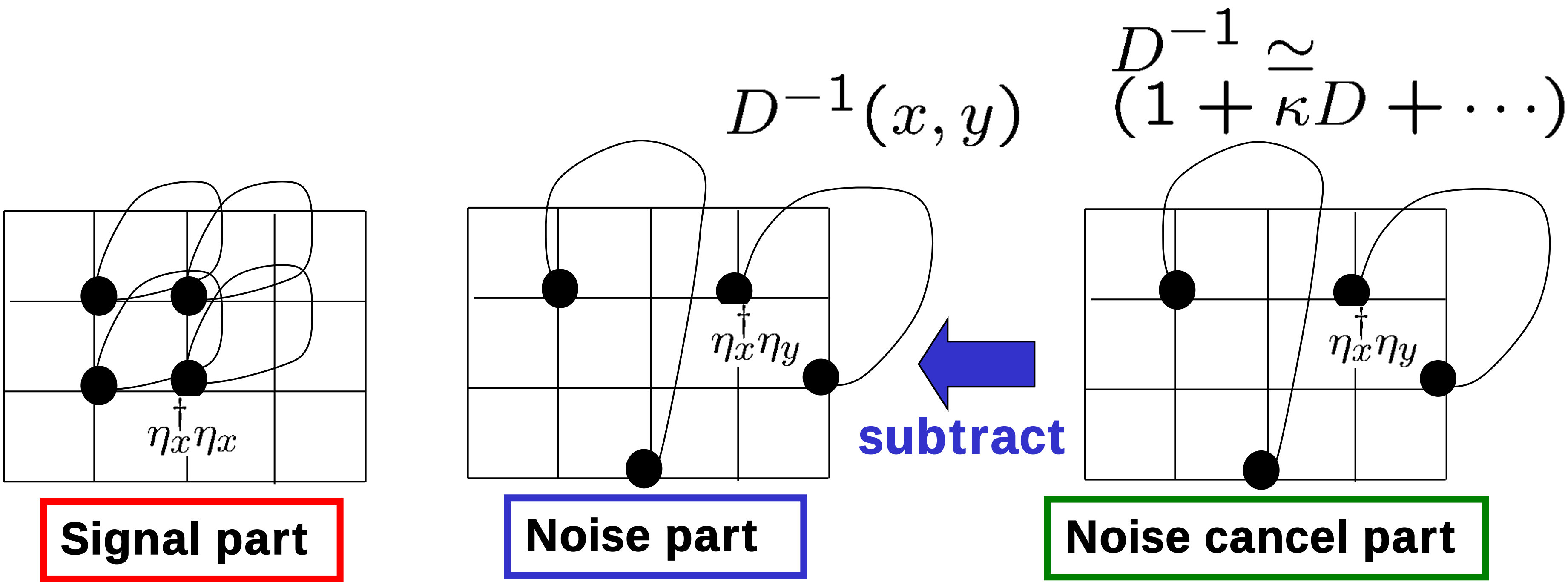
S.-J.Dong, K.-F.Liu,
PLB328(1994)130



Stochastic source **Signal part** **Noise part**

Improvement of DI calc

- The **unbiased subtraction** using **hopping parameter expansion (HPE)** to eliminate off-diagonal noises

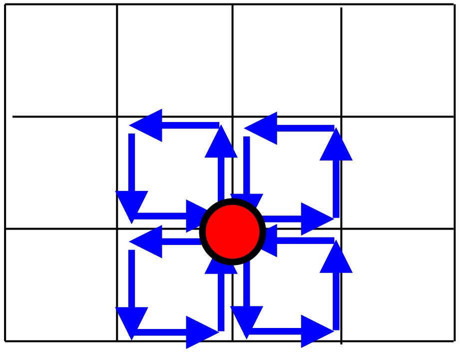


➔The error reduces by a factor of 2 or more

Challenges in Lattice QCD
(2) gluon matrix elements

- Gluon operator

$$T_G^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha}$$



  – Implementation is simple w/ link variables

    Fμν ←→ clover term w/ link Uμ

  – In practice, S/N is known to be notoriously noisy
    - Gluon DoF fluctuate too much in high-freq mode

M. Gockeler et al.,
Nucl.Phys.Proc.Suppl.53(1997)324

The approach for Glue

- Field tensor constructed from overlap operator

$$F_{\mu\nu}(x) \longleftarrow \mathrm{Tr}_{\text{(spinor)}}\left[\sigma_{\mu\nu} D_{ov}(x,x)\right]$$

$$(a \to 0)$$

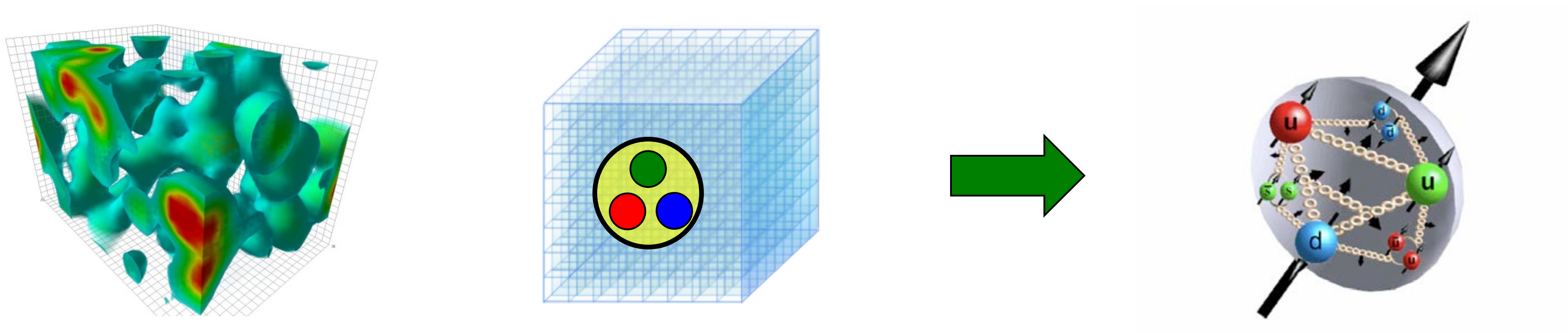
K.-F.Liu, A.Alexandru, I.Horvath
PLB659(2008)773

$$D_{ov} = \rho\left(1 + X\frac{1}{\sqrt{X^\dagger X}}\right), \; X = -\rho + D_W$$

  – Ultraviolet fluctuation is expected to be suppressed (automatic smearing)
  – In order to estimate $D_{ov}(x,x)$, stochastic method is used w/ color/spinor & (some) spacial dilution

$$D_{ov}(x,x) \Leftarrow \langle \eta_x^\dagger (D_{ov}\eta)_x \rangle$$

c.f. other approaches
Smearing (Meyer et al., 2008)
Change Action & response (Horsley et al., 2012)
Wilson-Flow (H.Suzuki, 2013)



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Lattice Setup

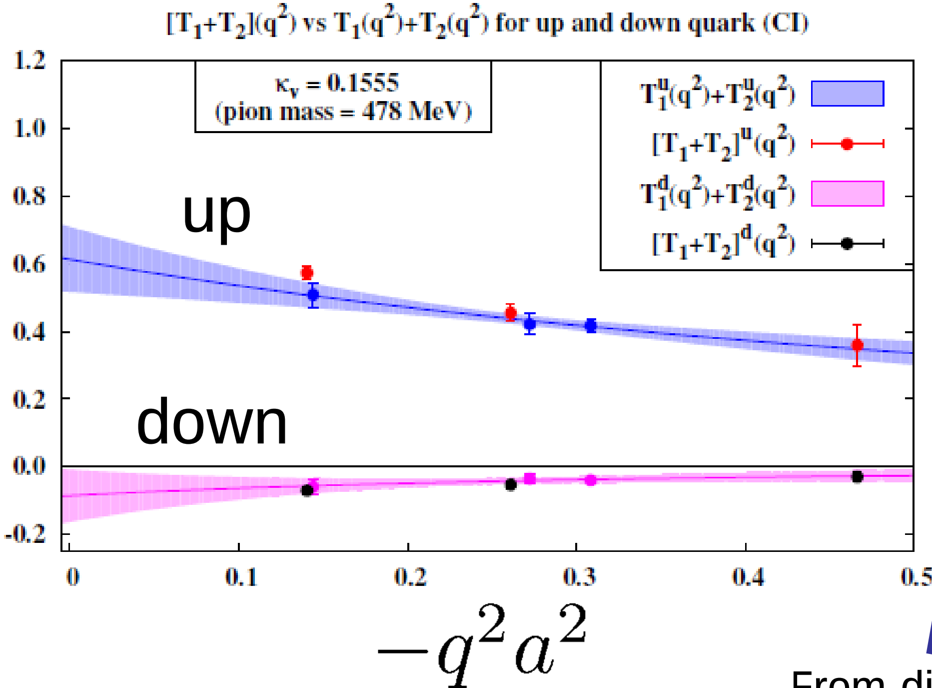
- Wilson Fermion + Wilson gauge Action
  - 500 configs with Quenched approximation
  - 1/a=1.74GeV, a=0.11fm (beta=6.0)
  - 16^3 x 24 lattice, L=1.76fm
  - kappa(ud) = 0.154, 0.155, 0.1555
    - m(pi) = 0.48, 0.54, 0.65 GeV
    - m(N) = 1.09, 1.16, 1.29 GeV
    - kappa(s)=0.154 , kappa(critical)=0.1568

Lattice Setup (cont'd)

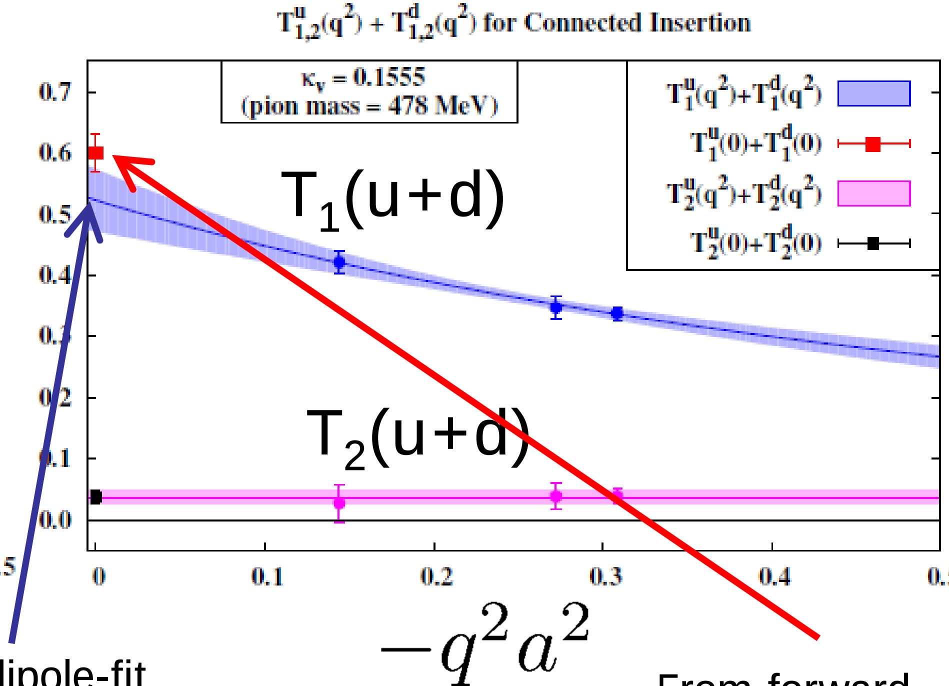
- Disconnected Insertion (DI)
  - Z(4) stochastic method, #noise=500
  - Unbiased subtraction w/ up to 4th HPE
- Glue matrix element
  - Overlap operator $D_{ov}(x,x)$
  - Z(4) stochastic method, #noise=2, w/ color/spinor dilution + spacial dilution (d=2 & even/odd ➔ taxi-distance=4)
- Improvement
  - Many nucleon sources, #src=16
  - CH, H and parity symmetry:
    - (3pt)=(2pt) X (loop)➔(3pt) = Im(2pt) X Re(loop) + Re(2pt) X Im(loop)

Results for CI: q^2-dependence

T_1+T_2



$T_1(u+d)$, T_2 $(u+d)$



From dipole-fit

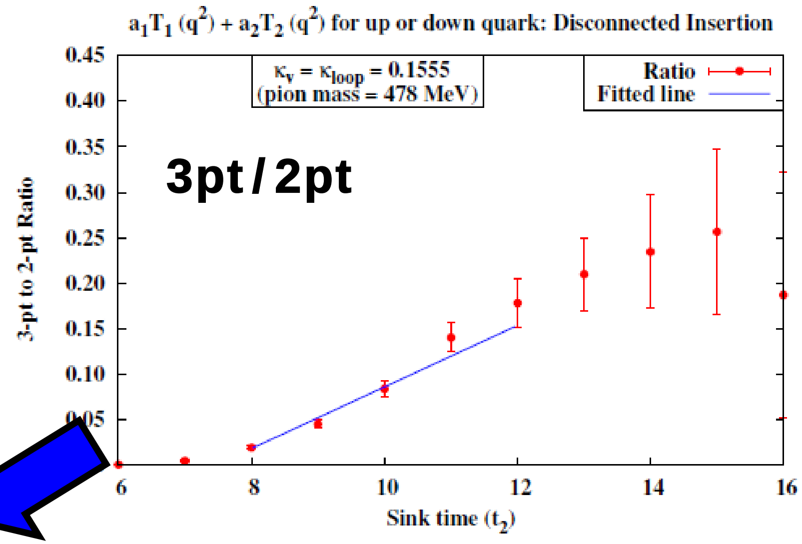
From forward matrix element

<u>Dipole-fit performed</u>

m_π =0.48GeV

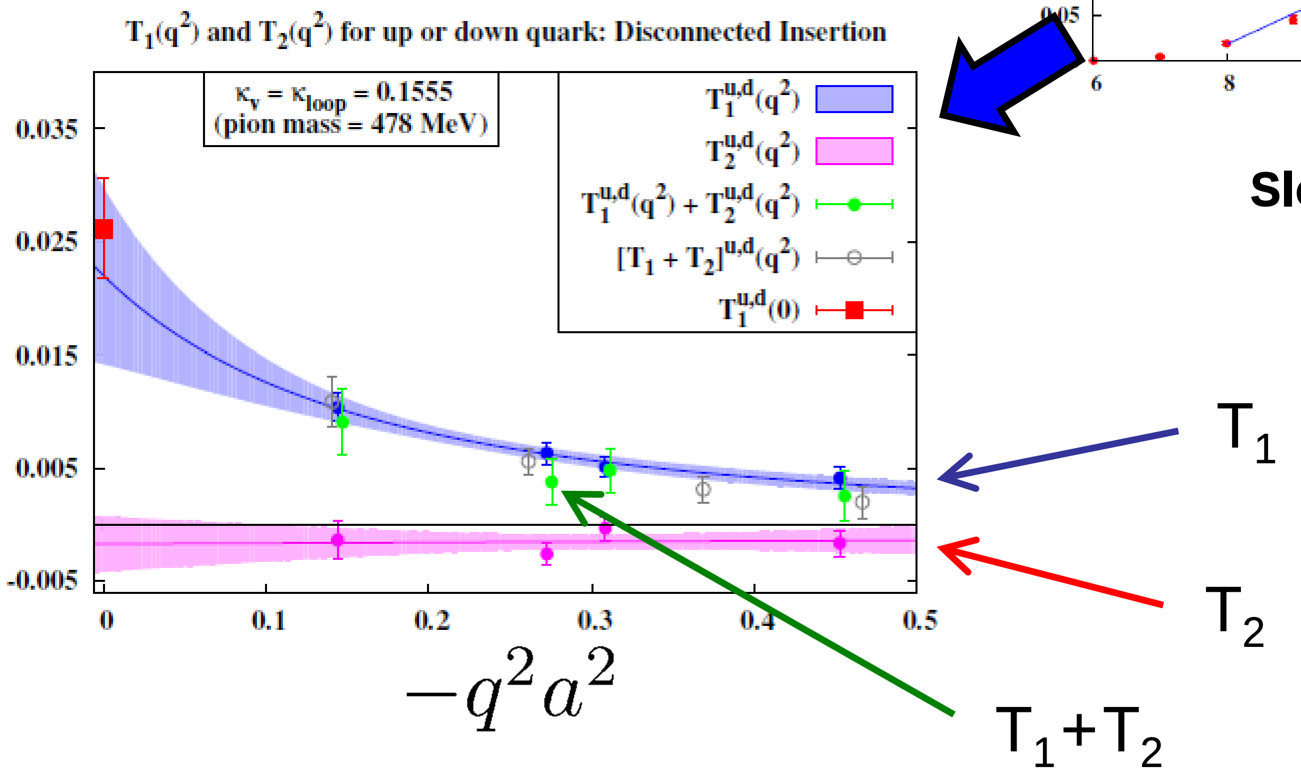
Different q^2 extrapolation
(1) $T_1(q^2)+T_2(q^2)$
(2) $[T_1+T_2](q^2)$
gives consistent results

Results for DI: q^2-dependence



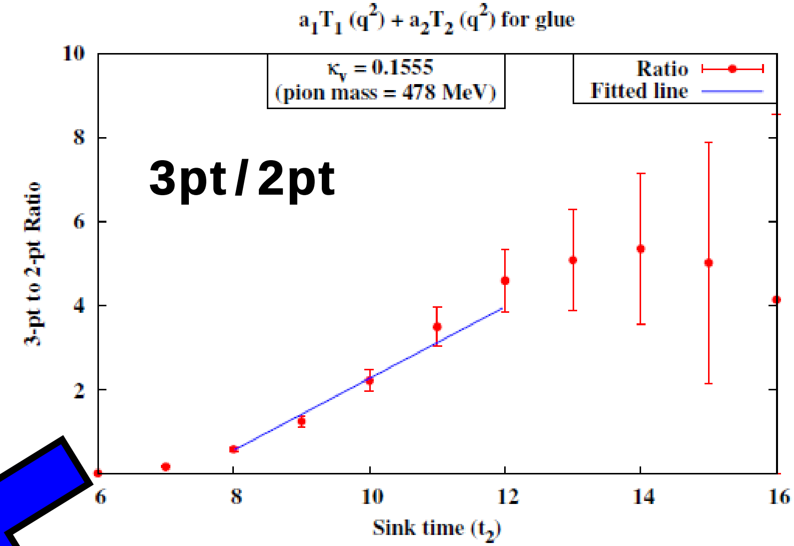
Slope ←→ Signal

T_1, T_2 (for u/d)



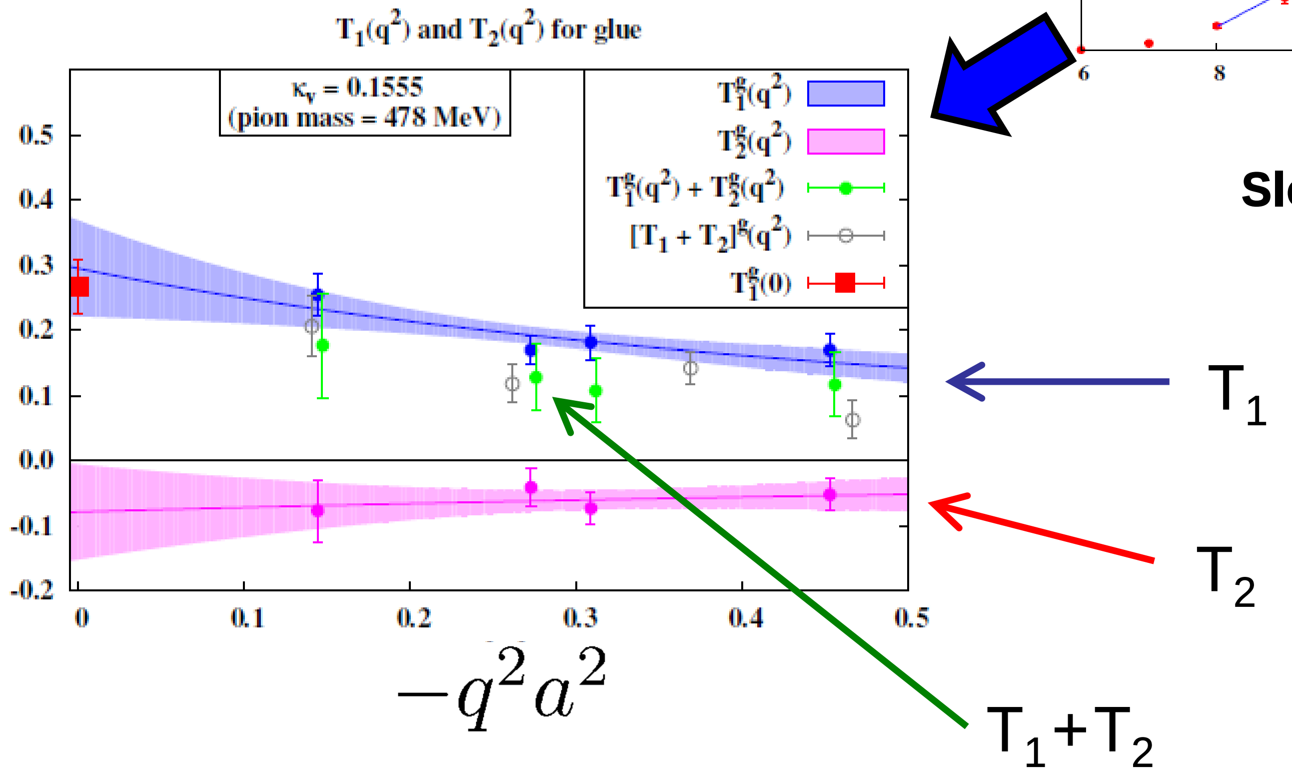
m_π = 0.48GeV

Results for Glue: q^2-dependence



Slope ←→ Signal

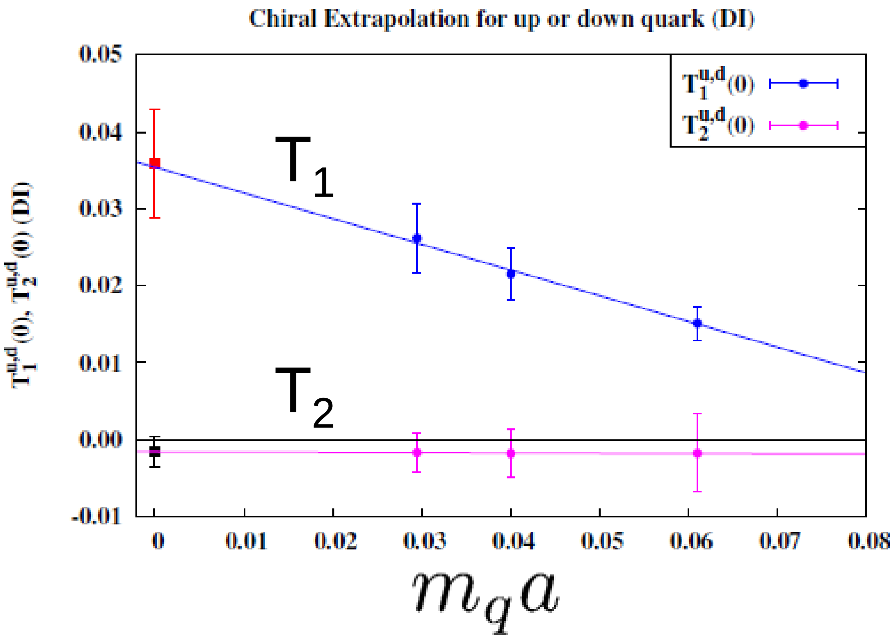
T_1, T_2 (for glue)



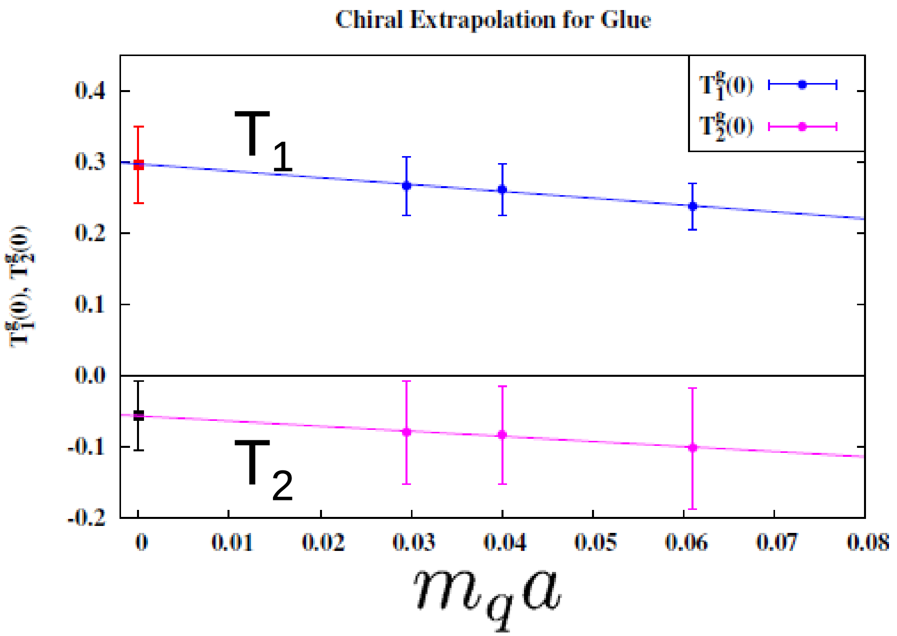
$m\pi = 0.48$GeV

Chiral Extrapolation

T_1, T_2 (DI) (for u/d)



T_1, T_2 (for glue)



Simple Linear-extrapolation is performed

Renormalization

- Quark-glue mixing

$$\begin{pmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_G^{\overline{MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{qq}(a\mu, g_0) & Z_{qG}(a\mu, g_0) \\ Z_{Gq}(a\mu, g_0) & Z_{GG}(a\mu, g_0) \end{pmatrix} \begin{pmatrix} \langle x\rangle_q^{lat} \\ \langle x\rangle_G^{lat} \end{pmatrix}$$

Check on Momentum sum rules for lat results

$$\langle x\rangle_q^{lat} + \langle x\rangle_G^{lat} = 0.95(7)$$

$$2(J_q^{lat} + J_G^{lat}) = 0.95(9)$$

$Z_{qG} = 0$ (quenched)

$$Z_{qq} = 1 + \frac{g_0^2}{16\pi^2} C_F \left(\frac{8}{3}\log(a^2\mu^2) + f_{qq}\right), \quad Z_{qg} = -\frac{g_0^2}{16\pi^2}\left(\frac{2}{3}N_f \log(a^2\mu^2) + f_{qg}\right),$$

$$Z_{gq} = -\frac{g_0^2}{16\pi^2} C_F \left(\frac{8}{3}\log(a^2\mu^2) + f_{gq}\right), \quad Z_{gg} = 1 + \frac{g_0^2}{16\pi^2}\left(\frac{2}{3}N_f \log(a^2\mu^2) + f_{gg}\right).$$

Lat PT calc (one-loop) ← M.Glatzmaier, K.-F.Liu, M.Ramsey-Musolf

$$f_{qq} = -7.60930 \quad f_{qG} = 0$$
$$f_{Gq} = -2.37600 \quad f_{GG} = -3.76900$$

$$\frac{1}{\sqrt{X^\dagger X}} = \int_{-\infty}^{\infty} \frac{d\sigma}{\pi} \frac{1}{\sigma^2 + X^\dagger X}$$

(Integral form for glue op.)

Renormalization

- "Sum-rule improved" version

$$\langle x\rangle_q^{lat,S} + \langle x\rangle_G^{lat,S} = 1 \qquad 2(J_q^{lat,S} + J_G^{lat,S}) = 1$$

"normalization-improvement" by imposing sum-rules to account for latt systematics

$$\langle x\rangle_q^{lat,S} = Z_q^L \langle x\rangle_q^{lat}$$

$$\langle x\rangle_G^{lat,S} = Z_G^L \langle x\rangle_G^{lat} \quad \text{etc.}$$

– We also have to modify matching coeffs

$$\begin{pmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_G^{\overline{MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{qq}(a\mu, g_0) & Z_{qG}(a\mu, g_0) \\ Z_{Gq}(a\mu, g_0) & Z_{GG}(a\mu, g_0) \end{pmatrix} \begin{pmatrix} \langle x\rangle_q^{lat,S} \\ \langle x\rangle_G^{lat,S} \end{pmatrix}$$

"Sum rule constraint" $Z_{qq} + Z_{Gq} = 1, \quad Z_{Gq} + Z_{GG} = 1$

➡ $\tilde{f}_{qq} = \tilde{f}_{Gq} = (f_{qq} + f_{Gq})/2 \qquad \tilde{f}_{qG} = \tilde{f}_{GG} = (f_{qG} + f_{GG})/2$

$$Z = \begin{pmatrix} 0.9641 & 0.0119 \\ 0.0359 & 0.9881 \end{pmatrix}$$

(ad-hoc solution w/ ~1% sys err)

(to Msbar mu=2GeV)

Results

$\overline{MS},\ \mu = 2\ \mathrm{GeV}$

(Stat. Error Only)

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
$\langle x \rangle$	0.416(40)	0.151(20)	0.567(45)	0.037(7)	0.023(6)	0.334(56)
$T_2(0)$	0.283(112)	-0.217(80)	0.061(22)	-0.002(2)	-0.001(3)	-0.056(52)
$2J$	0.704(118)	-0.070(82)	0.629(51)	0.035(7)	0.022(7)	0.278(76)
g_A	0.91(11)	-0.30(12)	0.62(9)	-0.12(1)	-0.12(1)	–
$2L$	-0.21(16)	0.23(15)	0.01(10)	0.16(1)	0.14(1)	–

Spin = 25(12)%

Glue = 28(08)%

Orbital = 47(13)%

DI part is important

From our old results:
S.-J.Dong et al.,
PRL75(1995)2096

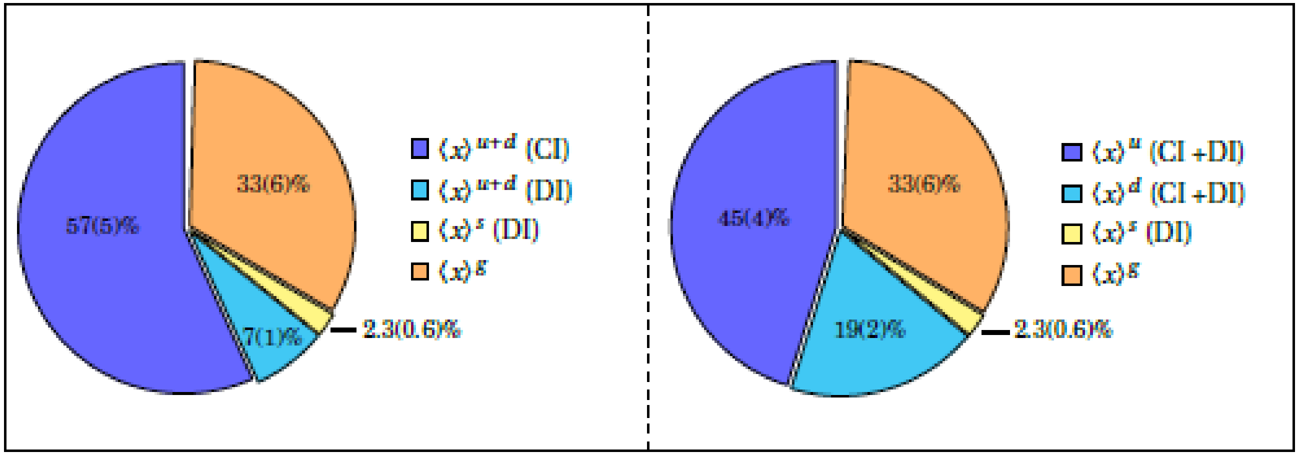
L(u) + L(d) [CI] ~= 0

J(u) >> J(d) [CI] ~= 0

(observed in other Lat)

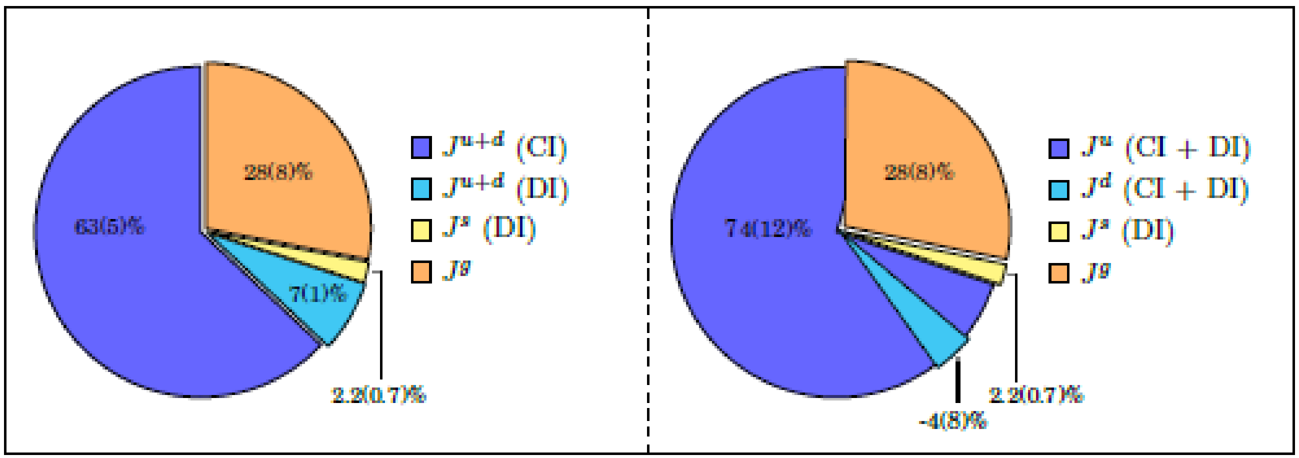
Results

<x>

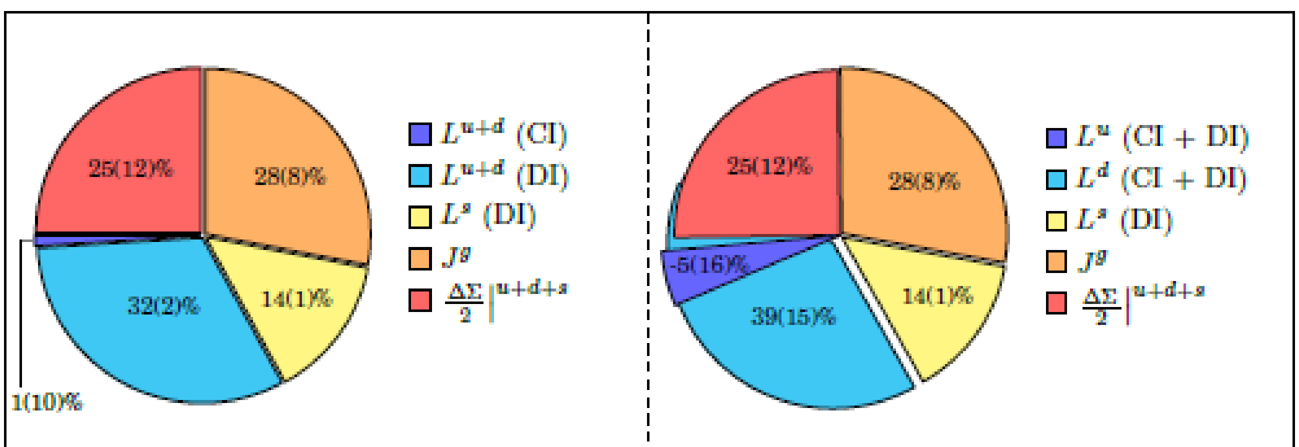


57(5)%
33(6)%
7(1)%
2.3(0.6)%
$\langle x\rangle^{u+d}$ (CI)
$\langle x\rangle^{u+d}$ (DI)
$\langle x\rangle^{s}$ (DI)
$\langle x\rangle^{g}$
45(4)%
33(6)%
19(2)%
2.3(0.6)%
$\langle x\rangle^{u}$ (CI +DI)
$\langle x\rangle^{d}$ (CI +DI)
$\langle x\rangle^{s}$ (DI)
$\langle x\rangle^{g}$

J



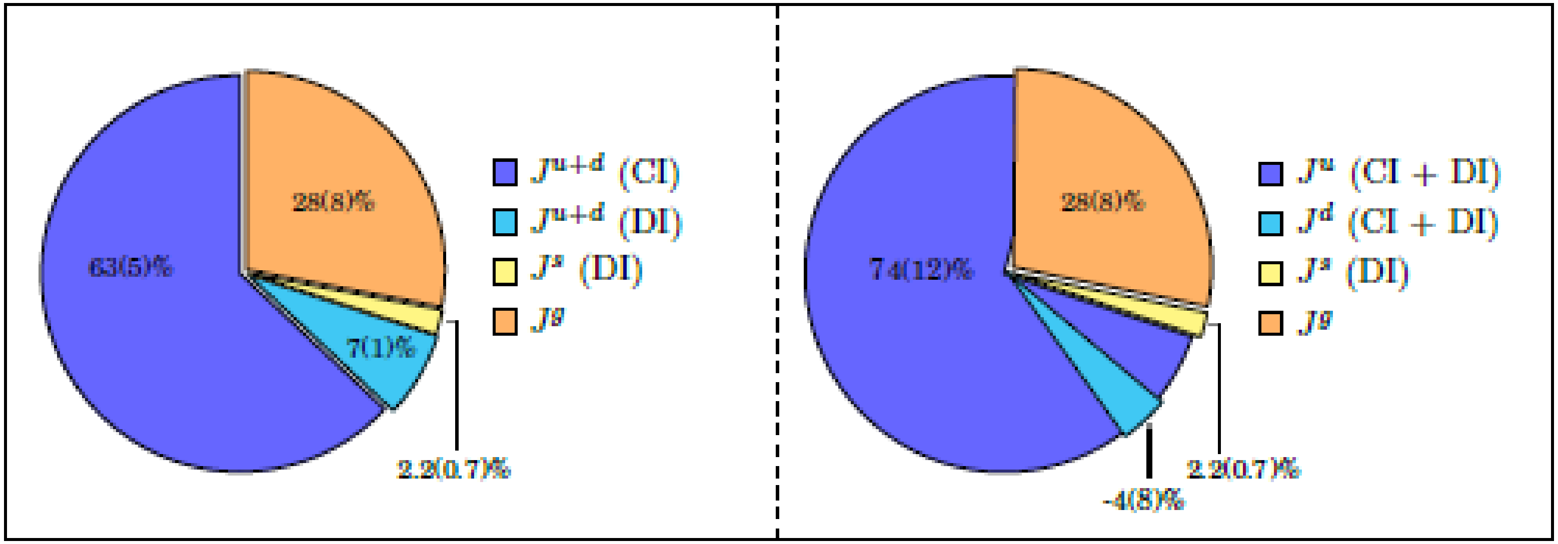
63(5)%
28(8)%
7(1)%
2.2(0.7)%
J^{u+d} (CI)
J^{u+d} (DI)
J^{s} (DI)
J^{g}
74(12)%
28(8)%
-4(8)%
2.2(0.7)%
J^{u} (CI + DI)
J^{d} (CI + DI)
J^{s} (DI)
J^{g}



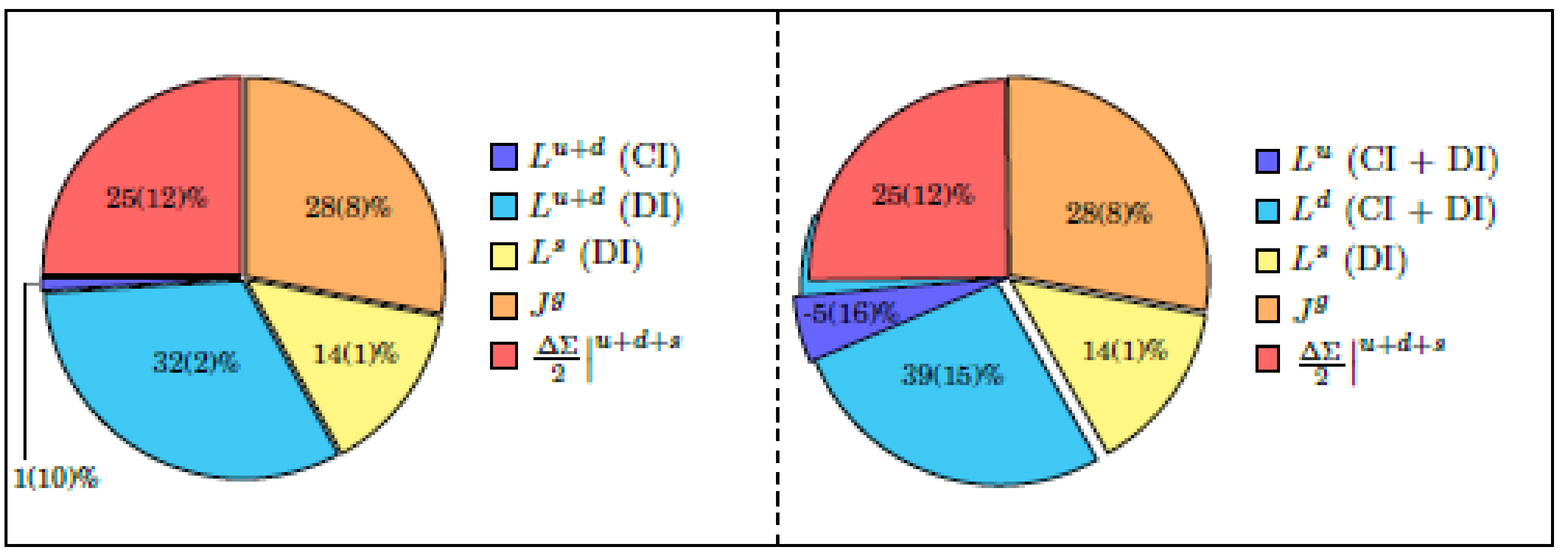
25(12)%
28(8)%
32(2)%
14(1)%
1(10)%
L^{u+d} (CI)
L^{u+d} (DI)
L^{s} (DI)
J^{g}
$\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$
25(12)%
28(8)%
-5(16)%
39(15)%
14(1)%
L^{u} (CI + DI)
L^{d} (CI + DI)
L^{s} (DI)
J^{g}
$\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$

Results

J



63(5)%
28(8)%
7(1)%
2.2(0.7)%
J^{u+d} (CI)
J^{u+d} (DI)
J^{s} (DI)
J^{g}
74(12)%
28(8)%
-4(8)%
2.2(0.7)%
J^{u} (CI + DI)
J^{d} (CI + DI)
J^{s} (DI)
J^{g}



25(12)%
28(8)%
32(2)%
14(1)%
1(10)%
L^{u+d} (CI)
L^{u+d} (DI)
L^{s} (DI)
J^{g}
$\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$
25(12)%
28(8)%
-5(16)%
39(15)%
14(1)%
L^{u} (CI + DI)
L^{d} (CI + DI)
L^{s} (DI)
J^{g}
$\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$

Systematic errors to be explored

- Dynamical quark effect (vs quenched calc.)
- Uncertainty in (long) chiral extrapolation
- Contamination from excited states
- Finite volume artifact, discretization artifact
  - m(pi) L >~ 4, a = 0.11fm
- Renormalization

- Ex.) quark spin

Quenched calc (1995) $\Delta\Sigma^{u,d}\ (\mathrm{DI}) \simeq \Delta\Sigma^{s}\ (\mathrm{DI}) \simeq -0.12$

Recent (preliminary) dynamical clac $\Delta\Sigma^{u,d}\ (\mathrm{DI}) \sim -0.05$, $\Delta\Sigma^{s}\ (\mathrm{DI}) \sim -0.03$

➔ **Smaller orbital mom?**

HOWEVER:

$$g_A^0 = (\Delta u + \Delta d)[CI] + (\Delta u + \Delta d + \Delta s)[DI] \sim 0.25$$
$$g_A^8 = (\Delta u + \Delta d)[CI] + (\Delta u + \Delta d - 2\Delta s)[DI] = 0.579(25)$$

➔ **Large DI & larger orbital ?**

Summary & Prospects

- The first study of **complete calc** of proton spin
  - Connected (CI), Disconnected (DI) & Glue
  - DI: stochastic method + unbiased subt. w/ HPE
  - Glue: overlap operator to improved S/N
- Quenched calc at heavy quark mass
  - J (u+d): 70(5)%, J(s): 2.2(7)%, J(glue): 28(8)% where L(u+d+s): 47(13)%
- **Future:**
  - Full QCD calc at lighter mass
  - New approach (next talk)